End-of-life Failure Modeling of Overhead Lines Considering Loading and Weather Effects

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Abstract—Aging failure modeling is a fundamental requirement in reliability assessment of any actual power system. The level of model detail can potentially reflect realistic means of reliability indices. In that context, this paper proposes an advanced end-of-life failure model for overhead lines (OHLs), incorporating the effects of loading and weather conditions into the aging failure probability calculation. The Arrhenius life-temperature relationship is used to model the lifetime of OHLs as a function of conductor temperature. The life measure of the Arrhenius model is adopted as the scale parameter of the Weibull probability distribution. The approach to estimate parameters of the resulting Arrhenius-Weibull distribution is described in detail. Unavailability calculations are performed using the proposed aging failure model for a distribution test system. The results show that if the maximum continuous operating temperature is exceeded, the unavailability of OHLs increases depending on both time period, where the conductor temperature is higher than the threshold, and conductor age.

Index Terms—Aging failure, Arrhenius model, overhead lines, thermal stress, Weibull distribution.

I. INTRODUCTION

EQUIPMENT aging is one of the main problems that many utilities are currently facing. There are several external factors responsible for accelerating the aging process of power system components. Corrosion, fatigue, and overheating are key factors that deteriorate physical and electrical properties of overhead conductors [1]. Irreversible degradation of oil/cellulose insulation of power transformers is caused by high temperatures, moisture, and oxygen [2]. Likewise, thermal stress and electrical stress affect cable insulation properties. The main consequence of equipment aging is to increase failure probabilities, and consequently to impute higher system risk [1].

Aging failure modeling plays a fundamental role in reliability assessment of power systems that have aged equipment [3]. Indeed, many utilities look at aging failures in their planning studies due to many assets are approaching their end-of-life stage [4], as well as equipment replacement costs can be very high. Wenyan Li introduced the first method to incorporate aging failures into power system reliability evaluation [5]. The method is based on probabilistic distribution functions, such as Normal and Weibull distributions that are generally derived from statistical data. That method allows estimating the unavailability of components for reliability studies that use the Non-sequential Monte Carlo simulation technique. Since then, some investigations have been carried out using this end-of-life failure model. For instance, a probabilistic approach to estimate the number of spare transformers and time requirement for each spare transformer is presented in [6]. To model aging failures, the authors of [6] calculated the Weibull distribution using historical statistic records.

In a recent study [7], improvements of the aging failure modeling were proposed through incorporating condition monitoring information and aging condition measurements of OHLs and power transformers, respectively. In that study, the author considered OHLs as non-aged equipment, and consequently the effects of high operating temperatures on the calculation of aging failure probabilities were not investigated.

OHLs can be exposed to thermal limit violations due to facts including increased penetration of renewable power generation and adverse weather conditions [8]. Nowadays, some utilities have increased the amount of power that their transmission lines can transfer by raising the maximum threshold operating temperature limit of conductors due to the improved knowledge of real-time operating and influential factors [9]. Although high temperatures at conductors exist in short time periods, the frequency of such events can potentially accelerate the aging process of conductors [9]. Certainly, exceeding the maximum continuous operating temperature leads to annealing and potentially results loss of strength in the aluminum wires of electrical conductors [1].

An innovative aging failure model for power transformers and cables that incorporates the effects of loading conditions into the reliability assessment is presented in [10] and [11]. Both studies combine the thermal models of power transformers and cables, the Arrhenius life-temperature relationship, and the Weibull distribution. Reliability assessments using the innovative reliability model along with the Monte Carlo simulation method were performed.

This paper proposes an advanced end-of-life failure model for OHLs that incorporates the effects of loading and weather conditions into the aging failure probability calculation. Since loading and weather conditions have a great influence on conductor temperature, the Arrhenius relationship is used to
model the lifetime of OHLs as a function of temperature. Then, the lifetime is used as the scale parameter of the Weibull probability distribution. The parameters of the resulting Arrhenius-Weibull distribution are estimated based on the procedure presented in [12] and lifetime information of Aluminum Conductor Steel-Reinforced (ACSR) conductors.

The remaining parts of the paper are organized as follows. Section II presents the proposed end-of-life failure model. Section III describes the procedure used to estimate the parameters of the Arrhenius-Weibull distribution. In Section IV, unavailability calculations of a test system are analyzed in detail. Finally, the conclusions are given in Section V.

