Behaviour and design of high-strength steel cross-sections under combined loading

Michaela Gkantou PhD
Research Fellow, Department of Civil Engineering, School of Engineering, College of Engineering and Physical Sciences, University of Birmingham, Edgbaston, Birmingham, UK (corresponding author: m.gkantou@bham.ac.uk)

Marios Theofanous PhD
Lecturer, Department of Civil Engineering, School of Engineering, College of Engineering and Physical Sciences, University of Birmingham, Edgbaston, Birmingham, UK

Jie Wang PhD
Research Associate, Department of Civil and Environmental Engineering, Imperial College London, London, UK

Charalampos Baniotopoulos PhD
Professor, Department of Civil Engineering, School of Engineering, College of Engineering and Physical Sciences, University of Birmingham, Edgbaston, Birmingham, UK

Leroy Gardner PhD
Professor, Department of Civil and Environmental Engineering, Imperial College London, London, UK

The behaviour of hot-rolled high-strength steel (HSS) tubular sections under combined compression and uniaxial bending was investigated both experimentally and numerically. The experimental programme encompassed a series of material coupon tests, initial geometric imperfection measurements, residual stress measurements and 12 tests on stub columns subjected to uniaxial eccentric compression. Numerical models were developed and validated against the experimental results. An extensive parametric study was then performed with the aim of generating further structural performance data over a wider range of cross-section slendernesses, aspect ratios and applied eccentricities. The results were utilised for an assessment of the applicability of relevant Eurocode provisions to HSS cross-sections under combined loading. Conclusions regarding the applicability of Eurocode interaction curves to S460 and S690 square and rectangular hollow sections are presented.

Notation

- $A$: cross-sectional area
- $A_c$: cross-sectional area of coupon
- $A_{eff}$: effective cross-sectional area
- $a_t$: eurocode coefficient used on the evaluation of the reduced cross-section moment resistance of hollow sections subjected to combined bending around minor axis and axial load
- $a_w$: eurocode coefficient used on the evaluation of the reduced cross-section moment resistance of hollow sections subjected to combined bending around major axis and axial load
- $b$: section width
- $c/l$: element slenderness
- $E$: Young’s modulus
- $e_0$: actual initial loading eccentricity
- $e_{0,n}$: nominal initial loading eccentricity
- $e$, $e'$: eccentricity generated due to second-order effect
- $e_u$: eccentricity at ultimate load generated due to second-order effect
- $f_a$: ultimate tensile strength
- $f_y$: yield strength
- $h$: section depth
- $I$: second moment of area
- $L$: length
- $M_d$: elastic moment
- $M_{pl}$: plastic moment
- $M_u$: failure moment
- $M_{u,exp}$: experimentally obtained failure moment
- $M_{u,FE}$: numerically obtained failure moment
- $N_u$: failure load
- $N_{u,exp}$: experimentally obtained failure load
- $N_{u,FE}$: numerically obtained failure load
- $R_{exp}/R_{pred}$: ratio of experimental capacity to predicted capacity
- $R_{FE}/R_{pred}$: ratio of finite-element model capacity to predicted capacity
- $r_i$: internal corner radius
- $t$: thickness
- $W_{eff}$: elastic modulus of effective section
- $W_{el}$: elastic section modulus
- $W_{pl}$: plastic section modulus
- $\beta$: coefficient for prediction of imperfection amplitude
- $e_{concave}$: strain on concave side of cross-section
- $e_{convex}$: strain on convex side of cross-section
- $e_{eng}$: engineering strain
- $e_f$: strain at fracture
- $e_{lg}$: logarithmic plastic strain
- $e_u$: strain at ultimate stress
- $\sigma_{cr}$: elastic local plate buckling stress
- $\sigma_{eng}$: engineering stress
- $\sigma_{true}$: true stress
- $\phi_u$: mean end-rotation at failure load
- $\psi$: ratio of stresses or strains across section depth
- $\omega_0$: measured initial local geometric imperfection
- $\omega_{DW}$: initial local geometric imperfection from the model of Dawson and Walker (1972)

1. Introduction

Over the last few decades, several studies have shown that there are potential benefits in using high-strength steels (HSSs) in
building and bridge applications (Bjorhovde, 2004; Höglund et al., 2005). However, given that most international structural design standards (ANSI/AISC 360-10 (AISC, 2010), AISI S100 (AISI, 2012), GB 50017-2003 (CABP, 2006), CAN/CSA-S16-01 (CSA, 2001) and AS 4100-01 (Standards Australia, 2012) either do not cover HSSs or adopt design methods identical to those for normal-strength steels, there is a clear need for the development of comprehensive design guidance for HSS structures.

The European provisions for HSS structural design are set out in EN 1993-1-12 (CEN, 2007), where additional rules for steels with yield strengths beyond 460 N/mm² and up to 700 N/mm² are specified. EN 1993-1-12 (CEN, 2007) relaxes the requirements imposed on the strain hardening and ductility characteristics of HSS material but, other than some specific restrictions (e.g. plastic design is not permitted), generally applies the same cross-section and member design rules as for conventional steel design by referring to EN 1993-1-1 (CEN, 2014). There is, however, a clear need to fully verify and further develop these rules, and to extend the experimental database on HSS structural elements beyond that available when EN 1993-1-12 (CEN, 2007) was published (Beg and Hladnik, 1996; McDermott, 1969; Rasmussen and Hancock, 1992, 1995; Ricles et al., 1998; Usami and Fukumoto, 1984; Yang and Hancock, 2004; Yang et al., 2004).

