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Alvarez Alvarado, Manuel; Jayaweera, Dilan

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Reliability Model for a Static Var Compensator

Manuel S. Alvarez-Alvarado  
Department of Electronics, Electrical and Systems Engineering  
University of Birmingham  
Birmingham, United Kingdom  
manuel.alvarez.alvarado@ieee.org

Dilan Jayaweera  
Department of Electronics, Electrical and Systems Engineering  
University of Birmingham  
Birmingham, United Kingdom  
d.jayaweera@bham.ac.uk

Abstract—This paper presents a reliability model of a Static Var Compensator (SVC) using an innovative algorithm based on sequential Monte Carlo simulation and Markov chains. The method employs the equivalent circuit of a SVC and takes the failure rate and repair time of each component as input in order to compute the failure rate and repair time of the whole SVC system. The specific contribution of this investigation is that it presents a mathematical pathway to model operating conditions of a SVC subject to individual operating states of its components, resulting in a comprehensive reliability model.

Keywords— failure rate; Markov Chain; Monte Carlo simulation; reliability model; repair time; static var compensator

I. INTRODUCTION

The Static Var Compensators (SVCs) are employed very often in power systems, due to its versatile and dynamic responses at the need of reactive power demand. Several studies related to its optimal operation and control have been proposed [1-3]. Nevertheless, its reliability is in fact not so far explored. Hence, there is a need to develop a model that characterizes its operational states.

The power systems possess different components and each of these are susceptible to failures. Some are more severe than others, for instance, lines have higher probabilities of failure than transformers as presented in the IEEE gold book [4]. This is defined by the component’s failure rate ($\lambda$) and repair time ($r$).

The determination of the $\lambda$ and $r$ values for a system is not an easy task, since they required periodic operational records. These operational data can be transformed into statistical analysis in order to estimate $\lambda$ and $r$ values. Reference [5] presents an approach to evaluate the failure rate of a power transformer based on inspections. Reference [6] propose a methodology to calculate wind dependent failure rates for overhead transmission lines using reanalysis data records and a Bayesian updating scheme. Reference [7] employs a grey linear regression model with recorded operational data in order determine and predict the value of the failure rate of substation equipment. In the published literature, there is no comprehensive information related to $\lambda$ and $r$ values of a SVC, however, there are operational records of its components.

This paper proposes a new sequential Monte Carlo (MC) simulation that in combination with the Markov Chain, allows estimating the operational and failure states of a SVC. The paper has been organized as follows. Section II presents an extensive literature review related to the failure rates and repair hours of each component of the main circuit of a SVC. In section III, the theory related to the reliability models is presented. Section IV introduces the Markov chain as a pathway to develop a reliability model. Section V presents an advanced algorithm for a MC simulation applied to repairable components. In Section VI, the theories manifested in section III, IV and V are applied to develop the SVC main circuit reliability model. Finally, Section VII brings the conclusion based on the obtained results and the applied approach.

II. OPERATIONAL RECORDS OF THE COMPONENTS OF A STATIC VAR COMPENSATOR

The SVC has several functions such as control gain change, phase angle regulator, voltage support, power factor correction, loss reduction and more [8]. A SVC primarily has three systems: 1) Main circuit. 2) Auxiliary power supply; 3) Control and protection. This research focuses on the Main circuit which consists of thyristor controlled (TCR) and thyristor switched branches (TSC/TSR) together with filter branches for harmonic current absorption, medium voltage switchgear and the step down transformer. Fig. 1 shows a schematic diagram of the main circuit of a SVC.

In a reliability context, the more elements involved in a system the less reliability may get. Hence, applying this criterion to the SVC main circuit due to the number of components involved, the contribution of forced outages may be high. Nevertheless, these have lower failure rates. Moreover, the auxiliary power supply and the control and protection system contributes to failures more than the SVC main circuit, as presented in [9].

