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Smart Indicators for Monitoring Uncertainty Propagation in a Deterioration Rate of Track Quality Index

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***Abstract*:**
Track inspection is dedicated to detecting track geometry-related potential defects which require several visits to accomplish the objective due to uncertainties in a track deterioration model. Through updating belief about prior specifications of the model parameters using freshly available track inspection data, the current inspection design would gain an improvement in terms of the value of investment (not necessary concerning cost reduction) for next maintenance cycles. In the sense of maximising an opportunity of having uncertainty reduction, this study proposes a statistical-based method blending with information theory to assign a priority index to a discrete-time point (called a candidate) in a given time interval. High priority is, in general, dedicated for a candidate that expectedly offer large cut in uncertainties i.e. high informativeness time for inspection. To lowering a false positive rate in the index assignment bearings and size of the ratio of adjustment are sequentially analysed before the final ranking gets to publish. Analysis of the simulation results corresponds to various settings of linear geometry deterioration model establishes correlations between covariance and priority index convincingly. Also, the relationship between the time gap between consecutive inspections and a prior deterioration rate is found significant in this context. A detailed description of the proposed model development is presented in this paper.

***Keywords*:** Uncertainty propagation, Entropy, Track quality index, Rail track inspection, Risk analysis

1.Introduction

Condition of rail track deteriorates progressively over its service time that means there are always detectable symptoms or warning signs before the state of failure [1]. By inspecting the geometry condition of the track followed with a maintenance decision process, appropriate maintenance work can be assigned to the inspected track. In practice, the severity of track geometry condition is evaluated to define a maintenance scheme; either intermediate action limit, intervention limit or alert limit [2].

Realising that inspections are discrete events, it is vital to capture track irregularities and track defects before the track geometry condition passes maintenance limits to effectively maintain a track (i.e., avoid track failure which incurs higher maintenance costs) [3]. A common approach to reducing the likelihood of suffering late defect detection that positively correlates with unplanned maintenance tasks is performing periodic in-spections. Recent trends show that a periodic track inspection design problem is formulated and solved as a constrained optimisation problem subject to direct costs, risk, reliability and/or safety standard [4].

Under the batch-mode optimisation methodology in which epistemic uncertainty is presented in the model formulation, a track manager has no an opportunity/access point to accurately evaluate the effects of the new observation(s) towards the effectiveness of the final inspection. This situation makes sense because an exact time when the geometric condition of the inspected track will pass a maintenance limit is uncertain. Hence, track supervisors or planners predict their empirically designed track deterioration model to try their best to deliver an inspection at an adequate time. A key benefit of such deterioration models is the deposit of a significant reduction in the number of (predictable) inspections, which leads to cost-effectiveness in maintaining railway infrastructures [5, 6]. Despite this, the deterioration model itself holds some degree of uncertainty about its parameter(s), but this drawback could be managed in many ways [7]. To illustrate, see Figure 1.

Let w be an estimated inspection time, which is believed to be near (or exactly) the time of crossing the limit, w^*. At time w, a calculated condition index of a targeted track, y, is presumed to be near the limit, y^* by a decision error of e; $e = y - y^*$. A positive error indicates that a track does not require maintenance yet is associated with the limit. However, a large positive error indicates that the final inspection (we use the term $final$ to distinguish it from periodic inspections, which occur before w) is underestimated and leaves the planner with uncertainty about the index during the period (w, w^*). Alternatively, the planner could miss the opportunity to detect y^* before its occurrence. Thus, it is not surprising when a recent survey shows that up to 85% of inspections end with no failure detection results [8].

The high percentage of the so-called supplementary inspections before the final inspection (before maintenance works) gives an insight of re-evaluating the planned inspection work in order to update prior belief about the rate of track deterioration using unobserved quantities (track measurement and inspection data). In this sense, a benefit in terms of the value of in-formation from the existing quantities first needs a quantification which later being used to prioritise them based on the selected measure(s). A decision to replace and/or to persist any part of an inspection plan would use this priority index as an input.

