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A fuzzy reasoning and fuzzy-analytical hierarchy process based approach to the process of railway risk information: A railway risk management system

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ABSTRACT

Risk management is becoming increasingly important for railway companies in order to safeguard their passengers and employees while improving safety and reducing maintenance costs. However, in many circumstances, the application of probabilistic risk analysis tools may not give satisfactory results because the risk data are incomplete or there is a high level of uncertainty involved in the risk data. This article presents the development of a risk management system for railway risk analysis using fuzzy reasoning approach and fuzzy analytical hierarchy decision making process. In the system, fuzzy reasoning approach (FRA) is employed to estimate the risk level of each hazardous event in terms of failure frequency, consequence severity and consequence probability. This allows imprecision or approximate information in the risk analysis process. Fuzzy analytical hierarchy process (fuzzy-AHP) technique is then incorporated into the risk model to use its advantage in determining the relative importance of the risk contributions so that the risk assessment can be progressed from hazardous event level to hazard group level and finally to railway system level. This risk assessment system can evaluate both qualitative and quantitative risk data and information associated with a railway system effectively and efficiently, which will provide railway risk analysts, managers and engineers with a method and tool to improve their safety management of railway systems and set safety standards. A case study on risk assessment of shunting at Hammersmith depot is used to illustrate the application of the proposed risk assessment system.

1. Introduction

The risk, in the railway sector, can be defined in relation to accidents and incidents leading to fatalities or injuries of passengers and employees [31]. Recent structured hazard identification work within the industry has confirmed the high-risk

scenarios of the types of accidents such as collision, derailment and fire [11,25,30,32]. The statistic figures of accidents and incidents include not only workers, but also a significant number of people not employed in the industry, including children and members of the public. In the UK railway industry, many people have been injured and there even have been fatalities in relation to this industry over the past years [25,27,28,34]. This shows the dangerous nature of the railway industry and demonstrates the need for increased awareness and better safety management [2,29,33]. To assess how this can be effectively achieved, knowledge on the nature and causes of these accidents is fundamental. Therefore, risk analysis plays a central role in the railway safety and health management framework. The most common hazards in the railway depots identified by the railway industry over the years [18,25,27,28,32,34] provide very useful information for risk analysis, for example, derailment hazards, collision hazards, fire hazards, electrocution hazards, falls hazards, train strike hazards, slip/trip hazards, and platform train interface hazards. The requirement of risk analysis is to demonstrate that if risks associated with a railway system are high, risk reduction measures must be applied or operation and maintenance have to be reconsidered to reduce the occurrence probabilities or to control the possible consequences. If risks are negligible, no actions are required but the information produced needs to be recorded for audit purpose [2,11,25]. Therefore, railway engineers, managers and safety analysts need to develop and employ risk assessment approaches for their safety management and set safety standards.

Many of risk assessment techniques currently used in the railway industry are comparatively mature tools, which have been developed on the basis of probabilistic risk analysis (PRA), for example, fault tree analysis, event tree analysis, Monte–Carlo simulation, consequence analysis and equivalent fatality analysis (EFA) [8,25,27,28,32,34]. The results of using these tools heavily rely on the availability and accuracy of the risk data [2,3,8,32]. However, in many circumstances, these methods often do not cope well with uncertainty of information. Furthermore, the statistic data does not exist and it must be estimated on the basis of expert knowledge and experience or engineering judgement. Therefore, railway risk analysts often face the circumstances where the risk data are incomplete or there is a high level of uncertainty involved in the risk data [2,4]. Additionally, railways are a traditional industry, whose history extends for at least two centuries. Much of their safety record depends upon concepts developed many years ago and established practices over the whole of their history. However, the existing databases contain a lot of data and information, but the information may be both an excess of other information that cannot be used in risk analysis or a shortage of key information of major failure events. There are numerous variables interacting in a complex manner which cannot be explicitly described by an algorithm, a set of equations or a set of rules. In many circumstances, it may be extremely difficult to conduct PRA to assess the failure frequency of hazards, the probability and the magnitudes of their possible consequences because of the uncertainty in the risk data. Although some work has been conducted in this field, no formal risk analysis tools have been developed and applied to a stable environment in the railway industry [2,8,32]. Therefore, it is essential to develop new risk analysis methods to identify major hazards and assess the associated risks in an acceptable way in various environments where such mature tools cannot be effectively or efficiently applied. The railway depot safety problem is appropriate for examination by FRA and fuzzy-AHP.

The magnitude of a risk can usually be assessed by considering two fundamental risk parameters: failure frequency (FF) and consequence severity (CS) [2,29,32,34]. The FF defines the number of times an event occurs over a specified period, e.g. number events/year. The CS represents the number of fatalities, major injuries and minor injuries resulting from the occurrence of a particular hazardous event. However, it should be noted that the magnitude of a particular risk also highly depends on the probability that the effects will happen given by the occurrence of the failure. Therefore, the probability of current consequence caused by a particular failure should be taken into consideration in the risk assessment process to obtain a reliable result. In order to assess the risks associated with a railway depot efficiently and effectively, a new risk parameter, consequence probability (CP) is introduced in the proposed risk analysis model to determine the risk level (RL) of a hazardous event. The use of FRA allows imprecision or approximate information involved in the risk assessment process [3,5,18,22,24–26]. In this method, a membership function (MF) is regarded as a possibility distribution based on a proposed theory; and an apparent possibility distribution expressed by fuzzy set theory is transferred into a possibility measure distribution. FRA method provides a useful tool for modelling risks and other risk parameters for risk analysis involving the risks with incomplete or redundant safety information [2,8,18,34]. Because the contribution of each hazardous event to the safety of a railway depot is different, the weight of the contribution of each hazardous event should be taken into consideration in order to represent its relative contribution to the RL of the railway depot. Therefore, the weight factor (WF) is introduced, which indicates the magnitude of the relevant importance of a hazardous event or hazard group to its belongings in a risk tree. Fuzzy-AHP is then employed to calculate the WFs [2–4,6–9,14–17,35–41]. The outcomes of risk assessment are represented as the risk degrees, the defined risk categories of RLs with a belief of percentage and risk contributions that provide safety analysts, managers, engineers and decision makers useful information to improve their safety management and set safety standards. This article presents the development of a railway risk assessment system using FRA and fuzzy-AHP.

After the introduction, Section 2 presents a new risk model. The application of qualitative descriptors and corresponding MF to represents risk factors and RLs are discussed in Section 3 and the development of fuzzy rule base is also addressed to describe the relationship among risk factors and RL expressions. Section 4 introduces the application of the proposed risk assessment system and presents a case study on risk assessment of shunting at Hammersmith depot to demonstrate the proposed risk assessment methodology. Finally, Section 5 gives conclusions and a summary of the main benefits of using the proposed methodology in railway risk analysis.

2. Proposed safety risk assessment model

A risk assessment is a process that can be divided into five phases: problem definition phase; data and information collection and analysis phase; hazard identification phase; risk estimation phase and risk response phase [2,8,25,32]. This process provides a systematic approach to the identification and control of high-risk areas. A risk assessment model based on FRA and fuzzy-AHP approaches is proposed as shown in Fig. 1 in which EI and UFN stand for expert index and uniform format number, respectively. The proposed risk model consists of five phases: preliminary phase, design phase, FRA risk estimation phase, fuzzy-AHP risk estimation phase and risk response phase. The details of the proposed risk model are described in the following sections.

2.1. Preliminary phase

Risk assessment begins with problem definition which involves identifying the need for safety, i.e. specific safety requirements that should be made at different level, e.g. hazardous event level, hazard group level and the railway depot level. The requirements may include sets of rules and regulations made by the national authorities and classification societies, deterministic requirements for safety, reliability, availability, maintainability, and criteria referring to probability of occurrence of serious hazardous events and the possible consequences [2,8,25,32].

Once the need for safety is established, the risk assessment moves from the problem identification to the data and information collection and analysis. The aim of data and information collection and analysis is to develop a good understanding of
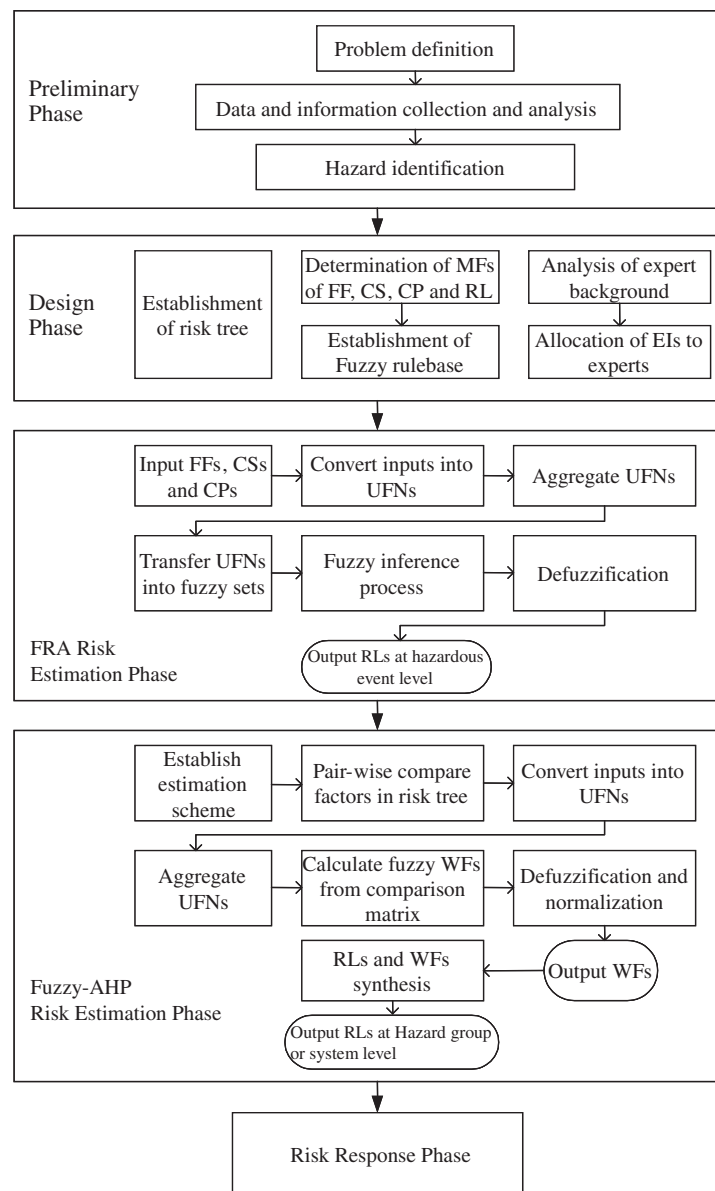

Fig. 1. The proposed risk assessment model.

what serious accidents and incidents occurred in a particular railway depot over the years and generate a body of information. If the statistic data does not exist, expert and engineering judgements should be applied. The information gained from data and information collection will then be used to define the standards of qualitative descriptors and associated MFs of risk parameters, i.e. FF, CS, CP and RL.

