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Dynamic Trajectory Optimization of Driver Advisory System for Mainline Railways

Zhu Li, Lei Chen, Clive Roberts

***Abstract*— The Driver Advisory System (DAS) provides a reliable and efficient driving instruction to the driver by optimal trajectory. This paper discusses trajectory optimization to achieve energy efficiency and punctuality for Driver Advisory Systems (DAS). Static optimization is studied, initially using the Genetic Algorithm (GA) to optimize the energy-efficient trajectory. The performance of static optimization will be evaluated by the application of three different driving styles. Dynamic optimization is constructed to dynamically adjust the trajectory when a delay has occurred. The combination of static optimization and dynamic optimization is integrated as a trajectory optimization system in Matlab, contributing to a practical solution for optimal trajectory for DAS systems in the future. An evaluation of energy saving and computation of efficiency for the trajectory optimization system is conducted based on a simulation of a real-scale route.**

Keywords*— *Driver Advisory System, Dynamic Optimization, Railway Energy Efficiency, Genetic Algorithm

I. Introduction

MINIMISING energy usage is a huge challenge against the benefits of the railway. Energy efficiency is achieved by methods such as regenerative braking and energy-optimized timetables [1], and a similar contribution could be made by the way in which we operate the railway. Using digital railway technology, the Driver Advisory System (DAS) informs the driver with driving instructions from optimal train trajectories in order to minimize energy consumption, maintain punctuality and increase capacity. [2] The DAS provides an eco-driving solution to reduce the energy consumption during train operation. Optimal train trajectory or eco-driving is one of the most promising strategies for saving energy in railway operations. For a given timetable (a certain train with a certain route), energy consumption could still be variable due to a time buffer which allows the train to recovery from delay [2]. In this case, the design of a speed profile that consumes the minimum energy is permitted.

Algorithms for trajectory optimization have been well studied in [3-10], which can be classified into two groups [3]. Approach I, which includes methods such as dynamic programming and direct search, uses a discrete search for speed, time and position. The discrete dynamic programming algorithm uses kinetic energy instead of speed to obtain an analytical solution for real time. [4] Approach I has two limitations: the driver can only follow the given trajectory exactly; the accuracy of calculation depends on the chosen discretization of the search space. Approach II derives the optimal regimes and computes the switching point of different regimes. Maximum principle can find speed profiles with minimum energy consumption without the ability of continuous control of the driving. [3] Genetic algorithm is used to find the optimal location of coasting points by a trade-off between energy usage and journey time. [5] The choice of algorithm depends on the available computation power, on the accuracy needed and on the number of allowed regime changes.

Train operation applies optimal trajectory for energy efficiency using various approaches. The trackside information system uses the pre-calculated optimal solution to advise on the targeted speed and switching point, while driver is trained to operate the train with eco-driving skills.[11] However, the DAS technique is raising increasing attention as a method to achieve energy efficiency. The real-time optimal driving instruction is provided in a more straightforward way to drivers by DAS. Furthermore, DAS has the potential to connect to the train operation system and communicate with the Train Control Centre (TCC) to realize the implementation of Automatic Train Operation (ATO).

The fundamental principles of a DAS system are: [11]

1. To determine the target arrival time for the train at stations and junctions that must be achieved to satisfy the published timetable and avoid conflicts with other trains.
2. To calculate an energy-efficient speed/ distance profile starting from the current train location and time, to achieve the target arrival times along the route.

3. To monitor the movement of the train and provide information to the driver so that the profile is followed and target arrival times are achieved.
4. To achieve the ability for real-time adjustment as the driver operates the train. Dynamic optimization to provide a recalculated optimal trajectory should be considered when the train is delayed or ahead of time.

In this study, both static optimization and dynamic optimization for the train trajectory are calculated for the DAS system. The genetic algorithm in Approach II is applied to optimize the regimes in the speed prolife in order to provide drivable advice aimed at the DAS. With regard to dynamic optimization, the discrete computation in Approach I should be used because it provides a state which contains speed, time and position to compare the current driving situation with the schemed driving situation.

In this article, a simulation of a train system with three driving styles is described in section II. Section III illustrates the energy efficiency for a simplified trajectory, including static optimization and dynamic optimization, with the development of a DAS platform based on GUI. Results and discussion will be given in section IV through a real route simulation. Section V gives the conclusions.

