Energy Evaluation for DC Railway Systems with Inverting Substations

Zhongbei Tian, Gang Zhang, Ning Zhao, Stuart Hillmansen, Pietro Tricoli, Clive Roberts

Abstract—Energy and environmental sustainability in transportation have received a great deal of attention in recent decades. Electrified railway systems play an important role in contributing to the reduction of energy usage and CO₂ emissions compared with other transport modes. For metro-transit systems with frequently motoring and braking trains, the effective use of regenerated braking energy is a significant way to reduce the net energy consumption. This paper presents a simulation method to evaluate the energy flow of DC railway systems. The network receptivity of railway systems with and without inverting substations are analyzed and compared. The power load in inverting substations is illustrated based on a case study. The results show that the inherent receptivity of a non-inverting system varies with the operation timetable. A shorter headway operation timetable could lead to a higher receptivity, but the headway is not the only factor. With the implementation of inverting substations, the receptivity can be improved. In addition, the global energy can be reduced by 10–40% with different timetables.

Index Terms—Traction power supply systems, inverting substation, regenerative braking, energy consumption, network receptivity

I. INTRODUCTION

DC-fed railway systems are widely used in many cities across the world. With the development of controllable power electronic devices, new technologies, for example, reversible substations and storage devices are introduced into the DC traction power systems. The energy-saving performance of the new implementations becomes a popular topic to study.

Power supply network modeling is a key capability in understanding railway system operation. The capability would allow current and future operations to be understood, managed and optimized. The energy consumption of a traditional traction power system with non-reversible substations is evaluated in [1]. Train driving styles and timetable operation are optimized to achieve minimum traction energy usage and maximum regenerative energy [2, 3].

A very early design of inverting substations for traction systems is presented in [4, 5]. The design principle and working performance evaluation are studied. A simulation method for a comprehensive DC railway network is presented in [6]. The models of the main components including inverting substations, energy storage devices and trains, are explained in detail. A modified current injection algorithm is used to solve power flow. The performance of this solver is compared with other solvers using Newton-Raphson and Backward/Forward Sweep methods. A reversible DC substation for efficient recovery of braking energy is designed in [7] and two prototypes were built and tested on a tramway route. The real operation test validates the performance of proposed reversible substations.

 NOMENCLATURE

\[ V_{\text{rec}} \] rectifier output voltage [V]  
\[ V_{\text{no-load}} \] rectifier no-load voltage [V]  
\[ r_{\text{rec}} \] rectifier equivalent resistance [Ω]  
\[ I_{\text{rec}} \] rectifier output current [A]  
\[ V_{\text{inv}} \] inverter output voltage [V]  
\[ V_{\text{trigger}} \] inverter trigger voltage [V]  
\[ r_{\text{inv}} \] inverter equivalent resistance [Ω]  
\[ I_{\text{inv}} \] inverter output current [A]  
\[ M \] train effective mass [kg]  
\[ s \] distance [m]  
\[ t \] time [s]  
\[ F \] tractive effort [N]  
\[ M \] vehicle mass [kg]  
\[ g \] acceleration due to gravity [m/s²]  
\[ \alpha \] the angle of the route slope [rad]  
\[ R \] vehicle resistance [N]  
\[ P_t \] traction power [kW]  

\[ v \] speed [km/h]  
\[ E_s \] global substation energy consumption [kWh]  
\[ E_{\text{rec}} \] rectified energy [kWh]  
\[ E_{\text{inv}} \] inverted energy [kWh]  
\[ E_{\text{td}} \] substation energy loss [kWh]  
\[ \eta_{\text{rec}} \] rectifier efficiency  
\[ \eta_{\text{inv}} \] inverter efficiency  
\[ E_t \] transmission loss [kWh]  
\[ E_{tr} \] train energy [kWh]  
\[ E_{\text{reg}} \] regenerative braking energy [kWh]  
\[ \eta_{\text{reg}} \] regeneration efficiency  
\[ E_{\text{eb}} \] electro-braking energy [kWh]  
\[ E_{\text{eb},r} \] electro-braking energy loss [kWh]

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Although the reversible substations have been studied for DC rail systems. The quantity of energy saved by the new power supply implements has not been fully studied. This paper presents a method to evaluate the energy consumption for DC railway systems with inverting substations. A simulation method to calculate the energy flow through the DC railway systems in Section II. A case study of an example route is presented in Section III. With the consideration of train driving styles and timetables, the global energy consumption with and without inverting substations is compared. Power loads of the substations are studied based on the simulation.