The purpose of hazard identification is to systematically identify all potential hazardous events associated with a railway depot at each required level, e.g. hazardous event level, hazard group level with a view to assessing their effects on railway depot safety. Various hazard identification methods such as brainstorming approach, check-list, 'what if?', HAZOP (Hazard and Operability), and failure mode and effect analysis (FMEA), may be used individually or in combination to identify the potential hazardous events of a railway depot [8,11,18,25,32]. The hazard identification can be initially carried out to identify hazardous events at hazardous event level, and then progressed up to hazard group level and finally to the depot level. The information from hazard identification will then be used to develop a risk tree.

2.2. Design phase

Once the risk information of a railway depot is obtained in preliminary phase, the risk assessment moves from preliminary phase to design phase. On the basis of information collection, the tasks in the design phase are to develop a risk tree and MFs of FF, CS, CP and RL and a fuzzy rule base.

2.2.1. Development of a risk tree

There are many possible causes of risks that impact on railway depot safety. The purpose of development of a risk tree is to decompose these risk contributors into adequate details in which risks associated with a railway depot can be efficiently assessed [3,8]. A bottom-up approach is employed for the development of a risk tree. Fig. 2 shows a typical risk tree that can be broken down into hazardous event level, hazard group level and the depot level. For example, hazardous events of E_1, E_2, ..., E_n at hazardous event level affect the RL of S-HG$_1$ at sub-hazard group level, the RLs of S-HG$_1$, S-HG$_2$, ..., S-HG$_n$ contribute to the RL of HG$_2$ at hazard group level and RLs of HG$_1$, HG$_2$, ..., HG$_n$ contribute to the overall RL of a railway depot at system level.

2.2.2. Development of a fuzzy rule base

A fuzzy rule base consists of a set of fuzzy if-then rules [5,13,23,26] that are often employed to capture the imprecise modes of reasoning, which plays an essential role in the human ability to make decisions in the environments of uncertainty and imprecision. The fuzzy rules can be derived from several sources such as historical data analysis, engineering judgement and expert knowledge. These approaches are mutually supporting and a combining of them is often the most effective way to determine the rule base. A fuzzy rule is developed in terms of qualitative descriptors of input parameters: FF, CP, CS and the output RL. The qualitative descriptors are characterised by fuzzy sets that are derived from historical incidents and accident information. The fuzzy sets are defined in the universe of discourse and described by MFs. Currently, there are several geometric mapping functions widely adopted, such as triangular, trapezoidal and S-shaped MFs. However, triangular and trapezoidal MFs are the most frequently used in railway risk analysis practice [3,8,24,28]. For example, in the railway risk assessment, input parameters of FF, CP, CS and output of RL are constructed by trapezoidal MFs, where the FF is defined as 'Remote', 'Rare', 'Infrequent', 'Occasional', 'Frequent', 'Regular', and 'Common'; CS is defined as 'Negligible', 'Marginal', 'Moderate', 'Critical', and 'Catastrophic'; and CP is defined as 'Highly unlikely', 'Unlikely', 'Reasonably unlikely', 'Likely', 'Rea-
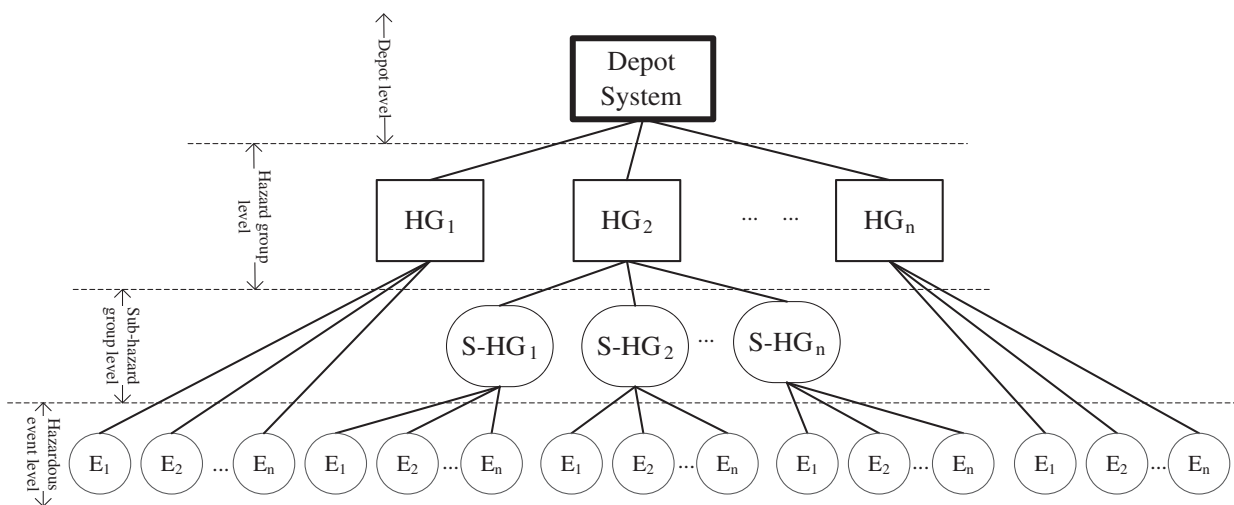

Fig. 2. A typical risk tree.

sonably likely', 'Highly likely' and 'Definite'; the RL is defined as 'Low', 'Possible', 'Substantial', and 'High'. The fuzzy rules are subjective defined on the basis of expert experience and engineering judgement [5,13–18,23,38–41]. For example, in railway risk analysis, the following rule is commonly used

If FF is 'Remote' and CS is 'Negligible' and CP is 'Highly unlikely', Then RL is 'Low'.

2.2.3. Allocation of EIs to experts

Railway risk analysis is a very complicated subject, which is determined by numerous relevant risks associated with a railway system. It is impossible for a single engineer or manager to consider all relevant aspects of a problem. Therefore, many decision making processes take place in group settings by introducing linguistic preference relations under uncertain linguistic environment [14–17,38–41]. In practice, risk assessment usually involves a number of experts with different background or discipline regarding the railway safety. Thus, these experts may have different impacts on the final decision. The EI is therefore introduced into the risk model to distinguish experts' competence. Suppose there are n experts involved in the risk assessment, the EI of the ith expert can be obtained by

$$EI_i = \frac{RI_i}{\sum_{j=1}^{n} RI_j} \tag{1}$$

where RI_i is relevant importance of the ith expert according to his experience, knowledge and expertise, which takes an value in the universe of 1 to 9. RI is defined in a manner that '1' means less importance, whereas '9' means most importance. Obviously, it is necessary to review EIs when the topic or the circumstance has been changed.

2.3. FRA risk estimation phase

In the FRA risk estimation phase each risk is assessed at hazardous event level based on FF, CS and CP to calculate its RL. However, railway risk analysts often face the circumstances where the risk data are incomplete or there is a high level of uncertainty involved in the risk data. A flexible method for expressing expert and engineering judgements is proposed [2,3,8]. The uniform format number (UFN) is introduced to capture and convert expert and engineering subjective judgements. As described earlier in this paper, this allows imprecision or approximate information in risk analysis process. There are six steps to calculate RLs of hazardous events that are described as below.

2.3.1. Step 1: input FFs, CSs, and CPs

The input data usually can gather from historical data, but, however, in many circumstances, the data may not exist or uncertainty involved in the risk data. Experts may provide their judgements on the basis of their knowledge and expertise for each hazardous event [14–16]. The input values can be a precise numerical value, a range of numerical values or a linguistic term. For example, if adequate information is obtained and the risk factor is quantitative measurable, an expert is likely to provide a precise numerical value. However, experts sometimes find that it is hard to give numerical values due to uncertainties involved or the hazardous event is quantitative immeasurable, then a range of numerical values, a linguistic term or a fuzzy number can be used in the proposed model [1–7,14–18,24,37–41], e.g. "CP is 60% to 70%", "CS is around 3 to 7 and most likely to be 5 in the universe of [0, 10]" and "FF is average".

2.3.2. Step 2: convert inputs into UFNs

Because the input values of hazardous events are crisps, e.g. a numerical value, a range of numerical value, a fuzzy number, or a linguistic term, the UFN is employed to convert these inputs into a uniform format for the composition of a final decision [2,3,8,14–22,37–41]. An UFN can be defined as $A = \{a, b, c, d\}$, and its corresponding MF indicates the degree of preference, which is defined as

$$\mu_A(x) = \begin{cases} (x-a)/(b-a), & x\varepsilon[a,b], \\ 1 & x\varepsilon[b,c] \\ (x-d)/(c-d), & x\varepsilon[c,d], \\ 0, & otherwise. \end{cases} \tag{2}$$

where four real numbers (a, b, c and d) with satisfaction of the relationship $a \leqslant b \leqslant c \leqslant d$ determine the x coordinates of the four corners of a trapezoidal MF. Table 1 shows the possible inputs and its corresponding UFNs.

Table 1
Expert judgement and the corresponding UFNs.

Description	Input Values	Input type	UFNs
"… is a"	$\{a\}$	A numerical value	$\{a, a, a, a\}$
"… is between a and b"	$\{a, b\}$	A range of number	$\{a, (a+b)/2, (a+b)/2, a\}$
"… is between a and c and most likely to be b"	$\{a, b, c\}$	Triangular fuzzy numbers	$\{a, b, b, a\}$
"… is between a and d and most likely between b and c"	$\{a, b, c, d\}$	Trapezoidal fuzzy numbers	$\{a, b, c, d\}$
"… is $RARE$"	RARE	A linguistic term	RARE MF $\{a, b, c, d\}$

2.3.3. Step 3: aggregate UFNs

The aim of this step is to apply an appropriate operator to aggregate individual judgement made by individual expert into a group preference of each hazardous event. Based on EIs of experts obtained in the design phase, the expert judgments can be aggregated according to weighted trapezoidal averaging operator [3,8,9,38–41]. Assume m experts involved in the risk assessment and n experts provide non-zero judgments for the ith hazardous event, the aggregated UFN A^i can be determined by

$$A^i = \{a^i, b^i, c^i, d^i\} = \left\{ \frac{\sum_{k=1}^m a_k^i EI_k}{\sum_{k=1}^n EI_k}, \frac{\sum_{k=1}^m b_k^i EI_k}{\sum_{k=1}^n EI_k}, \frac{\sum_{k=1}^m c_k^i EI_k}{\sum_{k=1}^n EI_k}, \frac{\sum_{k=1}^m d_k^i EI_k}{\sum_{k=1}^n EI_k} \right\}$$

$$A_k^i = \{a_k^i, b_k^i, c_k^i, d_k^i\} \tag{3}$$

where a_k^i, b_k^i, c_k^i and d_k^i are the numbers of UFN A_k^i that represents the judgement of the kth expert for the ith hazardous event and EI_k stands for the kth expert's EI.

2.3.4. Step 4: calculate fuzzy values of UFNs

Assume A_{FF}^i, A_{CS}^i and A_{CP}^i are three UFNs of FF, CS and CP of the ith hazardous event, respectively. Their corresponding fuzzy sets $\tilde{A}_{FF}^i, \tilde{A}_{CS}^i$ and $\tilde{A}_{CP}^i$ are defined as

$$\tilde{A}_{FF}^i = \{(u, \mu_{A_{FF}^i}(u))|u \in U = [0, u], \mu_{A_{FF}^i}(u) \in [0, 1]\} \tag{4}$$

$$\tilde{A}_{CS}^i = \{(v, \mu_{A_{CS}^i}(v))|v \in V = [0, v], \mu_{A_{CS}^i}(v) \in [0, 1]\} \tag{5}$$

$$\tilde{A}_{CP}^i = \{(w, \mu_{A_{CP}^i}(w))|w \in W = [0, w], \mu_{A_{CP}^i}(w) \in [0, 1]\} \tag{6}$$

where $\mu_{A_{FF}^i}, \mu_{A_{CS}^i}$ and $\mu_{A_{CP}^i}$ are trapezoidal MFs of A_{FF}^i, A_{CS}^i and A_{CP}^i, and u, v and w are input variables in the universe of discourse U, V and W of FF, CS and CP, respectively.