II. Train System modelling

A. *Principle of Train Motion*

In railway operation, generally a train model for motion is concerned with three forces, tractive force (or braking force), running resistance and component of gravitational acceleration. According to Newton's Low, the equation for train movement is below.

$$\mathrm{F} - \mathrm{R} = \mathrm{M} \cdot \frac{\partial^2 \mathrm{S}}{\partial \mathrm{t}^2} + \mathrm{Mgsin}\alpha \tag{1}$$

Due to the maximum tractive effort cannot exceed adhesion, the tractive force F at the wheel rims are delivered by the motors which is $\mathrm{F_{mot}}$ and limited by the force caused by adhesion f. The final tractive force is denoted as the minimum of $\mathrm{F_{mot}}$ and f.

$$\mathrm{F} = \min(\mathrm{F_{mot}}, \mathrm{f}) \tag{2}$$

The braking force $\mathrm{F_b}$ is a variable that changes with time when there is a brake applied by the driver [12]. The braking force can be calculated by the cylinder pressure.

The resistance to motion R is caused by a combination of internal forces within the vehicles and external forces due to the interaction of the vehicles and track.

$$\mathrm{R} = (\mathrm{A} + \mathrm{Bv})\mathrm{M} + \mathrm{Cv}^2 + \mathrm{Mg}\alpha + \frac{\mathrm{DMg}}{\mathrm{r}} \tag{3}$$

Due to the inertia of the train, the effective train mass is added. The ratio of additional energy to the vehicle kinetic energy is λ_m. And the accelerating mass is $M' = M(1 + \lambda_m)$. Since A, B, C, D are constants, the value of R is only affected by train velocity v for specific vehicle and route. The equation above could be transferred to Davis equation.

$$\mathrm{R} = \mathrm{A}' + \mathrm{B}'\mathrm{v} + \mathrm{C}'\mathrm{v}^2 \tag{4}$$

And the Equation (1) is as

$$\mathrm{M}\frac{\partial^2 \mathrm{S}}{\partial \mathrm{t}^2}(1 + \lambda_\mathrm{m}) = \mathrm{F} - (\mathrm{A}' + \mathrm{B}'\mathrm{v} + \mathrm{C}'\mathrm{v}^2) - \mathrm{Mg}\alpha \tag{5}$$

If the total journey length S for a train motion is divided into N distance slots and the distance slot is denoted as ΔS_i, the acceleration for each slot can be calculated by the force of the train.

$$a_i = \begin{cases} \frac{1}{M(1+\lambda_m)}\left(F_{t(i)} - R_{(i)}\right) & when\ motoring\ or\ coasting \\ -\frac{1}{M}\left(F_{b(i)} + R_{(i)}\right) & when\ the\ train\ is\ braking \end{cases}$$

(6)

where $F_{t(i)}$, $F_{b(i)}$ and $R_{(i)}$ are the tractive force, braking force and resistance at ΔS_i. The speed can be obtained by the speed of the previous slot.

$$v_{i+1} = \sqrt{{v_i}^2 + 2a_i\Delta S_i}$$

(8)

The distance for each slot can be calculated as

$$\Delta t_i = \frac{2\Delta S_i}{v_{i+1} + v_i}$$

(9)

Finally, the total travelled time is calculated by considering all of the time slots, which is shown below.

$$T = \sum_{i=1}^{N} \Delta t_i$$

(10)

The expression of total energy consumption is derived by the definition of energy ($E = P \cdot t$).

$$E = \sum_{i=1}^{N} \frac{P_{t(i)}}{\eta v_i} \Delta S_i$$

(11)

where η is the efficiency of power. The energy efficiency for DAS can be achieved by providing optimal driving style which has advantage such as easy to implement and can be widely used. For this reason, the energy consumption will be concentrated on traction energy by different driving modes and speed profiles.

B. Operation and Driving Styles

The train operation system usually contains four basic control regimes, including motoring, cruising at constant speed, coasting with no traction applied, and braking at stop [13]. A general speed profile containing the four regimes is shown below (Fig.1). The selection and arrangement of these four regimes can formulate various driving styles, leading to different energy usage for the same route.

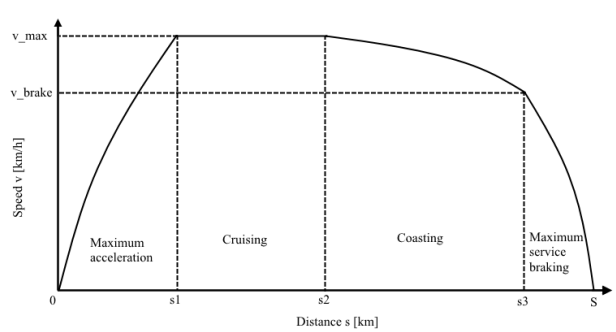

Fig.1. Four regimes in a typical speed profile

Driving styles have been studied as a way to achieve energy efficiency. Under the consideration of the application in a DAS system, three typical driving styles will be introduced, including maximum style, coasting style proposed in [14] and cruising plus coasting style proposed in [13]. The examples are shown by sequence of regimes in Table 1.