In our earlier works [9, 10], we define this situation as a positive disruption. Hypothetically, one or two planned additional inspection(s) might be avoided depending on the amount of infor-mation gathered by the latest inspection. Generally, railway infrastructure companies could save hundreds of thousands of dollars in their maintenance budgets if a single inspection is taken out of the plan for each track segment [11].

This study proposes a method to quantify the effect of a single supplementary inspection result on the estimation risk level of choosing time w to perform a final track inspection. The method integrates the benefits of Bayesian inferences and the entropy concept, and importantly, it works in a binary mode and requires only one input. For this, the Bayesian approach provides a theoretical inference framework for updating prior beliefs about uncertain quantities once additional information becomes available (if the decision-maker can make observations) from the tests and analyses conducted during the development program [11]. The computation of entropy characterises the value of inspection decision hence establishing that any inspection investment should reflect the effort of uncertainty reduction [12].

2.Uncertainty propagation: Rule

When a track manager decides to assign the final inspection at w, the manager understands that there is risk relating to a decision error x, which can be defined as the difference between y and y^*. In x lies outside an acceptable error tolerance, $\vec{\epsilon} = [\epsilon_i, \epsilon_h]$, a penalty cost c_p is imposed. Some may view the cost as an average of underestimation an overestimation costs.

For any t in the monitoring cycle, $t_o \leq t \leq w$, the expected value of c_p, $E(c_p)$ can be defined as:

$$E(c_p) = c_p \times (1 - p(\epsilon_l \leq x \leq \epsilon_h, t)) \quad \textbf{(1)}$$

where $p(x)$ is a probability density function of the random variable x. Intuitively, we can adopt Eq. (1) as a risk volume of trusting/adopting $p(x)$ deciding to deliver an inspection at time w. A proportional relationship between $Risk_t$ and $E(c_p)$ is given as follows:

$$\begin{aligned} Risk_{t=w} &\propto E(c_p, t = w) \\ &\propto c_p \times (1 - p(\epsilon_l \leq x \leq \epsilon_h, t)) \end{aligned} \quad \textbf{(2)}$$

Optimistically, we maintain the risk volume (i.e. $Risk_{t=t_o} \approx Risk_{t=w}$) along the monitoring cycle, while seeking opportunities to reduce the initial volume, which should be systematically performed. Consider c_p is fixed for any selection of function $p(x)$ and is applicable along the monitoring cycle, a desired risk volume $Risk_{t=w}$ might face an alteration in the presence of of changing values of parameters of $p(x)$. The changes can be observed at time w upon completion of the inspection, but risk mitigation might be ineffective in a tight time interval between w and w^*. Fortunately, n supplementary inspection(s), $G = \{g_1, g_2, \ldots, g_n\}$ can be arranged and performed on the inspected track prior to w, and in turn, their results might be informative to detect early changes in the shape and position of $p(x)$ early. Eq. (2) is redefined as:

$$Risk_{t=w} \propto c_p \times (1 - p(\epsilon_l \leq x \leq \epsilon_h, t|G)) \quad \textbf{(3)}$$

To investigate whether dependencies between a random variable x and G exists, a measure of the entropy of x and the entropy of $x|G$ is compared.

$$\begin{aligned} \Delta Ent &= Ent(x) - Ent(x|G) \\ &= \int_{\mathbb{R}} p(x) \ln p(x) dx + \iint_{\mathbb{R}\times\mathbb{R}} p(x|G) \ln p(x|G)\, dx\, dG \end{aligned} \quad \textbf{(4)}$$