2.3.5. Step 5: FRA evaluation

FRA evaluation is a process that is developed on the basis of Mamdani method [2,8,24] to determine which rules are relevant to the current situation for calculating the fuzzy output, i.e. RL. Relations between input parameters FF, CS, CP and output RL are presented in a form of if-then rules that have the following format

$$R_i: \text{ if } u \text{ is } \tilde{B}_{FF}^i \text{ and } v \text{ is } \tilde{B}_{CS}^i \text{ and } w \text{ is } \tilde{B}_{CP}^i, \text{ then } x \text{ is } \tilde{B}_{RL}^i, \; i = 1, 2, \ldots, n \tag{7}$$

where $u, v, w,$ and x are variables in the universe of discourse U, V, W and X of FF, CS, CP and RL, and $\tilde{B}_{FF}^i, \tilde{B}_{CS}^i, \tilde{B}_{CP}^i$ and $\tilde{B}_{RL}^i$ are qualitative descriptors of FF, CS, CP and RL respectively, The calculation of the fire strength of α_i of the ith rule with input fuzzy sets $\tilde{A}_{FF}^i, \tilde{A}_{CS}^i$ and $\tilde{A}_{CP}^i$ using fuzzy intersection operation is given by

$$\alpha_i = \min\left[\max\left(\mu_{A_{FF}^i}(u) \wedge \mu_{B_{FF}^i}(u)\right), \max\left(\mu_{A_{CS}^i}(v) \wedge \mu_{B_{CS}^i}(v)\right), \max\left(\mu_{A_{CP}^i}(w) \wedge \mu_{B_{cp}^i}(w)\right) \right] \tag{8}$$

Where $\mu_{A_{FF}^i}(u), \mu_{A_{CS}^i}(v)$ and $\mu_{A_{CP}^i}(w)$ are the MFs of fuzzy sets $\tilde{A}_{FF}^i, \tilde{A}_{CS}^i$ and $\tilde{A}_{CP}^i$, respectively, and $\mu_{B_{FF}^i}(u), \mu_{B_{CS}^i}(v)$ and $\mu_{B_{CP}^i}(w)$ are the MFs of fuzzy sets $\tilde{B}_{FF}^i, \tilde{B}_{CS}^i$ and $\tilde{B}_{CP}^i$ of qualitative descriptors in rule R_i. After the fuzzy implication, the truncated MF $\mu_{B_{RL}^i}'$ of the inferred conclusion fuzzy set of rule R_i is obtained by

$$\mu_{B_{RL}^i}'(x) = \alpha_i \wedge \mu_{B_{RL}^i}(x) \tag{9}$$

where α_i is the fire strength of rule R_i, $\mu_{B_{RL}^i}(x)$ is the MF of the qualitative descriptor $\tilde{B}_{RL}^i$ and x is an input variable in the universe of discourse X.

The $\mu_{B_{RL}}'(x)$ of output fuzzy set can then be calculated by using fuzzy union (maximum) operation

$$\mu_{B_{RL}}'(x) = \bigvee_{i=1}^n \mu_{B_{RL}^i}'(x) \tag{10}$$

where $\mu_{B_{RL}^i}'$ is the MF of conclusion fuzzy set of rule R_i and n is the total number of rules in the rule base.

2.3.6. Step 6: defuzzification

Defuzzification is to convert the aggregated result to a crisp number that represents the final result of the FRA evaluation. The centroid of area method, which determines the centre of gravity of an aggregated fuzzy set, is employed for defuzzification [2,8,24]. Assume the output fuzzy set obtained from the FRA evaluation is $\tilde{B}_{RL}' = \{(x, \mu_{B_{RL}}'(x))|x \in X, \mu_{B_{RL}}'(x) \in [0, 1]\}$, RL^i of the ith hazardous event can be calculated

$$RL^i = \frac{\sum_{j=1}^m \mu_{B_{RL}}'(x_j) \cdot x_j}{\sum_{j=1}^m \mu_{B_{RL}}'(x_j)} \tag{11}$$

where m is the number of quantization level of X and $x_j \in X$, x_j denotes the position of $\mu_{B_{RL}}'(x_j)$ in RL expression, RL^i is the aggregated output MF which indicates the position in the RL expression and also shows the extent to which RL belongs and the corresponding membership degrees.

2.4. Fuzzy-AHP risk estimation phase

As stated earlier in section 1, because the contribution of each hazardous event to the overall RL is different, the weight of the contribution of each hazardous event should be taken into consideration in order to represent its relative contribution to the RL of a railway depot. The application of fuzzy-AHP may also solve the problems of risk information loss in the hierarchical process in determining the relative importance of the hazardous events in the decision making process so that risk assessment can be progressed from hazardous event level to hazard group level, and finally to a railway depot level. A fuzzy-AHP is an important extension of the traditional AHP method [22,35,36], which uses a similar framework of AHP to conduct risk analysis but fuzzy ratios of relative importance replace crisp ratios to the existence of uncertainty in the risk assessment. An advantage of the fuzzy-AHP is its flexibility to be integrated with different techniques, for example, FRA techniques in risk analysis. Therefore, a fuzzy-AHP analysis leads to the generation of WFs for representing the primary hazardous events within each category. There are six steps to calculate WFs as described below.

2.4.1. Step 1: establish an estimation scheme

Fuzzy-AHP determines WFs by conducting pairwise comparison. The comparison is based on an estimation scheme, which lists intensity of importance using qualitative descriptors. Each qualitative descriptor has a corresponding triangular MF that is employed to transfer expert judgments into a comparison matrix [3,8,23,38–41]. Table 2 describes qualitative descriptors and their corresponding triangular fuzzy numbers for risk analysis at railway depots. Each grade is described by an important expression and a general intensity number. When two risk contributors are of equal importance, it is considered (1,1,2). Fuzzy number of (8,9,9) describes that one risk contributor is absolutely important than the other one. Fig. 3 shows triangular MFs (solid lines) with "*equal importance*"–(1,1,2), "*weak importance*"–(2,3,4), "*strong importance*"–(4,5,6), "*very strong importance*"–(6,7,8) and "*absolute importance*"–(8,9,9), respectively. The other triangular MFs (dash lines) describe the corresponding intermediate descriptors between them.

2.4.2. Step 2: compare risk contributors

"HG_1", "HG_2",...., and "HG_n" as shown in Fig. 2 are the risk contributors that contribute to overall RL of a railway depot. Assume two risk contributors HG_1 and HG_2, if HG_1 is of very strong importance than HG_2, a fuzzy number of (6,7,8) is then assigned to HG_1 based on the estimation scheme as shown in Table 2. Obviously, risk contributor HG_2 has fuzzy number of (1/8,1/7,1/6). Suppose n risk contributors, there are a total of $N = n(n-1)/2$ pairs need to be compared. The following classifications can be used in the comparison.

- A numerical value, e.g. "3"
- A linguistic term, e.g. "strong importance".
- A range, e.g. (2, 4), the scale is likely between 2 and 4.
- A fuzzy number, e.g. (2, 3, 4), the scale is between 2 and 4, most likely 3 or (2, 3, 4, 5), the scale is between 2 and 5, most likely between 4 and 5.
- 0, e.g. the two risk contributors cannot be compared at all.

2.4.3. Step 3: convert comparison pairwise into UFNs

As described in steps 1 and 2, because the values of risk contributors are crisps, e.g. a numerical value, a range of numerical value, a linguistic term or a fuzzy number, the FRA is employed again to convert these values into UFNs according to Table 1. A series of UFNs can be obtained to correspond to the scores and the scales of the defined risk contributors in the risk tree.

Table 2
Fuzzy-AHP estimation scheme.

Qualitative descriptors	Description	Parameters of MFs (triangular)
Equal importance (EQ)	Two risk contributors contribute equally	(1, 1, 2)
Between equal and weak importance (BEW)	When compromise is needed	(1, 2, 3)
Weak importance (WI)	Experience and judgment slightly favour one risk contributor over another	(2, 3, 4)
Between weak and strong importance (BWS)	When compromise is needed	(3, 4, 5)
Strong importance (SI)	Experience and judgment strongly favour one risk contributor over another	(4, 5, 6)
Between strong and very strong importance (BSV)	When compromise is needed	(5, 6, 7)
Very strong importance (VI)	A risk contributor is favoured very strongly over the other	(6, 7, 8)
Between very strong and absolute importance (BVA)	When compromise is needed	(7, 8, 9)
Absolute importance (AI)	The evidence favouring one risk contributor over another is of the highest possible order of affirmation	(8, 9, 9)
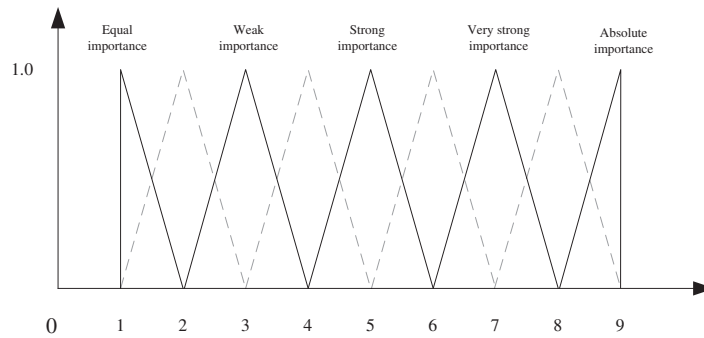
Fig. 3. MFs of qualitative descriptors in fuzzy-AHP estimation scheme.

2.4.4. Step 4: aggregate UFNs

Usually, there are a number of experts in the risk assessment group, their judgements may be different. Therefore, UFNs produced in Step 3 need to be aggregated into a group UFN for each risk contributor. The process is same as described in section 2.3 at step 3.

2.4.5. Step 5: calculate WFs

The aggregated UFN are then used to construct a comparison matrix. Suppose $C_1, C_2, \ldots, C_n$ are risk factors in a hazard group p, $A_{i,j}$ is the aggregated UFN representing the quantified judgement on C_i comparing to C_j and C_i is more important than C_j. The pairwise comparison between C_i and C_j in the hazard group p thus yields a $n \times n$ matrix defined as

$$M = [A_{i,j}] = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,n} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n,1} & A_{n,2} & \ldots & A_{n,n} \end{bmatrix} i,j = 1,2,\ldots,n \tag{12}$$

$$A_{i,j} = \{a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j}\}, A_{j,i} = \{1/a_{i,j}, 1/b_{i,j}, 1/c_{i,j}, 1/d_{i,j}\}$$

where $a_{i,j}, b_{i,j}, c_{i,j}$ and $d_{i,j}$ are the numbers of UFN $A_{i,j}$.