1. Maximum style: Operating the train with acceleration at the beginning to reach the speed limit with maximum traction. Then

the train travels at constant velocity, only changing the speed when the speed limit changes or the train is required to brake for the next station.

2. Coasting style: Operating the train with acceleration to reach the speed limit. Then coasting is applied to slow down the train. When the speed drops to coasting speed, acceleration is repeated before train reaches the speed limit then coasting is applied again. Braking is applied when the train reaches the next station.
3. Cruising plus coasting style: Operation of the train with acceleration to reach the speed limit. Then the train will cruise at the maximum permitted speed. Braking is applied when it meets a change of speed limit or at the next station.

TABLE I
THREE DRIVING STYLES IN SEQUENCE OF REGIMES

Driving styles	**Example of regimes applied**
Maximum style	M→CR→B→CR→B
Coasting style	M→C→M→C…→B
Cruising + coasting style	M→CR→C→CR→C→B

C. Single Train Simulator

A Single Train Simulator (STS) was developed in MATLAB 2014b to simulate the train movement and energy usage. The STS takes specific route information and properties of a train to simulate the speed profile of the train with three different driving styles. The calculation of energy usage and journey time is also provided. The structure of the STS (Fig.2) illustrates that the route model and vehicle model will be constructed initially, where the route model contains restrictions such as speed limit and timetable, as well as gradient information. Driving style and train specification gives traction, train physical properties and operating information. By the calculation of a motion model, the speed profile is obtained, with energy usage and running time.

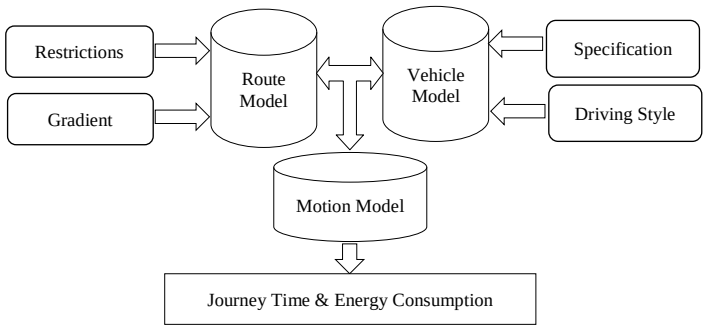

Fig.2.The structure of the STS system

The calculation is processed by two levels on a discrete-distance basis, where the higher level feeds data to the lower level based on [14]. By the calculation in every distance slot, the speed prolife can be expressed in a sequence of information including current journey time and current velocity for each distance slot.

III. TRAJECTORY OPTIMIZATION DEVELOPMENT

A. Static Optimization

The efficient driving system is performed by means of a simulation-based framework. Static optimization to produce an optimal speed profile is shown in Fig.3. Train infrastructure and route information is for the single train simulator, where genetic algorithm (GA) is used to find the best control variables for construction of the trajectory to achieve energy optimization.

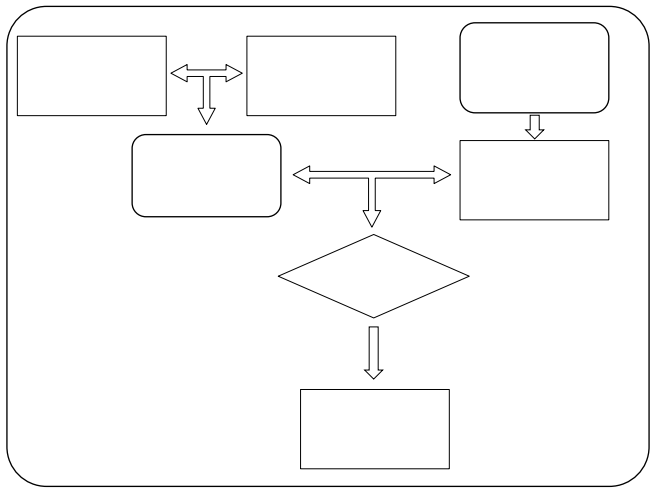

Fig.3. Static optimization flow chart

GA has been applied in the search for the coasting points, as well as the acceleration rate, the braking rate, and the cruising speed. The energy usage E and running time T are functions with variables including acceleration, deceleration, speed limit and location of coasting point L_{cp}. The energy minimization can be expressed as:

$$E = f(L_{cp}, v_{lim}, a_{tr}, a_{br})_{min}$$
$$if\ \ T_{min} < T = g(L_{cp}, v_{lim}, a_{tr}, a_{br}) < T_{max} \tag{12}$$

After introducing three variable factors to replace $L_{cp}, v_{lim}, a_{tr}, a_{br}$ withX_m, X_{cr}, X_c, X_b, the expression can be transferred to

$$E = f(X_m, X_{cr}, X_c, X_b)_{min}$$
$$if\ \ T_{min} < T = g(X_m, X_{cr}, X_c, X_b) < T_{max} \tag{13}$$

where X_m and $X_b$are the percentage of the maximum available traction effort and braking effort respectively, and coasting factor and cruising factor, which are shown in Table 2. Since the four factors have no influence on each other, energy consumption E and running time T can be calculated by applying different factors.

