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## Distributed admission control algorithm for random access wireless networks in the presence of hidden terminals

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**Abstract:** We address the problem of admission control for wireless clients in WLANs taking into account collisions between competing access points and considering explicitly the effect of hidden terminals, which play a prominent role in optimised client association. We propose an efficient, distributed admission control algorithm, where the wireless client node decides locally on which access point it will associate with in order to maximise its link throughput. The client can choose to optimise either its uplink or downlink throughput, depending on the type of traffic it predominantly intends to exchange with the network. The proposed approach takes into account the full contention resolution of the RTS/CTS IEEE802.11 medium access control protocol and leads towards an increase of the total throughput for the whole network. Finally, an algorithm is proposed, which can serve also as the basis for the development of efficient traffic offloading protocols in heterogeneous 5G networks.

**Keywords:** admission control, IEEE802.11, multi-cell, WiFi offloading.

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## 1 Introduction

Future mobile internet access will be based on 5G public mobile networks, which are an overarching set of technologies that will enable faster speeds from tens of Mbps (outdoors) up to 1 Gbps (indoors), improved coverage, enhanced signaling and spectral efficiency, and support for thousands of simultaneous connections to satisfy the communication requirements for multimedia applications and the Internet of Things (Panjanathan and Ramachandran, 2017). 5G will rely on new technologies (e.g., mesh networking) and will also have to incorporate existing network technologies such as WiFi, in order to keep infrastructure costs low. However, the restricted number of orthogonal WiFi channels available is known to lead to severe capacity limitations for dense deployments of multi-cell WLANs (Soldati and Koudouridis, 2015; Murty et al., 2008; Ergin et al., 2008). These limitations are due to partially overlapping cells and co-channel interference problems prevalent in uncoordinated or unplanned dense deployments. 4G cellular communications provide an expensive solution to meet the projected increase in traffic capacity through the adoption of ever smaller cell sizes. The 5G communication strategy recognises that such an approach is questionable as it will require a significant infrastructure investment, in addition to the considerable cost of deployment in expensive, licensed spectrum. The pragmatic solution incorporated in 5G is to adopt traffic offloading to WiFi hotspots (Singh et al., 2013; Dhillon et al., 2012). Thus multi-cell WLANs will play a prominent role in managing the explosive increase of mobile data traffic in the foreseeable future. However, many technical issues related to the design and operation of multi-cell WLAN networks (e.g. the impact of interference between access points on the system capacity, operations and planning) remain to be resolved fully. One of the key problems in developing such multi-cell WLAN networks is optimising the admission control of a client to the network, taking into account contention resolution and client throughput demand (Kumar and Kumar, 2005). It is desirable that such client association takes place in a distributed way and is controlled to yield optimised link throughput for the client and a stable and fair network resource allocation amongst all clients.

## 2 WiFi admission control: state-of-the-art

The IEEE802.11 association procedure for a wireless client to a network is based on a received signal strength indicator (RSSI) client measurement of broadcast frames by potential access points (AP). The client associates with the AP which provides the strongest RSSI. The disadvantage of this RSSI-based association scheme is that no information is provided to the client about the current traffic load of the AP or the contention environment the client will face (i.e., the level of medium access contention by the competing covered and hidden terminal node population). It is well known that this association policy can lead to inefficient use of network

resources (Xu et al., 2011; Arbaugh et al., 2003; Bejerano et al., 2007), because the RSSI level is only loosely correlated to achievable throughput. Moreover, it is worth remarking that in the standard implementation the RSSI is an indicator for the downlink, but not for the uplink channel conditions, since it is broadcasted by the APs. In general, taking into account the asymmetry of the wireless channel conditions between uplink and downlink, the RSSI based association cannot facilitate nodes intending to mainly upload traffic to the network.

A number of admission control schemes have been published, and these fall into two broad categories: measurement-based and model-based (Gao and Cai, 2005). In measurement-based schemes, admission control decisions are made based on measured network conditions such as throughput, delay, or average collision rate. Such measurement-based schemes may incur significant overheads and suffer from instabilities in rapidly changing network conditions (Xu et al., 2013; Xiao and Li, 2004; Yerima, 2011). At the other extreme, they can rely on sparse measurements in time, such as the IEEE802.11 association protocol, but are not sufficiently responsive to changing network conditions leading to inefficient use of network resources. Finding an appropriate time-scale for measurement-based adaptation is an exceedingly complex and challenging task.

In model-based schemes appropriate models are developed to construct performance metrics to estimate the state of the network. These fall roughly into three sub-categories: The first category (Kumar and Kumar, 2005; Bejerano et al., 2007; Gao and Cai, 2005; Mishra et al., 2006) abstracts away the contention and interference by adopting a model for the effective bandwidth available at each AP and then proceeds to allocate this resource to individual wireless nodes via an optimisation process which takes into account the load demand of each wireless node. The admission of an additional wireless node to an AP is determined by whether this violates the optimisation constraints, or reduces the overall network throughput. The second category (Shakkottai et al., 2006; Ercetin, 2008) extends this approach by employing a cell-wide MAC contention model but omits any inter-cell or any inter-AP interference in arriving at the overall available bandwidth to be optimally allocated to individual wireless nodes. The third category of model based association schemes takes a cross-layer approach and combines per node measurements of uplink and downlink effective bandwidth with parameters derived from a MAC protocol model into a cost function that is averaged for the uplink and downlink across each cell. The wireless nodes chose to associate with the access point with the lowest cost (Athanasίου et al., 2009). This approach implicitly incorporates contention and interference into its channel model, but only in a sense of a periodically updated spatially average value. Its drawback is that in reality each wireless node does not experience identical levels of contention within the coverage area of the same AP.