The WFs can be calculated by using geometric mean method [8,14,18,31,32]. The UFN geometric mean $\bar{A}_i$ of the ith row in the comparison matrix is defined as

$$\bar{A}_i = \{\bar{a}_i, \bar{b}_i, \bar{c}_i, \bar{d}_i\} = \left\{ \sqrt[n]{\prod_{j=1}^{n} a_{i,j}}, \sqrt[n]{\prod_{j=1}^{n} b_{i,j}}, \sqrt[n]{prod_{j=1}^{n} c_{i,j}}, \sqrt[n]{\prod_{j=1}^{n} d_{i,j}} \right\} \tag{13}$$

$$W_i = \{a_i, b_i, c_i, d_i\} = \left\{ \frac{\bar{a}_i}{\sum_{j=1}^{n} \bar{d}_j}, \frac{\bar{b}_i}{\sum_{j=1}^{n} \bar{c}_j}, \frac{\bar{c}_i}{\sum_{j=1}^{n} \bar{b}_j}, \frac{\bar{d}_i}{\sum_{j=1}^{n} \bar{a}_j} \right\} \tag{14}$$

where W_i is the fuzzy WF of C_i.

2.4.6. Step 6: defuzzification and normalization

Because the outputs of geometric mean method are fuzzy-WFs, a defuzzification is adopted to convert fuzzy-WFs to the corresponding crisp WF [22]. Suppose a fuzzy-WF of w_i'

$$w_i' = \frac{a_i + 2(b_i + c_i) + d_i}{6} \tag{15}$$

The WF of C_i can be calculated by

$$WF_i = \frac{w_i'}{\sum_{j=1}^{n} w_j'} \tag{16}$$

2.4.7. Step7: Calculate RLs of sub-hazard groups

Once the WFs of risk contributors are obtained, the overall RL at sub-hazard group can be calculated by synthesising of WF and RL of each hazardous event produced in FRA risk estimation phase. The RL of a sub-hazard group $S - HG_i$ is defined by

$$RL_{S-HG_i} = \sum_{i=1}^{n} RL_{C_i} WF_{C_i} \quad i = 1,2,\ldots,n \tag{17}$$

where RL_{C_i} and WF_{C_i} are the RL and WF of C_i.

Similarly, WF_{S-HG_i} of sub-hazard groups and WF_{HG_i} of hazard groups can be obtained by repeating steps 1 to 7. The RLs of hazard groups and the overall RL of a railway depot can be obtained by

$$RL_{HG_i} = \sum_{i=1}^{n} RL_{S-HG_i} WF_{S-HG_i} \quad i = 1, 2, \ldots, n \tag{18}$$

$$RL_{Depot} = \sum_{i=1}^{n} RL_{HG_i} WF_{HG_i} \quad i = 1, 2, \ldots, n \tag{19}$$

where RL_{HG_i} and WF_{HG_i} are the RL and WF of the ith hazard group HG_i, RL_{S-HG_i} and WF_{S-HG_i} are the RL and WF of the ith sub-hazard group and RL_{Depot} is overall RL of the railway depot.

2.5. Risk response phase

The results produced from the risk estimation phases may be used through the risk response phase to assist risk analysts, engineers and managers in developing maintenance and operation policies. If risks are high, risk reduction measures must be applied or the depot operation has to be reconsidered to reduce the occurrence probabilities or to control the possible consequences. However, the acceptable and unacceptable regions are usually divided by a transition region. Risks that fall in this transition region need to be reduced to as low as reasonably practicable (ALARP) [25,29,32]. In this study, the RLs are characterized into four regions, i.e. 'High', 'Substantial', 'Possible' and 'Low'.

3. A railway risk assessment prototype system

It has been noted that many risk assessment approaches currently used in the railway industry may have some problems in situations where there is a lack of confidence in risk assessment. A FRA-based approach combined with fuzzy-AHP technique may provide a solution as it emulates the reasoning process for synthesizing human expert judgements within a specific domain of knowledge, codes, and standards based on guidelines and company policy. A railway intelligent safety risk assessment system (RISRAS) has been developed based on the proposed risk analysis model. It has been designed to aid the railway risk assessment process via a user-friendly interface, hence requiring no knowledge of the manner in which the data are stored and manipulated. The system has been developed using Microsoft Visual C++ and operates under Microsoft Windows 2000, NT, XP or Vista. The architecture of RISRAS has been designed to ensure that the system is adaptable and upgradeable so that customisation of the system can be easily carried out. The current system is generic, but it is acknowledged that individuals or corporations will prefer specific systems based on the rules in the particular cases. In order to minimise system changes and maintain the flexibility and scalability of the system, the system is designed on the basis of the three-layer architecture [3,10]: i.e. presentation logic layer, application logic layer and database layer as shown in Fig. 4. The presenter can be thought of much like a web-browser that performs many functions involving user-interaction at presentation logic layer, but the bulk of the processing work is done behind the scenes, in this case, it is risk assessment server which consists of a number of modules at application logic and database layers. The risk assessment server application controls the data and information flow between the presenter and the data stores/sources, which effectively forms the heart of the RISRAS. This is particularly useful by using such a three-layer architecture in the system that allows other modules to be developed and added into the system conveniently. The main benefit by using such architecture is that the data and analytical process are completely separated from the user. This independence enables modifications to be made as each of the modules individually with little or no impact on others. The roles of each layer are discussed in the following sections.

3.1. Presentation logic layer

The presentation logic layer provides the user with a user-friendly interface to aid the railway risk assessment process. The presenter controls the graphical user interface, which the user can interact in order to perform the risk assessment process. The presenter application makes a call to the sever then communicates with other layers through several input/output functional modules, i.e. '*Risk Tree Module* (RTM)', '*Fuzzy Estimation Module* (FEM)', '*Fuzzy Weighting Module* (FWM)' and '*Output Module* (OM)' as shown in Fig. 4.

As stated earlier in this paper, the RLs of hazard groups affect the overall RL of a railway depot, which can be further broken into sub-hazard groups in order to identify all possible hazardous events. The risk assessment can initially be carried out from hazardous event level and then progressed up to sub-hazard group level, hazard group level and finally to depot level. RTM module provides the user to create a risk tree that links by risk nodes. In this case, it is hazardous events, sub-hazard groups, hazard groups with relevant information, i.e. belongings and descriptions.

FEM module consists of four sub-functional modules to process FRA evaluation. The '*Project Configuration*' defines the input risk parameters, i.e. FF, CP and CS. The '*Input/output configuration*' defines the output RL in the risk assessment. The '*MF Configuration*' defines the MFs to describe risk qualitative descriptors, fuzzy rules and fuzzy operations, which can be specified for a particular case. As there could be a large amount of risk data involved in the risk analysis, the system provides an '*Input Template*' functional module, which the user can easily input data manually or transfer data from an existing Microsoft Excel file to the system.
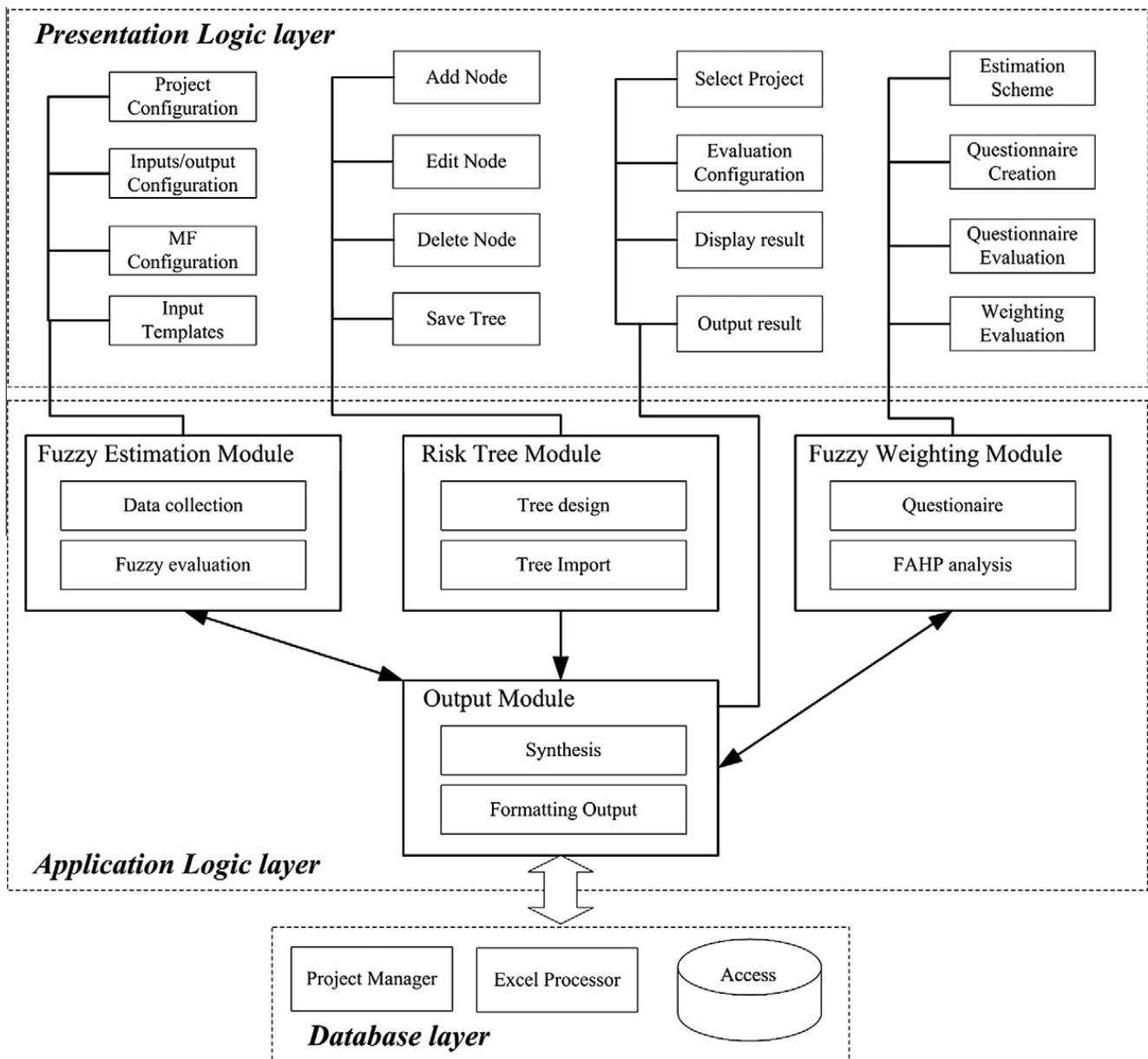
Fig. 4. The three-layer architecture of the railway risk assessment system.

As stated earlier in this paper, fuzzy-AHP is employed to determine the relative importance of risk factors in order to synthesize the contributions of risks at hazardous event level to hazard group level and finally to depot level. FWM module has four functional modules. The '*Estimation Scheme*' defines the estimation scheme as described in Section 2. It allows users to add or modify MFs to create new scheme for a particular case. The '*Questionnaire Creation*' can generate a Microsoft Excel file on the basis of selected risk factors of the risk tree and then the '*Questionnaire Evaluation*' creates a fuzzy-AHP comparison matrix automatically by the system. The WF of each risk factor can finally be derived from the '*Weight Evaluation*' functional module.

