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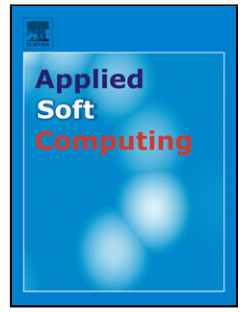

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Fuzzy Extended Kalman Filter for Dynamic Mobile Localization in Urban Area Using Wireless Network

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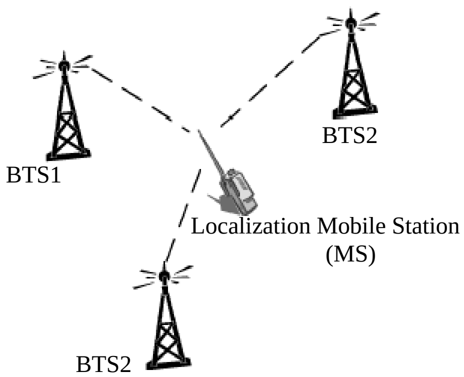
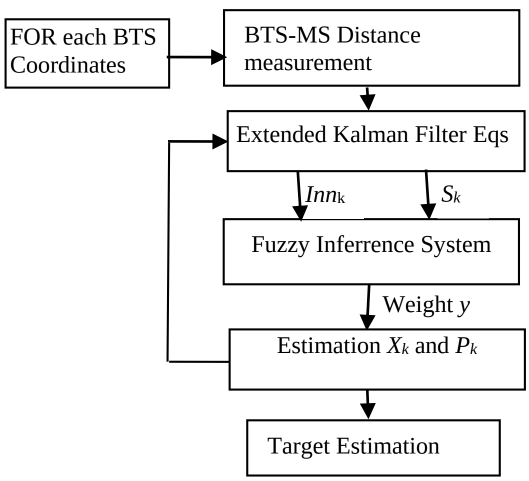
Graphical abstract





Highlights

- New proposal for hybrid estimator based on combination of extended Kalman filtering and zero-order Takagi-Sugeno inference system
- Comparison of the performance of the new estimator with classical estimators
- Application of the proposal to mobile positioning in wireless network
- Evaluation of the proposal using both simulation and real-time environmental settings.

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Abstract—The problem of accurate mobile positioning in cellular network is very challenging and still subject to intensive research, especially given the uncertainty pervading the signal strength measurements. This paper advocates the use of fuzzy based reasoning in conjunction with Kalman filtering like approach in order to enhance the localization accuracy. The methodology uses TEMS Investigation software to retrieve network information including signal strength and cell-identities of various base transmitter stations (BTS). The distances from the mobile station (MS) to each BTS are therefore generated using Walfish-Ikigami radio propagation model. The performances of the established hybrid estimator –fuzzy extended Kalman filter (FEKF)- are compared with extended Kalman filter approach and fuzzy-control based approach. Both simulation and real-time testing results demonstrate the feasibility and superiority of the FEKF localization based approach.

Index Terms—estimator, Fuzzy inference system, Kalman filter, mobile positioning, wireless network.

1. **Introduction**

1.1 Wireless Positioning

Motivated by the exponential increase of mobile applications including terthless access and the increased network quality both in terms of data rate and dependability, the deployment of wireless networks has exponentially increased across all cities. Indeed, nowadays wireless technology is not only restricted to standard mobile phone communication but extended to almost every aspect of our daily life including house appliances, consumer applications as well as transport, medical, industrial, ubiquitous computing and military applications [1]. On the other hand, the widespread of wireless systems increased the demand for accurate positioning of such wireless devices [2]. For instance, in sensor network, efficient position technology allows the operator to quickly identify the damaged sensor (s) [3]. Besides, it is typical that some tasks, e.g., object tracking, coordinating action (s) implicitly make use of device position. In mobile positioning, the US E-119 requirement forced the mobile operator to achieve a certain level of accuracy (50 meter accuracy in 67%) in order to handle emergency calls based only on wireless infrastructure, regardless of availability of GPS or others [4]. Location based advertisement enables users to selectively receive advertisement according to the promotions / services available nearby their locations, which also allows providers to provide additional features to subscribers. Social networking like Facebook, MySpace, among others, allows location-based social networking where friends located in nearby can be grouped and tracked. Roughly speaking, the knowledge of the physical location of mobile user devices, such as phones, laptops and PDAs, is crucial in applications involving, for instance, network planning, context aware network services, law enforcement and network performance analysis [5]. In this respect, the users can roam ubiquitously from various locations. Various technologies for wireless positioning have been put forward. These technologies differ according to the topology of the network, e.g., number of transmitters, their range and raw measurements. This includes, for instance, Infrared Radiation (IR), Radio Frequency Identification (RFID), Blue Tooth, Ultrasonic (US), Wi-Fi, ZigBee, Ultra Wide Band (UWB) frequency, GSM, Global Positioning System (GPS), Assisted GPS, Cellular technologies [3].
Positioning techniques can be classified according to different perspectives. First, regarding whether some prior knowledge is available or not. For instance, if a cartography of the environment is available, then fingerprinting like techniques with some machine learning like method, e.g., support vector machine, k-nearest neighbor, Bayesian classification, can be employed to match the current observation to the training phase where the mobile location is extrapolated from the (best) matching fingerprints [6-7]. Second, if there is no prior knowledge, then the position is inferred from the measurement according to a given geometrical principle. In this respect, one distinguishes triangulation, multilateration, hyperbolic like approaches, depending on the type of measurements (e.g., time difference of arrival (TDOA) entails hyperbolic principle, time of arrival (ToA) and read-signal strength (RSS) may entail triangulation, etc.) [3, 8-9]. From the perspective of the whereabouts of the software unit responsible for the mobile location determination, positioning systems are categorized into three groups: handset-based positioning; network-based positioning and; hybrid-positioning system [3]. In a handset-based positioning, the handset calculates its own location while in network-based positioning the network calculates the handset's location. In hybrid-positioning method, there is collaboration between the network and handset in order to measure and calculate the device's position. GPS is one of the examples of handset-based positioning in which position estimation will be done by handset and GPS based on signals received from at least four satellites. Examples of network-based positioning systems include cellular networks and Airborne Early Warning and Control System (AWACS). Lastly, Assisted GPS (A-GPS) is a good example for hybrid-positioning system. Throughout this paper, one shall mainly be interested in cellular technology [8]. In this respect, a device's location is usually estimated by monitoring a distance dependent parameter such as wireless signal strength or time of flight from the known base station (s) to the device. Such location is commonly estimated with respect to a reference location (s), often corresponding to that of (fixed) base stations. In order to explain the localization process, Figure 1 highlights the key elements involved in such process. The wireless device whose location is to be estimated is referred to a Mobile Station (MS), while the network entity with known location communicating with MS is referred to base transceiver station (BTS), or, base station for short.

Besides, from the hardware requirement, one distinguishes localization techniques requiring additional hardware that will be imposed on the top of existing infrastructure in order to ensure synchronization or performing specialized measurements, and those that do not require such burden additional costs. Techniques like Angle-of-Arrival based localization belong to the first class, while Cell-ID, Time-of-Arrival, Time difference of Arrival, Signal Strength belong to the second class [3, 8-9]. Because of the relatively low cost, localization techniques based on received signal strength (RSS) are particularly popular. In essence, this assumes a complete knowledge of network topology in terms BTS locations, where the use of radio propagation model allows us to bring the problem into determining the target position (mobile terminal) from a set of known distances. Several estimation based techniques can be employed to solve the underlying optimization problem. Nonlinear least squares [10-11] and Kalman

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filter [12] are quite popular for this purpose. Nevertheless, when the quality of measurements is degraded together with inadequate tuning of parameters of the filter, the risk of filter divergence is high with highly nonlinear systems [12]. This promotes non-conventional estimation theory including those based on fuzzy logic as potential candidates.