Model-based schemes recognise that offered traffic, the number of competing nodes and radio interference play a significant role in determining the optimised throughput. However, the details of the MAC layer contention are considered on a macroscopic, average cell level (mean field approach) through the formulation of the utility or cost function adopted in the optimisation. The limitation of this approach is that the actual network performance is strongly dependent on a microscopic level description of the network contention mechanism. For example, the precise timing of interfering node transmissions, especially during a vulnerable time period for collisions can dramatically alter the realised network throughput (Hung and Marsic, 2010). Therefore, a mean-field formulation of the utility maximisation problem lacks necessary detail and can be improved upon. Recognising that the presence of hidden terminals can potentially give rise to strong spatial variations in the admission control algorithm decisions indicates that a distributed approach with the algorithm running independently at each wireless node is to be preferred. Nevertheless, a purely local distributed approach has long been recognised as being capable of arriving at a globally optimal network-wide operation (Steenstrup, 2002).

### 3 Proposed approach to admission control with hidden terminals

The hidden terminal problem in wireless networks arises when a node A attempts to transmit a packet to node B without being able to sense a simultaneous transmission from a node C (hidden terminal) to node B. The hidden terminal problem should be explicitly considered in a reliable admission control model since it can result in a dramatic degradation of multi-cell network performance and unfair network access for some wireless nodes (Joon and Singh, 2015; Hung and Bensaou, 2009). The activation of the RTS/CTS mechanism can reduce the number of collisions due to hidden terminal transmissions, increase throughput and avoid retransmissions. It should be emphasised that even when the RTS/CTS mechanism in IEEE802.11 networks is activated, there is a vulnerable time period for a collision from a hidden terminal equal to the duration of the RTS+SIFS control packet, which continues to degrade network performance. This unwanted effect is expected to become more pronounced in large, congested, multi-cell wireless networks, where the population of hidden nodes is high compared to the number of the covered nodes. It is worth mentioning that the probability of a hidden terminal accessing the channel during the vulnerable time period has been calculated to be nearly three times higher than the corresponding probability for a covered node (Hung and Marsic, 2010). Thus, the deleterious effect of hidden terminals during the vulnerable time period must be explicitly addressed in improved admission control schemes. To the best of our knowledge, no currently proposed admission control schemes in multi-cell WLAN networks has taken the role of hidden terminal nodes into

account explicitly, while at the same time taking into account the full contention resolution of the MAC layer in IEEE802.11 protocol, and *this is a novel contribution of this paper*.

Our proposed admission control strategy takes the view that each node seeks association with an access point (AP) which will offer it the maximum link throughput. Only those APs are considered for association, which can support a bit error rate (BER) above a certain threshold for reliable communications. The node first receives RSSI indicators from the physical layer to construct a table of APs which can offer a BER above a threshold level. Next, it estimates its link throughput, exploiting well-established models (Hung and Marsic, 2010). The model we adopt is an accurate and detailed MAC model which includes the causal link between precise event timings, the probabilities of the channel being idle, busy due to successful transmissions, and busy due to packet collisions. Moreover, this model includes the collisions during the vulnerable time due to transmissions from hidden nodes. Intuitively, one would expect that estimating the mean back-off state accurately, or, alternatively, the size of the contention window locally would be necessary. As we shall show later, a crude constant contention window estimate suffices in making robust association decisions.

One of the attractive features of the proposed approach is that it has negligible overheads, is purely local and gives the node the extra degree of freedom to optimise its association based on the predominant type of traffic (uplink or downlink). Even in the worst network case, when the time required for proper admission decision exceeds a maximum limit, the node could have the alternative to immediately associate with an AP based on RSSI and subsequently refine its admission based on our algorithm.

### 4 Analysis of the system model

Consider a multi-cell WLAN network comprising a set of cells with an access point (AP) at the centre of each cell as shown in Figure 1. The worst case cell ‘planning’ scenario is a single channel to be shared by all APs in the network. Let  $A = \{a_1, a_2, \dots, a_N\}$  be the set of APs forming the multi-cell network. A client mobile node enters the area covered by the network and attempts to associate with an AP in order to start exchanging traffic (uplink and/or downlink) with the network (c.f. Figure 1). Each AP is assumed to be directly connected to a gateway router for access to the rest on internet. If a client node can sense  $K$  APs within its coverage area, we define as  $A_i = \{a_1^i, a_2^i, \dots, a_K^i\}$  the set of APs within the sensing area of client  $i$ . Once node  $i$  decides which AP it will associate with, it sends the appropriate connection request in order to start getting network service. A client  $i$  will choose to join that particular AP  $a_K^i \in A_i$  which will maximise the throughput of the link ( $i \rightarrow a_K^i$ ). In controlling its network admission, we allow an extra degree of freedom to client  $i$  by taking into account the type of traffic it is going to exchange with the network and

distinguish between uplink and downlink traffic throughput  $S_i^a$ . We consider a link ( $i \rightarrow a$ ) between a node  $i$  (transmitter) and an AP  $\alpha$  (receiver), with  $n_c$  being the total number of other (covered) nodes within the sensing area of node  $i$ , and  $n_h^a$  being the number of hidden nodes, when node  $i$  is associated to AP  $a$ . The saturated link throughput  $S_i^a$  of the wireless link from node  $i$  to AP  $a$  can be expressed (see Appendix for the details of the derivation) as:

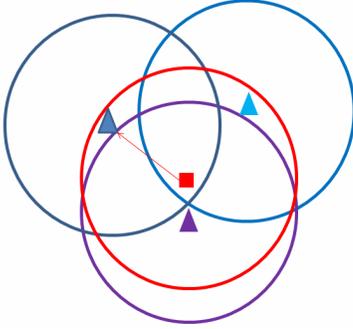
$$S_i^a = \frac{1}{A^a + (n_c + n_h^a + 1)B^a} \quad (1)$$

with  $A^a = \left( \frac{1 - P_i^r}{P_i^r} \right) \left( T_{\text{slot}} + \left( \frac{1}{P_{\text{idle}}} - 1 \right) T_c \right)$ ,  $B^a = T_s - T_c$  and

$$P_{\text{idle}} = (1 - P_i^r)^{n_c + 1} (1 - P_i^{\tau_h})^{n_h^a}, \quad (2)$$

where  $P_i^r$  is the channel access probability for node  $i$  and  $P_i^{\tau_h}$  is the probability for a hidden node to access the channel and transmit a packet during a vulnerable time period to have a collision.  $T_s$  is the average time a slot is sensed busy due to successful transmissions and  $T_c$  is the average time the slot is sensed busy due to packet collisions (it is given as an average of the contribution from both covered and hidden nodes).