Recently, several researchers have investigated the member buckling behaviour of HSS long columns (Ban et al., 2013; Rasmussen and Hancock, 1995; Shi et al., 2012; Wang et al., 2014; Yang et al., 2004), the cross-sectional behaviour of HSS beams (Beg and Hladnik, 1996; Lee et al., 2012; McDermott, 1969; Ricles et al., 1998; Usami and Fukumoto, 1984; Wang et al., 2016) and stub columns (Beg and Hladnik, 1996; Gao et al., 2009; Gkantou et al., 2017; Rasmussen and Hancock, 1992; Shi et al., 2014; Usami and Fukumoto, 1984; Yang and Hancock, 2004, 2006; Yoo et al., 2013), and have made recommendations regarding the structural design of HSS members, including revised slenderness limits, effective width equations and column buckling design curves. However, studies on HSS cross-sections under combined axial load and bending moment (Kim et al., 2014) remain scarce. Similar studies on the structural response of eccentrically loaded stub columns have been recently reported for stainless steel sections (Arrayago and Real, 2015; Zhao et al., 2015a, 2015b) and composite sections (Fujimoto et al., 2004; Sheehan et al., 2012).

A comprehensive experimental programme was undertaken in the Structures Laboratory at Imperial College London, focusing on the structural behaviour of hot-rolled HSS square hollow sections (SHSs) and rectangular hollow sections (RHSs). The overall programme comprised material coupon tests, geometric imperfection and residual stress measurements, stub column tests (Wang et al., 2017), three-point and four-point in-plane bending tests (Wang et al., 2016), and tests on cross-sections under combined loading, which are reported herein. In parallel with the experimental programme, a numerical study was also conducted. The first step of the numerical study was to develop reliable finite-element (FE) models capable of replicating the experimental findings; the second step was to use the validated models to generate further structural performance data over a wider range of local slendernesses and loading (i.e. combinations of axial load and bending moments). Finally, the combined experimental and numerical results were used to assess the accuracy of the design rules presented in EN 1993-1-12 (CEN, 2007), which refer to EN 1993-1-1 (CEN, 2014), for predicting the cross-section capacity of hot-finished HSS SHS and RHS under combined loading.

2. Experimental study

2.1 General

A total of 12 stub column specimens were tested under uniaxial eccentric compressive loads to assess their structural behaviour under combined axial load and bending moments. The tested cross-sections were SHS 50 × 50 × 5 in grade S460 steel and SHS 50 × 50 × 5 and SHS 90 × 90 × 5–6 in grade S690 steel. Both the S460 and S690 specimens were hot-rolled seamless tubular sections, hollowed out by a piercing mill to the final shape, after which the S460 sections were normalised, whereas the S690 were quenched and tempered. The chemical composition and tensile material properties of the tested specimens, as provided by the mill certificates, are presented in Tables 1 and 2, respectively. In addition to the eccentric compression tests, corresponding material coupon tests, initial geometric imperfection measurements and residual stress measurements were also conducted for each cross-section.

2.2 Material testing

A comprehensive coupon testing programme covering tensile flat (TF), tensile corner (TC) and compressive flat (CF)

<table>
<thead>
<tr>
<th>Chemical composition of tested specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S460 SHS</strong></td>
</tr>
<tr>
<td>50 × 50 × 5</td>
</tr>
<tr>
<td>Carbon: %</td>
</tr>
<tr>
<td>Silicon: %</td>
</tr>
<tr>
<td>Manganese: %</td>
</tr>
<tr>
<td>Phosphorus: %</td>
</tr>
<tr>
<td>Sulfur: %</td>
</tr>
<tr>
<td>Copper: %</td>
</tr>
<tr>
<td>Chromium: %</td>
</tr>
<tr>
<td>Nickel: %</td>
</tr>
<tr>
<td>Molybdenum: %</td>
</tr>
<tr>
<td>Vanadium: %</td>
</tr>
<tr>
<td>Titanium: %</td>
</tr>
<tr>
<td>Niobium: %</td>
</tr>
<tr>
<td>Boron: %</td>
</tr>
<tr>
<td>Aluminium: %</td>
</tr>
</tbody>
</table>
coupons was carried out on the studied cross-sections. The resulting material properties were used in the analysis of the combined loading test results and in the development of the numerical models of the tested specimens. For each cross-section, four flat coupons and one corner coupon were extracted from the locations indicated in Figure 1 and tested in tension. Additionally, one CF coupon was also cut from a flat face of each cross-section. The tests were conducted in accordance with ISO 6892-1 (BSI, 2009). Measured stress–strain curves from the coupon tests are shown in Figures 2(a) and 2(b) for the S460 SHS 50 × 50 × 5 and S690 SHS 50 × 50 × 5 specimens, respectively. It can be seen that both grades of material display a sharply defined yield point followed by a yield plateau; the S690 material generally exhibited less strain hardening and lower ductility than the S460 material. Key results from the TF, TC and CF coupon tests are summarised in Table 3; the material parameters reported are the Young’s modulus $E$, the upper yield strength $f_y$, the ultimate tensile strength $f_u$, the tensile-to-yield stress ratio $f_u/f_y$, the strain at the ultimate tensile stress $\varepsilon_u$ and the plastic strain at fracture $\varepsilon_f$, based on elongation over the standard gauge length equal to $5\sqrt{A_c}$, where $A_c$ is the cross-sectional area of the coupon (ISO 6892-1 (BSI, 2009)). Further details of the experimental procedure and results are reported by Wang et al. (2017).