With a view to increasing the reliability in the system, the design for an SVC is presented in [10]. Furthermore, in [11] the authors state that a value for failure and repair rate of a SVC. Nevertheless, the reliability is limited to the TCR and TSC avoiding other elements that composite the main circuit of the SVC such as the harmonic filters, medium voltage switchgear and the step down transformer. A detailed description of the SVC main circuit is given below.
A. Thyristor Controlled Reactor

The composition of a TCR is basically, reactors with thyristor valves connected in series, as shown in Fig. 1. The failure rate for a reactor depends on their operating voltage. For instance, [12] reports a failure rate (failure/year) of $2 \times 10^{-6}$, for low voltage level. The TCR can operate with a medium voltage and for the case of forced outages with a reactor of this feature, [13] reports 0.0344 failures per year and repair time of 627.8 hours. For the thyristor valves, the authors in [14] reported between 0.00283 and 0.07299 failures per year. On the other hand, a repair time of 6.10 hours was reported in [15].

B. Thyristor Switched Capacitor

Unlike the TCR, the TSC includes capacitor banks as shown in Fig. 1 the failure rate for capacitors as the reactors depends on their operating voltage. In [16] states that distribution capacitor bank (medium voltage) has 0.1744 failures per year and a repair time of 2.30 hours. However, the values reported in [4, 13] for a shunt capacitor bank that works up to 109 kV is 0.0037 failures per year and 251.2 repair hours.

C. Harmonics Filters

They are generally divided into two parallel banks in Y-Y connection with ungrounded neutrals tied together with internal fuses that protect the capacitor units and for the cooling system, it uses fans [17]. The data recorded in [12, 18] is 0.0438 failures per year and a repair time of 0.25 hours [18] for damages related to the capacitor and fans. There is no comprehensive information about the repair time for forced outages, nevertheless repair time can be varied from three to seven hours, based on the experiences reported by personal of PSEG [19].

D. Medium Voltage Switchgear

Their reliability depends on their location (indoor or outdoor); voltage level; and equipment sub-class, which can be insulated or bare. When switchgear is connected to a SVC, it is insulated due to the high voltage level of the switchgear and it is commonly placed indoor. IEEE gold book [4] and the Power Systems Reliability Subcommittee [20] reported that these switch gears have a 0.0017 failures per year and a 26.8 hours of repair time.

E. Step-down Transformer

From a data collection between 1960 and 1980, of 32 utilities from Germany, Austria, Swiss, France, United Kingdom, Spain, Denmark and Netherlands, it was reported that an average of 0.005 failure per years for transformer up to 500 kV [21] with a repair time between 173.2 and 308.9 hours [13, 22].

All the other components of a SVC follow an exponential distribution [4, 13]. However, the distribution of the transformer is different because it follows a two-parameter Weibull distribution [23]. Hence, the parameters needed to develop the reliability model are the shape parameter $\beta$ and the scale parameter $\eta$. The authors in [24] presents a study, which shows the distribution function and from there the values can be calculated using $\beta = 2.8076$ and $\eta = 55.14$ for sub-transmission transformers (63/20 kV). On the other hand, a detailed reliability model for power transformers was proposed in [25, 26] and reported the values of $\beta = 5$ and $50 \leq \eta \leq 80$. Reference [26] states that changing the value $\eta$ does not affect the shape of the function of the instantaneous failure rate versus age, hence $\beta$ is the predominant parameter in the distribution function.

III. RELIABILITY MODEL

The reliability model is based on operational records. This means, that when a failure occurs, all data related with it is recorded and then some analyses are done in order to get: 1. reliability index; 2. probabilistic models. Based on these, some preventive measure can be taken into account. Even a past behavior of a component can be gotten from a probabilistic model. In addition, a historical evaluation can be done with a reliability index. Hence, operational records allow performing reliability analysis with a view to finding the operational state for components in the past or future. An illustrative explanation is given in Fig. 2.