**Figure 1** A client node (square) can sense three APs (triangles) in its vicinity (see online version for colours)



To analyse the admission control problem in multi-cell WLANs we adopt the full contention resolution approach of IEEE802.11 protocol and describe the wireless nodes with a Markov chain model. We include the influence of hidden terminals during the vulnerable time-period within the framework of four-step handshaking mechanism. For mobile stations in the saturation condition the probabilities  $P_i^r$  and  $P_i^{\tau_h}$  and the packet collision probability  $p$  can be expressed as (Hung and Marsic, 2010):

$$P_i^r = \frac{1 - p^{m+1}}{1 - p} b_{00}, \quad (3)$$

$$P_i^{\tau_h} = \left[ \begin{array}{c} (\tau_v + 1) \left( \frac{1 - p^{m+1}}{1 - p} \right) - \left( \frac{\tau_v (\tau_v + 1)}{2W} \right) \\ \left( \frac{1 - (p/2)^{m+1}}{1 - (p/2)} \right) \end{array} \right] b_{00}, \quad (4)$$

with,

$$p = (1 - P_i^r)^{n_c} (1 - P_i^{\tau_h})^{n_h^a}, \quad (5)$$

and

$$b_{00} = \frac{2(1 - p)(1 - 2p)}{2(1 - p)(1 - 2p) + (1 - 2p)(1 - p^{m+1}) + W(1 - p)(1 - (2p)^{m+1})}. \quad (6)$$

$\tau_v$  is the vulnerable period time over which a transmission from a hidden terminal will result in a collision,  $W$  is the minimum contention window size and  $m$  the maximum binary exponential back-off stage used. Without loss of generality we consider typical values of  $m = 5$  and  $W = 32$  ( $\tau_v < W$  for the chosen network parameters). In practice the system of non-linear Eq. (3)–(6) with three unknowns  $P_i^r$ ,  $P_i^{\tau_h}$  and  $p$  needs to be solved numerically in order to compute  $P_i^r$ ,  $P_i^{\tau_h}$  in terms of network parameters and the numbers of covered and hidden terminals of client node  $i$ . Then, equation (2) yields  $P_{\text{idle}}$  and, finally, equation (1) yields the throughput of a particular link between a client node  $i$  and the AP  $a$ . The resulting throughput is given only in terms of network parameters and the numbers  $n_c$  and  $n_h^a$  of the covered and hidden nodes respectively. We note that the formalism we presented above can equally describe uplink and downlink traffic. This is because APs do not have any particular privilege when competing with the client nodes for medium access in order to transmit data.

#### 4.1 Uplink throughput in large congested network

If a client aims to join a large congested network by maximising its uplink throughput, it can do so by associating to an AP  $\alpha$ , which results in the lowest number  $n_h^a$  of hidden terminals. Thus, the node can avoid the complete computation of equation (1), resulting in a faster admission control process.

For the uplink traffic case the number of covered nodes  $n_c$  depends only on the wireless node density distribution and the sensing range of the client in question, and is independent of the AP the client eventually associates to. As a consequence, we consider  $n_c$  as a constant parameter irrespective of the node's association decision. We initially consider the efficacy of equation (1) as an optimal association criterion in a multi-cell network, in the decoupling approximation with no exponential backoff (Hung and Marsic, 2010; Bianchi, 2000). Solving equations (3)–(6) in this limit (i.e., using  $m = 0$ ), enables us to relate  $P_i^r$  to a constant contention window  $W$  for every

contending node according to  $P_i^r = 1/(3+W)$ . The decoupling approximation was shown to be quite accurate in congested networks with a large number of nodes (Kumar et al., 2007). In this large network limit, we invoke the idle sense method of Heusse et al. (2005) where throughput maximisation leads to an optimum probability for a time slot to be sensed idle. This can be written as,  $P_{\text{idle}} \approx e^{-\xi}$  with the parameter  $\xi$  satisfying the non-linear equation  $1 - \xi = \left(1 - \frac{T_s}{T_c}\right) e^{-\xi}$  (Heusse et al., 2005).  $T_s$  is the average

time a slot is sensed busy due to a successful transmission, and depends solely on the network parameters but not on the number of competing covered and hidden nodes. The time period  $T_c$ , which represents the average time a slot is sensed busy due to a packet collision, is given by,

$$T_c = \frac{n_c}{n_c + n_h} T_{\text{cov}}^c + \frac{n_h}{n_c + n_h} T_{\text{hid}}^c \text{ for the four-step handshaking}$$

mechanism.  $T_{\text{cov}}^c$  and  $T_{\text{hid}}^c$  are network parameters which in the IEEE802.11 protocol turn out to have numerical values close to each other, and differentiate the time period  $T_c$  for collisions arising from covered and hidden node transmissions, respectively. It is sufficient to point out that they have constant values and do not depend on the number of competing nodes (covered or hidden) in the network. If the uplink throughput were to be employed as a metric for network admission decision,  $n_c$  is the same for all AP association choices and only the number of hidden terminals  $n_h$  changes across the set of candidate APs. If, for fixed  $n_c$ , the limiting case of strong contributions from hidden terminals ( $n_h \gg 1$ ) is considered, the above expression for  $T_c$  can be approximately reduced to  $T_c \approx T_{\text{hid}}^c$ , a network parameter which is independent of the number of nodes. This finally implies that Eq. (1) can be written as:

$$S_i^\alpha = \frac{1}{A + (n_c + n_h^\alpha + 1)B}, \quad (7)$$

with the corresponding factors  $A$  and  $B$  depending solely on the network protocol parameters and not on the number of surrounding covered or hidden nodes associated to a particular AP  $\alpha$ . Eq. (7) holds in the limit of the decoupling approximation and of large networks, as is expected to be the case for public WLAN multi-cell networks deployed in dense urban areas. It is clear now from Eq. (7) that the only crucial parameter, which the client node should consider in order to associate to an AP maximising its uplink throughput, is the number of hidden nodes with which it will compete, since the corresponding link throughput behaves as  $S_i^\alpha \sim 1/n_h^\alpha$ , a monotonically decreasing function of  $n_h^\alpha$ . The number of hidden nodes  $n_h^\alpha$  depends on the particular AP with which the client considers to associate. If the client aims to maximise its link throughput, it must associate to that AP which results in the lowest number  $n_h^\alpha$  of hidden terminals, provided the corresponding link SNR is above a certain threshold. The crucial question remains as to how a client node can determine in a

distributed manner the number of hidden nodes it will experience through its potential association with every candidate AP within its sensing area. We address this issue in detail, and propose a distributed algorithm for its determination in Section 5.

#### 4.2 Algorithm feasibility and numerical implementation

We now examine how the link throughput can successfully be used as a metric for distributed admission control of a client node in an uncoordinated, multi-cell IEEE 802.11 network. This is a plausible metric to consider, since every wireless node in the network acts selfishly and prefers to control its network admission in such a way as to maximise its link throughput. The questions are whether the link throughput can be computed efficiently in a distributed manner, and whether this choice of metric leads to an increased throughput for the whole network. To address these questions we clarify first the contribution of all the relevant factors to link throughput  $S_i^\alpha$  with respect to AP association and traffic type (uplink or downlink).

To provide supporting evidence for the use of the link throughput  $S_i^\alpha$  as an association metric and pose the crucial parameters of the problem, we compute  $S_i^\alpha$  from equation (1) by solving numerically the system of non-linear equations (3)–(6). The numerical analysis for the system model presented in this section supports the argument that this model can serve as a consistent platform in realistic uncoordinated IEEE802.11 environments, with low complexity and ease of implementation of distributed admission control algorithms. We solve equation (1) to compute the saturated link throughput between two nodes (transmitter/receiver) in a multi-cell WLAN network. The role of the transmitter (AP or client node) determines whether we refer to uplink or downlink throughput, respectively. To simplify the analysis, we assume that all WLAN cells of the network are configured with the same network parameter values (c.f. Table 1). If network parameters are not the same across network cells, equation (1) should be solved with the appropriate network parameter values of each individual cell. We also assume the activation of a four-step handshaking mechanism (RTS/CTS) to ameliorate the hidden terminal problem, and use typical IEEE802.11 protocol parameters expressed as:

$$T_s = RTS + \delta + SIFS + CTS + \delta + SIFS + H \\ + E[P] + \delta + SIFS + ACK + \delta + DIFS,$$

$$T_{\text{cov}}^c = \frac{1}{2} \sigma + RTS + \delta + (SIFS + CTS + 2\delta),$$

$$T_{\text{hid}}^c = \frac{1}{2} (RTS + \delta) + RTS + \delta + (SIFS + CTS + 2\delta), \text{ and}$$

$$T_c = \frac{n_c}{n_c + n_h} T_{\text{cov}}^c + \frac{n_h}{n_c + n_h} T_{\text{hid}}^c.$$

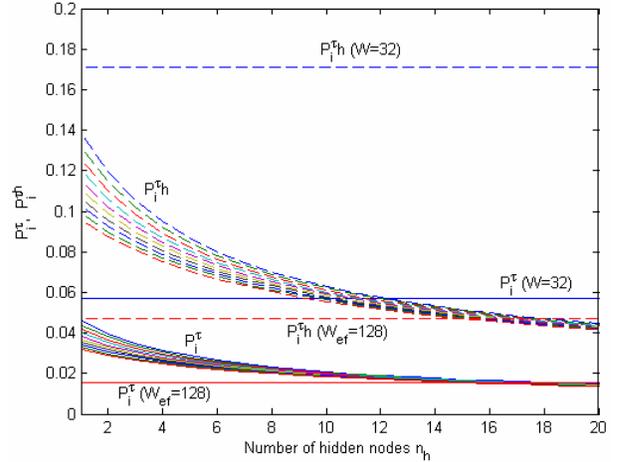
**Table 1** System parameters

MAC header	224 bits
PHY header	192 bits
RTS header	160 bits+PHY
CTS header	160 bits+PHY
ACK header	160 bits+PHY
Vulnerable period time ( $\tau_v$ )	RTS+SIFS
DIFS	50 $\mu$ s
SIFS	10 $\mu$ s
Slot time ( $T_{\text{slot}}$ )	20 $\mu$ s
Propagation delay ( $\delta$ )	1 $\mu$ s
Min. contention window ( $W$ )	32
Transmission rate	11 Mbps