II. MODEL FORMULATION

A. Power Supply Network

In modern railways, the DC traction substations are normally equipped with transformers and rectifiers, drawing electricity from local AC distribution network. Fig. 1 shows a typical DC traction supply network. A rectifier substation and a rectifier with inverting substation topology diagram are shown. Both substations transfer power to catenary systems and supply the motoring trains. When the train is braking, the regenerated power can be flow back to catenary systems and used by motoring trains. At the same time, the regenerated power can be inverted from DC to AC side and used by other loads in the AC distribution network.

B. Substation Modeling

A rectifier substation converts AC power to DC power to supply motoring trains. In modern rapid transit systems, the 12-pulse or 24-pulse rectifiers are commonly used. The voltage regulation characteristic of the rectifier is nonlinear [8]. However, the voltage regulation at a nominal working state can be simplified as linear, shown in Fig. 2 ‘A-B’. The voltage regulation of the rectifier is given by (1). \( r_{rec} \) is an equivalent coefficient to represent the voltage regulation. It is not a physical resistance and therefore there is no energy loss to associate it.

\[
V_{rec} = V_{no load} - r_{rec} \times I_{rec}
\]  

A rectifier blocks the negative current, which has the non-receptive capability. To capture the surplus regenerative braking power in the power network, an inverting substation is employed. The output voltage from an inverting substation is controlled, which can be maintained constant or made to follow a particular slope, as shown in Fig. 2 ‘C-D’. The red line ‘B-C’ is the deadband between the transition of inverting and rectifying. Point ‘B’ is the no-load voltage of the rectifier substation and point ‘C’ is the triggering voltage of inverting substation. The voltage regulation of the inverting substation can be expressed by (2). \( r_{inv} \) is obtained according to the inverter control scheme.

\[
V_{inv} = V_{trigger} - r_{inv} \times I_{inv}
\]

C. Train Modeling

A motoring train is a power load in the railway power network, while a braking train a power source. The forces on a motoring train on an uphill track is shown in Fig. 3. The movement of a train can be represented by the well-known Lomonossoff’s equation in (3).

\[
M_e \frac{d^2 s}{dt^2} = F - Mgsin(\alpha) - R
\]

The train power demand is calculated by multiplying train tractive effort (positive in traction and negative in braking) by train speed, as shown in (4).

\[
P_t = F \times v
\]
flow can be solved using a current injection iterative method [9]. Thus, the system energy consumption can be calculated by the integral of power over time.

D. Energy Calculation

An energy flow chart of a DC railway system is described in Fig. 4. There are four layers, namely AC network, substation level, catenary system level and train level. The substations collect electricity from the national electricity grid to feed the whole railway system.

Fig. 4 Energy flow chart of a DC railway system

The global substation energy consumption is rectified energy subtracted by the inverted energy, as shown in (5). The inverted energy is zero for DC rail systems without inverting substations.

\[ E_s = E_{rec} - E_{inv} \quad (5) \]

Substation losses include the losses during rectifying and inverting, which is given in (6). The efficiency of the rectifier and inverter is assumed as 97% and 95%, respectively.

\[ E_{st} = E_{rec} \times (1 - \eta_{rec}) + E_{inv} \times (1 - \eta_{inv}) \quad (6) \]

The rest of the rectified energy supplies trains by the catenary system. The transmission loss is calculated by the integral of power through conductors over time. The train received energy is dissipated by onboard conversion and motion resistance. Part of electro-braking energy is regenerated and reused by trains and inverted back to AC side. The surplus part of the electro-braking energy is dissipated by onboard braking rheostat for overvoltage protection.

The energy conservation equation of the whole system is given in (7). The global substation energy consumption equals the sum of substation loss, transmission loss and train energy deduced by the regenerative braking energy.

\[ E_{rec} - E_{inv} = E_{st} + E_{tr} + E_{tr} - E_{reg} \quad (7) \]

All these values have very comprehensive relations with each other. When a train is braking, the regenerated braking power can be used by adjacent motoring trains. If there is no motoring train nearby, the regenerative power increases the line voltage and some of the electro-braking power is dissipated by onboard braking rheostat for overvoltage protection. If the rail system is implemented with inverting substations, the regenerative braking power can be converted back to AC network.