It should be noted that the TF results are the average results of the four TF coupons. Since the corner coupons were observed to behave very similarly to their flat counterparts in terms of the shape of the stress-strain curve and the key material parameters, the average results from the flat coupon tests (TF results in Table 3) were used in the subsequent data analysis and numerical modelling of the combined loading tests.

### Table 2. Mechanical properties as stated in mill certificates

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>$f_{y,\text{mill}}$: N/mm²</th>
<th>$f_{u,\text{mill}}$: N/mm²</th>
<th>$\varepsilon_f$: %</th>
</tr>
</thead>
<tbody>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>473</td>
<td>615</td>
<td>26.5</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>797</td>
<td>838</td>
<td>22.4</td>
</tr>
<tr>
<td>S690 SHS 90 × 90 × 5</td>
<td>789</td>
<td>825</td>
<td>16.6</td>
</tr>
</tbody>
</table>

Figure 1. Locations of TF and TC coupons and definition of cross-section symbols

Figure 2. Measured stress–strain curves for TF, TC and CF coupons: (a) S460 SHS 50 × 50 × 5; (b) S690 SHS 50 × 50 × 5

2.3 Local imperfection and residual stress measurements

For structural elements prone to buckling, the presence of imperfections can have a strong influence on their behaviour and load-carrying capacity. Typical structural imperfections for steel members include geometric (global and local) imperfections and residual stresses. Including imperfections in FE simulations enables accurate modelling of the structural response of the tested specimens. Since global imperfections are very small compared with the applied eccentricity in the present study and are only important for member buckling, which is not relevant for stub columns, only local imperfections and residual stress measurements are reported here. The maximum recorded local geometric imperfections for the tested cross-sections, denoted $\omega_{\text{ini}}$, are reported in Table 4. The maximum measured longitudinal membrane residual stresses were $0.05f_y$ in...
tension and 0.031σ; in compression and their low values were attributed to the seamless fabrication procedure. Owing to their very small magnitudes compared with the material yield strength, the residual stresses were not explicitly introduced into the FE models. A detailed description of the initial geometric imperfection and residual stress measurements is provided elsewhere (Wang et al., 2016, 2017). Residual stress measurements on HSS box sections were also executed by Rasmussen and Hancock (1995) and Wang et al. (2012). Even though the aforementioned studies focused on welded sections, residual stresses and geometric imperfections to be present in the tested cross-section, thus enabling a representative pattern of residual stresses and geometric imperfections to be present in the tested member, while preventing global buckling.

The combined loading tests were conducted in a Satec 2000 kN hydraulic loading machine; Figure 3 shows the test setup. The specimens were welded onto end-plates at an offset from the centre to include the nominal eccentricities, and then installed in the testing machine by bolting the end-plates to the loading plates. The top and bottom loading plates were in contact with the loading rig through knife edges which provided pin-ended boundary conditions about the axis of bending and fixed-ended boundary conditions about the other axis. In terms of the instrumentation, two linear variable differential transformers (LVDTs) were placed horizontally at the mid-height of the specimens to measure lateral displacement, strains, which would be used for determination of the actual calculated initial loading eccentricity, as discussed later in the paper. Two inclinometers were attached to the end-plates (one at each end) to record the end-rotation of the specimens. The applied load was obtained from the loading machine. The stub columns were loaded under displacement control.

### 2.4 Eccentric stub column tests

To investigate the structural behaviour of HSS hollow sections under combined compression and uniaxial bending, 12 stub columns were tested under compression with different loading eccentricities to generate different ratios of axial load to bending moment. The average measured geometric dimensions of the test specimens (length of the specimen L, section depth h, section width b, thickness t and average internal corner radius r) are reported in Table 4, together with the maximum local geometric imperfection ω0 and the nominal initial loading eccentricity e0,n. In accordance with technical memorandum B3 (Ziemian, 2010), the length of the tested specimens was set equal to three times the largest dimension of the cross-section, thus enabling a representative pattern of residual stresses and geometric imperfections to be present in the tested member, while preventing global buckling.

The combined loading tests were conducted in a Satec 2000 kN hydraulic loading machine; Figure 3 shows the test setup. The specimens were welded onto end-plates at an offset from the centre to include the nominal eccentricities, and then installed in the testing machine by bolting the end-plates to the loading plates. The top and bottom loading plates were in contact with the loading rig through knife edges which provided pin-ended boundary conditions about the axis of bending and fixed-ended boundary conditions about the other axis. In terms of the instrumentation, two linear variable differential transformers (LVDTs) were placed horizontally at the mid-height of the specimens to measure lateral displacement, strains, which would be used for determination of the actual calculated initial loading eccentricity, as discussed later in the paper. Two inclinometers were attached to the end-plates (one at each end) to record the end-rotation of the specimens. The applied load was obtained from the loading machine. The stub columns were loaded under displacement control.