Variables x and G are independent if ΔEnt is zero, which means that the information of G is meaningless in estimating x. In the same vein, a reduction in an amount of uncertainty (randomness) in $p(x)$ cannot be gained using the conditional probability distribution, $p(x|G)$. In the case of $Ent(x|G) \neq Ent(x)$,it is worthwhile to assess summative components of $Ent(x|G)$ which are formulated in the following equation:

$$Ent(x|G) = Ent(x, G) - Ent(G) \quad \textbf{(5)}$$

where $Ent(x, G)$ represents the joint entropy of x and G. Intuitively, we assign a weak assumption, which states that only the last element of the right-hand side of Eq. (5) will be affected with an introduction of new observations to the sample space G. At this point, we could manipulate an information regarding level of deviation in $Ent(G)$ to establish a simple rule to detect unacceptable changes in prior $Ent(x|G)$ upon completion of a single periodic inspection.

The following subsection explains a method to identify which parameter(s) in $p(G)$ should be controlled to allow the track manager to smartly monitor the $Risk_{t=w}$ along the monitoring cycle. Further, a justification to either terminate, skip, or reschedule a remaining part of the supplementary inspections can be reached depending on the amount of fluctuation of risk volume in each completed inspection.

2.1. Parameter of Interest

Suppose G is a continuous random variable in $\mathbb{R}$ which is independent identically distrubuted (i.i.d) over the mean (i.e. $G \sim N(\mu_G, \sigma_G)$). Over $\mathbb{R}$, a maximum entropy of G can be determined by solving the following equation:

$$\begin{aligned} \{Ent_{max}(G) = &- \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)((G-\mu)/\sigma)^2} \\ &\left(-\log(\sqrt{2\pi}\sigma) - \frac{1}{2}\left(\frac{G-\mu}{\sigma}\right)^2 \right) dG \\ &= \frac{1}{2}(1 + \log(2\pi\sigma^2)) \leq \log(n)\} \end{aligned} \quad \textbf{(6)}$$

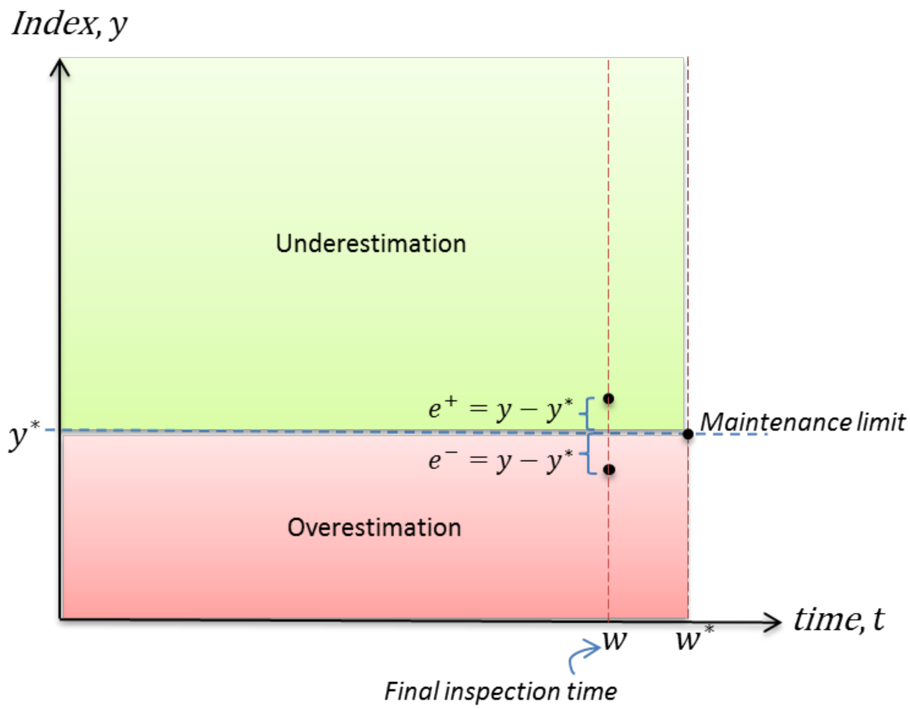

Figure 1. A region of an under-over estimation risk subject to a final track inspection time