II. END-OF-LIFE FAILURE MODEL OF OVERHEAD LINES

A. End-of-life Failure Modeling

Power system components undergo end-of-life failures that are characterized by being nonrepairable failures and randomly happen within the wear-out stage on the life basin curve [3], as depicted in Fig. 1. According to [3], the unavailability of a component due to end-of-life failure is the probability that the component fails during a time period \( t \), given that it has kept in service for \( T \) years. The unavailability due to aging failure (\( U_a \)) in a subsequent period \( t \) is calculated using (1) and (2) [5].

\[
U_a = \sum_{i=1}^{N} P_i \cdot \frac{U D_i}{t} \tag{1}
\]

\[
U D_i = t - (2i - 1) \cdot \frac{\Delta x}{2} \tag{2}
\]

where \( P_i \) is the failure probability, \( U D_i \) is the average unavailable duration, and \( N \) is the total number of equal intervals with length \( \Delta x \). Equation (3) is used to determine the value of \( P_i \).

\[
P_i = \frac{\int_{T}^{T+i\Delta x} f(t) dt - \int_{T}^{T+(i-1)\Delta x} f(t) dt}{\int_{0}^{\infty} f(t) dt} \tag{3}
\]

where \( f(t) \) is a failure density probability function.

Normal and Weibull distributions have been used to model end-of-life failures because the failure rate calculated from these distributions increases over time as in the wear-out stage of Fig. 1 [3]. If the normal distribution is used to model aging failures, an approximation of \( P_i \) is given by (4) [5].

\[
P_i = \frac{Q \left( \frac{T + (i - 1)\Delta x - \mu}{\sigma} \right) - Q \left( \frac{T + i\Delta x - \mu}{\sigma} \right)}{Q \left( \frac{T - \mu}{\sigma} \right)} \tag{4}
\]

where \( \mu \) and \( \sigma \) are the mean and standard deviation of the normal distribution, and \( Q \) is obtained as follows:

\[
Q(y) = \begin{cases} 
    w(y) & \text{if } y \geq 0 \\
    1 - w(-y) & \text{if } y < 0
\end{cases}
\]

\[
w(y) = z(y)(b_1s + b_2s^2 + b_3s^3 + b_4s^4 + b_5s^5)
\]

\[
z(y) = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{y^2}{2}\right) \quad s = \frac{1}{1 + ry}
\]

\[
r = 0.2316419 \quad b_1 = 0.31938153 \quad b_2 = -0.356563782
\]

\[
b_3 = 1.78147793 \quad b_4 = -1.82125597 \quad b_5 = 1.3302744
\]

Otherwise, if the Weibull distribution is used to model aging failures, \( P_i \) is given by (5) [5].

\[
P_i = \frac{\exp \left[-\frac{T + (i - 1)\Delta x}{\alpha}\right] - \exp \left[-\frac{T + i\Delta x}{\alpha}\right]}{\exp \left[-\frac{T}{\alpha}\right]} \tag{5}
\]

where \( \alpha \) and \( \beta \) are the scale (also known as characteristic life) and shape parameters of the Weibull distribution, respectively.

B. Arrhenius Life-temperature Relationship

The Arrhenius relationship has been widely used to model product life as a function of temperature [13]. Lifetime estimations of winding insulation, Aluminum-Zirconium alloy that is used for the outer strands in Aluminum Conductor Composite Reinforced (ACCR) conductors, and electrical connections have been carried out through constant-stress laboratory tests [14]- [15]. The Arrhenius life-temperature relationship is given by (6).

\[
L(\theta) = A \cdot \exp \left(\frac{B}{\theta}\right) \tag{6}
\]

where \( L \) is a quantifiable life measure, \( \theta \) is the temperature in Kelvin, and \( A \) and \( B \) are empirical constants [10].

The operating temperature of bare overhead conductors depends on electrical current and weather conditions, such as wind speed, wind direction, ambient temperature, and solar radiation [16]. Industrial standards, e.g., IEEE 738-2006, CIGRE working group 22.12, and IEC TR 61597, allow calculating the thermal rating and temperature of bare overhead conductors [17]- [18]. In this study, the standard IEEE 738-2006 has been used to estimate the steady-state conductor temperature \( (T_c) \) through an iterative process of (7).

\[
q_c + q_r = q_s + I^2 \cdot R(T_c) \tag{7}
\]

where \( q_c \) and \( q_r \) are the heat losses due to convection and radiation, \( q_s \) is the solar heat gain, \( I \) is the conductor current, and \( R \) is the conductor resistance.

C. End-of-life Failure Model Based on the Arrhenius-Weibull Distribution

The proposed end-of-life failure model combines a Weibull life distribution with an Arrhenius dependence of life on
temperature [13]. Aging failures of power transformers and cables have been analyzed using this unconventional reliability model in [10] and [11]. The Weibull distribution is used for the Arrhenius relationship since it only has one life measure (\( \alpha \)) [12].