### Table 3. Average measured material properties from coupon tests

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Coupon</th>
<th>E: N/mm²</th>
<th>fᵧ: N/mm²</th>
<th>fᵧ: N/mm²</th>
<th>εu: %</th>
<th>εf: %</th>
<th>fᵧ/fᵧ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>TF</td>
<td>211 100</td>
<td>505 208</td>
<td>620 209</td>
<td>14.9</td>
<td>31.0</td>
<td>1.23</td>
</tr>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>TC</td>
<td>208 000</td>
<td>481 209</td>
<td>631 209</td>
<td>12.7</td>
<td>26.0</td>
<td>1.31</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>TF</td>
<td>204 200</td>
<td>759 209</td>
<td>790 209</td>
<td>7.5</td>
<td>21.7</td>
<td>1.04</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>TC</td>
<td>209 000</td>
<td>782 209</td>
<td>813 210</td>
<td>6.9</td>
<td>18.0</td>
<td>1.04</td>
</tr>
<tr>
<td>90 × 90 × 56</td>
<td>CF</td>
<td>220 000</td>
<td>813 220</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>90 × 90 × 56</td>
<td>CF</td>
<td>205 000</td>
<td>774 205</td>
<td>790 205</td>
<td>7.4</td>
<td>20.1</td>
<td>1.02</td>
</tr>
</tbody>
</table>

### Table 4. Mean measured dimensions of eccentric stub column specimens

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>e0,n: mm</th>
<th>L: mm</th>
<th>h: mm</th>
<th>b: mm</th>
<th>t: mm</th>
<th>r: mm</th>
<th>e0,n: mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>5</td>
<td>149.98</td>
<td>50.03</td>
<td>49.86</td>
<td>4.94</td>
<td>3.00</td>
<td>0.054</td>
</tr>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>10</td>
<td>150.01</td>
<td>49.86</td>
<td>50.16</td>
<td>4.98</td>
<td>3.00</td>
<td>0.054</td>
</tr>
<tr>
<td>S460 SHS 50 × 50 × 5</td>
<td>20</td>
<td>149.95</td>
<td>50.32</td>
<td>50.11</td>
<td>4.90</td>
<td>3.00</td>
<td>0.054</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>30</td>
<td>149.97</td>
<td>50.07</td>
<td>50.36</td>
<td>4.95</td>
<td>3.00</td>
<td>0.054</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>5</td>
<td>149.91</td>
<td>50.24</td>
<td>50.60</td>
<td>5.03</td>
<td>3.00</td>
<td>0.076</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>10</td>
<td>149.96</td>
<td>50.27</td>
<td>50.39</td>
<td>4.94</td>
<td>3.00</td>
<td>0.076</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>15</td>
<td>149.84</td>
<td>50.45</td>
<td>50.52</td>
<td>4.96</td>
<td>3.00</td>
<td>0.076</td>
</tr>
<tr>
<td>S690 SHS 50 × 50 × 5</td>
<td>20</td>
<td>149.98</td>
<td>50.16</td>
<td>50.36</td>
<td>4.97</td>
<td>3.00</td>
<td>0.076</td>
</tr>
<tr>
<td>S690 SHS 90 × 90 × 56</td>
<td>5</td>
<td>269.07</td>
<td>89.56</td>
<td>89.81</td>
<td>5.68</td>
<td>4.50</td>
<td>0.089</td>
</tr>
<tr>
<td>S690 SHS 90 × 90 × 56</td>
<td>10</td>
<td>269.00</td>
<td>89.84</td>
<td>90.10</td>
<td>5.65</td>
<td>4.63</td>
<td>0.089</td>
</tr>
<tr>
<td>S690 SHS 90 × 90 × 56</td>
<td>25</td>
<td>268.96</td>
<td>90.21</td>
<td>90.65</td>
<td>5.72</td>
<td>4.88</td>
<td>0.089</td>
</tr>
<tr>
<td>S690 SHS 90 × 90 × 56</td>
<td>30</td>
<td>269.02</td>
<td>90.57</td>
<td>90.08</td>
<td>5.59</td>
<td>4.63</td>
<td>0.089</td>
</tr>
</tbody>
</table>
at a constant displacement rate of 0·2 mm/min and 0·4 mm/min for the SHS 50 × 50 × 5 and SHS 90 × 90 × 5·6 specimens, respectively. During testing, the load, lateral deflection at mid-height, longitudinal strains and end-rotations were all recorded at 1 s intervals using the data acquisition system DataScan.

After testing, the strain gauge readings were used to calculate the actual initial loading eccentricities applied to the tested cross-sections, since these have a strong influence on the behaviour of the specimens under combined compression and bending and are also required for the numerical replication of the tests. Under uniaxial bending and compression, the relationship between the moment \( M \) and the axial force \( N \) applied to a cross-section is

\[
M = N(e_0 + e')
\]

where the sum of the initial eccentricity \( e_0 \) and the eccentricity generated due to the second-order effect \( e' \) comprises the total eccentricity at the mid-height of the specimen. In the initial stages of loading, during which the specimens remain elastic, the theoretical relationships between the applied bending moment and compressive force and the strain gauge readings are given by Equations 1 and 2, respectively, where \( E \) is the Young’s modulus, \( I \) is the second moment of area, \( A \) is the area of the cross-section, \( h \) is the depth of the cross-section and \( e_{\text{convex}} \) and \( e_{\text{concave}} \) are the strains on the convex side and concave side of the cross-section.

1. \[
M = \frac{Eh(e_{\text{convex}} - e_{\text{concave}})}{h}
\]

2. \[
N = \frac{EA(e_{\text{convex}} + e_{\text{concave}})}{2}
\]

By substituting the above expressions for \( M \) and \( N \) into \( M = N(e_0 + e') \), the relationship between the strain gauge readings and the initial loading eccentricity \( e_0 \) can be established, according to Equation 3, where \( \psi \) is the ratio \( e_{\text{concave}}/e_{\text{convex}} \) and \( e' \) is the second-order eccentricity recorded by the two lateral LVDTs at the mid-height of the specimen.