where n is the sample size. From Eq. (6), it is obvious that the change in the amount of entropy carried out by G can be monitored directly from its variance. The calculation is no longer straightforward in the event of unknown variance, regardless of what the status of the mean is. To estimate σ^2 considering n data observations $\{g_1, g_2, ..., g_n\}$ we adopt a Bayes$'$ theorem which provides us the following relationship:

$$p(\theta|\{g_1, g_2, ..., g_n\}) \propto p_o(\theta) \times p(\{g_1, g_2, ..., g_n\}|\mu, \theta) \quad (7)$$

where a model precision θ is an inverse proportional of σ^2. For prior density function, $p_o(\theta)$, a flexible choice is a gamma distribution, $\theta \sim G(\alpha_o, \beta_o)$. Thus, a conditional likehood function of θ in Eq. (7) is also in terms of gamma parameters and is equated as follows:

$$\begin{aligned} \{p(\theta|\{g_1, g_2, ..., g_n\}) &\propto \left(\theta^{\alpha_o - 1} exp(-\theta\beta)\right) \\ &\left(\theta^{n/2} exp(-\theta/2 \sum_{i=1}^{n} (g_i - \mu)^2\right) \\ &\propto \theta^{(\alpha + n/2) - 1} exp(-\theta(\beta_o + \\ &1/2 \sum_{i=1}^{n} (g_i - \mu)^2)) \\ &\propto \theta^{\alpha_n} exp(\theta\beta_n)\} \end{aligned} \quad (8)$$

Further, we can apply a proportion of its expected value $E(\theta|\{g_1, g_2, ..., g_n\}) = \alpha_n\beta_n$ into Eq. (6) for entropy calculation. Following this, an effect of new observation, g_{n+1} on the initial amount of $Ent(G)$ could be quantified from a difference between $Ent(G|\theta_{n+1})$ and $Ent(G|\theta_n)$, where the corresponding formula as follows:

$$\Delta(Ent(G|\theta_{n+1}), Ent(G|\theta_n)) = \frac{1}{2} log(\frac{\alpha_n\beta_n}{\alpha_{n+1}\beta_{n+1}}). \quad (9)$$

Interestingly, the associated plot of Eq. (9) in Figure 2 indicates that there will be no entropy increment $\frac{\alpha_n\beta_n}{\alpha_{n+1}\beta_{n+1}} \leq 1$. Application of elements in the second line of Eq. (8) to the fraction component of Eq. (9) exhibits a boundary condition between g_{n+1} and the ratio of β_n to α_n as illustrated in Figure 3 . Important steps in the formulation of the boundary condition are presented below:

$$\begin{aligned} \alpha_n\beta_n &\leq \alpha_{n+1}\beta_{n+1} \\ \frac{\alpha_n}{\alpha_{n+1}} &\leq \beta_n + 0.5(g_{n+1} - \mu)^2 \\ (g_{n+1} - \mu)^2 &\leq \frac{\beta_n}{\alpha_n + 0.5} < \frac{\beta_n}{\alpha_n} \\ (g_{n+1} - \mu)^2 &\leq \frac{\beta_n}{\alpha_n} \end{aligned} \quad (10)$$

For a given μ, $\Delta(Ent(G|\theta_{n+1}), Ent(G|\theta_n))$ has positive values if g_{n+1} lies in the shaded region. Importanty, information of gamma parameters is not necessarily required at this stage.

