Small Signal Model of Modular Multilevel Matrix Converter for Fractional Frequency Transmission System

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ABSTRACT
Fractional frequency transmission is a promising technology for medium distance offshore wind power transmission. The key component in the fractional frequency transmission system (FFTS) is the modular multilevel matrix converter (M³C). It is regarded as the next generation AC/AC converter for high voltage and high power applications due to various advantages such as high-quality waveforms, scalability and controllability. It is important to fully study its impact on the power network. The key to the understanding and impact is the development of a suitable model, which is the focus of this paper. A small signal model of the M³C taking into account the dynamics of the capacitor voltage, AC currents and the control system is developed. Electrical quantities from both AC sides at different frequencies couple in the M³C since there is no DC link. The complicated nonlinear terms in ABC frame are isolated and transformed in DQ frame. The model is convenient to interface with the control system and external AC systems. Small signal analysis is carried out on the influence of the controller parameters and sub-module capacitance. The correctness of the proposed model is verified by comparing to a detailed electromagnetic transient model of the M³C simulated in RTDS.

INDEX TERMS
Fractional frequency transmission system, small signal model, modular multilevel matrix converter, AC/AC converter, energy storage.

I. INTRODUCTION
Offshore wind power develops rapidly in recent years. Its penetration keeps growing and the market witnesses increasing investments and decrease of product prices [1]. Away from the population center, offshore wind is advantageous not only because of not taking up city land, but also due to higher wind speed and suitability for large-scale exploitation. Traditional high voltage AC (HVAC) is not capable of long distance offshore transmission [2], while fractional frequency transmission (FFT) proposed in [3] is able to overcome its cable shortcoming and it is an economical solution for medium distance offshore wind power transmission [4]. The principle is to use a proportion of the system frequency, mostly 1/3, for the offshore system. Compared to high voltage DC (HVDC) transmission, another merit of FFT is the offshore grid forming capability, as AC technology is well developed and it is more reliable especially with a high penetration level of offshore wind power in the future.

The core component of a FFTS is the frequency changer, or the AC/AC converter. In early research, cycloconverter was often adopted since it has been used in driving applications [5]. However, it is not an ideal candidate for offshore wind connection. Because the grid code has stringent requirements on fault ride through capability and harmonics level of wind power conversion and transmission systems, but cycloconverter has poor controllability and heavy harmonics [6]. On the contrary, modular multilevel matrix converter (M³C) enjoys advantages including low loss and harmonics, full controllability and flexible scalability, making it particularly suitable for high voltage applications, e.g., offshore wind power transmission [7].

First introduced in 2001 [8], M³C has been proposed for various applications, mainly in wind power integration [9, 10] and motor driving [11, 12], and also in power quality enhancement as a unified power quality conditioner (UPQC) [13]. In [11], a decoupled current control method was proposed for M³C on medium voltage motor drives. A three-phase 400V 15kW experiment system was developed to verify the performance. [12] focused on optimizing the phase reactor of the M³C to reduce its size and weight. The effectiveness was proven by experimental results in a downscaled motor drive system. Motor driving is a vital application as it plays an important role in industry production [14]. But for offshore wind transmission, different from driving applications, the sub-module number is high to reach a high voltage rating, and the frequencies on both AC sides are controlled to be constant, giving a fixed frequency ratio. This paper focuses on offshore wind FFTS application. In [15], a space vector modulation (SVM) based control was used on M³C to realize wide frequency range operation. But SVM is not recommended to be used at high voltage level since the increasing number of sub-modules induces an exponential growth of the space vectors, making it time-consuming to compute and difficult to analyze. A widely used control method was developed in [16]. The concept of double $ABC - \alpha \beta 0$ transformation was used to realize fully decoupled current control. But
some variables after two transformations were not DC but with a mixture of frequency components. A multi-hierarchy control strategy in ABC frame was developed for $M^3C$ in [17]. However, this method would bring difficulties for small signal modelling as the variables were time-variant even at steady state. In this paper, the complicated nonlinear terms in ABC frame are isolated and transformed into DQ frame so that the model is convenient to interface with the control system and external AC systems. [18] focused on topologies and control of $M^3C$-based FFT but the sub-module voltage ripple was not considered. And in the case study, the $M^3C$ was consisted of only 3 sub-modules in each arm and each sub-module had an average voltage of 60 kV. On the contrary, the proposed model in this paper considers more detailed dynamics of the sub-modules by including the capacitor voltage ripples. As will be shown by the small signal analysis, this is necessary because the modes related to capacitor voltage ripples can have poor damping and adversely affect stability. Moreover, the $M^3C$ EMT model in this paper has a sub-module number of 40 and average voltage of 1.5 kV considering the IGBT capability nowadays. The mathematical model and the time-domain simulation are more realistic and accurate.