3. \[
e_0 = \frac{2I(1 - \psi)}{4h(1 + \psi)} - e'
\]

All four SHS 90 × 90 × 5·6 specimens and the S690 SHS 50 × 50 × 5 specimen that was loaded under an eccentricity of 5 mm displayed clear signs of local buckling at failure, as shown in Figures 4(b) and 4(c), while the remaining specimens failed with little visible local buckling, as shown in Figure 4(a). The differences in the observed failure modes can be explained by considering the effect of the yield strength and the stress gradient due to the applied loading eccentricity on the cross-section slenderness. For the same cross-section geometry, the S690 sections had a higher yield load but a similar elastic buckling load to their S460 counterparts and were thus more slender and more prone to local buckling prior to yielding.

With regard to the stress distribution, the cross-sections with the higher loading eccentricities had a steeper stress gradient in the webs, making the webs less prone to local buckling, in
turn meaning that they could also provide greater restraint against local buckling to the flanges on the concave side of the cross-section. The load–end-rotation relationships for all the tested specimens are shown in Figures 5(a)–5(c), while the load–longitudinal strain curves for typical cases are shown in Figure 6. The key test results are summarised in Table 5, where $N_u$ is the failure load, $e_0$ is the calculated initial loading eccentricity based on the strain gauge readings using Equation 3, $e_0^*$ is the recorded lateral deflection at the failure load, referred to as the second-order eccentricity, $M_u$ is the failure moment given by $M_u = N_u(e_0 + e_0^*)$ and $\phi_u$ is the mean end-rotation at the failure load.

3. **Numerical modelling**

In parallel with the experimental study, a numerical investigation using the general-purpose FE software Abaqus (Hibbitt, Karlsson & Sorensen, Inc., 2014) was performed in order to investigate further the structural response of HSS hollow sections under combined loading. The FE models were first validated against the test results and subsequently utilised for the execution of parametric studies, thus generating...
additional data over a wide range of cross-section slenderness and loading combinations, based upon which design recommendations could be made.

3.1 Modelling assumptions

The four-noded doubly curved shell element S4R with reduced integration and finite-membrane strains was adopted for discretisation of the modelled geometries as it has been shown to perform well in similar studies (Wang et al., 2016; Zhao et al., 2015b). An initial mesh convergence study was performed, resulting in an average element size equal to the material thickness.

The material stress–strain properties were incorporated into the FE models based on the results of the tensile coupon tests, in the form of an elastic–plastic multi-linear curve with the von Mises yield criterion and isotropic hardening. Since no significant differences in the stress–strain behaviour between the flat and corner coupon tests or between the tensile and compressive properties were observed, the average values of the material properties obtained from the TF coupon tests, as recorded in Table 3, were utilised for the material model. Abaqus requires the material properties to be input in the form of a multi-linear true stress–logarithmic plastic strain curve. Hence, the measured engineering stress–strain curves were converted into the true stress–logarithmic plastic strain curves by means of Equations 4 and 5, where \( \sigma_{\text{true}} \) and \( \varepsilon_{\text{ln}} \) are the true stress and logarithmic plastic strain, respectively.

\[
\begin{align*}
\sigma_{\text{true}} & = \sigma_{\text{eng}} \left( 1 + \varepsilon_{\text{eng}} \right) \\
\varepsilon_{\text{ln}} & = \ln \left( 1 + \varepsilon_{\text{eng}} \right) - \frac{\sigma_{\text{true}}}{E}
\end{align*}
\]

For modelling convenience and computational efficiency, the effect of the supports and the loading plates was introduced through appropriate boundary conditions and constraints, while only half of the cross-section was modelled, thus exploiting the symmetry with respect to the geometry, boundary conditions, applied load and failure mode of the test specimens. At each end, the degrees of freedom of all nodes were constrained to the degrees of freedom of a control-point node through rigid-body constraints, replicating the experimental conditions in which the ends of the specimens were welded to plates, thus preventing any deformation of the end cross-sections. In the initial validation against the experimental data,
the top and bottom control points were located in a plane perpendicular to the specimen axis and at a distance of 103 mm (equal to the thickness of the knife edges) from the end sections, while in the subsequent parametric studies the control points were located within the plane of the end sections of the stub columns. The load was applied incrementally as a prescribed displacement at the top control point. All other translational degrees of freedom were restrained at both control points, while all rotational degrees of freedom, except for those allowing flexure due to the eccentrically applied load, were also restrained. The eccentricity of the loading was introduced by offsetting the rigid-body control points from the centroid of the section along the symmetry axis. Appropriate symmetry boundary conditions were also applied.

Local geometric imperfections were introduced into the models in the form of the lowest elastic buckling mode shape, in line with previous studies (Gao et al., 2009; Gardner et al., 2011; Wang et al., 2016; Zhao et al., 2015b). In order to investigate the imperfection sensitivity of the models, five values of local imperfection amplitude were examined: 1, 2 and 10% of the section wall thickness, the maximum measured imperfection \( \omega_0 \) as given in Table 4 and an imperfection amplitude \( \omega_{DW} \) based on the predictive model developed by Dawson and Walker (Dawson and Walker, 1972; Gardner and Nethercot, 2004), as defined by Equation 6, where \( f_y \) is the yield strength of the plate material and \( \sigma_{el} \) is the elastic buckling stress of the most slender plate in the cross-section, which is a function of its width-to-thickness ratio.