3.Indicators

With respect to the track geometry condition index, the vertical scale in Figure 3 will have a limited range of val-

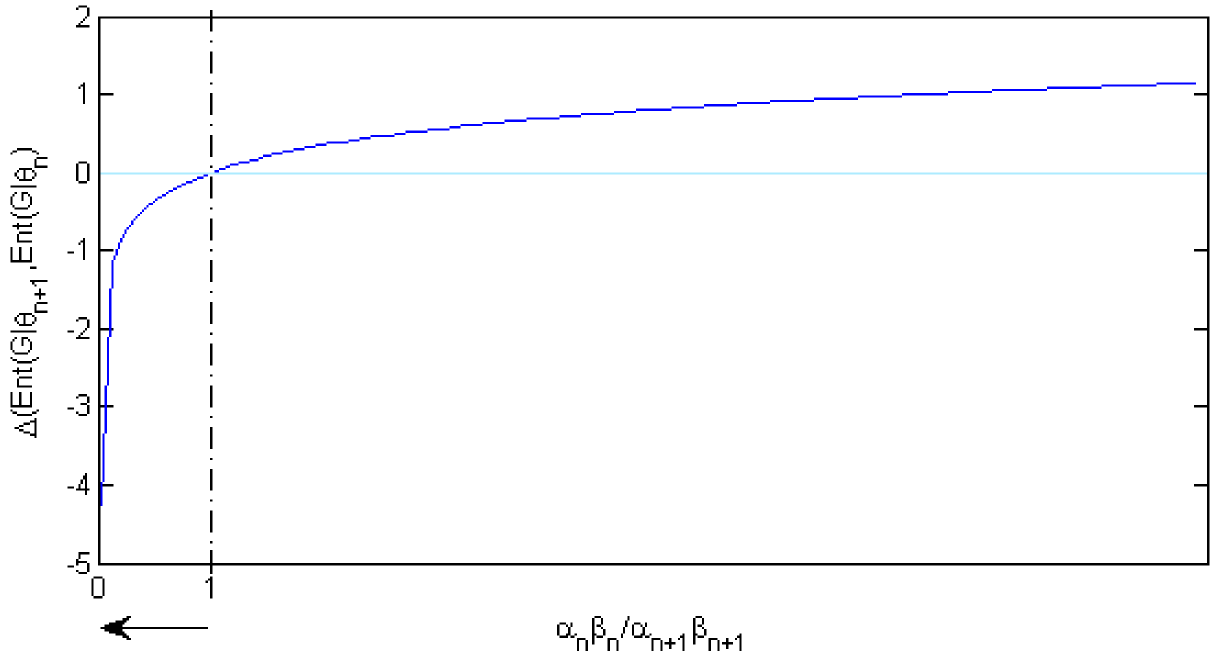

Figure 2. A cutting point between a safe and alert effect of a new observation to G