Although great effort has been spent on $M^3C$ control method development, very limited attention has been paid to $M^3C$ small signal modelling. The small signal model of $M^3C$ should be convenient to be interlinked to different AC networks for system study. Also, as discussed earlier in this section, different control algorithms exist in the literature so the model should have easy interface with control methods. In [5], the small signal stability of an FFTS with cycloconverter was analyzed. But the AC/AC converter was modelled using only a first order time delay neglecting the dynamics from the cycloconverter and therefore the potential interaction between the converter and AC systems could not be evaluated. Contrarily, the promising AC/AC converter $M^3C$ is modelled in this paper, taking into account its internal dynamics. Some work has been done on modular multilevel converter (MMC) HVDC in terms of small signal modelling [19, 20]. However, $M^3C$ is fundamentally different from its multilevel counterpart. In MMC-HVDC, AC quantities are rectified to DC and transmitted until being inverted back to AC. For different needs and requirements the DC terminal can be modelled in different levels of details. In recent years, increasing attention has been paid to developing a more accurate MMC model to explore the interaction between the converter and the AC power systems or even the interaction between converters [21]. In $M^3C$, there is no DC link so quantities at two frequencies from both AC sides couple together in nine arms of the converter. Such operation is not common and therefore it is of significant importance to understand the dynamics of the $M^3C$ and its impact on the power grids. However, to the best knowledge of the authors, a small signal model of $M^3C$ does not exist in the literature yet. The main contribution of this paper is to develop a small signal model of the $M^3C$ for FFTS which can be used for small signal dynamic studies and controller design. Besides, a small signal analysis is carried out, giving insights to system stability and parameter selection on controller and sub-module capacitors. Frequencies from both AC sides mingling in the $M^3C$ are isolated and decoupled. The model is developed in DQ frame and it can be interlinked simply with the external AC systems and the control system. It is shown that the model is with a reduced number of variables but maintains satisfactory accuracy.

The rest of the paper is organized as follows. Section II introduces the operating principle of the $M^3C$ for FFTS. The voltage and current equations are derived at steady state. Section III develops the small signal model of the $M^3C$ for FFTS, considering the dynamics of the capacitor DC and ripple voltage components, AC current and the control system including the vector control, PLL and signal measurement. In Section IV, the developed small signal model is verified by a comparison with the time domain simulation of a detailed EMT model in RTDS. The influences of the control parameters and sub-module capacitance on small signal stability are analyzed. Section V provides a summary of the paper.

### II. $M^3C$ OPERATING PRINCIPLE FOR FFTS

An offshore wind FFTS is shown in Fig. 1. The offshore wind farm generates power at 20 Hz and it is transmitted at fractional frequency until the onshore $M^3C$ station steps up the frequency back to the system frequency at 60 Hz.

#### FIGURE 1. Illustrative diagram of an offshore wind FFTS.

The schematic diagram of the $M^3C$ is shown in Fig. 2. It is a three-phase to three-phase AC/AC converter with a total of nine arms. The subscripts $a, b, c$ represent quantities at the generator side for voltage and current while subscripts $u, v, w$ represent quantities at the system side. Current direction is as shown in Fig. 2. In each of the nine arms, there are $N$ IGBT based full bridge sub-modules, a resistor representing internal converter losses and an arm reactor. The performance of a system is greatly affected by the switching devices [22]. Therefore, it is important to model the switching behavior of the converter precisely. Considering a single sub-module in an arm, the switching signal $s_{armi} (=1, 0, -1)$ determines the operation mode of the $i$th sub-module. When the switching signal equals to 1, the sub-module is positively inserted with the capacitor voltage $u_{dc}$. Contrarily, $-u_{dc}$ would be inserted if the signal is -1, while the sub-module would be bypassed if the
signal is 0. Refer $i_{\text{arm}}$ to the arm current and $C$ is the submodule capacitance. The current equation of one submodule and one arm can be given by (1) and (2):

$$S_{\text{arm}} i_{\text{arm}} = C \frac{du_{dc}}{dt}$$

$$\sum_{i=1}^{N} S_{\text{arm}} i_{\text{arm}} = N C \frac{du_{dc}}{dt}$$

Under normal operation, submodules are inserted in a same direction. Therefore, the magnitude of $n$ is the inserted number of sub-modules, with $n>0$ as sub-modules positively inserted and $n<0$ as sub-modules negatively inserted. Define the arm switching function $S_{\text{arm}} = n/N$, (2) becomes:

$$S_{\text{arm}} i_{\text{arm}} = C \frac{du_{dc}}{dt}$$

Equation (3) gives the current relation of a M3C arm, and the voltage relation can be expressed by (4), where $u_{\text{arm}}$ represents the arm voltage:

$$u_{\text{arm}} = N S_{\text{arm}} u_{dc}$$

Within an arm, the sub-module voltage balancing method is the same for MMC-HVDC [23], which has been widely researched and tested [24]. In this study, it is assumed that the sub-modules voltage balancing control performs satisfactorily. Equations (3) and (4) assume that the submodule voltages are balanced at steady state. The focus of this paper is to develop a compact and manageable M3C small signal model. But multilevel converters for transmission applications contain up to several hundred sub-modules, making it mathematically inefficient to consider dynamics on every single sub-module for a small signal model.

Quantities with the generator side frequency (20 Hz) and those with system side frequency (60 Hz) couple in the M3C. At balanced steady state, AC side current spreads equally into three arms [25]. Take arm current $i_{au}$ for instance, it contains one third of the phase current $i_a$ and one third of the phase current $i_u$. Current harmonics are not taken into consideration in the model due to the reasons below: 1) By carefully select the values of circuit components, for example capacitors and inductors, the magnitudes of the current harmonics are kept to a negligible level [26, 27]. 2) The small signal model focuses on the external characteristics of the M3C. For circulating currents flowing within the converter, their influence on the outer AC systems can be neglected. 3) Under AC system unbalance, current harmonics may be salient. But such situation is out of the scope of this paper. Consequently, the arm current can be expressed as:

$$i_{au} \approx I_a \sin(\omega_1 t + \beta_1) + I_u \sin(\omega_3 t + \beta_3)$$

where $I_a, I_u, \omega_1, \omega_3, \beta_1$ and $\beta_3$ are the magnitudes (equaling to one third of the AC system phase currents magnitudes), angular frequencies and phase angles of the 20 Hz and 60 Hz currents in the arm. Also, the arm switching function is given by:

$$S_{au} = \frac{E_a}{u_{dc}} \sin(\omega_1 t + \alpha_1) + \frac{E_u}{u_{dc}} \sin(\omega_3 t + \alpha_3)$$

When frequency equals to 20 Hz, $\omega = \omega_1$ and when frequency is 60 Hz, $\omega = \omega_3$. The dynamic phasors of the quantities in DQ frame can be expressed as a function of the magnitude and the phase angle in ABC frame. For example:

$$\begin{align*}
E_{d20} &= E_a \sin(\alpha_1 - \beta_1) + E_u \sin(\alpha_3 - \beta_3) \\
I_{d20} &= I_a \sin(\beta_1) + I_u \sin(\beta_3)
\end{align*}$$

Substitute (5) and (6) into (3), the DC component of the capacitor voltage can be extracted and expressed as:

$$C \cdot U_{dc,0} = \frac{E_{d20}}{u_{dc}} \cos(\alpha_1 - \beta_1) + \frac{E_{q20}}{u_{dc}} \cos(\alpha_3 - \beta_3)$$

Express the right side of (9) with DC components and rearrange the equation, the differential equation of $U_{dc,0}$ can be given by:

$$\dot{U}_{dc,0} = \frac{1}{2u_{dc} C} (E_{q20} I_{d20} + E_{d20} I_{q20} - E_{q60} I_{q60} + E_{d60} I_{d60})$$

Similarly, the 40 Hz component of the capacitor voltage (transformed into DQ) is modelled by the following (11) and (12). Detailed derivation can be found in the appendix.

$$\dot{U}_{dc,d2} = 2\omega_1 U_{dc,q2} - \frac{1}{2u_{dc} C} (E_{q20} I_{q20} - E_{d20} I_{d20})$$

---

**FIGURE 2.** Schematic diagram of a M3C. The ABC quantities are transformed into DQ using park’s transformation. The transformation matrix is denoted as $T$:

$$T = \begin{pmatrix} \cos(\omega t) & \cos(\omega t - \frac{2}{3}\pi) & \cos(\omega t + \frac{2}{3}\pi) \\
-\sin(\omega t) & -\sin(\omega t - \frac{2}{3}\pi) & -\sin(\omega t + \frac{2}{3}\pi) \end{pmatrix}$$