The results of FRA and Fuzzy-AHP analysis will be finally synthesised to obtain an overall RL of a railway depot. There are four functional modules in OM module, which enables the user to view and store the results into the database. The user can use '*Select Project*' functional module to manage risk trees and risk information established previously, which provides the user to choose and re-use these risk trees and risk information in the future analysis. The user can also determine whether WFs should or not be taken into consideration in the assessment process by selecting '*Evaluation Configuration*' functional module. The "*Output Result*" functional module provides the user to display and save the results in a Microsoft Excel file.

3.2. Application logic layer

The application logic layer consists of four modules to manipulate data and information flow. The FEM is developed based on the proposed risk model using FRA approach to computer the RLs of hazardous events as described in Section 2.3. The

'*Data Collection*' sub-module takes the input data from a Microsoft Excel file via Open DataBase Connectivity (ODBC) connection to database [10]. '*Fuzzy Evaluation*' sub-module performs the FRA risk estimation. RTM manages the risk trees established in the risk analysis. By using the '*Risk Tree Design*' sub-module, the user can create a risk tree for a particular case and the '*Risk Tree Import*'' sub-module delivers the risk trees to OM. The FWM is designed based on the fuzzy-AHP approach proposed in the risk model as described in Section 2, which is used to quantify expert judgements and calculate the WFs. The '*Questionnaire*' sub-module provides the user with a template in excel format via ODBC connection. The user can easily fill in the expert opinions on the template and then the '*FAHP Analysis*' sub-module calculates the WFs and produces a fuzzy-AHP comparison matrix. OM manages data flow in risk assessment process and presents the results. The '*Synthesis*' sub-module ensures that the assessment is performed from hazardous event level then progressed up to sub-hazard group level, hazard group level and finally to railway depot level according to the risk tree established in RTM by synthesizing the RLs of hazardous events derived from FEM and the WFs from FWM. The '*Formatting Output*' sub-module manages the format of the output of results to a Microsoft Excel file via ODBC connection.

3.3. Database layer

Data is clearly fundamental to the operation of the RISRAS system. The database stores most of data and information that RISRAS uses, such as risk tree, risk parameters, and WFs. The database chosen for the system is Microsoft Access. This package was selected based on several reasons, including, its popularity as a user-friendly relational database within industry, the fact that it is available for a reasonable cost and its easy access via ODBC connection [10]. The "*Excel Processor*" module provides the functions to input and output of the data from or to a Microsoft excel file. The "*Project Manager*" manipulates data of projects including the configurations of FRA and fuzzy-AHP in the RISRAS system as well the details of the risk trees.

4. A case study: Risk assessment of shunting at Hammersmith depot

An illustrated case example on risk assessment of shunting at Hammersmith depot is used to demonstrate the proposed risk assessment methodology. The case materials have been collected from the industry [27,28]. The input parameters are FF, CP and CS of hazardous events. The outputs of risk assessment are RLs of hazardous events, hazard groups, and the overall RL of shunting at Hammersmith depot with risk scores located from 0 to 10 and risk categorized as 'Low', 'Possible', 'Substantial' and 'High' with a percentage belief. The RLs of hazard groups are calculated using the FRA based on the aggregation results of each hazardous event belonging to the particular hazard group. The overall RL of shunting at Hammersmith depot is obtained on the basis of the aggregation of the RLs of each hazard group contribution weighted by using fuzzy-AHP method. The use of the RISRAS system in practice follows the risk assessment process described in Section 2.

4.1. Identification

Hammersmith depot is one of the largest depots in London Underground. Historical data of accidents and incidents have been recorded over the past ten years. In this case, the historical accident and incident databases have been reviewed in the Hammersmith depot. Seven hazard groups and 17 sub-hazard groups have been identified and defined, and each sub-hazard group consists of a number of hazardous events [27,28], which are described as follows:

(1) *Derailment hazard group* (DHG) includes two sub-hazard groups i.e. typical outcome (minor injury) and worst-case scenario (major injury) which have been identified based on the pervious accidents and incidents. Both sub-hazard groups consist of six hazardous events such as track related faults including mechanical failure of track e.g. broken rail and fishplates, signalling related faults including mechanical failure of signals and points, rolling stock faults including mechanical failure of rolling stock e.g. brakes, axles and bogies, structure failure including collapsed drain or civil structure beneath track leading to derailment, object from train including object falls from train (e.g. motor) leading to derailment (like Chancery Lane incident) and human errors including human error causing derailment e.g. overspending, incorrect routing.

(2) *Collision hazard group* (CHG) consists of four sub-hazard groups i.e. collision between trains of worst case scenario (fatality), collision between trains of typical outcome (multiple minor injuries), collision hazard of worst case scenario (fatality) and collision hazard of typical outcome (minor injury). Collision between trains involves three scenarios. For example, when MR train is moving out over infrastructure from OZ18 and LU train is moving out platform 3 due to track or signal failure. Collision hazards include, for example, collision with object on track and collision with terminal e.g. over running into buffer stop on road 24 due to excessive speed, brake failure or human error.

(3) *Train fire hazard group* (TfHG) has one sub-hazard group i.e. train fire typical outcome which covers minor injury because fires are believed that it would not be able to catch quick enough to endanger a driver more than through smoke inhalation. There are two hazardous events resulting in the train fire including arcing from conductor rail causing train fire, and electrical, oil or hydraulic failure leading to train fire.

(4) *Electrocution hazard group* (EHG) has two sub-hazard groups, typical outcome (fatality) and worst case scenario (major injury), which cover a number of hazardous events, for example, contacting with conductor rail while entering/leaving
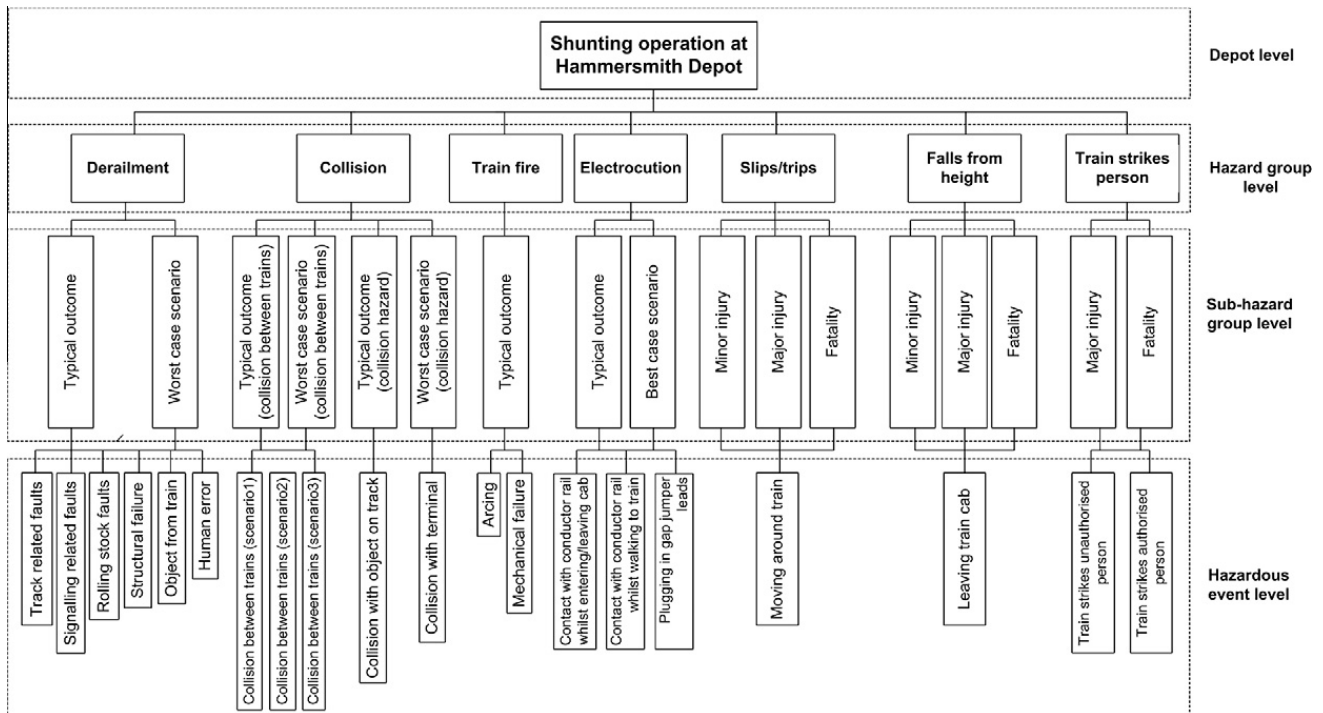
Fig. 5. Hazard identification at different levels for Hammersmith depot.

cab, contacting with conductor rail while walking to train and plugging in gap jumper leads if train is stalled/gapped. Due to high voltage direct current, fatality is most likely consequence. Therefore, even if injury is not fatal, it will still be serious.

(5) *Slips/trips hazard group* (SHG) includes three sub-hazard groups, i.e. minor injury, major injury and fatality. The hazardous events include, for example, instances when shunter is required to leave train, risks to other persons involved in move and instances when person is required to approach train when it is stalled/gapped. However, slips/trips are acknowledged as high frequency and low consequence events. Therefore, majority of slips and trips has been agreed as a minor injury and fatalities are very unlikely. There is only a chance of broken bones if slip or trip badly.

(6) *Falls from height hazard group* (FHG) consists of three sub-hazard groups i.e. minor injury, major injury and fatality which cover the events of falls from height agreed as when shunter leaves train cab. However, falls from height are much more likely than a slip/trip to result in major injury.

(7) *Train strikes person hazard group* (TsHG) has been identified based on the record in the past ten years into two sub-hazard groups – major injury and fatality. The hazardous events in these two sub-hazard groups include train strikes authorized person including other depot workers (e.g. ground shunter) or track side staff and train strikes unauthorized person e.g. trespassers, etc. Side swipe collision is considered non-fatal but still serious. Collision on head is considered fatal.

As described in Section 2, a risk tree has been developed for risk analysis of shunting at Hammersmith depot as shown in Fig. 5. Risk assessment is initially carried out from hazardous events and then progressed up to sub-hazard group level, hazard group level and finally to depot level.

The qualitative descriptors of FF, CS, CP and RL have been developed for the analysis of hunting at Hammersmith depot and the FRA is employed to estimate the RL of each hazardous event in terms of FF, CS and CP. The definition of FF defines the number of times of an event occurs over a specified period, e.g. number of events/year. The qualitative descriptors of FF are defined as 'Remote', 'Rare', 'Infrequent', 'Occasional', 'Frequent', 'Regular' and 'Common' and their meanings are presented in Table 3.

CS describes the magnitude of possible consequence in terms of number of fatalities, major and minor injuries resulting from the occurrence of a particular hazardous event. The qualitative descriptors of CS are defined as 'Negligible', 'Marginal', 'Moderate', 'Critical', and 'Catastrophic' and their meanings are shown in Table 4, where major and minor injuries are calculated in terms of equivalent fatalities. 10 major injuries or 200 minor injuries are considered equal to one equivalent fatality [25,32,34]. For example, qualitative descriptor 'Marginal' is defined to describe the consequence level of minor injury with an approximate numerical value of 0.005.