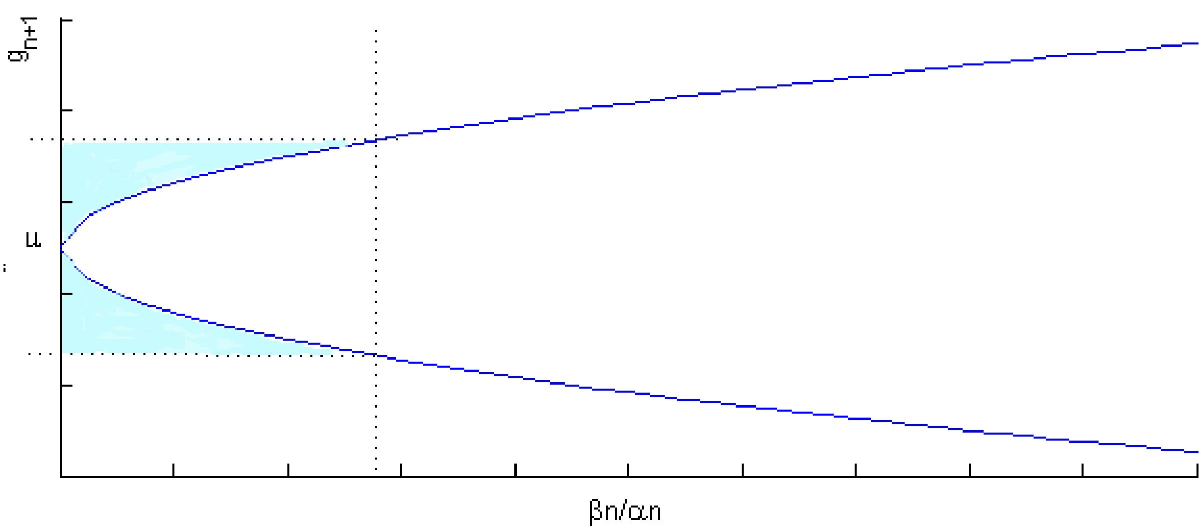

Figure 3. Boundary regions for track geometry condition index checking

ues. The minimum and maximum possible value of the selected index can be assigned as a lower and upper limit to the scale, respectively. By doing that, as illustrated with a dashed horizontal line below and above μ, we have established a separation point on the horizontal axis, c_x. For a given $\{g_1, g_2, \ldots, g_n\}$ with $b_n/a_n > c_x$, an additional geometric index reading brings no meaning to the prior entropy G. In this situation, the railway management team will waste their resources and lose profit for a valueless supplementary inspection. Conversely, the track manager can trace the separation line first, which automatically points out the location of both limits.

Figure 3 can be treated as a 0/1 decision map to determine whether $Ent(G)$ faces a change in value or not after a new inspection result is considered in the corresponding probability density function. In the former case, a calculation of conditional entropy $x|G$ in Eq. (5) required to quantify how a risk volume deviates from its initial value. To achieve that, $Ent(x, G)$ with an updated variance must be computed, which presumably requires extra computational effort. Establishment of decision rule(s) as done for $Ent(G)$ might accelerate the process.

In the event of G has non-Gaussian density function, the relationships in the Equations (8) to (10) might not be directly applicable. The statement is made due to the findings in [13], which point out the effects of the non-Gaussian prior on the observation value. Thus, redoing Section 2.1 will be our next agenda, aiming to identify a decision rule or boundary condition similar to that in Figure 3, specifically from a Poisson and exponential density function.

Apart from that, this study is keen to validate the proposed method with real data. A comparison analysis using different configurations of the dataset, such as track category, geographical properties, and environmental threats, might establish a set of separation points corresponding to perfect or imperfect data.

Lastly, this study generally assigns G to a single supplementary inspection without specifying the time duration from w. A decision to increase the time duration might cause G and X turns to be independent events. Consequently, the proposed method is invalid, which means that the next supplementary inspection in the schedule is mandatory. Despite the time-related issue, a proposal of considering multiple locations for G is an interesting direction to discover. An expansion of entropy equality and Bayes$'$ theorem can be expected when more than one inspection result is considered.

4.Conclusions

This study proposes an entropy-based method to determine changes in risk volume associated with a final track geometry inspection. The volume, which is a function of conditional probability of decision error, is unchanged during its monitoring cycle unless a supplementary inspection manifests it through a simple 2D-coordinate mapping $(b_n/a_n, g_{n+1})$ mapping. A ratio of scale to shape parameter, both from the gamma distribution, and the latest track condition reading gained from a supplementary inspection represent the $x-$ and $y-$ coordinates, respectively. The prescribed risk volume is unaffected along a period of $(w, w*)$ for any coordinates that lie on the non-shaded regions in Figure 3 . In this situation, several suggestions can be made, such as rescheduling and/or removing the remaining supplementary inspections scheduled between $(w, w*)$. A cost-benefit ratio of the track geometry inspection programme can be reduced under a controlled and safe environment.

Generalising the proposed model to cope with variability in a configuration of random variable G will be the main agenda to forward this research. Upon completion of the model generalisation, effects of rescheduling and or removal can be studied and analysed thoroughly where a system approach will be integrated. The whole package is expected to be validated using real track inspection data.

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