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\[ U_{dc,q2} = -2\omega_1 U_{dc,d2} + \frac{1}{2\omega C} (E_{d20} I_{q20} + E_{q20} I_{d20}) \]  

(12)

Only the 40 Hz capacitor voltage ripple is considered and higher order components are neglected in this model, as the 40 Hz component has the largest magnitude and dominates in voltage ripples. It will be further discussed and verified by the time domain simulation in Section IV. In situations where higher order voltage ripples are preferable to be included, similar approach can be applied to model ripples (at 80, 120 Hz...) easily. But in this model:

\[ u_{dc} \approx U_{dc,0} + U_{dc,2} \sin(2\omega_1 t + \theta_2) \]  

(13)

### B. Dynamics of the AC Current

Substitute (6) and (13) into (4), the 20 Hz and 60 Hz arm voltages can be calculated as:

\[
\begin{align*}
U_{arm,20a} &= \frac{N E_a}{U_{dc}} \sin(\omega_1 t + \alpha_1) + \frac{N L_2 E_a}{2U_{dc}} \cos(\omega_1 t + \theta_2 - \alpha_1) \\
&+ \frac{N L_6 E_a}{U_{dc}} \cos(\omega_2 t + \alpha_2) \\
U_{arm,60a} &= \frac{E_v U_{dc}}{U_{dc}} \sin(\omega_3 t + \alpha_3)
\end{align*}
\]  

(14)

Rewriting (14) in DQ components yields:

\[
\begin{align*}
U_{arm,20d} &= \frac{N L_2}{U_{dc}} E_{d20} + \frac{N}{2U_{dc}} (U_{dc,q2} E_{q20} + U_{dc,d2} E_{d20}) \\
U_{arm,20q} &= \frac{N L_2}{U_{dc}} E_{q20} - \frac{N}{2U_{dc}} (U_{dc,q2} E_{q20} - U_{dc,d2} E_{d20}) \\
U_{arm,60d} &= \frac{E_v U_{dc}}{U_{dc}} E_{d60} \\
U_{arm,60q} &= \frac{E_v U_{dc}}{U_{dc}} E_{q60}
\end{align*}
\]  

(15)

Apply Kirchhoff’s law to M$^3$C, equations at 20 Hz and 60 Hz can be given by:

\[
\begin{align*}
\dot{e}_a &= U_{arm,20a} + L \cdot i_{au,20} + R \cdot i_{au,20} + e_u \\
0 &= U_{arm,60a} + L \cdot i_{au,60} + R \cdot i_{au,60} + e_u
\end{align*}
\]  

(17)

Again transform voltage equations into DQ coordinate. The differential equations of the AC currents are computed as:

\[
\begin{align*}
\dot{i}_{d20} &= \frac{1}{L} U_{d20} - \frac{R}{L} i_{d20} + \omega_1 i_{q20} - \frac{1}{L} U_{arm,20d} \\
\dot{i}_{q20} &= \frac{1}{L} U_{q20} - \frac{R}{L} i_{q20} - \omega_1 i_{d20} - \frac{1}{L} U_{arm,20q} \\
\dot{i}_{d60} &= -\frac{1}{L} U_{d60} - \frac{R}{L} i_{d60} + \omega_3 i_{q60} - \frac{1}{L} U_{arm,60d} \\
\dot{i}_{q60} &= -\frac{1}{L} U_{q60} - \frac{R}{L} i_{q60} - \omega_3 i_{d60} - \frac{1}{L} U_{arm,60q}
\end{align*}
\]  

(18)

(19)

To combine, the M$^3$C itself can be modelled with the state and input variables below:

\[
\begin{align*}
x_m3c &= [U_{dc,0}, U_{dc,d2}, U_{dc,q2}, I_{d20}, I_{q20}, I_{d60}, I_{q60}] \\
u_m3c &= [E_{d20}, E_{q20}, E_{d60}, E_{q60}, \omega_1, \omega_3, U_{d20}, U_{q20}, U_{d60}, U_{q60}]
\end{align*}
\]