CP defines the FF that failure effects that will happen given the occurrence of the failure. One may often use such qualitative descriptors as 'Highly unlikely', 'Unlikely', 'Reasonably unlikely', 'Likely', 'Reasonably likely', 'Highly likely' and 'Definite'. Table 5 shows the evaluation criteria of CP and the corresponding qualitative descriptors.

Table 3
Definitions of qualitative descriptors of FF.

Index	Qualitative descriptors	Description	Mid-point of the estimated frequency	Approximate numerical value (event/yr)	Parameters of MFs (trapezoid)
1	Remote	< 1 in 175 years	1 in 500 years	0.002	0.00, 0.00, 2.00E−3, 6.00E−3
2	Rare	1 in 35 years to 1 in 175 years	1 in 100 years	0.01	2.00E−3, 6.00E−3, 1.50E−2, 4.00E−2
3	Infrequent	1 in 7 years to 1 in 35 years	1 in 20 years	0.05	1.50E−2, 4.00E−2, 8.00E−2, 2.00E−1
4	Occasional	1 in 1 ¼ years to 1 in 7 years	1 in 4 years	0.25	8.00E−2, 2.00E−1, 5.00E−1, 1.25
5	Frequent	1 in 3 months to 1 in 1 ¼ years	1 in 9 months	1.25	5.00E−1, 1.25, 2.25, 5.25
6	Regular	1 in 20 days to 1 in 3 months	1 in 2 months	6.25	2.25, 5.25, 10.25, 31.25
7	Common	1 in 4 days to 1 in 20 days	1 in 12 days	31.25	10.25, 31.25, 1.00E2, 1.00E2

Table 4
Definitions of qualitative descriptors of CS.

Index	Qualitative descriptors	Description	Numerical values	Parameters of MFs (trapezoid)
1	Negligible	No injury and/or negligible damage to the system	0–0.009	0.00, 0.00, 9.00E−3, 2.00E−2
2	Marginal	Minor system damage and/or minor injury	0.009–0.1	9.00E−3, 2.00E−2, 1.00E−1, 2.00E−1
3	Moderate	Failure causes some operational dissatisfaction and/or major injury	0.1–0.4	1.00E−1, 2.00E−1, 4.00E−1, 5.00E−1
4	Critical	Major system damage and/or severe injury	0.4–0.9	4.00E−1, 5.00E−1, 9.00E−1, 2.00
5	Catastrophic	System loss and/or fatality	>0.9	9.00E−1, 2.00, 5.00, 5.00

Table 5
Definitions of qualitative descriptors of CP.

Index	Qualitative descriptors	Description	Parameters of MFs (trapezoid)
1	Highly unlikely	The occurrence likelihood of accident is highly unlikely	0.00, 0.00, 1.50E−1, 2.00E−1
2	Unlikely	The occurrence likelihood of accident is unlikely but possible given the occurrence of the failure event	1.50E−1, 2.00E−1, 2.50E−1, 3.00E−1
3	Reasonably unlikely	The occurrence likelihood of accident is between likely and unlikely	2.50E−1, 3.00E−1, 3.50E−1, 4.25E−1
4	Likely	The occurrence likelihood of accident is likely	3.50E−1, 4.25E−1, 5.75E−1, 6.50E−1
5	Reasonably likely	The occurrence likelihood of accident is between likely and highly likely	5.75E−1, 6.50E−1, 7.00E−1, 7.50E−1
6	Highly likely	The occurrence likelihood of accident is very likely	7.00E−1, 7.50E−1, 8.00E−1, 8.50E−1
7	Definite	The accident occurs given the occurrence of the failure event	8.00E−1, 8.50E−1, 1.00, 1.00

Table 6
Definitions of qualitative descriptors of RL.

Qualitative descriptors	Description	Parameters of MFs (trapezoid)
Low	Risk is acceptable	0, 0, 1, 2
Possible	Risk is tolerable but should be further reduced if it is cost-effective to do so	1, 2, 4, 5
Substantial	Risk must be reduced if it is reasonably practicable to do so	4, 5, 7, 8
High	Risk must be reduced to safe in exceptional circumstances	7, 8, 10, 10

The qualitative descriptors of RL are defined as 'Low', 'Possible', 'Substantial', and 'High'. Their definitions, which are generally similar to those described in EN50126, EN50129, and GE/GN8561 [12,13,32], are listed in Table 6. The risk score is defined in a manner that the lowest score is 0, whereas the highest score is 10. For example, qualitative descriptor, 'Low', is defined on the basis of the risk score ranging from 0 to 2. Similar to the input qualitative descriptors of FF, CS and CP, the trapezoidal MFs are used to describer the RL. The results of RLs can be expressed either risk score located in the range from 0 to 10 or as risk category with a belief of percentage.

Because three parameters of FF, CP, and CS are used to determine the RLs of hazardous events, the rule base therefore consists of 245 if-then rules for this study. Fig. 6 shows five rule matrices. As can be seen that each matrix consists of 49 rules with a particular qualitative descriptor of CS. For example, the rule at the top left of the matrix of CS = Negligible would be expressed as follows:

IF FF is *Remote* and CP is *Highly unlikely* and CS = *Negligible*, THEN RL is *Low*

The RISRAS provides the user a design panel to develop the rule bases, which enable the user to update or modify rules depending on particular cases easily.
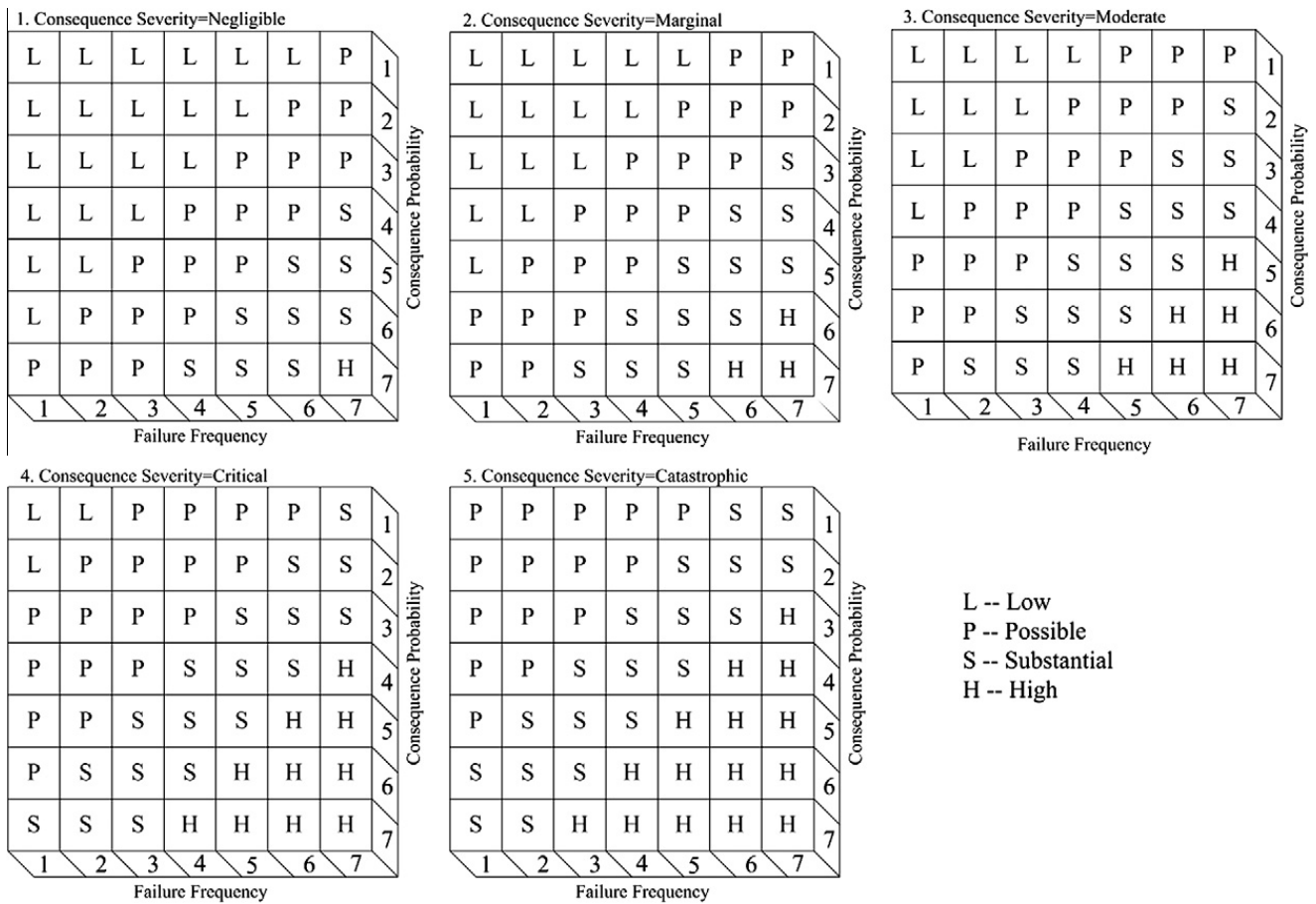
Fig. 6. Fuzzy rule base matrices.

In this case, five experts with high qualification regarding this subject are involved in the risk assessment group. EIs are allocated to experts by Eq. (1) on the basis of their background and experience as shown in Table 7. For example, the expert E5 has less experience. Therefore, he has a lowest $EI = 0.16$.

4.2. FRA risk estimation

Table 8 shows the inputs of FF, CS and CP of sub-hazard groups. For example, the FF, CS and CP of 'derailment (typical outcome)' sub-hazard group are '3.33E−2', '9.90E−1', and '5.00E−3' respectively. These crisp values are then converted into corresponding UFNs according to Table 1 and Eq. (2). Because the numerical data have been obtained from the historical records, there is no need to aggregate expert judgements. In this case, $A_{FF}^{i} = A_{FF,k}^{i}$, $A_{CP}^{i} = A_{CP,k}^{i}$, $A_{CS}^{i} = A_{CS,k}^{i}$.

According to Eqs. (4)–(6), UFNs are then transferred into fuzzy sets to obtain $\tilde{A}_{FF}$, $\tilde{A}_{CS}$ and $\tilde{A}_{CP}$. For example, the fuzzy sets $\tilde{A}_{FF}, \tilde{A}_{CP} and \tilde{A}_{CS}$ of 'derailment (typical outcome)' sub-hazard group are:

$$\tilde{A}_{FF} = \left\{ (u, \mu_{A_{FF}^{i}}(u)) | u \in [0, 100], \mu_{A_{FF}^{i}}(u) = \left\{ \begin{array}{ll} 1 & u = 0.0333 \\ 0 & otherwise \end{array} \right\} \right\}$$

$$\tilde{A}_{CP} = \left\{ (v, \mu_{A_{CP}^{i}}(v)) | v \in [0, 100], \mu_{A_{CP}^{i}}(v) = \left\{ \begin{array}{ll} 1 & v = 0.99 \\ 0 & otherwise \end{array} \right\} \right\}$$

$$\tilde{A}_{CS} = \left\{ (w, \mu_{A_{CS}^{i}}(w)) | w \in [0, 100], \mu_{A_{CS}^{i}}(w) = \left\{ \begin{array}{ll} 1 & w = 0.005 \\ 0 & otherwise \end{array} \right\} \right\}$$

In the fuzzy inference process, the fire strength of each rule with input fuzzy sets can be calculated by Eq. (8). Then, the fuzzy implication is applied to obtain the conclusion fuzzy sets of the fired rules. The truncated MF of the conclusion fuzzy set of each rule can be obtained by Eq. (9). Finally, the conclusion fuzzy sets are aggregated by Eq. (10) to form a single fuzzy set which represents the fuzzy output of RL.