6. \[ \omega_{DW} = \beta \left( \frac{f_y}{\sigma_{el}} \right)^{0.5} t \]

The coefficient \( \beta \) can be determined through regression analysis of measured imperfection data but, due to the limited available imperfection data for HSS sections, the value of \( \beta = 0.028 \), as proposed by Gardner et al. (2010) for normal-strength carbon steel hot-finished SHS and RHS, was adopted.

Owing to their very small magnitude (see Section 2.3), it was decided not to explicitly incorporate residual stresses into the numerical models. A non-linear static analysis, accounting for both material and geometric non-linearities, using the modified Riks procedure (Hibbitt, Karlsson & Sorensen, Inc., 2014) was performed in order to trace the full load–deformation response path of the modelled specimens.

3.2 Validation of the FE model

Utilising the modelling assumptions described above, the response of the tested specimens was simulated for the purposes of model validation. Typical comparisons of the test and FE load–end-rotation curves for S460 and S690 specimens are shown in Figures 7(a) and 7(b), respectively. As can be observed, the initial stiffness and the overall structural response were accurately captured. As anticipated, for the more stocky S460 section (Figure 7(a)), which failed without noticeable local deformation, variation in the initial local imperfection amplitude did not have significant influence on the observed response whereas, for the more slender S690 section (Figure 7(b)), which displayed clear evidence of local buckling, sensitivity to the local geometric imperfection amplitude was more pronounced. The failure modes were accurately captured in all cases, as indicated by the typical comparisons shown in Figures 8(a) and 8(b).

For all specimens, the ratios of numerical to experimental ultimate loads \( (N_{u,FE}/N_{u,exp}) \) and moments \( (M_{u,FE}/M_{u,exp}) \) for the different considered imperfection amplitudes are summarised in Table 6. It can be concluded that, overall, very good agreement between the experimental and numerical results was achieved, with the FE predictions being slightly on the conservative side in most cases. The best agreement was obtained...
when the measured imperfection amplitudes $\epsilon_0$ were employed in the FE models, with a mean value of $N_{u,FE}/N_{u,exp}$ equal to 0.92 and a mean value of $M_{u,FE}/M_{u,exp}$ equal to 0.98. However, very similar results were also achieved when an initial geometric imperfection amplitude of $t/50$ was employed; this imperfection amplitude was therefore adopted in the subsequent parametric study.

### 3.3 Parametric study

Upon successful validation of the FE models against the test results, an extensive parametric study was performed in order to generate data over a wide range of cross-section slendernesses and initial loading eccentricities corresponding to different ratios of axial load to bending moments. The average material properties of the TF coupon tests were incorporated in the models, whereas an initial local geometric imperfection amplitude of $t/50$, which gave the closest agreement with the test results, was used in all numerical models. Similar to the experiments, the length of the modelled stub columns was set to be three times the largest cross-sectional dimension, while the internal radius was set equal to half the cross-sectional thickness.

The loading eccentricities applied to the modelled stub columns were varied to generate a range of initial stress ratios $\psi$ over the cross-section depth from $-0.75$ to 1.00; the stress ratio $\psi$ was defined, as in EN 1993-1-5 (CEN, 2006), as the ratio of the stress on the most heavily compressed side of the cross-section to that on the least heavily compressed (or most tensioned) side, assuming elastic material behaviour, with $\psi = 1.00$ corresponding to pure compression and $\psi = -1.00$ corresponding to pure bending.

Three cross-section aspect ratios ($h/b$) of 1.00, 2.00 and 2.44, with varying thickness, were considered. The cross-section aspect ratio $h/b$ was set to be three times the largest cross-sectional dimension, while the internal radius was set equal to half the cross-sectional thickness.

### Table 6. Comparison of numerical and experimental results for the different considered imperfection amplitudes

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>$\epsilon_0$, mm</th>
<th>$\epsilon_{DW}$, mm</th>
<th>$t/100$</th>
<th>$t/50$</th>
<th>$t/10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S460 SHS 50 x 50 x 5</td>
<td>5</td>
<td>0.92</td>
<td>0.93</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>S460 SHS 50 x 50 x 5</td>
<td>10</td>
<td>0.91</td>
<td>0.93</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>S460 SHS 50 x 50 x 5</td>
<td>20</td>
<td>0.92</td>
<td>0.95</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>S460 SHS 50 x 50 x 5</td>
<td>30</td>
<td>0.87</td>
<td>0.86</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>5</td>
<td>0.94</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>10</td>
<td>0.89</td>
<td>0.97</td>
<td>0.89</td>
<td>0.97</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>15</td>
<td>0.93</td>
<td>1.01</td>
<td>0.93</td>
<td>1.01</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>20</td>
<td>0.94</td>
<td>0.84</td>
<td>0.94</td>
<td>0.84</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>25</td>
<td>0.96</td>
<td>1.09</td>
<td>0.96</td>
<td>1.12</td>
</tr>
<tr>
<td>S690 SHS 90 x 90 x 5 &amp; 6</td>
<td>30</td>
<td>0.94</td>
<td>1.21</td>
<td>0.94</td>
<td>1.23</td>
</tr>
<tr>
<td>Mean</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.03</td>
<td>0.10</td>
<td>0.03</td>
<td>0.11</td>
<td>0.03</td>
</tr>
</tbody>
</table>
slenderness was taken as the $c/t\varepsilon$ ratio of the most slender plate element in accordance with the current cross-section classification practice adopted in EN 1993-1-1 (CEN, 2014). The cross-section aspect ratio of 2.44 represents the case where the web and the flange of an RHS subjected to pure bending about the major axis, allowing for their respective stress distributions, are of the same non-dimensional plate slenderness $\bar{\lambda}_p$, as defined in EN 1993-1-5 (CEN, 2006) (Wang et al., 2016).