1.2 Fuzzy Kalman filtering for estimation

Several works focused on the application of fuzzy logic to strengthen standard estimators or to compensate for the lack of information regarding the model or noise statistics. A common example is when the covariance of the corrupting noise is unknown requiring the estimation of the (state) noise variance-covariance matrix. For this purpose, fuzzy rules can be applied. These are based on the difference between the measurement and its predicted value, since, if the filter works correctly, the residual should be zero-mean Gaussian. Examples of typical rules are [13-14]: “If residual is OK then Q is unchanged”; “If residual is very near to zero then Q is reduced”; “If residual is very far from zero, then Q is increased”. Another covariance estimation case is when the state transition noise is known or previously estimated, but the sensors state have changed during the process, requiring re-estimation of measurement noise variance-covariance matrix. To handle this issue, Loebis et al., [15] suggested fuzzy rules based on the difference between the theoretical and computed values of the covariance where a predefined difference is added (resp. subtracted) to (resp. from) an initial covariance. Similarly, [16-19] suggested a two-filter based approach where a correction gain were computed from other filter’s residual. Sasiadek et al. [20] introduced the Fuzzy Logic Adaptive System (FLAS) for adapting the process and measurement noise covariance matrices in navigation data fusion design. Abdelnour et al. [21] used the exponential- weighting algorithm for detecting and correcting the divergence of the Kalman filter. Kobayashi et al. [22] proposed a method combining Kalman filter and fuzzy inference system in order to compensate for initialization problems and inconsistent measurement using innovation information. A similar approach has also been investigated by Mostove and Soloviev [23] for enhancing accuracy of kinematic GPS. Simon [24] investigated a fuzzy Kalman filtering by forcing the outcome of Takagi-Sugeno rules coinciding with discrete Kalman filtering equations. Caron et al. [25] suggested to define the validity domains of the sensors using fuzzy sets. Simon [26] advocated the use of Kalman filtering in order to train and, subsequently, tune the parameters of the underlying fuzzy system; especially, the authors described a possible application, when the input and output membership functions are symmetric triangles, and an extended Kalman filter is used to estimate their centroid and half-widths.
Fuzzy Kalman filters can also be used for model selection/mixing. This occurs in applications such as tracking a maneuvering target where the maneuver detection is usually reduced to the estimation of a (possibly discrete) variable [27], or to computing the likelihood of different models, which may depend on some parameter [28] where a set of bank filters can also be employed. In the latter fuzzy rules may be applied to adapt a model set for a better coverage of possible maneuvers [29], or to adjust parameters of the models, or even to reinitialize the model if the track is lost [30], or to calculate the substitute a fuzzy measure to likelihood measure [31].
Fuzzy Kalman filters have also been investigated from the fusion architecture perspective. In [32], the authors used a fuzzy logic based Kalman filter to build adaptive, centralized, decentralized and federated Kalman filters for adaptive multisensory data fusion where the adaptation is carried out in the sense of adaptively adjusting the measurement noise covariance of each local filter in order to fit the actual statistics of noise profile present in the incoming measured data.
Several of the above works reported a better results in terms of estimation accuracy of the (adaptive) fuzzy Kalman filtering when compared to the extended Kalman filtering [18, 19] mainly using a set of Monte Carlo simulations. Nevertheless, it is acknowledged that the lack of wide scale comparative studies and the possible sensibility of the embedded linguistic variables prevented, at large extent the application of adaptive fuzzy Kalman filter in critical applications such as aircraft flight control systems [33]. This opens the door for further analysis both from theoretical and application oriented perspectives in order to demonstrate the effectiveness and usefulness of the underlying concept.

1.3 Contribution and paper organization

This work contributes to ongoing research activity on fuzzy Kalman filtering for estimation process with an application to cellular mobile positioning field. It builds on previous work, see Oussalah et al. [7] where an optimized fuzzy inference system is put forward in order to accurately locate a target using fingerprinting in WiFi based environment. In essence, a new fuzzy extended Kalman filtering (FEKF) is put forward. The proposal overlaps with the previously aforementioned work of tuning noise variance-covariance matrix using a dedicated fuzzy inference system. However, in contrast to previous works, the proposal advocates a decentralized like approach where single measurements are assumed to yield *local solutions* that will then be combined to output a global solution. More specifically, the FEK employs a zero-order Takagi-Sugeno system whose inputs are the innovation and its associated variance-covariance matrix and whose outputs consists of the weight attached to each local solution generated by a single BTS. The parameters of the underlying fuzzy systems have been optimized using ANFIS system. The performances of the developed FEKF are compared to both Extended Kalman Filter (EKF) and Fuzzy Control (FC) system where both simulated and real time measurements were employed. Besides, several theoretical results are pointed out.
This paper is organized as follows. In Section II, discusses the theoretical background and practical realization of measuring the distance between MS and BTS. The proposed strategy of the FEKF approach is introduced in Section III. Several parameters for determining the degree of divergence (DOD) are introduced for identifying the degree of change in mobile dynamics based on the innovation information. In Section V, simulation experiments on mobile dynamic localization are carried out to evaluate the performance of the approach in comparison to those by conventional EKF and FEKF. Conclusions are given in Section VI.

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2. BACKGROUND

Considering the typical positioning problem highlighted in Figure 1, let d_k^i be the (measured or inferred) distance from the i[th] base station to mobile station (MS) at time k. Let X_k and B_k^i be the 3D coordinates of the MS and ith BTS, respectively at time k. Assuming MS is almost fixed when the measurements are triggered, the state and measurement models are therefore described by:

$$\begin{cases} X_{k+1} = X_k + w_k \\ d_{k+1} = \begin{bmatrix} \|X_k - B_k^1\| \\ . \\ \|X_k - B_k^m\| \end{bmatrix} + v_k \end{cases} \quad (1)$$

Where ||.|| stands for Euclidean norm, w_k and v_k are independent zero mean Gaussian noise with known variance-covariance matrices Q and R, respectively. m is the total number of BTS triggered at time k.
Since the norm ||.|| is non-linear, the solution of the estimation problem in (1) requires a linearization around the target estimate X_k, which forms the essence of the extended Kalman filtering (EKF). The latter yields $\hat{X}_k$ an estimation of X_k as well as its associated variance-covariance matrix P_k. More specifically, the filter equations boil down to the prediction and update stages:

Prediction

$$\hat{X}_k^- = \hat{X}_{k-1} \quad (2)$$

$$P_k^- = P_{k-1} + Q_k \quad (3)$$

Update

Filter gain: $K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R_k\right)^{-1}$ (4)

$$\hat{X}_k = \hat{X}_k^- + K_k(d_k - \hat{d}_k) \quad (5)$$

$$P_k = (I - K_k H_k) P_k^- \quad (6)$$

Where $\hat{d}_k$ is the predicted measure given by $\|X_k - B_k\|$.
H_k is the measurement Jacobian matrix given by, for the i[th] measurement:

$$H_k^i = \begin{bmatrix} \frac{\partial \hat{D}_i(k)}{\partial x_k} & \frac{\partial \hat{D}_i(k)}{\partial y_k} & \frac{\partial \hat{D}_i(k)}{\partial z_k} \end{bmatrix} \quad (7)$$

$$\frac{\partial \hat{D}_i(k)}{\partial x_k} = \frac{(x_k - x_i)}{\sqrt{(x_k-x_i)^2+(y_k-y_i)^2+(z_k-z_i)^2}} \quad (8)$$

$$\frac{\partial \hat{D}_i(k)}{\partial y_k} = \frac{(y_k - y_i)}{\sqrt{(x_k-x_i)^2+(y_k-y_i)^2+(z_k-z_i)^2}} \quad (9)$$

$$\frac{\partial \hat{D}_i(k)}{\partial z_k} = \frac{(z_k - z_i)}{\sqrt{(x_k-x_i)^2+(y_k-y_i)^2+(z_k-z_i)^2}} \quad (10)$$

Where (x_i, y_i, z_i) stands for the 3D coordinate of the i[th] BTS.