TABLE II
THE DRIVING VARIABLES TO OPTIMIZE

Operating Regime	**Driving Variable Formula**
Motoring	$X_m = \frac{\text{Accelaration}}{\text{Max Accelaration}}$
Cruising	$X_{cr} = \frac{\text{Cruising Speed}}{\text{Speed Limit}}$
Coasting	$X_c = \frac{\text{Coasting Speed}}{\text{Cruising Speed}}$
Braking	$X_b = \frac{\text{Braking Effort}}{\text{Max Braking}}$

According to GA, the chromosome for energy minimization equation with three genomes X_m, X_{cr}, X_c, X_b is defined. The range for X_{cr}, X_c and discrete sets for X_m, X_b are given, as well as the initial factor set(X_m, X_{cr}, X_c, X_b). During the calculation of E and T for different factor sets based on equations (12) and (13), the ranking of the solution is determined by a fitness function, which decides the opportunity for the chromosome to survive in the next generation. After a repeat of new sets, the best solution of (X_m, X_{cr}, X_c, X_b) will be obtained, by which the optimal speed profile will be achieved.

The fitness function mentioned above is derived from the objective function. It is constructed to estimate the performance of the optimal trajectory. The weight of time and energy is considered as $\omega_E$and ω_T.

$$\mu(E,T) = \mu(E)\omega_E + \mu(T)\omega_T \tag{14}$$

$\mu(E)$ and $\mu(T)$ are two fitness functions which are constructed in fuzzy sets. Detail of the fitness functions can be found in [3].

After applying the static optimization to three driving styles mentioned in Section II, Fig.4 shows the optimized speed profiles respectively. For journeys with a considerable journey time margin, the coasting scheme and maximum scheme fail to find a smooth trajectory. There is no extended cruising phase for the coasting scheme and the speed changes frequently. For the maximum scheme, the change of train control variable is less, but there is a considerable difference between the maximum and minimum speeds when the control variable changes. However, the cruising plus coasting scheme gives a better solution, which performs better with a more constant cruising speed below the line speed constraint.

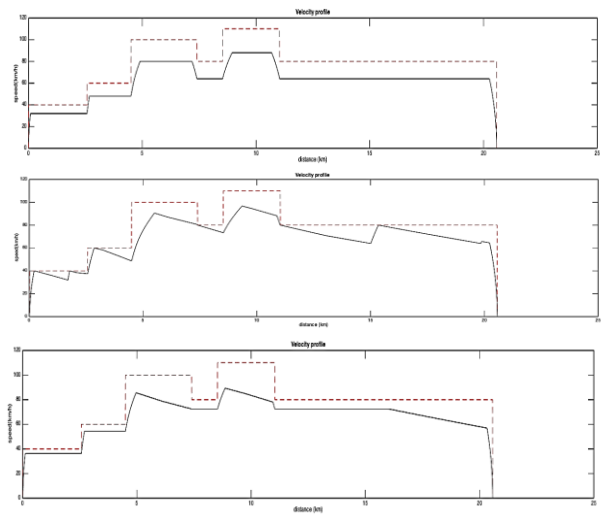

Fig.4. static optimal train trajectories for (a) maximum driving style (b) coasting driving style (c) cruising + coasting driving style

As for the energy aspect, Fig.5 shows the energy consumption against journey time of these three schemes for the route simulated above. When the journey time constraint is small, all three schemes perform well and their energy consumption is similar at 1072 s. As the journey time increases, the energy consumption in the cruising plus coasting scheme drops dramatically. The coasting scheme also shows an energy reduction which is not good as the cruising plus coasting scheme, while the maximum scheme is less energy efficient. The cruising plus coasting driving style is the most energy-efficient.