The cases of compression plus major axis bending and compression plus minor axis bending were considered in the parametric study. In total, 720 analyses of eccentrically loaded stub columns were performed using the validated FE models. Typical elastic buckling mode shapes and failure modes of the eccentrically loaded stub column FE models are depicted in Figures 9(a) and 9(b), respectively. The ultimate load-bearing capacity $N_u$ and the corresponding moment at mid-height accounting for second-order effects $M_u$ were determined for each analysis; the full moment–end-rotation responses for some typical cases are shown in Figures 10(a) and 10(b). The results of the experiments and the FE parametric study were analysed and used to assess European design provisions, as described in the next section.

4. **Analysis of results and design recommendations**

4.1 **Introduction**

Based on the obtained test and FE results, the Eurocode $N$–$M$ interaction curves for HSS SHS and RHS (EN 1993-1-1 (CEN, 2014)) are assessed in this section. The test and FE results are compared with the corresponding codified $N$–$M$ interaction curves in Figures 11–13 for class 1 and 2, class 3 and class 4 cross-sections, respectively. In the figures, the axial compressive force at failure and the second-order bending moment at failure are normalised by their respective resistances according to the cross-section class. Depending on the
cross-section properties and the applied loading conditions, each specimen was classified in accordance with table 5.2 of EN 1993-1-1 (CEN, 2014).

Comparisons between the test/FE results and the Eurocode design predictions are presented in Table 7 for all cross-sections. The assessment was based on the utilisation ratio of the test or FE to the predicted capacity ($R_{exp}/R_{pred}$ or $R_{FE}/R_{pred}$), which is graphically defined in Figure 14.

4.2 Assessment of the Eurocode interaction curve for class 1 and 2 cross-sections

The interaction curves for determining the resistance of class 1 and class 2 cross-sections under combined axial load and bending are provided in clause 6.2.9.1(5) in EN 1993-1-1 (CEN, 2014) and are presented in Equations 7 and 8 for major axis and minor axis bending, respectively.

7. $M_{N,M,Rd} = M_{pl,y,Rd}(1 - n)/(1 - 0.5\alpha_{w})$ but $M_{N,M,Rd} \leq M_{pl,y,Rd}$

8. $M_{N,z,Rd} = M_{pl,z,Rd}(1 - n)/(1 - 0.5\alpha_{f})$ but $M_{N,z,Rd} \leq M_{pl,z,Rd}$

where $n = N_{Ed}/N_{pl,Rd}$, $N_{Ed}$ is the design axial compressive load, $N_{pl,Rd}$ is the cross-section yield load ($Af_y$), $M_{N,Rd}$ is the reduced cross-section moment resistance to allow for the presence of axial load, $M_{pl,Rd}$ is the cross-section plastic moment capacity ($W_{pl,fy}$), $\alpha_w = (A - 2bt)/A$ but $\alpha_w \leq 0.5$ and $\alpha_f = (A - 2ht)/A$ but $\alpha_f \leq 0.5$. The subscripts $y$ and $z$ in Equations 7 and 8 denote the major and minor axis, respectively.
The codified $N$–$M$ curves are compared with the test and FE results obtained for classes 1 and 2 (i.e. those that can develop their full plastic moment capacity) SHS and RHS in compression plus major axis bending (Figure 11(a)) and RHS in compression plus minor axis bending (Figure 11(b)). The test and FE results can be seen to generally follow the trend of the Eurocode 3 interaction equation, although the predictions are very conservative in the case of the stocky cross-sections (low $c/t_\varepsilon$ ratios), particularly for the S460 steel. This conservatism stems principally from the neglect of strain hardening in the Eurocode interaction equations, and is therefore most pronounced for those cross-sections that are most resistant to local buckling (i.e. low local slenderness) and hence have high deformation capacity and for material that exhibits a high degree of strain hardening, which is more prominent in lower strength steel grades. It should be noted that at the high bending moment end of the interaction curves, some of the S690 tests and FE results fell marginally below $M_{pl}$. This was also observed by Wang et al. (2016) and, again, attributed principally to the lower degree of strain hardening that the higher grades of steel exhibit. For the S460 RHS specimens under compression and minor axis bending, there was an apparent change in the response of the stockier (i.e. lower $c/t_\varepsilon$) specimens at the higher axial load levels (see Figure 11(b)). In fact, the response of the specimens did not change significantly, but the value of the second-order moment at failure $M_u$ was sensitive to where the peak load arose on the rather flat load–lateral deflection curves. Overall, the graphical comparisons indicate that the existing interaction curves are generally applicable to HSS material, and similar conclusions were reached from the numerical comparisons presented in Table 7.

### 4.3 Assessment of the Eurocode interaction curve for class 3 cross-sections

The linear $N$–$M$ interaction expression for class 3 cross-sections specified in EN 1993-1-1 (CEN, 2014) is given by

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{x,Ed}}{M_{x,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

where $M_{x,Rd}$ and $M_{z,Rd}$ is the elastic moment capacity ($W_{e,el}$) of the cross-section and all other symbols are as previously defined.