### C. Control System for the M$^3$C

The control method of the M$^3$C in this study adopts the widely used vector control. Among different control methods [9, 15, 28, 29], the vector control has merits of easy implementation and satisfactory transient performance. The generator side of the M$^3$C is responsible for controlling active power and the system side of the M$^3$C is responsible for controlling capacitor voltage. The Q axis of the outer loop can be used to control voltage or reactive power. For the sake of simplicity, the Q axis current reference is given to zero to maximize the active power transmitting capability. The control diagram is as shown in Fig. 3. As the vector control algorithm has been well documented in the literature [30, 31]. The differential equations are given here directly:

\[
\begin{align*}
\dot{x}_1 &= P_{20,ref} - P_{20,mea} \\
\dot{x}_2 &= k_{p1} P_{20,ref} - k_{p1} P_{20,mea} + k_{i1} x_1 - I_{d20} \\
\dot{x}_3 &= I_{q20,ref} - I_{q20} \\
\dot{x}_4 &= U_{dc,0} - U_{dc,ref} \\
\dot{x}_5 &= k_{p4} U_{dc,0} - k_{p4} U_{dc,ref} + k_{i4} x_4 - I_{d60} \\
\dot{x}_6 &= I_{q60,ref} - I_{q60}
\end{align*}
\]  

(20)

The PLL provides angle reference and its dynamics should be included. The modelling method is the same as proposed in [32]. Four variables are added to model two PLLs at 20 Hz and 60 Hz sides. \(x_7\) and \(x_8\) are the time integration of the Q axis voltages. And \(x_{pll}\) represents the output of the PLL. Its control diagram is shown in Fig. 4.
the high frequency fluctuation. The differential equation is
given by the following (22), where \( P_{20} \) is the power
transmitted from the 20 Hz side, which is a function related
to \( U_{d20}, I_{d20}, U_{q20} \) and \( I_{q20} \). \( P_{20\text{mea}} \) is the measured power
at the 20 Hz side, and \( T_{\text{mea}} \) is the first order time constant.

\[
P_{20\text{mea}} = \frac{P_{20} - P_{20\text{mea}}}{T_{\text{mea}}} \tag{22}
\]

In total, the combined control system has the following
state and input variables:

\[
x_{\text{ctrl}} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_{p120}, x_8, x_{p60}, P_{20\text{mea}}]
\]

\[
u_{\text{ctrl}} = [U_{dc,0}, I_{d20}, I_{q20}, I_{d60}, I_{q60}, U_{d20}, U_{q20}, U_{d60}, U_{q60},
              P_{20\text{ref}}, I_{q20\text{ref}}, U_{d\text{cref}}, I_{q60\text{ref}}]
\]

### D. COMBINED SMALL SIGNAL MODEL

To get the small signal model of the complete system, the
abovementioned two systems: the M\( ^3 \)C system and its
control system are merged. In the first system, \( E_{d20}, E_{q20}, E_{d60} \) and \( E_{q60} \)
are the outputs from the vector control in the second system. And \( \omega_1 \) and \( \omega_3 \) are
the outputs from the PLLs. \( U_{d20}, U_{q20}, U_{d60} \) and \( U_{q60} \)
provide interfaces with the 20 Hz and 60 Hz side AC systems
respectively. The combined model can be expressed in the
form of:

\[
\dot{x} = Ax + Bu \tag{23}
\]

where the expression of the matrix \( A \) and \( B \) is given in
the appendix. The small signal model can be derived by
linearizing (23). The final model is of 18th order and all
variables are listed below:

\[
\Delta x = [\Delta U_{dc,0}, \Delta U_{dc,d}, \Delta U_{dc,q}, \Delta I_{d20}, \Delta I_{q20}, \Delta I_{d60}, \Delta I_{q60},

\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4, \Delta x_5, \Delta x_6, \Delta x_7, \Delta x_{p120}, \Delta x_8, \Delta x_{p60}, \Delta P_{20\text{med}}]
\]

\[
\Delta u = [\Delta U_{d20}, \Delta U_{q20}, \Delta U_{d60}, \Delta U_{q60}, \Delta P_{20\text{ref}}, \Delta I_{q20\text{ref}}, \Delta U_{d\text{cref}}, \Delta I_{q60\text{ref}}]
\]

### IV. MODEL VERIFICATION AND STABILITY ANALYSIS

To validate the proposed small signal model, the combined
model discussed in Section III is implemented in
MATLAB/Simulink. Also, a non-linear detailed model of
M\( ^3 \)C is developed in RTDS for EMT simulation. For
the EMT model, a small simulation time step of 3 \( \mu s \) is adopted
to precisely simulate the switching dynamics of the power
electronics. Unlike most existing research in the literature
where only several sub-modules are considered in each arm,
this model contains as many as 40 sub-modules in each arm
for all 9 arms in the M\( ^3 \)C, presenting an accurate
representation of the AC/AC multilevel converter. The
simulated system topology is as shown in Fig. 2 with
the AC system modelled as a voltage source behind a Thevenin
impedance. For the IGBT switches, the build-in module in
RTDS is used. Implementation of IGBT-based switches
modelling in RTDS for modular multilevel converters is
described in [33], with more technical details in [34]. The
‘MMCS’ model is adopted since for small signal study, the
exact firing pulse for each individual sub-module is not
concerned. The control system was developed according to
the control diagram Fig. 3. Full details of the system
parameters can be found in Table I in the appendix.