The typical system parameter values employed are shown in Table 1. We depict in Figure 2 the channel access probability  $P_i^r$  and the probability for transmission  $P_i^{rh}$  by a hidden terminal within the vulnerable time period  $\tau_v$ , as a function of the number of hidden terminals  $n_h^a$  for varying numbers of covered nodes. It can be observed that these probabilities approach a constant value as the number of hidden nodes increases, for any number of covered nodes. This limit reflects the validity of the decoupling approximation, with the channel access probability approaching a constant value. If no exponential back-off is considered, this limit can be derived from the minimum contention window  $W$  employed through the relation  $P_i^r = 1/(3+W)$ . This result for  $P_i^r$  is a straight-forward solution of the system of equations (3)–(6), when  $m = 0$ . Once the contention resolution protocol of the MAC layer is realised in its full functionality with the binary exponential back-off activated to resolve congestion ( $m > 0$ ), the above expression for the decoupling approximation fails to quantitatively predict the correct channel access probability. It is clear from Figure 2, that when  $m = 5$ , the use of  $W = 32$  leads to  $P_i^r = 0.6$ , which is three times higher than the value predicted by the exact numerical solution of equation (3)–(6). This difference is even higher for  $P_i^{rh}$ . We can restore the quantitative accuracy of the above expression between  $P_i^{rh}$  and  $W$  in the large network limit, even with the exponential back-off mechanism if we introduce a higher effective contention window  $W_{\text{eff}}$  to better account for the effect of node contention. From the relation  $P_i^r = 1/(3+W_{\text{eff}})$  we see in Figure 2 that an effective window as large as  $W_{\text{eff}} \sim 4W$  can accurately fit the behaviour of  $P_i^r$  in the congestion limit (large number of contending nodes), even when a maximum exponential back off stage  $m = 5$  is considered. For a different maximum back-off stage  $m$ , a different value of  $W_{\text{eff}}$  should be determined to accurately fit the exact numerical solution. However, according to the IEEE 802.11 standard, in actual WiFi networks  $m$  is recommended to take only a small range of integer values, up to  $m = 5$ . Thus, an admission

control algorithm using the link throughput as a decision metric could become more efficient if the client node employs a look-up table of  $m$  values stored with the corresponding  $W_{\text{eff}}$  ones, which can be used according to the value of  $m$  it receives from AP advertisements. In this case the full solution of the non-linear equations (3)–(6) may be avoided, thus reducing the algorithm complexity.

**Figure 2** Channel access probability  $P_i^r$  and  $P_i^{rh}$  for channel access by a hidden terminal during vulnerable time period respectively as a function of number of hidden terminals, for various numbers of covered nodes  $n_c$  (1–10) with  $m = 5$  calculated from equations (3)–(6) (see online version for colours)



Note: Corresponding probabilities calculated within decoupling approximation with no exponential back off are also plotted for  $W = 32$  and  $W_{\text{eff}} = 128$  respectively.

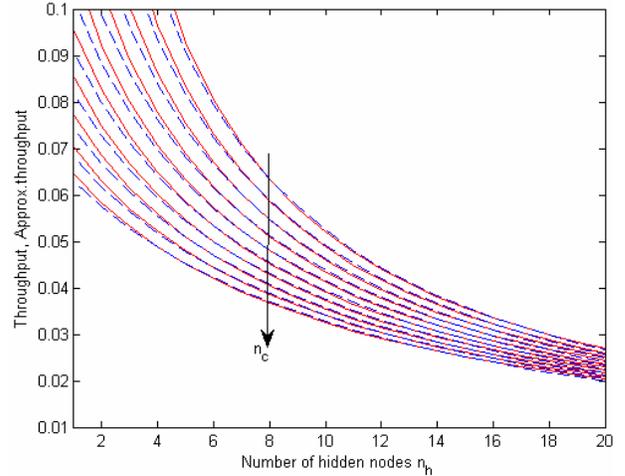
In Figure 3 we present numerical results for the saturated link throughput as a function of the number of hidden terminals  $n_h^a$  for various numbers of covered nodes,  $n_c^a$ , and network parameters  $m = 5$  and  $W = 32$ . For comparison we also show similar results obtained when we calculate the link throughput in the decoupling approximation without exponential back-off. In this approximation we get  $P_i^r = 1/(3+W_{\text{eff}})$  for the channel access probability and

$$P_i^{rh} = \left( \tau_v + 1 - \frac{\tau_v(\tau_v + 1)}{2W_{\text{eff}}} \right) P_i^r$$

for the probability a hidden terminal to access the channel within the vulnerable period. The numerical results of Figure 3 assume  $W_{\text{eff}} = 4W$ . From Figure 3 we note the following: First, there is excellent agreement of the approximate and exact calculations for the link throughput. This is very important for the client node during its admission control procedure, because it can directly use this approximate scheme to calculate the link throughput instead of solving a set of nonlinear equations. This can greatly reduce the complexity of the admission control algorithm we propose in the next section. Second, we note that as we change the number of covered nodes  $n_c^a$ , we have no cross-overs in the curves of the link throughput for the whole range of the  $n_h^a$  variation. This property of the