The FE results for class 3 cross-sections are compared against the Eurocode 3 linear interaction $N$–$M$ equation in Figure 12. The interaction equation yielded generally safe predictions and without excessive conservatism ($R_{FE}/R_{pred} = 1.15$), but improved predictions and reduced scatter were achieved using the linear transition (see Figure 15) between $M_{el}$ and $M_{pl}$ for class 3 cross-sections ($R_{FE}/R_{pred} = 1.09$) proposed by

### Table 7. Assessment of Eurocode design predictions based on utilisation ratios of the test or FE to the predicted capacities

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Aspect ratio</th>
<th>Bending axis</th>
<th>Number of test or FE results</th>
<th>$R_{exp}/R_{pred}$ or $R_{FE}/R_{pred}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Classes 1 and 2</td>
<td></td>
<td>Class 3 (linear transition)</td>
<td></td>
</tr>
<tr>
<td>S460 SHS-test</td>
<td>1.00</td>
<td>N/A</td>
<td>4</td>
<td>1.29</td>
</tr>
<tr>
<td>S690 SHS-test</td>
<td>1.00</td>
<td>N/A</td>
<td>8</td>
<td>1.07</td>
</tr>
<tr>
<td>S460 SHS-FE</td>
<td>1.00</td>
<td>N/A</td>
<td>72</td>
<td>1.11</td>
</tr>
<tr>
<td>S690 SHS-FE</td>
<td>1.00</td>
<td>Major</td>
<td>72</td>
<td>1.05</td>
</tr>
<tr>
<td>S460 RHS-FE</td>
<td>2.00</td>
<td>Major</td>
<td>72</td>
<td>1.16</td>
</tr>
<tr>
<td>S690 RHS-FE</td>
<td>2.00</td>
<td>Minor</td>
<td>72</td>
<td>1.07</td>
</tr>
<tr>
<td>S460 RHS-FE</td>
<td>2.00</td>
<td>Minor</td>
<td>72</td>
<td>1.05</td>
</tr>
<tr>
<td>S460 RHS-FE</td>
<td>2.44</td>
<td>Major</td>
<td>72</td>
<td>1.02</td>
</tr>
<tr>
<td>S690 RHS-FE</td>
<td>2.44</td>
<td>Major</td>
<td>72</td>
<td>1.10</td>
</tr>
<tr>
<td>S460 RHS-FE</td>
<td>2.44</td>
<td>Minor</td>
<td>72</td>
<td>1.03</td>
</tr>
<tr>
<td>S690 RHS-FE</td>
<td>2.44</td>
<td>Minor</td>
<td>72</td>
<td>1.02</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
</tr>
</tbody>
</table>

Coefficient of variation is 0.09 0.07 0.05 0.04 (Class 1 and 2), 0.04 (Class 3 and 4).

Figure 14. Definition of utilisation ratio of the test or FE to predicted capacities.

![Diagram of the Eurocode interaction curve](image-url)
The FE results for class 4 cross-sections are compared against the Eurocode 3 linear interaction $N-M$ equation in Figure 13 – data points were normalised based on their respective effective section properties calculated according to EN 1993-1-5 (CEN, 2006). The results shown in Figure 13 closely follow the design predictions, indicating that both the effective section properties and interaction curve are appropriate for HSS.

5. Conclusions

A comprehensive study into the structural behaviour of hot-rolled HSS (S460 and S690) hollow sections under compression and uniaxial bending has been reported. After the execution of 12 tests on eccentrically loaded stub columns together with complementary measurements of geometric and material properties, an extensive numerical programme was conducted in order to generate additional data over a wide range of cross-section slendernesses and loading eccentricities, generating different proportions of axial compression and bending moment at failure. The results were utilised for an assessment of the design provisions specified in EN 1993-1-1 (CEN, 2014) for cross-sections under combined compression and uniaxial bending moment. The Eurocode interaction curve for class 1 and 2 sections generally provided safe predictions, but was found to be rather conservative for the stockier cross-sections and lower steel grade. The linear interaction curve for class 3 sections gave accurate, although again slightly conservative, design predictions, while the use of a linear transition between $M_d$ and $M_{pl}$, as proposed by Taras et al. (2013), reduced this conservatism. The effective width equations were shown to be generally applicable to S460 and S690 square and rectangular hollow sections subjected to compression and uniaxial bending. Overall, the design provisions of EN 1993-1-1 (CEN, 2014) are deemed suitable for HSS sections.

Acknowledgement

The research leading to these results received funding from the Research Fund for Coal and Steel (RFCS) under grant agreement RFCS CT 2012-00028.

REFERENCES


AISI (American Iron and Steel Institute) (2012) AISI S100: North American specification for the design of cold-formed steel structural members. AISI, Washington, DC, USA.


Downloaded by [University of Birmingham] on [12/10/17]. Copyright © ICE Publishing, all rights reserved.


How can you contribute?

To discuss this paper, please email up to 500 words to the editor at journals@ice.org.uk. Your contribution will be forwarded to the author(s) for a reply and, if considered appropriate by the editorial board, it will be published as discussion in a future issue of the journal.

Proceedings journals rely entirely on contributions from the civil engineering profession (and allied disciplines). Information about how to submit your paper online is available at www.icevirtuallibrary.com/page/authors, where you will also find detailed author guidelines.