On the basis of the aggregated fuzzy set, by using Eq. (11) defuzzification calculates the defuzzified value, which is a crisp value, standing for the final result of fuzzy inference as shown in Table 8. For example, the RL of "derailment (typical outcome)' sub-hazard group is 3 which indicates that risk score of 3 and risk categories of 'possible' with a belief of 78% and

Table 7
Expert Index (EIs).

Experts	Years of experience	EIs
E_1	10 years experience	0.21
E_2	15 years experience	0.24
E_3	10 years experience	0.21
E_4	8 years experience	0.18
E_5	5 years experience	0.16
$E_1 + E_2 + E_3 + E_4 + E_5 = 1.00$		

Table 8
Shunting at Hammersmith depot.

Operation	Hazard Groups	Index	Sub−hazard Groups	Failure Frequency	Consequence Probability	Consequence Severity	Risk Score	Risk Category	EFA Risk Score
Shunting at Hammersmith depot	Derailment	1	Derailment (typical outcome)	3.33E−2	9.90E−1	5.00E−3	3.00	Possible: 78% Substantial: 22%	7 (Medium)
		2	Derailment (worst case scenario)	3.33E−2	1.00E−2	1.25E−1	0.81	Low: 100%	4 (Low)
	Collision	3	Collision between trains (worst case scenario)	3.00E−2	2.00E−1	6.25E−1	2.52	Possible: 100%	6 (Low)
		4	Collision between trains (typical outcome)	3.00E−2	8.00E−1	2.50E−2	3.00	Possible: 100%	4 (Low)
		5	Collision hazard (typical outcome)	1.20E−1	9.90E−1	5.00E−3	4.04	Possible: 96% Substantial: 4%	4 (Low)
		6	Collision hazard (worst case scenario)	1.20E−1	1.00E−2	1.25E−1	0.82	Low: 100%	4 (Low)
	Train fire	7	Train fire (typical outcome)	1.50E−2	1.00	5.00E−3	3.00	Possible: 100%	6 (Medium)
	Electrocution	8	Electrocution (typical outcome)	5.71E−2	7.50E−1	6.25E−1	6.00	Substantial: 100%	8 (High)
		9	Electrocution (best case scenario)	5.71E−2	2.50E−1	1.25E−1	0.80	Low: 100%	5 (Low)
	Slips/trips	10	Slips / trips (minor injury)	2.00E−1	8.00E−1	5.00E−3	3.00	Possible: 100%	5 (Low)
		11	Slips / trips (major injury)	2.00E−1	1.50E−1	1.25E−1	0.80	Low: 100%	5 (Low)
		12	Slips / trips (fatality)	2.00E−1	5.00E−2	6.25E−1	3.00	Possible: 100%	6 (Medium)
	Falls from height	13	Falls from height (minor injury)	1.43E−2	1.50E−1	5.00E−3	0.75	Low: 100%	2 (Low)
		14	Falls from height (major injury)	1.43E−2	8.00E−1	1.25E−1	3.00	Possible: 100%	5 (Low)
		15	Falls from height (fatality)	1.43E−2	5.00E−2	6.25E−1	0.75	Low: 100%	5 (Low)
	Train strikes person	16	Train strikes person (major injury)	1.00E−1	5.00E−1	1.25E−1	3.00	Possible: 100%	6 (Medium)
		17	Train strikes person (fatality)	1.00E−1	5.00E−1	6.25E−1	3.54	Possible: 100%	7 (Medium)

substantial of 22%. Similarly, the RLs of other sub-hazard groups can be calculated. RLs of sub-hazard groups are summarised in Table 8. The results of sub-hazard groups of EFA analysis [27,28] are also given in Table 8. The EFA value of each sub-hazard group and each hazardous event can be calculated as

$$EFA_{j-subgroup} = \sum_{i=1}^{n} EFA_i \quad (i = 1, 2, \ldots, n) \tag{20}$$

$$EAF_i = FF_i \times CS_i \tag{21}$$

where $EFA_{j-subgroup}$ is the EFA value of the jth sub-hazard group that has n failure events and EFA_i is the EFA value of the ith failure event. The EFA method only has two input parameters in its analysis process and this method cannot determine the contribution weight of each sub hazard groups to the overall RL of a railway depot. Therefore, the assessment can only be carried out at sub-hazard group level. By comparing both results from proposed risk model and EFA, it is worth to notice that although EFA method has a different output format comparing with the proposed method, but, however, both of methods determine that "electrocution (typical outcome)" sub hazard group has higher risk than other sub hazard groups, and also the proposed method is able to provide distinct risk rankings for the assessment comparing to the EFA method. Moreover, the proposed method can provides more reliable and comprehensive results that provide railway engineers, maintainers and managers with valuable information for risk response decision making.

Table 9
Expert judgements.

Index	Comparison	E1 Judgment	E1 Converted UFNs	E2 Judgment	E2 Converted UFNs	E3 Judgment	E3 Converted UFNs	E4 Judgment	E4 Converted UFNs	E5 Judgment	E5 Converted UFNs	Aggregated UFNs
1	CHG vs. DHG ($A_{1,2}$)	WI	1, 1, 1, 2	BWS	3, 4, 4, 5	4,6	4, 5, 5, 6	4,5	4, 4.5, 4.5, 6	BWS	3, 4, 4, 5	2.94, 4.00, 4.00, 5.00
2	DHG vs. TfHG ($A_{1,3}$)	EQ	1, 1, 1, 2	1,3	1, 2, 2, 3	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	1,2	1, 1.5, 1.5, 1	1.00, 1.32, 1.32, 2.24
3	DHG vs. EHG ($A_{1,4}$)	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	1.00, 1.00, 1.00, 2.00
4	DHG vs. SHG ($A_{1,5}$)	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	WI	2, 3, 3, 4	2, 3, 5	2, 3, 3, 5	WI	2, 3, 3, 4	2.45, 3.45, 3.45, 4.63
5	DHG vs. FHG ($A_{1,6}$)	BEW	1, 2, 2, 3	WI	2, 3, 3, 4	WI	2, 3, 3, 4	BEW	1, 2, 2, 3	WI	2, 3, 3, 4	1.61, 2.61, 2.61, 3.61
6	TsHG vs. DHG ($A_{1,7}$)	WI	2, 3, 3, 4	WI	2, 3, 3, 4	SI	4, 5, 5, 6	WI	2, 3, 3, 4	WI	2, 3, 3, 4	0.23, 0.31, 0.31, 0.45
7	CHG vs. TfHG ($A_{2,3}$)	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	SI	4, 5, 5, 6	BWS	3, 4, 4, 5	2.38, 3.18, 3.18, 3.99
8	CHG vs. EHG ($A_{2,4}$)	WI	2, 3, 3, 4	WI	2, 3, 3, 4	WI	2, 3, 3, 4	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	2.20, 3.20, 3.20, 4.20
9	CHG vs. SHG ($A_{2,5}$)	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	WI	2, 3, 3, 4	WI	2, 3, 3, 4	2.45, 3.45, 3.45, 4.45
10	CHG vs. FHG ($A_{2,6}$)	WI	2, 3, 3, 4	WI	2, 3, 3, 4	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	2.34, 3.34, 3.34, 4.34
11	TsHG vs. CHG ($A_{7,2}$)	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	1.00, 1.00, 1.00, 2.00
12	TfHG vs. EHG ($A_{3,4}$)	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	1.00, 1.00, 1.00, 2.00
13	TfHG vs. SHG ($A_{3,5}$)	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	3.03, 4.03, 4.03, 5.03
14	TfHG vs. FHG ($A_{3,6}$)	BWS	3, 4, 4, 5	SI	4, 5, 5, 6	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	3.03, 4.03, 4.03, 5.03
15	TsHG vs. TfHG ($A_{7,3}$)	BSV	5, 6, 6, 7	SI	4, 5, 5, 6	BWS	3, 4, 4, 5	SI	4, 5, 5, 6	BWS	3, 4, 4, 5	3.70, 4.76, 4.76, 5.88
16	EHG vs. SHG ($A_{4,5}$)	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	BEW	1, 2, 2, 3	2.47, 3.47, 3.47, 4.47
17	EHG vs. FHG ($A_{4,6}$)	BWS	3, 4, 4, 5	BSV	5, 6, 6, 7	SI	4, 5, 5, 6	BWS	3, 4, 4, 5	WI	2, 3, 3, 4	3.53, 4.53, 4.53, 5.53
18	TsHG vs. EHG ($A_{7,4}$)	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	EQ	1, 1, 1, 2	1.00, 1.00, 1.00, 2.00
19	SHG vs. FHG ($A_{5,6}$)	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	SI	4, 5, 5, 6	WI	2, 3, 3, 4	BWS	3, 4, 4, 5	3.03, 4.03, 4.03, 5.03
20	TsHG vs. SHG ($A_{7,5}$)	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	SI	4, 5, 5, 6	BWS	3, 4, 4, 5	BWS	3, 4, 4, 5	3.13, 4.17, 4.17, 5.26
21	TsHG vs. FHG ($A_{7,6}$)	BSV	5, 6, 6, 7	SI	4, 5, 5, 6	BSV	5, 6, 6, 7	BSV	5, 6, 6, 7	SI	4, 5, 5, 6	4.55, 5.56, 5.56, 6.67

Table 10
Pairwise comparison matrix *M*.

A_{ij}	1	2	3	4	5	6	7
1	1, 1, 1, 1	0.20, 0.25, 0.25, 0.34	1.00, 1.32, 1.32, 2.24	1.00, 1.00, 1.00, 2.00	2.45, 3.45, 3.45, 4.63	1.61, 2.61, 2.61, 3.61	0.23, 0.31, 0.31, 0.45
2	2.94, 4.00, 4.00, 5.00	1, 1, 1, 1	2.38, 3.18, 3.18, 3.99	2.20, 3.20, 3.20, 4.20	2.45, 3.45, 3.45, 4.45	2.34, 3.34, 3.34, 4.34	0.50, 1.00, 1.00, 1.00
3	0.45, 0.76, 0.76, 1.00	0.25, 0.31, 0.31, 0.42	1, 1, 1, 1	1.00, 1.00, , 1.00, 2.00	2.79, 3.79, 3.79, 4.79	3.03, 4.03, 4.03, 5.03	0.17, 0.21, 0.21, 0.27
4	0.50, 1.00, 1.00, 1.00	0.24, 0.31, 0.31, 0.45	0.50, 1.00, 1.00, 1.00	1, 1, 1, 1	2.47, 3.47, 3.47, 4.47	3.53, 4.53, 4.53, 5.53	0.50, 1.00, 1.00, 1.00
5	0.22, 0.29, 0.29, 0.41	0.22, 0.29, 0.29, 0.41	0.21, 0.26, 0.26, 0.36	0.22, 0.29, 0.29, 0.40	1, 1, 1, 1	3.03, 4.03, 4.03, 5.03	0.19, 0.24, 0.24, 0.32
6	0.28, 0.38, 0.38, 0.62	0.23, 0.30, 0.30, 0.43	0.20, 0.25, 0.25, 0.33	0.18, 0.22, 0.22, 0.28	0.20, 0.25, 0.25, 0.33	1, 1, 1, 1	0.15, 0.18, 0.18, 0.22
7	2.22, 3.23, 3.23, 4.35	1.00, 1.00, 1.00, 2.00	3.70, 4.76, 4.76, 5.88	1.00, 1.00, 1.00, 2.00	3.13, 4.17, 4.17, 5.26	4.55, 5.56, 5.56, 6.67	1, 1, 1, 1

Table 11
RLs of hazard groups at Hammersmith depot.