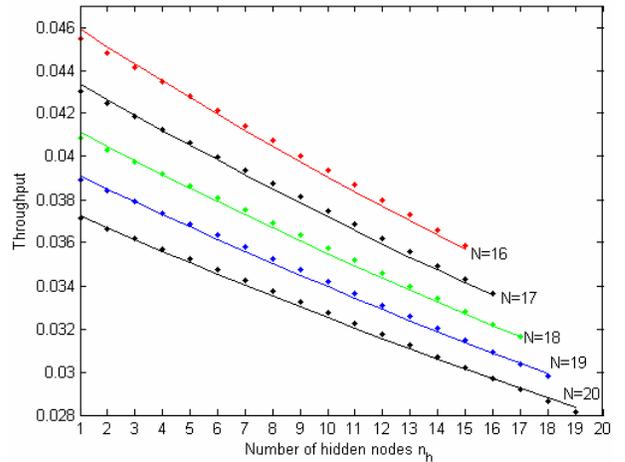
link throughput variation with  $n_c^a$  and  $n_h^a$ , will prove to be very useful for admission control purposes: A client node can find the maximum link throughput only by considering the relevant  $n_c^a$  and  $n_h^a$  of the corresponding contending nodes, without having to compute the actual throughput values. From Fig. 3 a node association to an AP results in a lower number of hidden nodes, leading to a higher throughput, for fixed  $n_c^a$ . For the same number of hidden nodes, the throughput increases as the number of covered nodes decreases. For admission control purposes it is useful to determine which link throughput is larger simply by knowing the number of covered and hidden nodes for each AP association respectively. Detailed analysis is required, however, to compare link throughputs when the two alternative APs have different covered and hidden nodes, even though their total number of nodes is the same. To clarify this point, we consider the link throughput as a function of the number of hidden nodes for various values of the total number of nodes  $N$  (covered and hidden). In Fig. 4 we plot results for possible AP associations, which result in differing total number of contending node configurations. Among various client associations which lead to the same number of hidden terminals  $n_h^a$ , the resulted link throughput is always lower for associations with a smaller total number  $N$  of competing nodes, as naively expected. However, we also notice that a client node during its association with an AP, may decide to join an AP with a larger total number  $N$  of competing nodes, because this results to a higher link throughput compared to an alternative one with a lower number of  $N$ . This possibility indeed occurs, provided the association with higher  $N$  comprises a low enough number of hidden terminals. For example, from Fig. 4 an association to an AP with  $N = 20$  and  $n_h^a = 4$  results in a lower throughput than an association with the same number of hidden terminals ( $n_h^a = 4$ ) but lower number of total nodes (i.e.,  $N = 17$ ), as expected. However, the second AP association with  $N = 17$ ,  $n_h^a = 15$ , and  $n_c^a = 2$ , counter-intuitively results in a significantly lower throughput than the case with  $N = 20$ ,  $n_h^a = 4$ , and  $n_c^a = 16$ . This result not only indicates how inefficient it is to design an admission control protocol based only on the total number  $N$  of competing nodes, but also reveals the deleterious effect the hidden terminals can have in multi-cell network admission. Note that this result is valid with the RTS/CTS protocol activated, as even in this case it is possible for a hidden terminal to access the channel with a probability  $P_i^{r_h}$  during the vulnerable time period of RTS+SIFS and result in a collision. This undesirable influence of the hidden terminals needs to be embedded into network admission control schemes. The proposed admission control algorithm in Section 5 addresses this issue, to the best of our knowledge, for the first time and in a distributed manner, with low complexity.

**Figure 3** Link throughput as a function of number of hidden terminals  $n_h$  for various numbers of covered nodes  $n_c$  (1–10) with  $m = 5$  (see online version for colours)



Note: The dashed lines depict the throughput for the no-backoff decoupling approximation with  $W_{\text{eff}} = 128$ .

**Figure 4** Link throughput as a function of the number of hidden terminals  $n_h$  for various total numbers  $N$  of contending nodes with  $m = 5$  (see online version for colours)



Note: The points depict the throughput for the no-backoff decoupling approximation with  $W_{\text{eff}} = 128$ .

## 5 Distributed admission control algorithm with hidden terminals

### 5.1 Distributed determination of covered and hidden terminals

The number of covered and hidden terminals is crucial in determining link throughput as a metric for admission control. Before presenting an algorithm, it is imperative to explain how covered and hidden terminals can be determined in a distributed manner for both uplink and downlink traffic.

A client node intending to join a multi-cell WLAN network, initially observes the wireless medium in order to identify all APs with which it can exchange traffic with an SNR above a predetermined threshold. This SNR threshold is chosen in such a way as to satisfy a required BER for the intended application. Thus, the RSSI criterion of admission control in IEEE802.11 is preserved, but only insofar as it is employed to identify the set of APs which are candidates for the network client association. The APs exchange appropriate network parameters (see Table 1), which are used in their respective cells with the client. The final decision, as to which AP the client node will associate with, will be taken independently by the client node itself in a distributed manner. The client considers first what type of traffic (uplink or downlink) is going to predominantly exchange with the network. This consideration is important for the admission control process, since the client node has to correctly record the numbers of contending covered  $n_c^a$  and hidden  $n_h^a$  terminals associated with each candidate AP  $\alpha$ . Only then can the client determine the appropriate AP link with the maximum throughput. Once the client node decides its preferred AP association, it forwards an admission request packet to the selected AP to complete network admission and start exchanging data.

Upon receiving a request by a newly entering the network client node, an access point (AP), broadcasts the total number of nodes  $N_a$  residing within its cell region (nodes associated with it and nodes in overlapping cell regions associated with other access points). An AP can record the number of nodes in its overlapping cell regions by carefully listening to these nodes when broadcasting MAC layer frames. At the same time, every node in the network can determine within its sensing area the number of covered nodes  $n_c^\mu$  associated to an AP  $\mu$  by promiscuously decoding MAC layer transmissions. For each successful reception of a data packet, the IEEE802.11 protocol dictates that the receiver broadcasts an ACK control packet. Taking advantage of the ACK packet broadcasts, we can include in them the address of one of the  $K$  APs the node senses, but is not associated with, as additional information. In the case, a node senses  $K$  APs,  $K$  consecutive ACK packets are sufficient to inform its surrounding competing nodes of all the  $K$  APs it senses. In this way every client node entering the network can build a table with the numbers  $n_c^{\mu,v}$  of nodes within its coverage area which are associated to AP $\mu$  and sense the AP $v$ . This minor, additional overhead in the transmitted ACK packets will not overload the network significantly, but will enable every client node to determine accurately the number of the hidden terminals potentially affecting its throughput. The admitting node can set an observation window time to collect the information for its surrounding environment in order to calculate  $n_c^a$  and  $n_h^a$ . At the end of this time it makes its admission request. Meanwhile, it continues to monitor periodically its contending environment, and, when it computes a “better” link with another AP, it has the option to initiate the procedure for updating its network association. This

network update association can be initiated also when the node decides to change the type of traffic (uplink or downlink) it predominately exchanges with the network. For example, a node which joined the network initially to upload a large video file to an internet server, decides to continue with downloading internet material.