The performance of the extended Kalman filter as indicated in Equations (2-6) is widely dependent on the behavior of the linearization carried out around the estimate $\hat{X}_k$, in addition to the quality of the initialization stage. This renders the convergence of the filter quite vulnerable [11-12]. For this purpose, several works have focused on monitoring the innovation of the measurements in order to decide on any corrective action in case where the deviation from its theoretical value is deemed to be important [12]. More specifically, the innovation is measured as:

$$Inn_k^i = d_k^i - \hat{D}_k^i \quad (11)$$

with its associated variance-covariance matrix:

$$S_k = H_k P_k H_k^T + R \quad (12)$$

A solution based on the use of fuzzy logic to tune the system and measurement noise has been proposed in [13-14], which allows the filter to reduce the risk of divergence. The key idea is to use an adaptive fuzzy logic based controller to continuously adjust the P and Q noise intensity depending on the gap between the theoretical and measured innovation. The covariance and mean of the residuals were used as input of the fuzzy controller, and the output is the degree of the divergence of the filter. In essence, it

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assumes that if the statistical covariance of the residuals is larger than its theoretical value in EKF formulating (12) and the mean value of the residuals is moving away from zero, then the Kalman filter is becoming unstable and will potentially diverge, therefore, the noise covariance get reduced according to the parametric expressions $R_k = \mathrm{R}\alpha^{-(2k+1)}$; $Q_k = \mathrm{Q}\alpha^{-(2k+1)}$, where R and Q are constant and α is the output of the fuzzy system.
Nevertheless, relaxing the noise constraint is also very debatable and could significantly impact the quality of the estimation as demonstrated in other studies, see, e.g., [12, 34]. This motivates the current proposal where the focus is rather shifted towards the quality of the measurement through monitoring the innovation sequence.

3. Hybrid Kalman filter and Fuzzy logic

3.1 Motivation and overview

The proposed FEKF relies on two main stipulates:

- First, inspired by the idea of decentralized Kalman filter [35] where the contribution of individual measurement to the global solution is explicitly quantified, a decoupling of the distinct measurements in the framework of Kalman filter is achieved. In other words, if a set of measurements are available, then instead of handling these measurements as a single measurement input vector, which would require handling large size innovation matrices, individual Kalman filter (local solution) is rather created for each single measurement. The outputs of the local filters are then combined via some master filter, see Figure 2. Under some linearity conditions, Felter [36] showed that such decentralized Kalman filtering is equivalent to centralized Kalman filtering. In our case, a single measurement corresponds to the case of a single base station. Therefore, the key is to generate local solutions induced by individual base stations, whose outcomes will next be combined in the same spirit as master-filter of decentralized Kalman filtering but involving fuzzy entities.
- Second, in order to deal with the divergence of non-linear filtering and in the same spirit as [20-21], the innovation and its associated variance-covariance matrix is monitored for each individual measurement, yielding a sort of reliability factor or confidence value to each local solution. This is achieved through the use of a Takagi-Fuzzy inference system [37] whose inputs are the innovation and its associated variance-covariance matrix and whose output is the reliability or weight factor as will be detailed in Section 3.2.
- Finally, the global estimate is therefore estimated as a weighted combination of the local solutions, say X_k, P_k, generated by individual measurements (base stations), making use of the weight factors (y_k) outputted by the fuzzy inference system. The detail of the combination process is provided in Section 3.3. Figure 3 provides a generic synoptic of the FEKF solution.

3.2 Fuzzy inference system

Fuzzy logic has been introduced by Zadeh in the sixties in order to represent imprecise and uncertain data. It provides an appealing framework to represent complex, ill-known and ambiguous entities. Especially fuzzy inference system [38] allows us to map fuzzy input variables to output variables. We particularly focused on zero-order Takagi-Sugeno fuzzy controller [37]. As pointed out earlier, we considered a two-inputs and one output system. The input variables or premises are innovation and its variance-covariance matrix, say, Inn_k^i and S_k^i, for the ith measurement. The output $y_i \in [0,1]$ quantifies the confidence attached to the ith local solution, where the $y_i = 0$ indicates a full inconsistency or lack of soundness of the ith solution, while $y_i = 1$ corresponds to a fully reliable (local) solution. In other words, y_i is the outcome of the fuzzy inference system when only the ith measurement was employed. The jth fuzzy rule reads as (we shall denote by y_{ij} the output according to the jth rule when the ith measurement is employed):

$$R^j \text{ IF } Inn_k^i \text{ is } A_{1j} \text{ AND } S_k^i \text{ is } A_{2j} \text{ THEN } y_{ij} = c_j$$

Where A_{1j} , A_{2j} are the fuzzy set describing the property attached to input variables Inn_k^i and S_k^i, respectively, and $c_j \in [0,1]$ are discrete weights outputted by the fuzzy system (j=1,m). The fuzzy sets A_{1j}, A_{2j} (j=1,m) are characterized by their membership functions $\mu_{A_{1j}}$ and $\mu_{A_{2j}}$. The aggregated output from the m rules is calculated as the weighted mean of the outputs of individual rules; namely,

$$y_i = \frac{\sum_{j=1}^{m} c_j . \mu_{A_{1j}}(Inn_k^i)\mu_{A_{2j}}(S_k^i)}{\sum_{j=1}^{m}(Inn_k^i)\mu_{A_{1j}}(Inn_k^i)\mu_{A_{2j}}(S_k^i)} \tag{13}$$

The term $\mu_{A_{1j}}(Inn_k^i)\mu_{A_{2j}}(S_k^i)$ is interpreted as the degree of fulfilment of the jth rule.

Trivially, the number of fuzzy rules m and the values of the discrete weight factors c_j are very much dependent on the number of partitions ascribed to fuzzy sets A_{1j} and A_{2j} within their associated universe of discourse. This will be detailed in the optimization subsection later on.

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Intuitively, one expects that the reliability of the local solution to be high if the innovation and its associated variance-covariance matrix are deemed to be sufficiently small where the quantification "sufficiently small" is captured by the underlying fuzzy set (e.g., A_{1j} for the innovation part and A_{2j} for the variance-covariance matrix). This translates the scenario where the prediction is sufficiently close to the actual measurement (small innovation) with high accuracy (small variance-covariance matrix). On the hand, the reliability should be deemed to be low whenever the innovation and its associated variance-covariance matrix are deemed to be high where the quantification high is also captured by the fuzzy sets A_{1j} and A_{2j}.