The Weibull cumulative distribution function \( (F) \) is given by (8). The parameter \( \alpha \) represents the age at which 63.2% of the population will fail [12]. Therefore, to introduce the effects of loading and weather conditions that OHLs experience into the aging failure probability calculation, the parameter \( \alpha \) in (8) is substituted by \( L(\theta) \) from the Arrhenius relationship given by (6) [10], [11]. Equation (9) represents the resulting Arrhenius-Weibull cumulative distribution function.

\[
F(t) = 1 - \exp \left( - \left( \frac{t}{\alpha} \right)^\beta \right) \\
F(t) = 1 - \exp \left( - \left( \frac{t}{A \exp (\frac{E}{T})} \right)^\beta \right)
\]

III. ESTIMATION OF THE ARRHENIUS–WEBULL DISTRIBUTION PARAMETERS

The parameters of the Arrhenius–Weibull distribution are \( A, B, \) and \( \beta \). Accelerated life tests with constant stress are usually performed to find the values of \( A \) and \( B \) [13]. Meanwhile, the parameter \( \beta \) may be obtained from statistic data [5]. A method for determining the Weibull distribution parameters \( (\alpha, \beta) \) of a power system component with limited aging failure data is presented in [19]. This method has been applied for components, such as reactors, power transformers, and underground cables, and it was used in this study for the case of OHLs.

First, it requires to estimate the parameters \( \mu \) and \( \sigma \) of a normal distribution that characterizes the lifetime of OHLs. It was necessary to make some assumptions because historical aging failure information of OHLs is not available in the open literature. The data on in-service and retired years of the 500-kV reactor group provided in [19] was used for the estimation of \( \mu \) and \( \sigma \). Considering that the mean life of OHLs with ACSR conductors is approximately 54 years [20], the chosen value of the reference year, needed for the estimation method, is 2010. It should be noticed that there are several factors influencing the lifetime of OHLs, e.g., climate, environment, corrosion, loading, etc. TABLE I shows the data used for the estimation of \( \mu \) and \( \sigma \). After applying the least square method, the calculated values are \( \mu = 44.06551 \) years and \( \sigma = 3.95922 \) years.

The optimal parameters of the Weibull distribution were obtained using the values of \( \mu \) and \( \sigma \), data specified in TABLE II, and the gradient descent technique [19]. The resulting scale and shape parameters are 46.9324 and 15, respectively. Fig. 2 illustrates the normal and Weibull density functions obtained using these estimated parameters. One may notice that the Weibull density curve is equivalent to the normal density curve since the scale and shape parameters were calculated from the mean and standard deviation.

The unavailability of OHLs due to aging failure considering both normal and Weibull distributions was calculated using (1), (2), (4), and (5), within an age range from 0 to 40 years. Fig. 3 shows that the unavailability curves increase over time because of the age increment, and they follow the same behavior of the normal and Weibull density functions of Fig. 2.

Two methods for estimating the parameters of the Arrhenius relationship \( A \) and \( B \) are described in [10] and [11]. Both methods use historic failure data, loading data, and lifetime information of power transformers and cables. The method proposed in [10] was used in this study, and it was modified for the case of OHLs with ACSR conductors.

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**TABLE I**

<table>
<thead>
<tr>
<th>Age</th>
<th>Cumulative Failure Probability</th>
<th>( z )</th>
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<tbody>
<tr>
<td>33</td>
<td>0.00100</td>
<td>-3.10</td>
</tr>
<tr>
<td>34</td>
<td>0.01767</td>
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<td>0.09879</td>
<td>-1.29</td>
</tr>
<tr>
<td>41</td>
<td>0.09879</td>
<td>-1.29</td>
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</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Age</th>
<th>Survival Probability</th>
</tr>
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<tbody>
<tr>
<td>33</td>
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</tr>
<tr>
<td>34</td>
<td>0.98333</td>
</tr>
<tr>
<td>36</td>
<td>0.95702</td>
</tr>
<tr>
<td>37</td>
<td>0.92999</td>
</tr>
<tr>
<td>40</td>
<td>0.90221</td>
</tr>
<tr>
<td>41</td>
<td>0.90221</td>
</tr>
</tbody>
</table>

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![Fig. 2. Normal and Weibull density functions of overhead lines](image1)

![Fig. 3. Unavailability due to end-of-life failure for overhead lines using normal and Weibull distribution parameters](image2)
The shape parameter of the Arrhenius-Weibull distribution is considered the same as the one of the Weibull distribution, i.e., $\beta = 15$. The estimations of $A$ and $B$ are based on two assumptions respect to $\alpha$ and the solution of (6). The first assumption is that an average conductor temperature ($T_{ca}$) can yield an $\alpha = 46.9324$ years. To find the value of $T_{ca}$, it is necessary to define a test system, create a multistep load model, perform load flow simulations, and apply the thermal model of OHLs.