Operation	Index	Hazard groups	Risk score	Risk category	WF	Contribution
Shunting at Hammersmith depot (3.29, Possible: 100%)	1	Derailment	2.31	Possible: 100%	0.12	8%
	2	Collision	3.3	Possible: 100%	0.27	27%
	3	Train fire	3	Possible: 100%	0.11	10%
	4	Electrocution	4.47	Possible: 53% Substantial: 47%	0.14	19%
	5	Slips/trips	2.4	Possible: 100%	0.06	4%
	6	Falls from height	2.17	Possible: 100%	0.04	2%
	7	Train strikes person	3.54	Possible: 100%	0.27	30%

4.3. Fuzzy-AHP risk estimation

In order to obtain the overall RL of shunting at Hammersmith depot, the relative importance of the contribution of hazard groups are taken into account. Therefore, the WFs of hazard groups are calculated and then synthesised with the RLs of hazard groups to determine the RL at railway depot level as described in Section 2.

The estimation scheme has been agreed and established by the experts in the risk assessment group as shown in Table 2. Table 9 shows the expert judgements of relative importance of hazard groups. Experts can use linguistic terms, numerical numbers, ranges and fuzzy numbers to express their opinions. For example, when comparing CHG with DHG, experts agree that CHG is more important than DHG. But they have different judgement on the importance degree, which experts E1, E2, E3, E4 and E5 chose 'WI', 'BWS', '4, 6', '4, 5' and 'BWS', respectively. According to Table 1, the judgements are converted into UFNs. The converted UFNs are then aggregated with respect to EIs by using Eq. (3). The aggregated UNFs are shown in Table 9.

The comparison matrix M is then established with these aggregated UNFs by Eq. (12). The completed comparison matrix M is shown in Table 10.

The UFN geometric mean $\bar{A}_i$ of the ith row in the comparison matrix is calculated by Eq. (13). Then fuzzy W_i of WFs of hazard groups can be calculated by Eq. (14). The crisp value w_i' of fuzzy WF W_i can be obtained by Eq. (15). The final WF of hazardous groups WF_i is determined by Eq. (16) as shown in Table 11.

Once the WFs of hazard groups are obtained, the RL at railway depot level RL_{Depot} can be derived from the synthesis of hazard group's WFs and RLs by Eq. (17) as shown in Table 11. As can be seen that the overall RL of shunting at Hammersmith depot is 3.29 which indicates 'Possible' with a belief of 100 percent.

4.4. Risk response phase

The overall RL of shunting at Hammersmith depot is 3.29 belonging to risk category of 'Possible' with a belief of 100%. This requires risk reduction measures to reduce the overall RL of depot to ALARP. Seven hazard groups effect the overall RL estimation at the Hammersmith depot. It should be noted that each hazard group contributes a different weight value to the overall RL of depot. As can be seen from Table 11 that the major contributions are from the hazard groups of 'Train strikes person', 'Collision' and 'Eelectrocution', which contributed 30%, 27% and 19%, respectively, to overall RL of shunting at Hammersmith depot. Each hazard group consists of a number of hazardous events. For example, in this case, there are two main hazardous events in 'Train strikes person' hazard group such as 'Train strikes authorized person' and 'Train strikes unauthorized person', which result in personal injury or fatality. Based on the accident and incident reports and statistics, majority of train striking person risk is put down to human errors. Therefore, in order to reduce RLs of 'Train strikes authorised person' such as track side staff, reference manual procedures and official walkways should be provided to avoid possible accidents. At the same time, depot familiarization and depot track accustomed certification are necessary to reduce the frequency of the hazardous events and possibility to a certain consequence. For example, PPE could be provided to reduce the frequency of the hazardous event and possibility to serious consequence and the possible loss from the accidents. With regard to 'Train strikes unauthorized person' such as trespassers, various security measures should be carried out such as CCTV, controlled access and egress to depot and platform end barriers. Furthermore, other potential control measures to reduce train striking person risks including staff training, limiting train speed and use of brakes and whistles on trains, which are effective measures to reduce RLs of both hazardous events. The potential control measures to reduce other hazardous events in other hazard groups and the proposed strategy for each addition control measures have been recommended to industry.

The hazard groups of 'Train fire', 'Derailment', 'Slips/trips' and 'Falls from height' contribute less than the above hazard groups with 10%, 8%, 4% and 2%, respectively. Although these hazard groups have relatively minor contribution to the overall RL of shunting at Hammersmith depot, the control measures are still done to reduce those hazardous events whose RLs fall in the transition region, i.e. 'Possible' and 'Substantial'. For example, the hazard group of 'Train fire' contributes 10% to the overall RL of depot. As the major fire related hazardous events leading to system failure or personal injury and health hazards include arcing and mechanical failure, therefore, the suggested control measures are to provide maintenance and inspection of fleet and track assets regularly. In addition, installed fire extinguishers in cab could mitigate consequence and reduce the chance to severe outcomes.
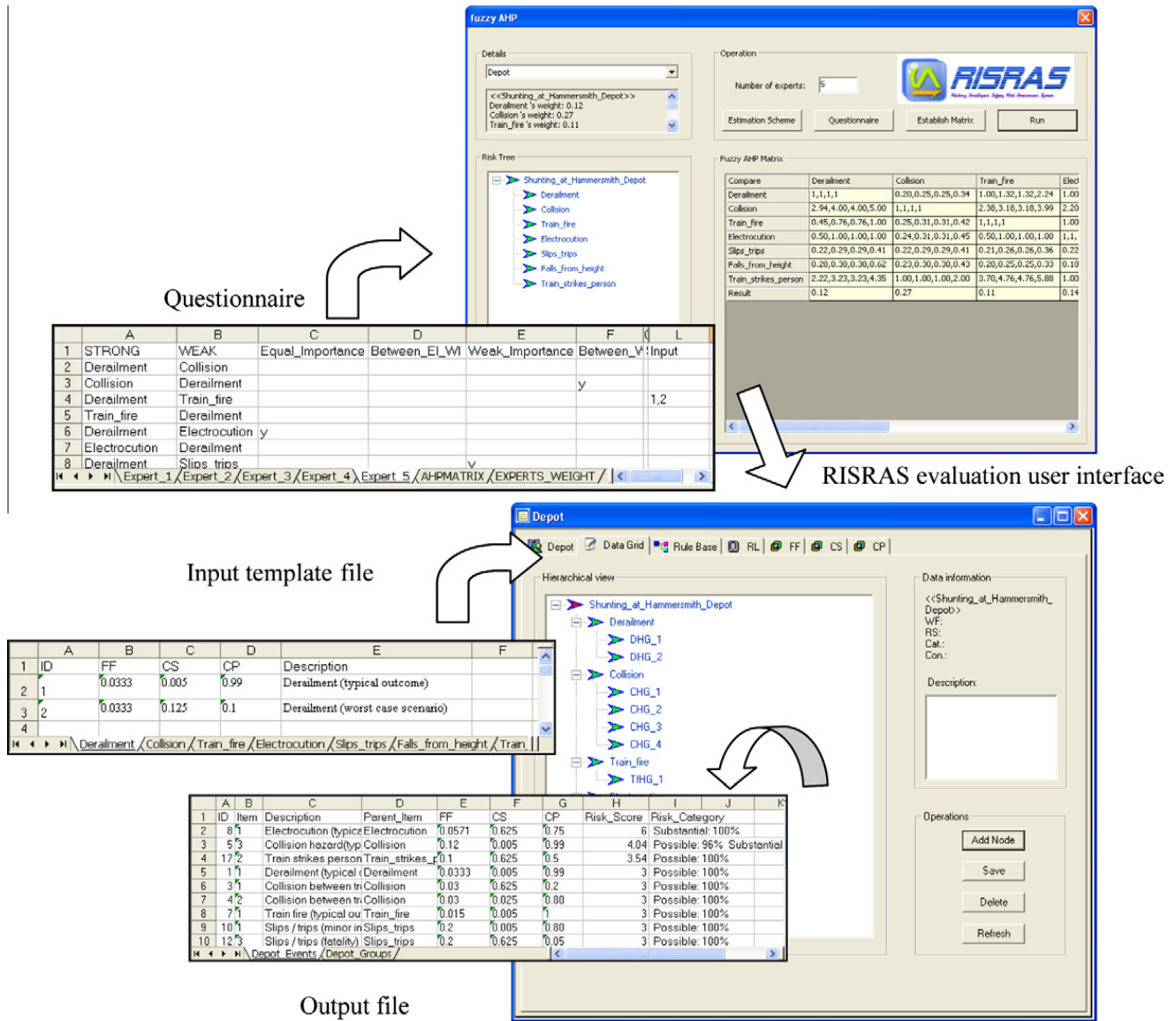
Fig. 7. Proposed RISRAS system.

5. Conclusions

Traditionally risk assessment techniques currently used in the railway industry have adopted a probabilistic approach, which heavily rely on the availability and accuracy of data, sometimes they are unable to deal adequately incomplete or uncertain data. This paper presents a new systematic railway risk assessment model using FRA and fuzzy-AHP, which is able to deal with both quantitative and qualitative risk data and information. A new risk parameter of consequence probability (CP) is introduced in order to obtain more reliable results of risk assessment, which the hazardous events with high CP can be properly assessed and identified. The outcomes of risk assessment are the RLs of hazardous events, hazard groups and a railway system and corresponding risk categories as well as risk contributions. It will provide railway risk analysts, managers and engineers with useful information to improve safety management and set safety standards.

A prototype railway safety risk assessment system is presented in this paper. Some screen shots of the proposed railway risk assessment system are shown in Fig. 7. This system consists of a user-friend interface that controls the risk assessment process including a project manager, an Excel processor, and a database management system, which is easy to use and update. A case study of risk analysis of shunting at Hammersmith depot is used to demonstrate the application of the proposed risk analysis methodology. Comparing with the conventional methods, that advantages of the proposed risk assessment system can be summarized as: (1) it can handle expert knowledge, engineering judgments, and risk historical data for the railway risk assessment in a consistent manner, (2) it can use imprecise, ambiguous and uncertainty information in the assessment, (3) the risk can be evaluated directly using linguistic expressions which are employed in the risk assessment, (4) the risk can assessed effectively on the basis of knowledge base built by transforming information from various sources,

and (5) it provides a more flexible structure for combining failure frequency, consequence and consequence probability in risk analysis.

Acknowledgements

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