The efficiency of usable regenerative braking energy can be used to evaluate the regeneration receptivity of the rail systems, which is defined in (8). When \( \eta_{reg} = 100\% \), all of the regenerated braking energy by electro-braking is transferred back to the catenary network, and used by motoring trains or inverted back to AC network. However, 100% receptivity does not mean the minimum energy consumption. The transmission loss must be considered in global energy consumption.

\[ \eta_{reg} = \frac{E_{reg}}{E_{rec}} = \frac{E_{reg}}{E_{rec} + E_{reg}} \quad (8) \]

III. ENERGY EVALUATION CASE STUDY

A. Network Data

A case study is presented based on a typical DC-fed railway line. The main parameters of the trains and networks are shown in TABLE I. This route covers 10 km with 6 stations. A rectifier substation is located at each station. The rated power is 4000 kW. The inverter can be fitted in each substation, with a triggering voltage of 800 V and maximum power of 2000 kW. In the following case studies, the systems without and with inverting substations are evaluated and compared.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS OF A DC RAILWAY LINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Route distance [km]</td>
<td>10</td>
</tr>
<tr>
<td>Number of substations</td>
<td>6</td>
</tr>
<tr>
<td>Train parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Train mass [ton]</td>
<td>250</td>
</tr>
<tr>
<td>Maximum operation speed [km/h]</td>
<td>80</td>
</tr>
<tr>
<td>Maximum traction power [kW]</td>
<td>3000</td>
</tr>
<tr>
<td>Maximum braking power [kW]</td>
<td>-3000</td>
</tr>
<tr>
<td>DC network parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Contact line resistivity [m(\Omega)/km]</td>
<td>10</td>
</tr>
<tr>
<td>Rail track resistivity [m(\Omega)/km]</td>
<td>10</td>
</tr>
<tr>
<td>Overvoltage protection [V]</td>
<td>1000</td>
</tr>
<tr>
<td>Rectifier substation parameters</td>
<td>Value</td>
</tr>
<tr>
<td>No-load voltage [V]</td>
<td>750</td>
</tr>
<tr>
<td>Rated voltage [V]</td>
<td>717</td>
</tr>
<tr>
<td>Rated power [kW]</td>
<td>4000</td>
</tr>
<tr>
<td>Equivalent resistance [m(\Omega)]</td>
<td>5.9</td>
</tr>
<tr>
<td>Inverting substation parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Triggering voltage [V]</td>
<td>800</td>
</tr>
<tr>
<td>Maximum power [kW]</td>
<td>2000</td>
</tr>
<tr>
<td>-------------------</td>
<td>------</td>
</tr>
<tr>
<td>Equivalent resistance [mΩ]</td>
<td>45</td>
</tr>
</tbody>
</table>

B. Train Driving Profile

An example train driving profile against the distance on up-track is shown in Fig. 5. The route speed limits and gradients are considered. The train driving controls include motoring, coasting, cruising and braking. The details of the train driving control modes can be found in [10].

![Fig. 5 Train driving profile against distance](image)

The corresponding train speed trajectory against the time is shown in Fig. 6. The total journey time is 940 s and the dwell time is 40 s at each station. The running time for each interstation is around 150 s and the average speed is around 39 km/h.

![Fig. 6 Train speed trajectory against time](image)

C. Network Receptivity

A number of factors can influence the line receptivity of a rail system, for example, the driving styles, operation timetables, train and substation control design, etc. The train driving styles for both directions are fixed in this paper. The timetable is obtained by the headway and turnaround departure time. The headway determines the distance between the adjacent trains in the same direction. When the headway is 600 s, the distance between two adjacent trains in the same direction is around 6.5 km. The regenerative braking power is difficult to be transferred between the adjacent trains in the same direction. Therefore, the use of regenerative energy mainly depends on the locations of the trains in the opposite direction. The turnaround departure time can adjust the synchronization of trains in opposite directions. The turnaround departure time of 0 s means the first train from each side departs at the same time.

The influence of train operation timetable on regeneration efficiency of a non-inverting system is shown in Fig. 7. Headways of 50 to 600 s are selected in this study with a step of 50 s. Since the journey time for each interstation is around 150 s, the maximum turnaround departure time is selected as 150 s to cover most cases with different train driving synchronization. The turnaround departure time is selected from 0 to 150 s with a step of 1 s. Each point in Fig. 7 represent the regeneration efficient of a selected timetable. Also, the average value is marked. The following features can be found in the results.

- The overall trend of regeneration efficiency decreases with the increase of headway. The highest average value is 0.87 when headway is 50 s, while the lowest one is 0.32 when headway is 550 s.
- The highest efficiency is 1.0 when headway is 50 s, while the lowest efficiency is 0.12, when headway is 600 s.
- When the headway is the same, the regeneration efficiency varies with different turnaround departure time. The range of regeneration efficiency difference with the same headway is around 0.3, but the highest difference could be 0.5 when headway is 200 s.
- The results denote that regeneration efficiency is very sensitive to the timetable.