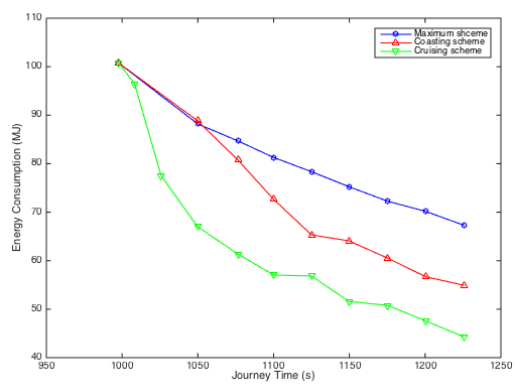

Fig.5. Energy efficiency compared for different driving styles

B. Dynamic optimization

From the static optimization, the driving profile is obtained with minimum energy consumption for a fixed running time. The optimal driving profile denotes as $(D_1, D_2, D_3 \dots D_M)$, where each state of train D_i contains information of train location, current speed and running time from the beginning until the location:

$$D_i = (L_i, v_i, t_i) \tag{15}$$

where $\Delta L = L_{i+1} - L_i$ is the precision of the profile which is usually fixed; D_M is the location of the next station.

A current profile is constructed during the train running, which is denoted as$(C_1, C_2, C_3 \dots C_M)$, where

$$C_i = (L_i, v_{ci}, t_{ci})$$

(16)

The structure of dynamic optimization is shown below. The pre-defined driving state is compared with the current state. If the train is delayed by a certain time, a dynamic optimization will be conducted by the system.

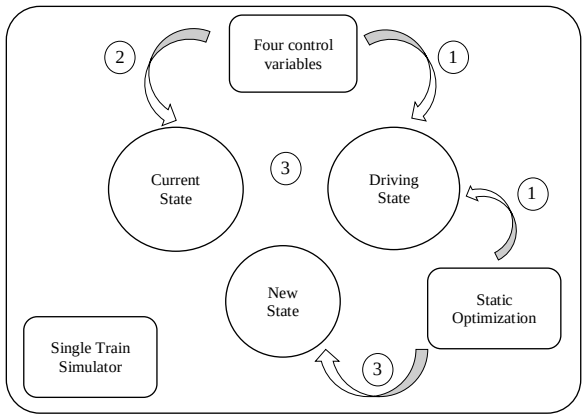

Fig.6.The procedure of dynamic optimization

The process to achieve dynamic optimization is concluded in the main steps below:

Step 1: Using the GA program to construct an energy-efficient driving instruction based on route and journey time from the timetable.

Step 2: Construct a current state with delay to optimize by the four control variables (or get information of a current state from the navigation system).

Step 3: Compare the driving state with the current state. Calculate a new state for driving after dynamic optimization.

The flow chart in Fig.7 illustrates the algorithm for dynamic optimization in step 3. The input of current state C_i (L_i, v_{ci}, t_{ci}) is compared to the state at the same location in the driving profile, which is D_i (L_i, v_i, t_i). The difference in time between the two states is

$$\Delta t = |t_i - t_{ci}|$$

(17)

The initialization of dynamic optimization will be preceded when $\Delta t > 15$. A new state N_1 replaces C_i, while the remaining time and distance T', S' will be defined as:

$$T' = T - t_{ci} - t_{comput}$$

$$S' = S - L_{ci} - s_{predict}$$

(18, 19)

where the Computing time t_{comput} means computing time should be considered, as dynamic optimization provides real-time information; Predicting distance $s_{predict}$ is the train running during the computing time.

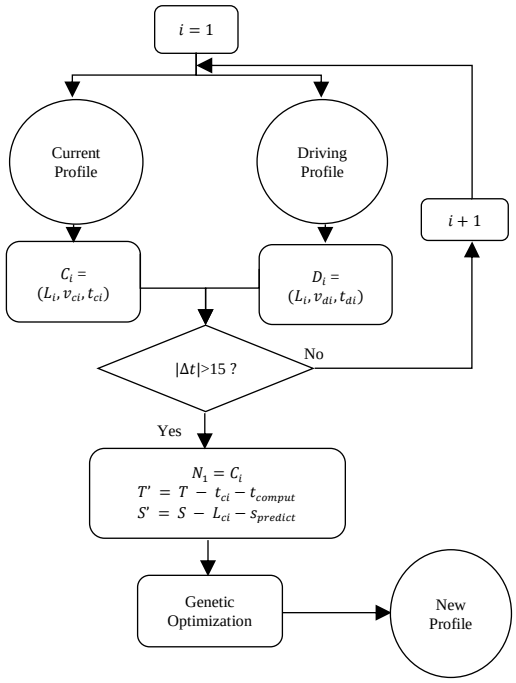

Fig.7.The dynamic programming

Genetic algorithm is applied to optimize the remaining route after the initialization. Fig.8 shows an example of the result after optimization, which recovers the delay of the current state at 2.9 km.