3.3 Combination of local solutions

3.3.1 Formulating of the combined state and variance-covariance estimates.

The process of estimating the state variable X_k and its variance covariance matrix P_k using the output of fuzzy inference system involves the use of local Kalman filter solutions provided at each individual single measurement. The global estimate is therefore calculated as a weighted average of the local solutions where the weights correspond to the outcome of the fuzzy inference system. More specifically,

$$\hat{X}_k^i = \hat{X}_{k-1} + K_k^i(Inn_k^i) \tag{14}$$

is the local (Kalman filter) estimation obtained when only the i[th] BTS is employed, where Inn_i corresponds to the innovation from the i[th] measurement and K_k^i is Kalman gain as in expression (4) for H_k^i . For N measurements issued from N BTS, the global estimation is calculated as, provided $\sum_{i=1}^{N} y_i > 0$:

$$X_k^{est} = \frac{\sum_{i=1}^{N} \hat{X}_k^i\, y_i}{\sum_{i=1}^{N} y_i} \tag{15}$$

Notice that since there is at least one fuzzy rule which is activated at a given time, it holds that for at least one measurement $y_i > 0$, which makes the expression (15) always definite.

(15) assumes that the global estimate of the target corresponds to the weighted estimate of the local estimates in view of (14). This agrees with the intuition pertaining to the outcome of the fuzzy inference system. On the other hand, (15) also resembles to the concept of federated Kalman filter [35] where the different subsystems were obtained by partitioning the measurement vector into individual single measurements. Especially, the following holds

Next, the calculus of the variance-covariance matrix associated to the estimate X_k^{est} is carried out using the statistical definition of the variance-covariance matrix. In this respect, the following holds.

Proposition 1

The variance-covariance matrix associated to X_k^{est} is given by

$$P_k^{est} = P_k^- - \frac{\sum_{i=1}^{N} P_k^- {H_k^i}^T {K_k^i}^T y_i}{\sum_{i=1}^{N} y_i} - \frac{\sum_{i=1}^{N} K_k^i H_k^i P_k^{-T} y_i}{\sum_{i=1}^{N} y_i} + \frac{\sum_{i=1}^{N} K_k^i S_k^i (y_i)^2 {K_k^i}^T}{(\sum_{i=1}^{N} y_i)^2} \tag{16}$$

Proof

First one uses the definition of the variance-covariance matrix

$$P = E[(\mathrm{X} - E[X])(\mathrm{X} - E[X])^T] \tag{17}$$

Therefore, for X_k^{est}, we have:

$$P_k^{est} = E[(X_k - X_k^{est})(X_k - X_k^{est})^T] \tag{18}$$

Where X_k^{est} can be rewritten as

$$\begin{aligned} X_k^{est} = \frac{\sum_{i=1}^{N} \hat{X}_k^i\, y_i}{\sum_{i=1}^{N} y_i} &= \frac{\sum_{i=1}^{N}\left(\hat{X}_{k-1} + K_k^i(Inn_k^i)\right) y_i}{\sum_{i=1}^{N} y_i} \\ &= \frac{\sum_{i=1}^{N}\left(\hat{X}_k^- + K_k^i(Inn_k^i)\right) y_i}{\sum_{i=1}^{N} y_i} \\ &= \hat{X}_k^- + \frac{\sum_{i=1}^{N} K_k^i\, Inn_k^i\, y_i}{\sum_{i=1}^{N} y_i} \end{aligned} \tag{19}$$

Substituting in (18), it holds that

$$P_k^{est} = E\left[\left(\left(X_k - \hat{X}_k^-\right) - \frac{\sum_{i=1}^{N} K_k^i Inn_k^i y_i}{\sum_{i=1}^{k} y_i}\right)\left(\left(X_k - \hat{X}_k^-\right)^T - \frac{\sum_{i=1}^{N} {Inn_k^i}^T {K_k^i}^T y_i}{\sum_{i=1}^{k} y_i}\right)\right] \tag{20}$$

Next one notices that:

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$$P_k^- = E\left[(X_k - \hat{X}_k^-)(X_k - \hat{X}_k^-)^T\right] \tag{21}$$
$$Inn_k^i = H_k^i\left(X_k - \hat{X}_k^-\right) + \nu_k \tag{22}$$

Using the linearity of the expectation operator and the zero mean of the noise ν_k:

$$E\left[\left(X_k - \hat{X}_k^-\right)\left(Inn_k^i\right)^T\right] = E\left[\left(X_k - \hat{X}_k^-\right)\left(X_k - \hat{X}_k^-\right)^T\right] H_k^{i\,T} \quad = P_k^- H_k^{i\,T} \tag{23}$$

Similarly, using the zero-mean and decorrelation property of the noise, it holds that

$$E\left[\sum_{i=1}^{N} \mathrm{K_k} Inn_k^i y_i \sum_{i=1}^{N} Inn_k^{i\,T} K_k^{i\,T} y_i\right] = \sum_{i=1}^{N} K_k^i S_k^i (y_i)^2 \, K_k^{i\,T} \tag{24}$$

Substituting the above in (20), the expression (16) of Proposition 2 follows straightforwardly. □

Expression (15-16) therefore constitutes the basis for the combination rule that combines the local solutions generated by individual sensors (BTS). From an algorithmic perspective, Table II summarizes the FEKF.

3.3.2 Properties of the combination rule

On other hand, it should be interesting to investigate the properties of the solution provided by (15-16) in some boundary cases. In this respect, one notices the following.

Proposition 2

The target estimation X_k^{est} is always situated within the n-ary convex polygon formed by the N local solutions $\hat{X}_k^i$ (i=1,N).

The proof of Proposition 1 follows straightforwardly by construction given that (15) can be seen as a linear convex combination of the $\hat{X}_k^i$, (i=1, N). This is due to the fact that (15) can be rewritten as $X_k^{est} = \sum_{i=1}^{N} \hat{X}_k^i \, y'_i$, with $y' = \frac{y_i}{\sum_{i=1}^{N} y_i}$ where $\sum_{i=1}^{N} y'_i = 1$. Using the geometrical properties of convex linear combination [11], the result is straightforward.

Since the local estimates $\hat{X}_k^i$ are typically close to B_k^i, Proposition 1 reinforces the natural result that the target estimate is within the region obtained by the set of base stations activated at time k. A special case of three-local solutions is shown in Figure 4 where the target position estimate is highlighted using the star mark (*). Especially, it is shown that when one weight, say, y_2=0, then the target estimate is located on the edge formed by the local solutions $\hat{X}_k^1$ and $\hat{X}_k^3$. Otherwise, the triangle formed by vertices 1, 2 and 3 represent the geometrical area where all possible solutions X_k^{est} lie in.

Interestingly, one may also notices some boundary cases.

- If all local solutions are fully unreliable, then no solution is induced by our FEKF as expression (15) does not hold, which is in full agreement with the intuition.
- If one solution is fully unreliable; that is, $y_i = 0$ for some ith solution, then the latter is fully discarded from the estimation of the combined estimate X_k^{est} in view of expression (15). This is also in full agreement with the intuition as one expects a non-reliable input not to be taken into account by the fusion scheme.
- If all local solutions provide the same output in terms of reliability factor and estimate X_k^i, then the output of the fusion process also yields the same input X_k^i regardless the value of the reliability factor. In other words, any agreement among the local solutions is made preserved by the underlying fusion operation.
- Another interesting case arises when only one local solution is full reliable, say, $y_j = 0$ and all others are fully unreliable; that is, $y_i = 0$ for all i≠j. In such case, it is easy to notice that the application of (15) yields a global estimate which exactly coincides with the fully reliable local solution $\hat{X}_k^j$.
- If all local solutions are fully reliable $y_i = 1$ for all i=1,N, the global estimate corresponds to the geometric centroid of all local solutions (arithmetic mean).
- Finally, it is worth investigating the influence on the variance-covariance matrix of local solutions, say P_k^i on the variance-covariance matrix of the combined estimate P_k. For this purpose, we first require to write expression (16) in terms of local variance covariance matrices. In this respect the following holds.