In time domain simulation, Fig. 5 (a) and (b) show the
sinusoidal voltage and current waveforms at the 20 Hz and
60 Hz AC sides. The arm voltages and currents are shown
in Fig. 5 (c), which are regular superposition waveforms of
20 Hz and 60 Hz sinusoidal components as discussed in
Section II. The arm voltage ripple and current ripple are
small at only 0.096% and 1.59% respectively. It is shown
that M\( ^3 \)C performs well as the AC/AC converter and it has
the advantage of low ripple level. In RTDS, the actual
measured sub-module voltage is plotted in Fig. 6, together
with the sub-module voltage added up only by the DC
component and 40 Hz ripple. It can be seen from the figure
that the 40 Hz component takes up the largest magnitude
of the ripples and two curves match closely. The dominant
ripple at 40 Hz has a magnitude of 0.05 kV or 3.33% of the
M\( ^3 \)C component. High order ripples have negligible amounts
that are less than 1%. Thus, the discrepancy brought by
neglecting high order capacitor voltage ripples is acceptable.
If required, the proposed approach is capable of modelling
high order ripple components.

#### A. DYNAMIC RESPONSE OF STEP CHANGE ON ACTIVE POWER REFERENCE

At initial state, 30 MW of active power is transmitted from
the 20 Hz side to the 60 Hz side. At \( t = 0.4s \), a step change of
\( P_{20\text{ref}} \) is applied from 30 MW to 32 MW. The dynamic
response of the developed small signal model is compared with the detailed time domain simulation. As can be seen in Fig. 7, the active power at 20 Hz side and the capacitor voltage show great consistency. For AC current waveforms, overall there is good agreement, except that the detailed simulation model contains minor high frequency fluctuations. Step change dynamic response validates the small signal model.

B. INFLUENCE OF THE OUTER LOOP CONTROLLER

The developed small signal model is helpful on the selection of controller parameters. In this sub-section, the outer loop PI controller parameter $k_{i1}$ is studied. The root locus of the related modes is plotted in Fig. 8. As $k_{i1}$ increases, the eigenvalues move towards the right half of the complex plane and the modes finally become unstable. In RTDS, a step change of $k_{i1}$ from 15 to 150 is applied at $t=0.25s$. Since this controller is responsible for controlling the active power at the 20 Hz side, the waveform of the measured $P_{20}$ is shown in Fig. 9. As can be seen, the system loses stability and the active power begins to oscillate with a period of 0.0047s. According to the eigenvalue analysis, the oscillation period of this mode is calculated as $2\pi/\omega = 0.0048s$, which agrees with the simulation result. Also in frequency domain, the bode plots are shown as Fig. 10 when $k_{i1}$ is small and Fig. 11 when $k_{i1}$ is large, with the input as the active power reference and the output as the measured active power at the 20 Hz side. As can be seen, when $k_{i1}$ is small, the system is stable, while when $k_{i1}$ is large, a resonant point is spotted at 208 Hz, which exactly matches the eigenvalue analysis ($1/0.0048s=208$ Hz) and the time domain simulation. The effectiveness of the proposed model is again validated. It is shown that the increasing outer loop integral gain has a negative effect on the small signal stability and therefore should be limited within a certain range.
C. INFLUENCE OF THE PLL

In this sub-section, the control parameters of the PLL are analyzed. If the eigenvalue of a mode is denoted as $\lambda = \sigma \pm j\omega$, the damping ratio of the mode is defined as $\xi = -\sigma/\sqrt{\sigma^2 + \omega^2}$. When the damping ratio is less than 5%, the mode is regarded as poorly damped. Fig. 12 plots the damping ratio of the mode related to PLL as the proportional gain grows from 0 to 2. It is shown that the damping ratio increases and then remains at 1. This mode has poor damping when $k_{p_pll} < 0.06$. In addition, the root locus is plotted in Fig. 13 when $k_{p_pll}$ varies from 0-20 and $k_{i_pll}$ varies from 1-20. When $k_{p_pll}$ raises, the eigenvalues firstly move towards and then get onto the real axis. After that, one eigenvalue moves further away from the imaginary axis while the other gets closer to the right half pane. As a result, if $k_{p_pll}$ adopts a large value, the system may be vulnerable to small signal instability. For the integral gain, the mode trajectory is more straightforward. As $k_{i_pll}$ increases, the eigenvalue gets more negative and therefore the small signal stability enhances. To sum up, the selection of $k_{p_pll}$ should be careful as it cannot be too small or too large. While a large $k_{i_pll}$ is preferred since that would bring more damping to the mode.