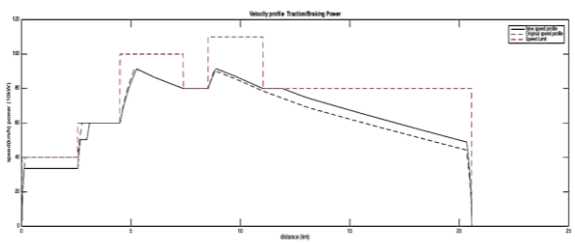

Fig.8. Trajectory after dynamic optimization

C. Integration

The static optimization and dynamic optimization are integrated when applied into DAS systems. By setting the required journey time, static optimization produces an optimized driving profile. During the operation, the current train state (including current position, speed and running time) is compared to the driving state. A dynamic optimization is preceded when the difference in running time is larger than 15 s, providing an updated driving profile.

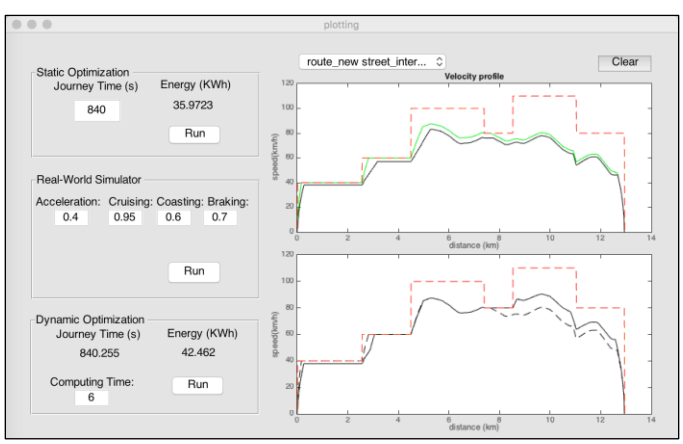

Fig.9.trajectory optimizer

A trajectory optimizer is constructed by GUI based on Matlab (Fig.9). The input of the current train state is replaced by a

real-world simulator to produce a trajectory with delay to optimize.

IV. Simulation & Result

The proposed trajectory optimization system has been tested to simulate a real-scale situation based on a suburban railway from Birmingham New Street Station to Birmingham International. This line is one of the most densely operated lines around Birmingham and has a route length of 12.9 km. The route information is shown in Fig.10 with an assumed speed limit profile. The vehicle model was chosen as British Rail Class 323, which is operated on the Cross-City Line. The parameters of vehicle Class 323 used in the simulation are given in Table III.

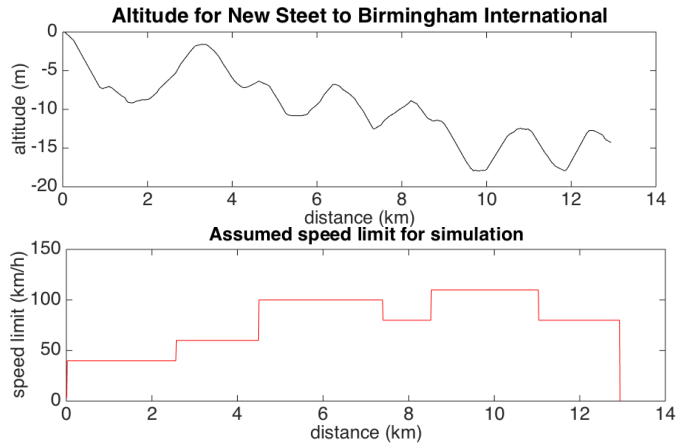

Fig.10. Route information for the simulated trajectory

TABLE III
Parameters for Vehicle Model

Symbol	Parameter	Value
η	Efficiency of Generated Traction	85%
η_r	Regenerative Braking Rate	80%
m	Mass of train	132.1t
P	Power output	1.168MW
V_M	Maximum speed	145km/h

In particular, the minimum running time for the journey above is 729.5 s. Due to a time slack of 7% added to the minimum running time, the journey time for the following analysis is defined as 780 s (13 min). All experiments are carried out on an Intel Core i7 2.3 GHz processor. The computing time for a single route is about 0.05 seconds.

A. *Static Optimization Evaluation*

The first study was the evaluation of the performance of static optimization using three driving styles. Fig.4 (a) shows that a maximum driving style maintains the running time of 780 s. This situation, which is without optimization, leads to a large energy consumption of 64.63 kWh. By setting the same running time, two optimized trajectories are calculated by static optimization with coasting style and cruising plus coasting style respectively. Comparing the three graphs in Fig.4, the energy for braking is reduced dramatically due to the application of coasting before the train needs to decelerate for the speed limit or for arrival at the station. However, the coasting style applies more traction due to the overuse of coasting. The total energy consumption for optimized trajectory by coasting style in Fig.4 (b) is reduced to 51.79 kWh, and the optimized trajectory by cruising plus coasting style in Fig.4 (c) is 42.50 kWh. Energy efficiency is 19.87% and 34.24% respectively, while the same running time is kept. The cruising plus coasting style is the most energy-efficient.