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Proposition 3
An equivalent expression of (16) is given by

$$P_k^{est} = \frac{P_k^{-}\left(P_k^{-T}\right)^{-1}\sum_i\left(P_k^{i^T} y_i\right)}{\sum_i y_i} + \frac{\left(\sum_i\left(P_k^i y_i\right)\right)\left(P_k^{-}\right)^{-1}P_k^{-T}}{\sum_i y_i} + \frac{P_k^{-}\left(\sum_i H_k^{i^T}\left(S_k^{i^T}\right)^{-1}H_k^i\left(y_i\right)^2\right)P_k^{-T}}{\left(\sum_i y_i\right)^2} - P_k^{-T} \qquad (25)$$

Proof
Using the expressions of the filter gain and variance-covariance matrix (4) and (6), we have:

$$P_k^i = \left(I - K_k^i H_k^i\right)P_k^{-} \quad \text{and} \quad K_k^i = P_k^{-}H_k^{i^T}\left(S_k^i\right)^{-1} \qquad (26)$$

This entails, using the properties of inverse and transpose matrices

$$K_k^i H_k^i = I - P_k^i\left(P_k^{-}\right)^{-1} \quad \text{and} \quad H_k^{i^T}K_k^{i^T} = \left(I - P_k^i\left(P_k^{-}\right)^{-1}\right)^T = I - \left(P_k^{-T}\right)^{-1}P_k^{i^T} \qquad (27)$$

Similarly, we have

$$K_k^i S_k^i K_k^{i^T} = P_k^{-}H_k^{i^T}\left(S_k^i\right)^{-1}S_k^i\left(S_k^{i^T}\right)^{-1}H_k^i P_k^{-T} \qquad (28)$$

Substituting (27) and (28) in (15) and after some manipulations of the matrix sum operations, the result is straightforward. □

Interestingly, if P_k^{-} is symmetric so that $P_k^{-} = P_k^{-T}$, then (25) reduces to

$$P_k^{est} = \frac{\sum_i\left(P_k^{i^T} y_i\right)}{\sum_i y_i} + \frac{\left(\sum_i\left(P_k^i y_i\right)\right)}{\sum_i y_i} + \frac{P_k^{-}\left(\sum_i H_k^{i^T}\left(S_k^{i^T}\right)^{-1}H_k^i\left(y_i\right)^2\right)P_k^{-}}{\left(\sum_i y_i\right)^2} - P_k^{-} \qquad (29)$$

Now it order to see the influence of the variance covariance inputs P_k^i (i=1, N) on the variance-covariance matrix of the estimate P_k^{est}, a rational criterion that defines the extent of the matrix is its trace. In this respect, the following holds.

Proposition 4
$Trace\left(P_k^{est}\right)$ is monotonic increasing with respect to $Trace\left(P_k^i\right)$.

The proof follows straightforwardly from the linearity property of the trace matrix when applied to expression (25) and the invariance of P_k^{-} entity. Besides using differentiation properties of trace matrices, we have:

$\frac{\partial Trace\left(P_k^{est}\right)}{\partial P_k^i} = P_k^{-}\left(P_k^{-T}\right)^{-1}\frac{y_i}{\sum_i y_i} + \frac{y_i\left(P_k^{-}\right)^{-1}P_k^{-T}}{\sum_i y_i} \approx \frac{2y_i}{\sum_i y_i} \quad if\ P_k^{-}\ symmetric$. The latter is positively valued, which justifies the monotonicity property. □

More the application of trace matrix to expression (29) yields

$$Trace\left(P_k^{est}\right) = \frac{2\sum_i y_i Trace\left(P_k^i\right)}{\sum_i y_i} + Trace\left[\frac{P_k^{-}\left(\sum_i H_k^{i^T}\left(S_k^{i^T}\right)^{-1}H_k^i\left(y_i\right)^2\right)P_k^{-}}{\left(\sum_i y_i\right)^2}\right] - Trace\left(P_k^{-}\right) \qquad (30)$$

Since the last two entities of the right hand side of equation (30) are constant with respect to P_k^i, the linearity of $Trace\left(P_k^{est}\right)$ with respect to $Trace\left(P_k^i\right)$ is demonstrated.

As a consequence of the above, if the local solutions induce estimates P_k^i of smaller order of magnitude in the sense of trace matrix, then the outcome of the combination rule will also induce an estimate whose variance-covariance matrix presents a lower order of magnitude. This is in full agreement with the intuition that an increase in accuracy of inputs would increase the accuracy of the outcome as well.

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From (25), one should also notice that if the local solutions have all the same variance-covariance matrix P_k^i , the outcome of the combination rule does necessarily have the same variance-covariance matrix as it is much dependent on the quality of the prediction P_k^- as well as the parameters of the individual systems.

In probabilistic setting, the combination of the local solutions is such that [39]

$$X = P \sum_i X_k^i \left(P_k^i\right)^{-1}$$
$$P = \left[\sum_i \left(P_k^i\right)^{-1}\right]^{-1} \tag{30}$$

In contrast to our FEKF, the variance-covariance matrix of the outcome of the probabilistic combination model is only dependent on the variance-covariance matrices of the individual inputs P_k^i , which appears to be problematic in case of presence of non-reliable sources.

3.4 Optimization of the fuzzy system parameters

The fuzzy inference system with (fuzz) inputs *Inn* and *C-Inn* corresponding to the innovation and its associated variance-covariance matrix for each measurement and time increment. While the (crisp) output corresponds to the discrete constant values c_i (with $0 \le c_i \le 1$), i=1,p, because of the use of zero-order Takagi-Sugeno controller, like:

IF Inn_k^i is Small AND S_k^i is Large THEN $y_i = c_j$

The number of fuzzy rules as well as the number of constants c_i is directly related to the number of partitions of input variables Inn_k^i and S_k^i. For this purpose, ANFIS system proposed by Jang [40] was employed. The latter is based on neuro-fuzzy system that makes use of a combination of least squares and gradient back-propagation algorithms. Such optimization is carried out on training dataset in order to find the optimal configuration of the various parameters; namely, the number of partitions of Inn_k^i and the number of partitions of S_k^i, the number of constants c_i, shape of membership functions associated to Inn_k^i and S_k^i, type of defuzzification method. Matlab implementation of ANFIS was employed. On the other hand, in order to speed up the optimization process and reducing the search domain, we deliberately chosen triangular membership functions. Besides, the training database was generated by choosing a simulated linear trajectory of the target as will be detailed in the next section. In order to get an order of magnitude of the range of values attached to the input variables
Inn_k^i and S_k^i, an extended Kalman filter based method is applied. In this respect, it was observed that Inn_k^i takes values in range [0, 1.5 Km], while S_k^i takes values in [0.22 0.32]. This was especially helpful to identify the universe of discourse of the fuzzy sets associated to the input variables. On the other hand, during the training phase, the output y_i is chosen proportional to the confidence attached to the position of the target given the full knowledge of the target positioning. The result of the ANFIS optimization is summarized in Table I in terms of fuzzy rules, and Figure 5 in terms of membership functions of premises Inn_k^i.

Similarly, the ANFIS based optimized membership function of the 2nd input S_k^i is provided in Figure 6.