D. ANALYSIS OF THE SUB-MODULE CAPACITOR MODE

The small signal stability of the mode related to the capacitor voltage ripple is analyzed in this subsection. The damping ratio is plotted against the sub-module capacitance in Fig. 14.

As can be seen, a larger value of the capacitor would result in poorer damping of the mode. In other small signal studies for instance for two-level VSC or MMC, capacitor ripples are often omitted and it is assumed that the capacitor voltage is DC [35]. However, the M$^3$C model proposed in this paper takes into account the sub-module voltage ripples. Therefore, it is able to analyze the possible poorly damped mode and the small signal analysis can help select the sub-module capacitance. It is found that including capacitor voltage ripple in the small signal modelling is necessary as their modes can have poor damping and affect the small signal stability of the system. Based on the results in this case, the sub-module capacitance should not be larger than 4 mF to avoid poor damping.

V. CONCLUSION

As M$^3$C based FFTS is a promising solution for offshore wind power integration, there is need of a model to study its influence on the existing power systems. The basic achievements of this paper are to develop a small signal model of M$^3$C and to conduct a small signal analysis giving insights to system stability and parameter selection. The model provides easy interfaces with both the external AC systems and the control system. In the model, the dynamics of AC currents and the DC and ripple components on sub-module capacitor voltage have been considered. The control system has included dynamics of the outer and inner loop PI controllers, PLL and measurement delay. According to the small signal analysis, it has been found that increasing the integral gain of the D axis outer loop control has an adverse effect and can induce power oscillation, while a larger integral gain of the PLL improves the small signal stability. The choice of the proportional gain of the PLL should be within a certain range as the damping ratio can be poor when a very small gain is chosen but a very large gain would result in poor damping. Also, the damping ratio of the capacitor ripple voltage mode decreases as capacitance grows.

The performance of the proposed model is satisfactory. Based on the comparison between the detailed EMT M$^3$C model and the small signal model, a very good matching on both dynamic response and stability analysis has been shown, validating the accuracy of the proposed model. The
assumption of balanced sub-converters and the discrepancy brought by neglecting high order sub-module voltage ripples have been analyzed by the time domain simulation. For future work, the proposed model can be enhanced by considering unbalanced operating conditions and system harmonics resonances. Also, it would be beneficial to develop a prototype of M\(^{\circ}\)C and carry out field tests.

**APPENDIX**

**A. DETAILED DERIVATION OF THE SMALL SIGNAL MODEL**

The Full Kirchhoff’s law equations can be expressed as:

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} =
\begin{bmatrix}
v_{au} & v_{av} & v_{aw} \\
v_{bu} & v_{bv} & v_{bw} \\
v_{cu} & v_{cv} & v_{cw}
\end{bmatrix}
\begin{bmatrix}
t_a \\
t_b \\
t_c
\end{bmatrix} +
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
\]

\[
(R + L \frac{d}{dt})
\begin{bmatrix}
t_a \\
t_b \\
t_c
\end{bmatrix}
+ \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
= \begin{bmatrix}
u_{da} \\
u_{db} \\
u_{dc}
\end{bmatrix}
(11)

\]

Applying the \(ABC - \alpha\beta\) transformation to A1-A3, A4-A6, and A7-A9 respectively and extract the zero sequence equations:

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} =
\begin{bmatrix}
v_{d0} \\
v_{b0} \\
v_{c0}
\end{bmatrix} +
\begin{bmatrix}
u_{d0} \\
u_{b0} \\
u_{c0}
\end{bmatrix}
(M - A3)
\]

\[
(R + L \frac{d}{dt})
\begin{bmatrix}
t_a \\
t_b \\
t_c
\end{bmatrix}
+ \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
= \begin{bmatrix}
u_{d0} \\
u_{b0} \\
u_{c0}
\end{bmatrix}
\]

When the AC system is balanced, there is no zero sequence voltage so \(u_{d0}\) can be omitted. Further apply the \(ABC - DQ\) transformation to A10-A12:

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} =
\begin{bmatrix}
v_{d0} \\
v_{b0} \\
v_{c0}
\end{bmatrix} +
\begin{bmatrix}
u_{d0} \\
u_{b0} \\
u_{c0}
\end{bmatrix}
\]

\[
\begin{bmatrix}
u_{d0} \\
u_{b0} \\
u_{c0}
\end{bmatrix}
\]

\[
\begin{bmatrix}
u_{d0} \\
u_{b0} \\
u_{c0}
\end{bmatrix}
\]

where \(E_{d0}\) and \(E_{q0}\) are the DQ arm voltage for 20 Hz. In this form, decoupled control can be applied to form the inner current loop. The outer loop at 20 Hz is selected to control active power. The 60 Hz side can be derived using similar approach. Or alternatively, after the \(ABC - \alpha\beta\) transformation, apply Park’s transformation to three sets of equations in \(\alpha\beta\) frame. At symmetrical state, the cluster voltages are balanced [28], so \(V_{adq}, V_{bdq}\) and \(V_{cdq}\) can be denoted as \(E_{d0}\) and \(E_{q0}\) uniformly:

\[
\begin{bmatrix}
E_{d0} \\
E_{q0}\end{bmatrix} = - \begin{bmatrix}
U_{d0} \\
U_{q0}\end{bmatrix} - (R + L \frac{d}{dt}) \begin{bmatrix}
I_{d0} \\
I_{q0}\end{bmatrix} + \begin{bmatrix}
\omega_3 L I_{d0} \\
\omega_3 L I_{q0}\end{bmatrix}
\]

\[
(15)
\]

\[
(16)
\]

Outer loop is selected to balance the capacitor voltage of three clusters respectively. Take cluster A for instance, the differential equation of the 40 Hz capacitor voltage ripple can be calculated as:

\[
C \dot{U}_{dc,a2} = - \frac{E_a I_a}{2U_{dc}} \cos(2\omega_1 + \alpha_1 + \beta_1)
\]

(17)

Transform the equation into DQ frame, (11) and (12) in II Section III can be derived.

**B. CIRCUIT AND CONTROL PARAMETERS**

**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>Fractional frequency</td>
<td>20 Hz</td>
</tr>
<tr>
<td>(I_3)</td>
<td>System frequency</td>
<td>60 Hz</td>
</tr>
<tr>
<td>(V_{L1})</td>
<td>Rated AC system voltage</td>
<td>33 kV</td>
</tr>
<tr>
<td>(N)</td>
<td>Sub-module number each arm</td>
<td>40</td>
</tr>
<tr>
<td>(L)</td>
<td>Inductance</td>
<td>15 mL</td>
</tr>
<tr>
<td>(C)</td>
<td>Sub-module capacitance</td>
<td>5 mF</td>
</tr>
<tr>
<td>(R)</td>
<td>Arm resistance</td>
<td>0.25 Ω</td>
</tr>
<tr>
<td>(U_{dc,ref})</td>
<td>Capacitor voltage reference</td>
<td>1.5 kV</td>
</tr>
<tr>
<td>(P_{d,ref})</td>
<td>Active power reference 20 Hz</td>
<td>30 MW</td>
</tr>
<tr>
<td>(I_{q,ref})</td>
<td>Q axis current reference</td>
<td>0 kA</td>
</tr>
<tr>
<td>(k_{p1}, k_{i1})</td>
<td>PI controller 1 parameters</td>
<td>0.025, 1</td>
</tr>
<tr>
<td>(k_{p2}, k_{i2})</td>
<td>PI controller 2 parameters</td>
<td>100, 20</td>
</tr>
<tr>
<td>(k_{p3}, k_{i3})</td>
<td>PI controller 3 parameters</td>
<td>100, 20</td>
</tr>
<tr>
<td>(k_{p4}, k_{i4})</td>
<td>PI controller 4 parameters</td>
<td>0.5, 10</td>
</tr>
<tr>
<td>(k_{p5}, k_{i5})</td>
<td>PI controller 5 parameters</td>
<td>50, 50</td>
</tr>
<tr>
<td>(k_{p6}, k_{i6})</td>
<td>PI controller 6 parameters</td>
<td>50, 50</td>
</tr>
<tr>
<td>(k_{p_{pll}}, k_{i_{pll}})</td>
<td>PLL parameters</td>
<td>5, 100</td>
</tr>
<tr>
<td>(\tau_{mea})</td>
<td>First order time constant of measurement</td>
<td>0.01 s</td>
</tr>
</tbody>
</table>

**C. EXPRESSION OF STATE SPACE MODEL MATRICES**

Please see below (Left: A matrix; Right: B matrix).
REFERENCES


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