- *Determine  $n_h^a$  for uplink:* Suppose a client node examines the possibility to associate with a candidate AP  $\alpha$  (see Figure 1), with a known (broadcast)  $N_a$ . As stated earlier, by promiscuous sensing the wireless frames the client node can determine the numbers  $n_c^{\beta,a}$  of the covered nodes associated to AP  $\beta$  and sense AP  $\alpha$ . The number of hidden nodes  $n_h^a$  is computed as:

$$n_h^a = N_a - \sum_{\beta} (n_c^{\beta,a}).$$

- *Determine  $n_h^a$  for downlink:* The AP  $\alpha$  becomes now the transmitter and the client node the receiver (Figure 1). Once every client node  $i$  has populated the table  $n_c^{\beta,v}$ , it can calculate the number of hidden terminals affecting the downlink throughput for its association to AP $\alpha$  as:

$$n_h^a = \sum_{\beta \neq \alpha, v \neq \alpha} n_c^{\beta,v}.$$

## 5.2 Algorithm to upload traffic

A client node  $i$  wanting to join a multi-cell WLAN network executes the following steps:

- 1 Observe the wireless medium and identify all APs  $A_i = \{\alpha_1^i, \alpha_2^i, \dots, \alpha_a^i\}$  with corresponding links having a SNR above a certain application-specified threshold, by reading the appropriate RSSI.
- 2 For each AP  $\alpha \in A_i$ :
  - a Request network parameters and the total number of nodes  $N_a$  within its cell coverage.
  - b Populate the table  $n_c^{\beta,a}$  by reading MAC frames and ACK packets.
  - c Determine the number of hidden terminals using  $n_h^a = N_a - \sum_{\beta} (n_c^{\beta,a})$ .
- 3 Send association request to AP  $\alpha$  with the minimum  $n_h^a$ .

## 5.3 Algorithm to download traffic

A client node  $i$  wishing to join a multi-cell WLAN network executes the steps:

- 1 Observe the wireless medium and identify all APs  $A_i = \{\alpha_1^i, \alpha_2^i, \dots, \alpha_a^i\}$  with corresponding links having a SNR above a certain application-specified threshold, by reading the appropriate RSSI.

- 2 For each AP  $\alpha \in A_i$ :
  - a Request the total number  $N_a$  of nodes within cell A.
  - b Populate the table  $n_c^{\beta, v}$  by reading MAC frames and ACK packets.
  - c Determine the overheard number of covered nodes,  $n_c^a$ , from step B).
  - d Determine the number of hidden terminals from  $n_h^a = \sum_{\beta, v \neq \alpha} n_c^{\beta, v}$ .
  - e Compute  $S_i^a$  from (1) in the approximation  $P_i^r = 1 / (3 + W_{\text{eff}})$  and  $P_i^{rh} = \left( \tau_v + 1 - \frac{\tau_v(\tau_v + 1)}{2W_{\text{eff}}} \right) P_i^r$  using appropriate values for network parameters.
- 3 Request association to AP  $\alpha$  with the maximum  $S_i^a$ .

#### 5.4 Global network considerations

The above algorithm leads not only to a maximum link throughput for the individual client node, but also increases the total throughput of the global network. To elucidate this claim, we consider the uplink case. Suppose the client has decided to associate with AP  $a$  because  $S_a > S_\beta \forall \beta \in A_i$ . Since the number of covered nodes with which the client  $i$  has to compete is the same, independent of the particular AP of association, the total throughput of the network for association with AP  $j$  is:

$$S_j = S_i^j + S - n_c \Delta S_c - n_h^j \Delta S_h, \quad (8)$$

where  $S_i^j$  is the uplink throughput of client  $i$  associated with AP  $j$ ,  $S$  is the throughput sum of all the nodes of the network before the client  $i$  enters the network, and the throughput correction factors  $\Delta S_c$  and  $\Delta S_h$  for the  $n_c$  covered and  $n_h^j$  hidden nodes, which are affected by the presence of client  $i$  in their competition to access the medium. The factors  $\Delta S_c$  ( $\Delta S_h$ ) are the throughput reduction, which a mobile node experiences, when an extra covered (hidden) node is added in its vicinity. Ignoring physical layer effects and taking into account only MAC layer contention, this throughput reduction should be the same for every covered (hidden) node affected by the presence of the additional client by symmetry. This is valid in the congested network limit. Suppose that client follows the proposed admission control algorithm and joins AP  $\alpha$  to maximise its link throughput  $S_i^a$ . The difference  $\Delta S$  in total network throughputs between node association to AP  $\alpha$  and any other AP  $\beta \in A_i$  becomes:

$$\Delta S = S_a - S_\beta = (S_i^a - S_i^\beta) + (n_h^\beta - n_h^a) \Delta S_h. \quad (9)$$

The algorithm dictates that the client is associated with AP  $\alpha$  and not AP  $\beta$  if the uplink throughput  $S_i^a > S_i^\beta$ , with

the condition  $n_h^\beta > n_h^a$  applied. From equation (9),  $\Delta S > 0 \forall \beta \in A_i$ . This implies that our admission control algorithm maximises not only the link throughput of the individual client but also increases the total throughput  $S_a$  of the global network.