3.5 Alternative fuzzy controller based approach

On the other hand, in order to assess the contribution of the Kalman filtering process in FEKF, a comparison with a more conventional fuzzy controller has been carried out. For this purpose, a fuzzy controller whose inputs are the distance measurements BTS-MS and the outputs are the weights associated to such measurements. Namely, to draw analogy with FEKF approach, the concept of local and global solution is preserved where the combination rule (14-15) is still employed. However, instead of a two-input fuzzy inference (and involving Kalman filtering parameters) as in FEKF, a *cheap* fuzzy inference system where the input is only constituted by the distance BTS-MS, so that a confidence factor is outputted for each distance. The rationale behind such process is to assign higher weights to measurements yielding smaller distances and smaller weights for those yielding higher distances. This agrees with practical considerations as larger distances are subject to non-light-of-sight effect and other random perturbations. Besides, in wireless communications, the mobile station is likely to get connected to BTS which is situated nearby instead of those situated far away. One shall refer to this alternative cheap fuzzy estimator, “Fuzzy controller system” (FCS). Similarly to FEKF fuzzy inference system, an ANFIS based system has been employed to tune the parameters of FCS (number of partitions of input variable –distance-, and weights). Table III summarizes the outcome of this optimization process.

For example,

If the distance « d » is very small, then the weight “c” is equal to 1
If the distance "d" is small then the weight “c” is equal to 0.75
If the distance “d” is Very large then, the weight “c” is null.

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The membership function associated to the distance input as outputted by the ANFIS system is shown in Figure 7.

The purpose of this alternative proposal is to challenge the usefulness of FEKF whose fuzzy inference system employs innovation and its variance-covariance matrix.
It should be noted that the process of optimizing the parameters of FCS has been investigated considering an urban like environment where the average radius of cells is around 100 meters. The generic scheme of the FEKF is kept unchanged. More specifically, similarly to FEKF, Table IV provides a generic synoptic of the FCS from an algorithmic perspective.

4. Experimentation

4.1 General Framework

In order to evaluate the performance of the developed FEKF in the context of mobile cellular positioning, experimentation in urban/suburban Algiers area has been conducted. The network topology and base stations characteristics have communicated by the mobile operator Mobilis.

Ericsson TEMS Investigation 8.0.3 Data Collection Software [41] together with GPS enabled handset have been employed as shown in Figure 8 for a particular drive test image. The software also allows us to force handover in order to communicate with surrounding base stations, which enable the user to get information from several base stations at the same time.

On the hand, the use of GPS devices in handset allows us to quantify the positioning error with relatively high accuracy; namely, one uses the root mean square error metric:

$$\mathrm{RMSE}(k) = \sqrt{(x_{k,real} - x_{k,est})^2 + (y_{k,real} - y_{k,est})^2 + (z_{k,real} - z_{k,est})^2} \qquad (25)$$

Where $X_{k,est}$ and $X_{k,real}$ are the estimated and real position of the target, respectively.
Prior to real time experimentation, a simulation study has been carried. Especially, the availability of several drive test images through TEMS allows us to build consistent simulation dataset.

4.2 Simulation

We considered two simple, but realistic, scenarios of a linear trajectory and a turning manoeuver of two-linear trajectories where three BTS at fixed known locations were active at each time increment.
The distance BTS-MS is therefore augmented with a zero-mean Gaussian noise of fixed variance-covariance corresponding to a noise of moderate intensity. That is,

$$d(BTS,MS) = d_{real}(BTS,MS) + noise \qquad (26)$$

with

$$noise \sim \mathcal{N}(0, \sigma),$$

where $\sigma = 0.001$

The estimated position is therefore calculated over a set of Monte Carlo simulations (100 Monte Carlo simulations) in order to take into account the various realizations of the noise.
Figures 9 and 10 illustrate the performance of the FEKF in terms of real path and RMSE. The results were compared to EKF and FCS. The initial parameters of the filter were chosen as

$$R_0 = 0.222$$

$$P_0 = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 0.020 & 0 & 0 \\ 0 & 0.020 & 0 \\ 0 & 0 & 0.002 \end{bmatrix}.$$

From results shown in figures 9, 10, the superiority of the FEKF is clearly highlighted. The results also show that FCS yields the worse performance among the three positioning algorithms. This can partly be explained by the inadequacy of the distance measurements for calculating the weights attached to individual measurements, in case of relatively low noise influence. Table V provides the overall average results in terms of RMSE over the whole trajectory for cases of both single linear trajectory and two-linear trajectory with a manoeuver.

Table V shows that FEKF outperforms the EKF by 30.5% and 32.5% in case of single linear and two-linear trajectory, respectively. While it outperforms FCS by 78% and 84% in case of single linear and two-linear trajectory.

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In order to quantify the statistical significance of the results pointed out in Table 5 given the fluctuation of the result from one Monte Carlo simulation to another, we carried out a (student) t-test [42] in order to test the null hypothesis that the RMSE-mean of FEKF technique and that of alternative technique (either EKF or FCS) are similar. At default 5% significance level, a *p*-value of the t-test of less than 0.05 ($p \leq 0.05$) indicates a rejection of the null hypothesis, and therefore a confirmation of the observation

RMSE-mean (FEKF) < RMSE-mean (FEKF)

and

RMSE-mean (FEKF) < RMSE-mean (FCS)

The results of this statistical test are summarized in Table VI. Since the p-values of the t-test are in both single linear and two-linear trajectories confirm the hypotheses that RMSE of FEKF is less than RMSE of EKF and RMSE of FCS. On the other hand, the analysis of the standard deviation results shown in Table V indicates that FEKF exhibits less fluctuations of the results when compared to both EKF and FCS. We believe the observed enhanced performances of the FEKF are rooted back to the efficiency of the centralized architecture of the filter as well as the optimized parameters of the underlying fuzzy inference system obtained using the ANFIS system. The established theoretical foundations of the filter also contributed to the stability of the outcome.

We have also investigated the influence of the noise intensity in the overall performance of the FEKF. In this respect, Table VII provides the RMSE for various noise intensities averaged over the whole trajectory.

Results of Table VII testify of the superiority of the FEKF in case of low-medium noise intensity. Nevertheless, when the noise intensity becomes higher, the FCS becomes progressively competitive. This again can be explained by the growing influence of the distances in case of higher noise intensity, which possibly translate into important non-light of sight factors. However, this result should be taken with cautious as with high noise intensity, the signal becomes blurred, which may question the usefulness of the result.

The influence of the initial parameters of the filter; namely, R_0, P_0 and Q_0 have also been investigated as illustrated in figures 11-14. A comparison between EKF and FEKF has been carried out. Note that the FCS does not use such parameters. On the other hand, we deliberately took the diagonal elements in P and Q of equal quantity in order to simply the analysis. This is also in agreement with previous studies in this context, see, e.g., [12] and references therein.

Especially, figures 11-13 show the existence of a range of values for which the performance of the filters, either EKF or FEKF, is stable. For instance, values of Q_0 (diagonal elements) between 0.02 and 5050 provide relatively similar performance, which also can act as good guess to initialize the filter (s). Similarly, values of P_0 between 0.001 and 0.1 provide a good guess to initialize the filter (s). While Figure 11 indicates rather a relatively good robustness of FEKF with respect to some initial values of R_0 as compared to EKF.

4.3 Real time experiment

In contrast to simulation data, in real time experiment, the distances BTS-MS are not directly available. For this purpose, TEMS investigation 8.0.3 software was first used to retrieve the network key components, including, cell identity (CI) of surrounding base stations communicating with the mobile station, as well as their physical characteristics, i.e., frequency, height, channel number, transmitted power.

By forcing the hand over in order to communicate directly with a specific CI (or BTS) using the channel number of such cell as pointed by the software, TEMS investigation also allows us to display the received signal strength (Rx) transmitted by each of the surrounding BTS pointed out at previous test. The determination of the location of the BTSs is accomplished using the data base of the sites in the public operator *Mobilis Ltd*., which updates the location of GSM base stations throughout Algiers area. It is therefore possible to measure the latitude/longitude positioning of all the surrounding base stations. An instance of drive test experience as outputted by TEMS is shown in Figure 14.