Most of the probabilistic models focus on determining the reliability, maintainability and availability function of a system. The reliability $R(t)$ is the probability of a system performing its intended function under stated conditions without failure for a given period of time. If the time to failure is defined by $T$, then the reliability can be mathematically expressed as [27]:

$$R(t) = P(t > T); \ t \geq 0$$

![Fig. 2 Reliability models based on time t.](image-url)
Using the probability density function \( f(t) \) (time to failure), the reliability can be written as [27]:

\[
R(t) = \int_0^t f(t) \, dt
\]

(2)

On the other hand, maintainability \( M(t) \) is the probability of performing a successful repair action within a given time \( t \). Mathematically can be expressed as follows:

\[
M(t) = P(0 < t < \tau)
\]

(3)

The maintainability in terms of the renewal density function \( g(t) \) (repair time) can be written as [27]:

\[
M(t) = \int_0^t g(t) \, dt
\]

(4)

The combination of high reliability and high maintainability lead to high system availability. The term availability is typically measured as a factor of reliability. The availability is very similar to the reliability function in that it gives a probability that a system will function at the given time, \( t \). Unlike reliability, however, the instantaneous availability measure incorporates maintainability information. At a given time, \( t \), the system will be operational if one of the following conditions is met [28]: 1. The system functioned properly from 0 to \( t \). This means the probability of the event happening is \( R(t) \). 2. The system was working properly since the last repair at time \( u \), such that \( 0 < u < \tau \). The probability of this condition is defined as \( \int_0^t R(t-u) \, g(u) \, du \). Consequently, the availability can be expressed as:

\[
A(t) = R(t) + \int_0^t R(t-u) \, g(u) \, du
\]

(5)

IV. MARKOV CHAIN

The reliability models of some components are not easy to deal with, since the mathematical models may be complex to solve. Nevertheless, a simple way to model is by applying Markov chain, which is a representation of all possible states in a diagram connected between them by variables called transition rates given by the failure rate \( \lambda(t) \) and the repair rate \( \mu(t) \) (the repair rate is defined as the inverse of the repair time \( r(t) \)). For instance, Fig. 3 shows a transition state of a repairable component with two possible states: operational and failure.

With a view to representing the model, consider a time interval \( \Delta t \), which is very small in such a way that the occurrence probability of more than one fault or repair is very small and therefore the occurrence of these events can be neglected. Then:

- Probability of a failure in time \( t = probability of a failure in time (t + \Delta t) = \lambda(t) \Delta t \)
- Probability of a repair in time \( t = probability of a repair in time (t + \Delta t) = \mu(t) \Delta t \)

The probability of being in the operational state after a time interval \( \Delta t \) is equal to the probability of being operative

Operating

\[
\lambda(t)
\]

\[
\mu(t)
\]

Failure

\[
\text{Fig. 3 Operational states.}
\]

At time \( t \) and not having failed in \( \Delta t \) plus the probability of being failed at time \( t \) and having been repaired in \( \Delta t \):

\[
P_1(t + \Delta t) = P_1(t)[1 - \lambda(t) \Delta t] + P_2[\mu(t) \Delta t]
\]

(6)

On the other hand, the probability of being in the repair state (failed) after a time interval \( \Delta t \) is equal to the probability of being failed in \( t \) and not having been repaired in \( \Delta t \) plus the probability of being non-failed in \( t \) and having failed in \( \Delta t \):

\[
P_2(t + \Delta t) = P_2(t)[1 - \mu(t) \Delta t] + P_1[\lambda(t) \Delta t]
\]

(7)

Solving (6):

\[
P_1(t + \Delta t) = P_1(t) - \lambda(t)P_1(t) \Delta t + \mu(t) \Delta t
\]

\[
P_1(t + \Delta t) - P_1(t)\]

\[
\frac{\Delta t}{\Delta t} = \lambda(t)P_1(t) + \mu(t)P_2(t)
\]

\[
\frac{dP_1(t)}{dt} = -\lambda(t)P_1(t) + \mu(t)P_2(t)
\]

(8)

Solving (7):

\[
P_2(t + \Delta t) = P_2(t) - \mu P_2(t) \Delta t + \lambda P_1(t) \Delta t
\]

\[
P_2(t + \Delta t) - P_2(t)\]

\[
\frac{\Delta t}{\Delta t} = \lambda(t)P_1(t) - \mu(t)P_2(t)
\]

\[
\frac{dP_2(t)}{dt} = \lambda(t)P_1(t) - \mu(t)P_2(t)
\]