The 33-bus, 11-kV radial distribution system specified in [21] is chosen as the test system. It consists of 32 overhead line sections and 32 load buses. The maximum demand of the network is 3.715 MW and 2.3 Mvar [21], and it is assumed that all sections have a length of 0.7 km. A six-step load model for the test system was calculated through the cluster technique [22] and considering the annual load profile specified in [23]. The load model results are shown in TABLE III.

Then, an annual equivalent current ($I_{eq}$) was obtained for each line section using (10) and performing load flow simulations.

\[
I_{eq} = \frac{\sum_{n=1}^{N} I_n \cdot L_n}{\sum_{n=1}^{N} L_n}
\]

(10)

where $I_n$ is the line section current, $L_n$ is the time length of each step, and $N$ is the total number of steps.

Now, it is possible to calculate an equivalent yearly conductor temperature ($T_{ca}$) based on the results of $I_{eq}$ and average weather conditions. Average values of wind speed, wind direction, and ambient temperature for 2010 were 2 (m/s), 45°, and 10 °C, respectively [24], [25]. Finally, the average conductor temperature of the test system is found as $T_{ca} = 15 °C$, which was then used as the conductor temperature corresponding to $\alpha = 46.9324$ years.

According to [26], the maximum normal operating temperature of ACSR conductors is approximately 90 °C. Consequently, the calculated value of $T_{ca}$ should yield a characteristic life longer than 46.9324 years because aluminum wires do not experiment annealing at low temperatures. Indeed, the radial configuration and load disposition of the test system as well as the average weather conditions are responsible for the low average conductor temperature.

Line sections located at the end of a branch, e.g., L17-18 or L32-33, carry lower currents than line sections located at the beginning of the same branch, e.g., L6-7 or L6-26. Otherwise, some branches of the test system have fewer load buses, e.g., B3-25 and B2-22, and consequently their line sections have lower loading levels. Average weather conditions also play an important role on the average conductor temperature because they affect heat losses due to convection and radiation [17]. Fig. 4 shows the influence of ambient temperature on the steady-state conductor temperature. The ratio between the variations of the conductor temperature and ambient temperature is approximately 1, as given by (11). It can also be seen that the conductor temperature keeps almost constant when current levels are less than 20 (A).

\[
\frac{\Delta T_C}{\Delta T_a} \approx 1
\]

(11)

The second assumption for the parameter estimation is based on the lifetime information of ACSR conductors presented in [27]. The authors of that report indicate that aluminum wires begin to anneal slowly at around 93 °C. They also mention that at 100 °C, 125 °C, and 150 °C aluminum wires lose 10% of their ultimate tensile strength in 1 year, 2 weeks, 12 hours, respectively. It should be noticed that these temperature levels do not deteriorate steel wire strength [27]. Thus, it is assumed that $T_c = 110 °C$ corresponds to $\alpha = 20$ weeks.

![Fig. 5. Characteristic life of overhead lines as a function of temperature (°C) (a) $A = 1.8 \times 10^{-7}$, $B = 5.58 \times 10^3$ and (b) $A = 3.48 \times 10^{-42}$, $B = 3.62 \times 10^4$](image-url)
Solving (6), the values of $A$ and $B$ are $1.8 \times 10^{-7}$ and $5.58 \times 10^{5}$, respectively. Fig. 5 (a) illustrates the relationship between $\alpha$ and $T_c$ based on the estimated parameters $A$ and $B$. This curve indicates that the characteristic life of OHLs decreases when the conductor temperature increases. Even though the behavior of the characteristic life curve is correct, its values are not consistent with the fact that lifetime of ACSR conductors decreases at temperatures above 93 °C [28]. Therefore, it is necessary to improve the estimation of $A$ and $B$.

Quantifiable life $L(\theta)$ has been used in [10] and [29] to model transformer and cable insulation life as a function of the hot-spot temperature (HST) and conductor temperature ($T_c$), respectively. In both studies, the authors assumed that power transformers and cables may live 40 years operating with constant temperatures $HST = 80$ °C and $T_c = 90$ °C, respectively. An accelerated thermal aging study of ACCR conductors using the Arrhenius model is described in [14]. Its experimental results indicate that ACCR conductors can live approximately 787 years operating at the maximum (continuous) temperature rating of 210 °C. For temperatures lower than 210 °C, the estimated life is much longer [14]. Based on the experiment results, it is assumed that at $T_c = 90$ °C, $\alpha$ equals 70 years.