The distance between each BTS and the handset is determined using one of the empirical propagation models. We focused in this paper on Walfish-Ikigami propagation model [43], see [44-45] for an exploration of alternative models. Basically, the model provides an expressing of the path loss of the signal transmitted by the BTS (Tx) and the received signal at the MS receiver (Rx), as a function of the distance between BTS and MS and the carrier frequency f, also determined using TEMS investigation displayed parameters.

The carrier frequency f is determined by the following expression [46-47]:

$$f = 1805 + 0.2\,(\mathrm{ARFCN} - 511)$$

where ARFCN stands for BTS carrier channel number as displayed by TEMS investigation software. The mathematical detail of the application of the Walfish-Ikigami model is reported to the Appendix of this paper. Table VIII provides an example of the BTS-MS calculus at specific locations of the above drive test data.

Similarly to the simulation results, the outcome of a single drive test is reported in figures 15 and 16 in terms of itinerary and RMSE performances. Comparison with EKF and FCS is also highlighted.

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The results shown in figures 15-16 agree with the simulation results and demonstrate the usefulness, feasibility and attractiveness of the FEKF where its superiority over both FCS and EKF is clearly highlighted. RMSE results averaged over the whole trajectory are shown in Table IX.

Similarly to the simulation study, we also investigated the sensibility of the result using t-test testing while repeating the real-time experiment hundred of times in order to accommodate, at some extent, the various noise realizations. The results summarized in Table X demonstrate the validity of the observation exhibited in Table IX about the superiority of the FEKF in both urban and semi-urban environment.

In order to evaluate the agreement between the simulation and real time results in terms of the influence of initial parameters, we have also investigated the influence of initialization R_0, Q_0 and P_0 on the RMSE results. In this respect, figures 17-19 provide direct counterparts to figures 11-13.

5. Conclusion

This paper presented a new estimation algorithm based on a hybrid combination of extended Kalman filter and zero-order Takagi-Sugeno fuzzy inference system. The latter takes the innovation and its variance-covariance matrix as inputs, and outputs the weight attached to each local solution obtained when a single measurement in terms of BTS was employed. Inspired from the idea of federated Kalman filter, the same approach has been adopted in order to combine local solutions generated by taking individual measurement separately. The global estimate is therefore calculated as a weighted mean of these local solutions, while theoretical estimation of its variance-covariance matrix is determined by making use of statistical definition of the covariance. Properties of the underlying combination scheme is investigated and original results were laid down. The elaborated fuzzy extended Kalman filter is next applied to localization of mobile station in cellular network with known topology. A simulation platform has been employed to optimize the parameters of the fuzzy inference system using ANFIS like approach. The performances of the FEKF have been evaluated using root mean square error metric in both simulated and real time dataset. The results in both cases confirmed the usefulness of the FEKF and its superiority to standard extended Kalman filter as well as a standard fuzzy controller whose inputs are the BTS-MS distance measurements, and outputs the weight or confidence associated to the measurement. We believe that part of the increased performances brought by the FEKF are due to its decentralized architecture, which allows it to deal with complex non-linearity and correlation issues that restricted the performances of the EKF. Besides, its optimized parameters using ANFIS system conveys an edge compared to its possible competitors.

On the other hand, the promising theoretical and experimental results open new era for future fuzzy Kalman filter designs embedding, for instance, concepts from particle filters or unscented filtering in order to handle more efficiently the non-linearity issues while enhancing the computational efficiency of the proposal.

From the cellular mobile positioning perspective, this work also opens new opportunities to add extra sources of information as part of *local solution* of the filter, issued, for instance, from any known or recognized landmark, WiFi signal, among others, in order to strengthen the global solution induced by the filter. Together with theoretical investigations of the convergence properties of the filter, this will would constitute part of our perspective work in the near future.

Appendix

The Walfish-Ikigami model defines a set of parameters intervening in the expression of the model, see [22, 24]:

- Loss=Tx−Rx: Path loss (dB)
- f: Frequency bearer (MHz): $800 \leq f \leq 2000$.
- h_b: Height of antenna (m) of the base station in relation to soil: $4 \leq h_b \leq 50$.
- h_m: Height of antenna (m) of the mobile station in relation to soil: $1 \leq h_m \leq 3$.
- h_r: Middle height (m) of the buildings: $h_r \geq h_{hm}$.
- W: Width of the road (m) where the mobile is situated
- b: Distance (m) between the centers of buildings
- d: Distance (Km) between the BS and the mobile: $0.2 \leq d \leq 5$.
- α: Angle (in degrees) that makes the journey with the axis of the road

$\Delta h_b = h_b - h_r$: Height of BS to the cover of the roofs.
$\Delta h_m = h_r - h_m$: Height of MS below the roofs.

- Case of Line Of Sight LOS

$$L_p = 42.64 + 26\log(d) + 20\log(f)$$

- Case of Non Line Of Sight NLOS

$$L_{oss}(dB) = \begin{cases} L_{fs} + L_{rts}\ (dB) + L_{msd}(dB) \\ L_f \qquad If\ \ L_{rts} + L_{msd} \leq 0 \end{cases}$$

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With:

L_{fs}: the attenuation in free space

L_{rts}: the attenuation due to the diffraction on the roofs of the buildings.

L_{msd}: the attenuation due to the multiple diffractions

- The attenuation in free space:
  L_{fs}= 32,44+20Log(d) +20Log(f)
- The attenuation due to the diffraction on the roofs of the buildings:

  L_{rts}= -16,9 – 10 Log (w) + 10 Log(f) +20 Log (Δh_m) + L_{ori}

L_{ori}: is a term that depends on the orientation of the road in relation to the emitter.

$$L_{ori} = \begin{cases} -10 + 0{,}3574\alpha & 0 \leq \alpha \leq 35° \\ 2{,}5 + 0{,}075(\alpha - 35) & 35° \leq \alpha \leq 55° \\ 4 - 0{,}1004(\alpha - 55) & 55° \leq \alpha \leq 90° \end{cases}$$

- The attenuation due to the multiple diffractions:
  L_{msd}= L_{bsh}+K_a+K_dLog (d) +K_fLog (f) – 9Log (b)

K_a and K_d **:** are two factors of empiric correction of the height of the antenna.

K_f: is a factor of adaptation of the different densities of the buildings.

With :

$$L_{bsh} = \begin{cases} -18Log(1 + \Delta h_b) & h_b > h_r \\ 0 & h_b \leq h_r \end{cases}$$

$$K_a = \begin{cases} 54 & h_b > h_r \\ 54 - 0{,}8\Delta h_b & d \geq 0{,}5 \;\; nd \; h_b \leq h_r \\ 54 - 0{,}8\Delta h_b \left(\frac{d}{0{,}5}\right) & d < 0{,}5 \;\; and \; h_b \leq h_r \end{cases}$$

$$K_d = \begin{cases} 18 & \Delta h_b > 0 \\ 18 - 15\left(\frac{\Delta h_b}{\Delta h_m}\right) & \Delta h_b \geq 0 \end{cases}$$

$$K_f = \begin{cases} -4 + 0{,}7\left(\frac{f}{925} - 1\right) & Average\ city \\ -4 + 1{,}5\left(\frac{f}{925} - 1\right) & Big\ city \end{cases}$$

In the absence of detailed data on the structure of the buildings, the Cost231 recommends the following values:

20 ≤b ≤50m, w=b/2.