![Fig. 7 Regeneration efficiency of a non-inverting system](image)

A simulation test is conducted for a system with inverting substations. The efficiency of regeneration can be improved to nearly 1.0, as shown in Fig. 8. The 100% regeneration efficiency is achieved with any headway and turnaround departure time. The regeneration efficiency becomes non-sensitive to the timetable.

![Fig. 8 Regeneration efficiency of an inverting system](image)
D. Energy Consumption

The substation energy consumption results with different timetables in a non-inverting system are shown in Fig. 9. The substation energy consumption has an opposite trend with the regeneration efficiency in Fig. 7. A higher regeneration efficiency could lead to a low substation consumption. The minimum energy consumption is 10.3 kWh/train-km, while the maximum is 20.6 kWh/train-km. Around 50% of energy can be saved with the highest receptivity.

The substation energy consumption results in an inverting system are shown in Fig. 10. Although the regeneration efficiency is 100% for an inverting system, the substation energy consumption varies with different timetables. The minimum substation is 10.3 kWh/train-km, which occurs at a headway of 50 s. The maximum substation is 11.2 kWh/train-km, which occurs at a headway of 600 s. The difference between substation energy consumption is mainly because of the difference in transmission losses. The different ratio is not large, which is around 8.7%.

Compared with the energy consumption of the non-inverting system, the percentage of energy saved using inverting substations is shown in Fig. 11. The energy saving by inverting substations rises with the increase of headway. The saving ratio at a headway of 50 s is between 0 and 0.23, with an average of 0.1. The saving ratio at a headway of 600 s increase to an average of 0.38. By using the inverting substations, the global substation energy consumption could be reduced by around 10-40%.

The energy flow results of some selected timetables are shown in TABLE II. The rectified energy of the non-inverting system and inverting system are very close. The substation losses and transmission losses of the inverting system are higher than the non-inverting system. The trend of regeneration energy is consistent with the analysis above.

![Fig. 9 Substation energy consumption of a non-inverting system](image)

![Fig. 10 Substation energy consumption of an inverting system](image)

![Fig. 11 Energy saved by inverting substations in comparison with non-inverting substations](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Network</th>
<th>Non-inverting</th>
<th>Inverting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Headway</strong></td>
<td><strong>50s</strong></td>
<td><strong>100s</strong></td>
</tr>
<tr>
<td><strong>Turnaround depart time</strong></td>
<td><strong>Substation energy total</strong></td>
<td>13.82</td>
</tr>
<tr>
<td></td>
<td><strong>Substation energy rectified</strong></td>
<td>13.82</td>
</tr>
<tr>
<td></td>
<td><strong>Substation energy inverted</strong></td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td><strong>Substation losses</strong></td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td><strong>Transmission losses</strong></td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td><strong>Train traction energy</strong></td>
<td>20.80</td>
</tr>
<tr>
<td></td>
<td><strong>Electric braking energy</strong></td>
<td>11.45</td>
</tr>
<tr>
<td></td>
<td><strong>Regenerated braking energy</strong></td>
<td>8.03</td>
</tr>
<tr>
<td></td>
<td><strong>Efficiency of regeneration</strong></td>
<td>70%</td>
</tr>
</tbody>
</table>
E. Power Loads of Substations

Fig. 12, Fig. 13 and Fig. 14 describe the power of each substation with different headways. The maximum rectified power of the substations with a headway of 100 s is 5000 kW, which is much higher than it with longer headway. The maximum inverted power has the same feature, even though the inverted energy with shorter headway is lower than it with longer headway shown in TABLE II. Therefore, to design the capacity of inverting substations must take the timetable into consideration.

IV. CONCLUSION

The paper introduces a method to evaluate the system energy flow of DC-fed railway systems. A simulation study is illustrated based on an example route. The simulation results denote:

- The network receptivity is very sensitive for a non-inverting system. The timetable is one of most significant factors on the network receptivity. The network receptivity with a shorter headway is higher than it with a longer headway.
- With the implement of inverting substations in each station, the network receptivity is improved to nearly 100%.
- The benefit of using inverting substation depends on the train operation. Compared the energy consumption in a non-inverting system, the energy is reduced by 10-40% using inverting substations.
- The capacity of inverting substations should be designed with the consideration of timetables.

The simulation and evaluation method would also allow future requirements, such as timetable changes or rolling stock or infrastructure upgrades, to be assessed before implementation. The inverter can operate with different control schemes. The energy efficiency with different control schemes can be further studied.

V. REFERENCES