(9)

Expressing (8) and (9) in matrix form:

\[
\begin{bmatrix}
\frac{dP_1(t)}{dt} \\
\frac{dP_2(t)}{dt}
\end{bmatrix} =
\begin{bmatrix}
-\lambda(t) & \mu(t) \\
\lambda(t) & -\mu(t)
\end{bmatrix}
\begin{bmatrix}
P_1(t) \\
P_2(t)
\end{bmatrix}
\]

(10)

where \( \dot{P}(t) \) is the time derivatives vector of the probabilities of each of the states, \( \dot{P}(t) \) the probabilities vector of each of the states and \( H \) the stochastic matrix of transition states. Then, (10) can be written as:

\[
P(t) = H \dot{P}(t)
\]

(11)

Applying Laplace transform:

\[
sP(s) - P(0) = H \dot{P}(s)
\]

(12)

\[
\dot{P}(s) = \frac{P(0)}{s - H}
\]

(13)

Applying inverse Laplace transform:

\[
\dot{P}(t) = \dot{P}(0) e^{\mu t}
\]

(14)
The solution for the system still being complicated due the exponential matrix involved. To simplify the solution, the Putzer’s spectral formula is applied [29], in which the term $e^{-HT}$ can be expressed as a function of the eigenvalues $\nu_i$ and eigenvectors $\vec{v}_i$ of the stochastic matrix of transition states $H$, as follows:

$$e^{HT} = \sum_{i=1}^{n} \vec{v}_i e^{\nu_i t}$$

(15)

Replacing (15) in (14) and knowing that the $P(0)$ will bring a constant $C_i$ for each term of the sum, the general solution for the Markov chain is given by:

$$P(t) = \sum_{i=1}^{n} C_i \vec{v}_i e^{\nu_i t}$$

(16)

Finally, the availability of the system can be calculated as the probability of all states that are in the set of operational states of the system defined in $\varphi$.

$$A(t) = \sum_{k=0}^{k=\varphi} P_k(t)$$

(17)

V. MONTE CARLO SIMULATION

Monte Carlo method is a broad class of a computational algorithm that relies on repeated random sampling to obtain numerical results [30, 31]. The method allows to: (1) obtain a solution of complicated or impossible mathematical models; (2) develop experiments that are not possible to do directly due time involved, which can be very long; (3) get observations (data) of a random variable or process.

Sometimes it is not possible to get the reliability function of a system by employing analytical methods. This is due to the mathematical complexity involved as presented in section III. However, by employing MC simulation, the solution can be gotten.

In order to estimate the failure and repair rate of a system, an improved MC simulation architecture is employed. It uses the reliability parameters of each component as input data. The algorithm is divided into two parts, one to get the reliability function and the other to obtain the maintainability function of the system. The random number generation is done based on the failure rate and repair rate of each independent component. When all components are operating, the reliability is considered to be one, otherwise is zero. For the case of maintainability, it is considered as a success only if the generated number (time to repair) of all components, is less than the maximum time for restoration. Then, the values are saved and the experiment is repeated several times for each time slot defined. Finally, the mean value of the reliability and maintainability for each hour is gotten. For more details about the process, Fig. 4 presents the complete algorithm.

The developed algorithm has the pathways to scale down a complex problem to a manageble level with the aim of reducing the processing time and mathematical burden in comparison with the conventional MC simulation.

VI. STATIC VAR COMPENSATOR RELIABILITY MODEL

The SVC that is used to incorporate in this study has the features shown in TABLE I.