According to the speed profile, the optimized trajectory by cruising plus coasting style shows the smoothest curve, which means it can provide a comfortable and safe journey for passengers. The application of optimization with cruising plus coasting style also has the potential to minimize energy consumption for train operation.

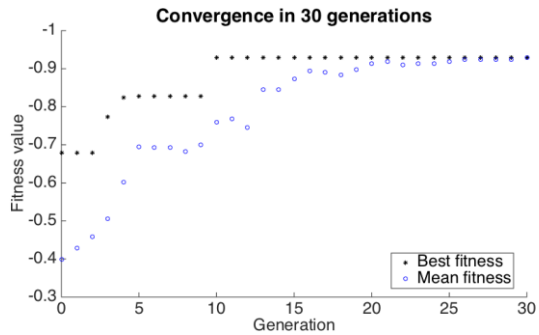

Fig.11. Convergence for static optimization

The convergence for optimization of cruising plus coasting driving style is shown in Fig.11. The initial population size for GA is set as 30. At the 10th generation, the best fitness value reaches the peak of 0.92. The mean fitness values get closer to the best fitness values as the generation increases. The mean value is convergent to the best value at the 27th generation. In this case, the static-optimal driving profile is obtained, which applies cruising plus coasting style with control variables [0.95, 1.00, 0.80, 0.80].

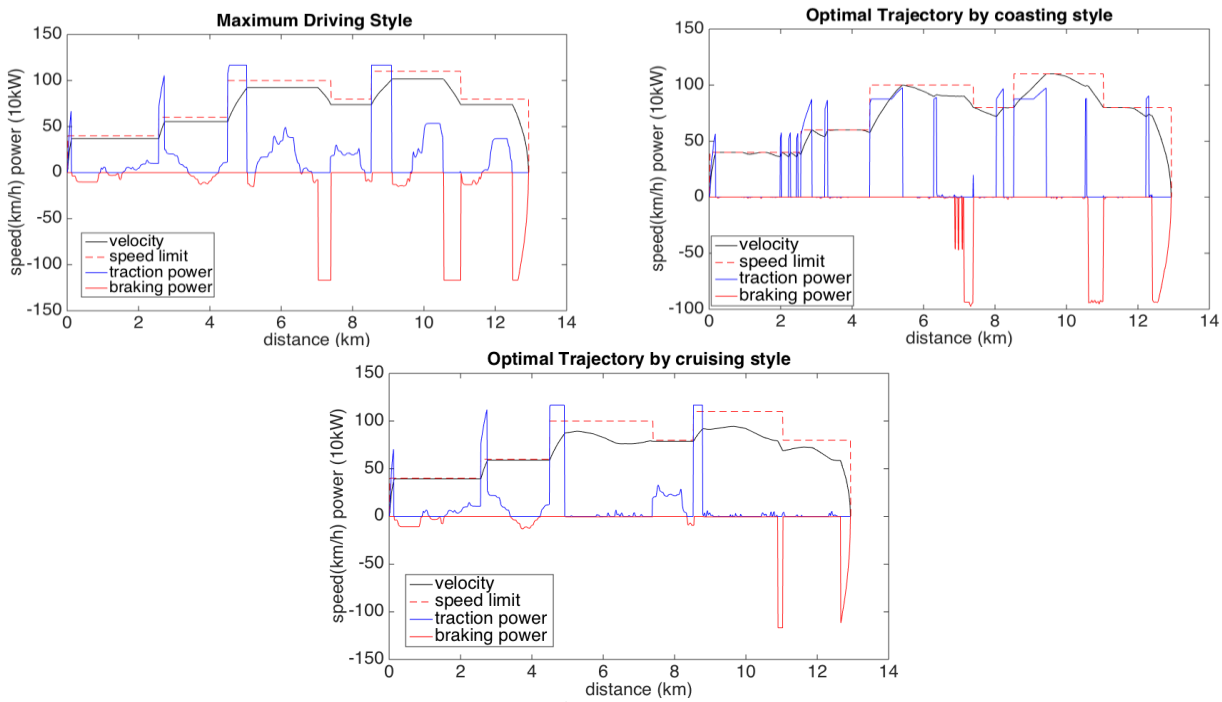

Fig.12. Static optimization for the simulated trajectory in diving styles
(a) Without optimization (b) Optimal coasting style (c) Optimal cruising + coasting style

B. Dynamic optimization for a delay which has occurred

Static optimization provides an energy-efficient trajectory. However, applied in real-world operation, it is possible that the train driver could fail to keep to the trajectory. Since the system has no communication with the train at the simulation stage, the input of a current state for dynamic optimization is not available. An assumed speed profile with delay was used to replace the input of current state, called delayed speed profile. The delayed speed profile is constructed by variables [0.80, 0.95, 0.80, 0.70]. The dynamic optimization is preceded at the location where the train is delayed by 15 s.