In the downlink case, client  $i$  is assumed to sense  $A_i$  APs and compete with them on equal grounds to access the wireless medium. From the point of view of the IEEE802.11 MAC layer protocol, the downlink throughput of all  $A_i$  APs will be on average equally affected by a factor  $\Delta S$  due to the presence of the client  $i$ , no matter from which AP the client decides to receive its network service. The total downlink network throughput is given by,

$S_j = S_i^j + \sum_{k=1}^K (S_k - \Delta S) + S$ , where  $S_i^j$  is the individual downlink throughput of client associated to AP  $j$ ,  $S_k$  is the downlink throughput within the cell of AP  $k$ , before the client  $i$  enters the network, corrected by the factor  $\Delta S$ . The throughput sum of all other cells beyond the sensing area of client  $i$  is  $S$  and this is not affected by the presence of the client node in question. The difference  $\Delta S$  in total network throughput between node association with AP  $\alpha$  and any other AP  $\beta \in A_i$  becomes,  $\Delta S = S_a - S_\beta = S_i^a - S_i^\beta > 0$ , since we assumed that client  $i$  joins AP  $\alpha$  to maximise its downlink throughput,  $S_i^a$ . Thus, for the case of downlink throughput the proposed algorithm also increases the total network throughput, in addition to local link throughput maximisation.

## 7 Conclusions

A low complexity, fully distributed admission control algorithm, has been proposed where a wireless client node decides locally to which access point it will associate to maximise either its uplink or downlink throughput, depending on the type of traffic it predominantly intends to exchange with the network. This protocol introduces a minimal communication overhead, based on promiscuous eavesdropping of MAC frames in vicinity of the client, and leads to an increase of the total throughput of the global network. This protocol takes into account the full contention resolution of the RTS/CTS IEEE802.11 medium access control protocol through well-established theoretical models of the MAC layer, and does not require computation of network statistics. The proposed algorithm can also serve as the basis for the development of efficient traffic offloading protocols in heterogeneous 5G networks.

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## Appendix

The saturated link throughput  $S_i^a$  of the wireless link from node  $i$  to AP  $\alpha$  can be regarded as the ratio of the average amount of payload information (average packet load size) successfully transmitted in a time slot (t.s.) over the link ( $i \rightarrow a$ ) divided by the average duration of the slot time:

$$S_i^a = \frac{E[\text{payload info transmitted in t.s. on link } (i-a)]}{E[\text{length of t.s.}]} \quad (\text{A1})$$

The probability of a particular node  $i$  successfully transmitting a packet to an AP  $\alpha$  during a time slot, provided there is one transmission in this time slot, is:

$$P_{sc}^i = \frac{P_i^r (1 - P_i^r)^{n_c} (1 - P_i^{r_h})^{n_h}}{P_{tr}} \quad (\text{A2})$$

where  $P_i^r$  is the channel access probability for node  $i$  and  $P_i^{r_h}$  is the probability for a hidden node to access the channel and transmit a packet during the vulnerable time period, giving rise to a collision. Equation (A2) can be interpreted as a transmission being successful if node  $i$  has a packet to transmit, while at the same time none of its surrounding covered nodes and none of its hidden terminals transmit during the vulnerable time for collisions discussed earlier. Adopting a Markov chain model of the wireless medium random access process, we calculate the

probabilities  $P_i^r$  and  $P_i^{rh}$  based only on the network parameters, under the assumption of the decoupling approximation with constant packet collision probability  $p$  (Joon and Singh 2015; Hung and Marsic, 2010; Bianchi, 2000; Kumar et al., 2007). This is a reasonable assumption to make and is particularly valid as we approach the congested network regime (Bianchi, 2000; Kumar et al., 2007). The probability to have at least one transmission from any contending (covered or hidden node) to the client terminal  $i$  during the time slot we consider is:

$$P_r = 1 - (1 - P_i^r)^{n_c + 1} (1 - P_i^{rh})^{n_h^\alpha} = 1 - P_{idle}, \quad (\text{A3})$$

where  $P_{idle}$  is the probability to have the time slot sensed idle by the client node  $i$ . The conditional successful probability that any one node transmits during a time slot, provided there is at least one transmission in that time slot, is given by:

$$P_{sc} = \frac{(n_c + n_h^\alpha + 1) P_i^r (1 - P_i^r)^{n_c} (1 - P_i^{rh})^{n_h^\alpha}}{P_r} \quad (\text{A4})$$

From equations (A2)–(A4) we finally have:

$$P_r P_{sc}^i = \frac{P_i^r}{(1 - P_i^r)} P_{idle} \quad (\text{A5})$$

and

$$P_r P_{sc} = \frac{(n_c + n_h^\alpha + 1) P_i^r}{(1 - P_i^r)} P_{idle}. \quad (\text{A6})$$

Denoting the average packet payload size by  $E[P]$ , the average amount of payload information successfully transmitted through link  $i \rightarrow \alpha_k^i$  in one time slot is  $P_r P_{sc}^i E[P]$ . If the time slot is empty (idle) with probability  $1 - P_r$ , or contains a successful transmission with probability  $P_r P_{sc}$ , or undergoes a collision with probability  $P_r (1 - P_{sc})$ , we finally rewrite (A1) as:

$$S_i^\alpha = \frac{P_r P_{sc}^i E[P]}{(1 - P_r) T_{slot} + P_r P_{sc} T_s + (1 - P_r) P_r T_c}, \quad (\text{A7})$$

where  $T_{slot}$  is the duration of a time slot,  $T_s$  the average time a slot is sensed busy due to a successful transmission and  $T_c$  is the average time a slot is sensed busy due to a packet collision (taken as an average of the contribution from both covered and hidden nodes). Equation (A7) with the help of equations (A2)–(A6), yields:

$$S_i^\alpha = \frac{1}{A^\alpha + (n_c + n_h^\alpha + 1) B^\alpha} \quad (\text{A8})$$

with

$$A^\alpha = \left( \frac{1 - P_i^r}{P_i^r} \right) \left( T_{slot} + \left( \frac{1}{P_{idle}} - 1 \right) T_c \right), \quad B^\alpha = T_s - T_c$$

and

$$P_{idle} = (1 - P_i^r)^{n_c + 1} (1 - P_i^{rh})^{n_h^\alpha} \quad (\text{A9})$$