<table>
<thead>
<tr>
<th>Component</th>
<th>$\lambda$ [failure per year]</th>
<th>$r$ [repair hours]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor air core</td>
<td>0.0344</td>
<td>628.1</td>
</tr>
<tr>
<td>Thyristor valve</td>
<td>0.0050</td>
<td>6.10</td>
</tr>
<tr>
<td>Capacitor bank</td>
<td>0.0037</td>
<td>251.2</td>
</tr>
<tr>
<td>Harmonic filters</td>
<td>0.0438</td>
<td>7.00</td>
</tr>
<tr>
<td>Switchgear</td>
<td>0.0017</td>
<td>26.8</td>
</tr>
<tr>
<td>Step down transformer</td>
<td>0.0050</td>
<td>200</td>
</tr>
<tr>
<td>Step down transformer</td>
<td>0.0344</td>
<td>5</td>
</tr>
</tbody>
</table>

TABLE I. DATA FOR SVC RELIABILITY ASSESSMENT

Fig. 4 Advanced Monte Carlo simulation algorithm for reparable components
A. SVC Reliability parameters

There is no data recorded about the $\lambda$ and $\mu$ values for the TCR and TSC, hence they are to be estimated. MC simulation is applied by following the algorithm shown in Fig. 4. The input data for the simulation are the recorded data of the capacitor, reactor and thyristor valve of TABLE I. The results are shown in TABLE II.

<table>
<thead>
<tr>
<th>TABLE II. TCR AND TSC RELIABILITY PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [failure per year]</td>
</tr>
<tr>
<td>0.0599</td>
</tr>
<tr>
<td>$r$ [hours repair]</td>
</tr>
</tbody>
</table>

Now, combining all components of the SVC main circuit and employing again the developed MC algorithm, the reliability parameters of the SVC are gotten. This is presented in TABLE III.

<table>
<thead>
<tr>
<th>TABLE III. SVC RELIABILITY PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [failure per year]</td>
</tr>
<tr>
<td>0.0906</td>
</tr>
</tbody>
</table>

Finally, the algorithm allows describing the reliability of the SVC as a function of time. This is shown in Fig. 5.

B. SVC Operational States

The results in Fig. 5 reveal that the SVC reliability model follows an exponential distribution function, then the failure rate ($\lambda$) and repair rate ($\mu$) becomes time-independent variables.

Now, the stochastic matrix of transition states is as follows:

$$H = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix}$$

(18)

Later, the eigenvalues and eigenvectors are as follows respectively:

$$\psi_1 = 0; \psi_2 = -\lambda - \mu$$

(19)

$$\omega_1 = \begin{pmatrix} \mu/\lambda \\ 1 \end{pmatrix}; \omega_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix};$$

(20)

Knowing that at $t = 0$ the component is in operational state ($P_1|t=0 = 1; P_2|t=0 = 0$), then (16) can written as:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = C_1 \begin{pmatrix} \mu/\lambda \\ 1 \end{pmatrix} e^{0 \lambda} + C_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{0 \mu}$$

(21)

Solving for $C_1$ and $C_2$:

$$C_1 = \frac{\lambda}{\mu + \lambda}; C_2 = -\frac{\lambda}{\mu + \lambda}$$

(22)

Finally, the solution of a Markov chain for a repairable component that follows an exponential distribution function with two operational states is:

$$P_1(t) = \frac{\mu}{\mu + \lambda} e^{-(\lambda + \mu) t}$$

(23)

$$P_2(t) = \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu) t}$$

(24)

Replacing $\lambda$ and $\mu$ values given in TABLE III, the probabilities of operational and failure state for a SVC is respectively:

$$P_1(t) = 0.9817 + 0.0183 e^{-4.9096 t}$$

(25)

$$P_2(t) = 0.0183 - 0.0183 e^{-4.9096 t}$$

(26)

The state “1” defines the availability of the SVC, while the state “2” defines its unavailability.

VII. CONCLUSIONS

This paper proposes a systematic methodology for modelling and quantification of the reliability for a SVC. The advanced MC simulation proposed in this paper allows determining the reliability parameters of a SVC system based on the operational records of components that are integrated into the SVC.

A general solution for Markov chains is presented and employed to describe the availability and unavailability of a SVC. The methodology can also be extended to the other FACTS devices.

The proposed approach presents a technical pathway for assessing the reliability performance of a SVC integrated in a power grid.

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REFERENCES