The trajectory in Fig.12 (a) shows the optimized speed profile when the train runs without the instruction of the driving profile at the beginning. When the train was delayed by 15 s, the train was at 2.53 km according to the assumed speed profile, where the dynamic optimization started. After optimization, the train recovered 15 s delay when it arrived at the station. The energy consumption rose from 42.50 kWh to 60.45 kWh after dynamic optimization. Fig.13 (b) shows the running time against distance for dynamic profile and static profile. It can be seen the train arrived at the same time of 780 s, while there is a delay of 15 s at 2.53 km.

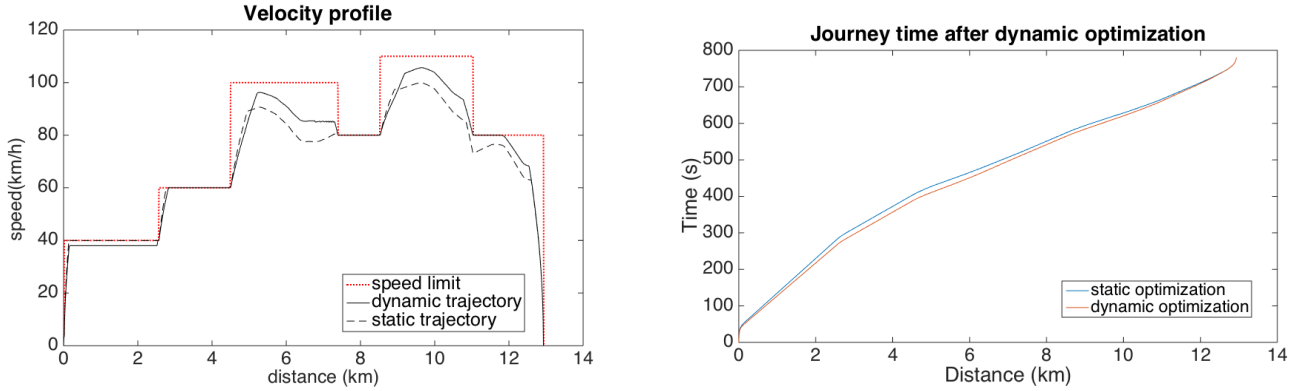

Fig.13.The speed profile of dynamic optimization

The computing time of dynamic optimization has been studied. The optimization program was repeated 600 times by setting a random computing time from 1 s to 6 s. Fig.14 illustrates the convergence of fitness value with increasing computing time. A computing time of less than 3 s gives a significant variation in fitness values. When the computing time is over 5 s, the fitness value is convergent to around 0.92.

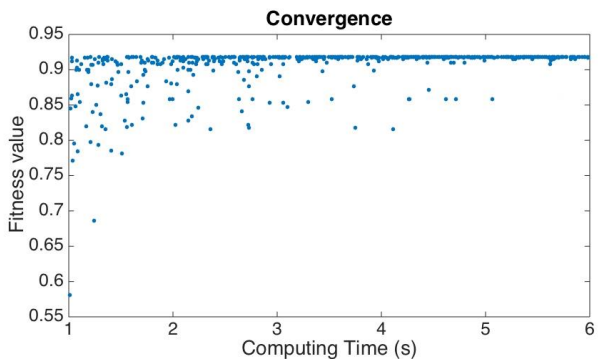

Fig.14.The convergence

V. Summary & Conclusions

A. Overview

In this paper, a dynamic optimization method to generate optimized train trajectory for DAS systems has been studied. A trajectory optimizer in Matlab is derived by using a combination of static optimization and dynamic optimization. Different driving styles are applied and simulated in the static optimization, including maximum style, coasting style and cruising plus coasting style. An assumed speed profile with delay is tested by dynamic optimization to recover the delay.

B. Conclusion

1. Driving styles can have considerable influence on the energy consumption for train operation. The cruising plus coasting driving style is more promising than the other two styles in this research as it has less energy consumption. The cruising plus coasting driving style has benefits for energy efficiency as well as passenger comfort.

2. The trajectory optimizer could minimize the energy consumption by providing an optimized trajectory,which could be transferred as driving instruction in DAS systems. Dynamic optimization also contributes to a real-time profile update when delay occurs, which achieves energy efficiency and punctuality at the same time.

3. Considering the computing performance of the trajectory optimizer, the computation time for calculating the dynamic trajectory is short, which is reasonable for real-world train operation.

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