The new values of the parameters $A$ and $B$ are $3.48 \times 10^{-42}$ and $3.62 \times 10^{4}$, respectively. Fig. 5 (b) shows the characteristic life curve based on these new parameters. This curve establishes that at temperatures above 90 °C the characteristic life of OHLs decreases due to the annealing of the aluminum wires. For temperatures lower than 90 °C, the characteristic life is very long, i.e., aluminum wires do not experiment damage. Through this curve, it is possible to obtain that at $T_c = 91.5$ °C, $\alpha$ equals 46,9324 years, which is a better approximation than the one obtained with $T_{c,a} = 15$ °C.

IV. UNAVAILABILITY ESTIMATION USING THE END-OF-LIFE FAILURE MODEL BASED ON THE ARRHENIUS-WEIBULL DISTRIBUTION

The end-of-life failure model based on the Arrhenius-Weibull distribution establishes that the unavailability of OHLs depends not only on conductor age but also loading and weather conditions, i.e., conductor temperature. Fig. 6 shows the influence of constant conductor temperature on unavailability within an age range from 0 to 45 years. If the conductor temperature is less or equal than 90 °C, the unavailability values are approximately 0 during the entire period. However, when the conductor temperature is greater than 90 °C, the failure probability starts increasing. Certainly, higher continuous temperatures increase failure probabilities due to the faster degradation of aluminum wires.

A. Influence of Age, Loading and Weather Conditions on Unavailability due to End-of-life Failure

The influence of age, loading, and weather conditions on the unavailability is analyzed with the end-of-life failure model based on the Arrhenius-Weibull distribution. The information provided by the six-step load model, the test system, and assumed weather conditions are used to calculate the conductor temperature for each load level of the line section located between the buses 1 and 2 (L1_2), as shown in TABLE IV.

The unavailability calculation considers two different ages (20 and 35 years) of the line section L1_2. TABLE V shows the obtained failure probabilities for both ages. The influence of the ambient temperature can be appreciated at load levels 1 and 2 by noticing that two different currents yield the same conductor temperature (93 °C). However, the unavailability values are not the same at age of 20 years because load levels 1 and 2 have different time lengths, as specified in TABLE III. The influence of the conductor age can be observed by comparing, for each load level, the unavailability values at 20 and 35 years. For the load levels 5 and 6, age does not affect the unavailability since the conductor temperatures are less than 90 °C.

Otherwise, the unavailability values of load levels 3 and 4 are quite different because of the wind direction reduction from 22° to 5°. In this case, the conductor temperature (95 °C) yields a high aging failure probability (1.0) due to the fact that the effect of high temperatures on the unavailability is more critical for aged conductors.

### TABLE IV

<table>
<thead>
<tr>
<th>Description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current (A)</td>
<td>217</td>
<td>193</td>
<td>168</td>
<td>147</td>
<td>123</td>
<td>101</td>
</tr>
<tr>
<td>Wind speed (m/s)</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>Wind direction (°)</td>
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<td>45</td>
<td>22</td>
<td>5</td>
<td>45</td>
<td>0</td>
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<tr>
<td>Ambient temperature (°C)</td>
<td>30</td>
<td>42.5</td>
<td>42.5</td>
<td>42.5</td>
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<td>40</td>
</tr>
<tr>
<td>Conductor temperature (°C)</td>
<td>93</td>
<td>93</td>
<td>91</td>
<td>95</td>
<td>62</td>
<td>72</td>
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### TABLE V

<table>
<thead>
<tr>
<th>Age</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>20</td>
<td>$4.9 \times 10^{-5}$</td>
<td>$8.5 \times 10^{-5}$</td>
<td>$3.2 \times 10^{-8}$</td>
<td>0.116</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>0.115</td>
<td>0.184</td>
<td>7.9 \times 10^{-5}</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

V. CONCLUSION

The paper proposes an advanced end-of-life failure model for overhead lines based on the Arrhenius-Weibull distribution, that incorporates the effects of loading and weather conditions.
into the failure probability calculation. It is demonstrated that loading and weather conditions can have a great influence on the aging failure probability of overhead lines if the conductor temperature is higher than the maximum normal operating temperature. The application of the proposed reliability model can improve both reliability assessment of power systems and management of overhead conductors since thermal stress effects originated during normal operation are taken into account.

VI. REFERENCES