In our test data, we used the following:

Distance (m) between the centers of buildings b= 50 m, Width of the road w=25 m, the angle (in degrees) who makes the journey with the axis of the road α = 30°, the middle height (m) of the buildings h_r =15 m.

The carrier frequency *f* is determined using the already pointed out formula:

f = 1805 + 0.2 (ARFCN *-511)*

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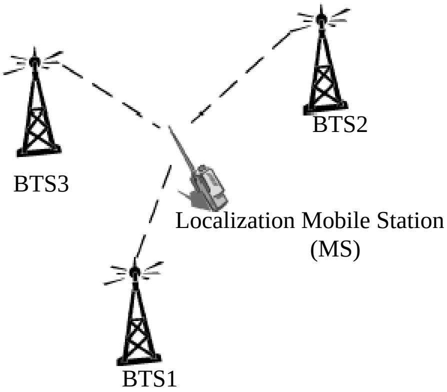

Fig. 1 Localization framework

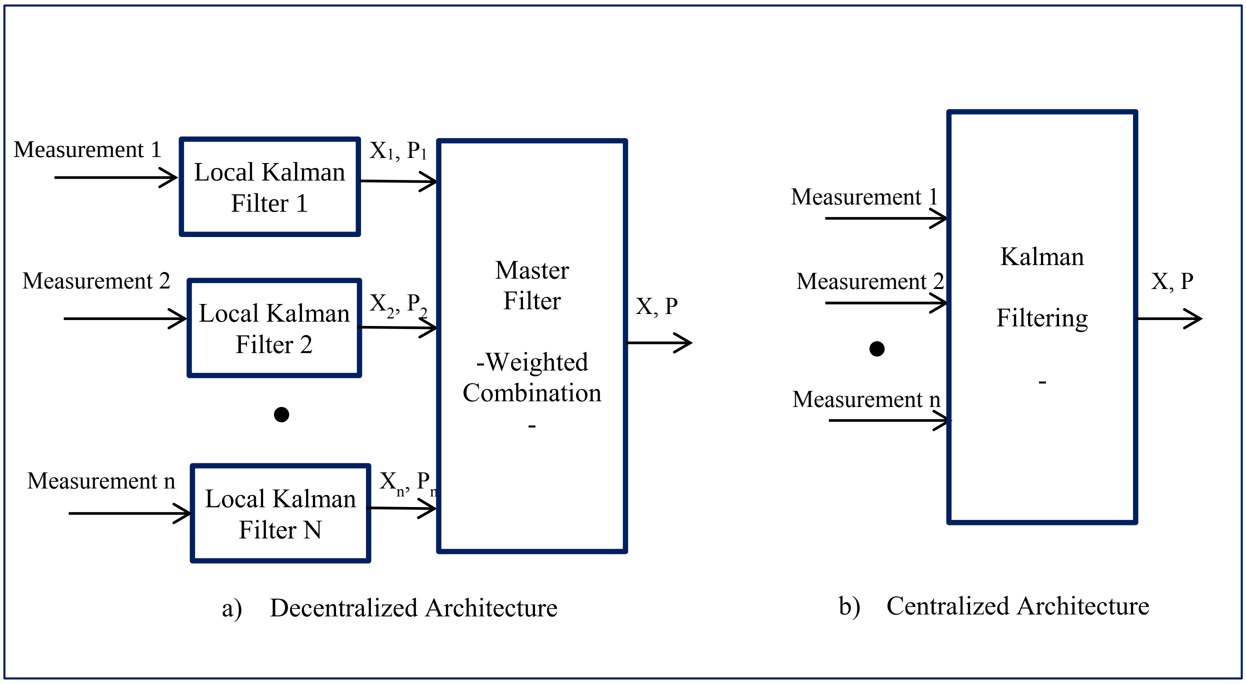

Figure 2. Centralized versus decentralized Kalman Filtering architectures

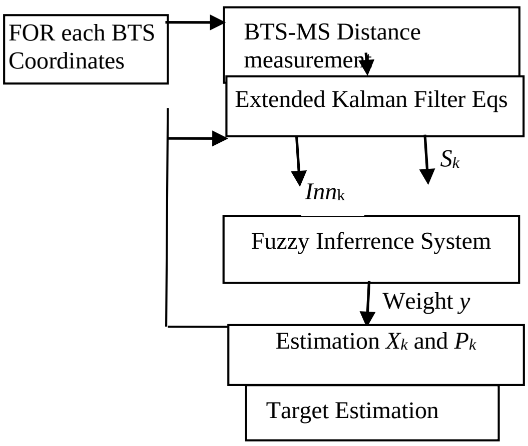

Figure 3. Generic scheme of FEKF

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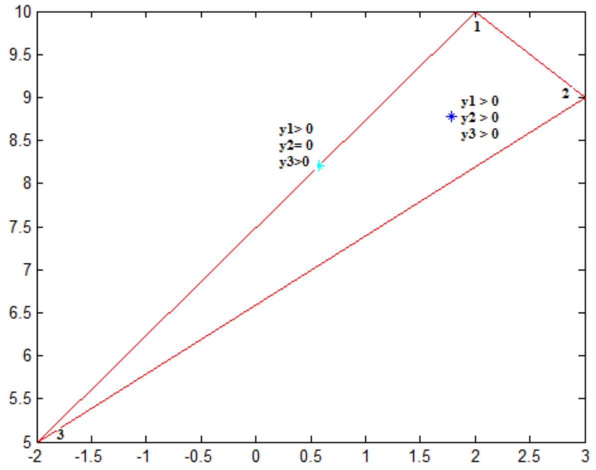

Figure 4. Example of target positioning with respect to weights y_i.

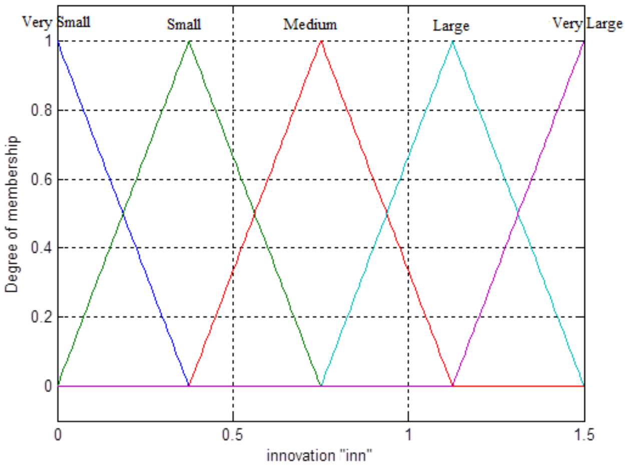

Figure 5. Membership function of input variable "*innovation"*

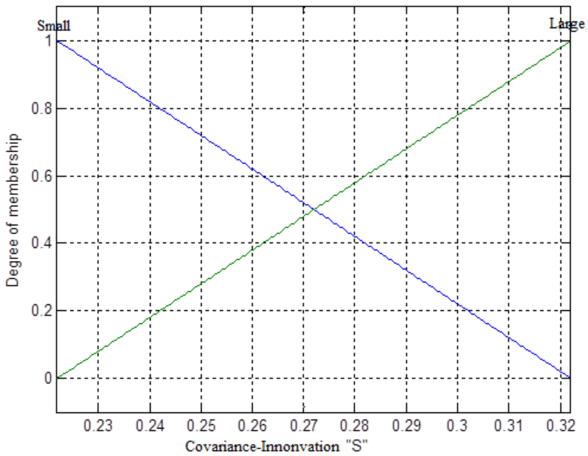

Figure 6. Membership function of input variable "Covariance-*innovation"*

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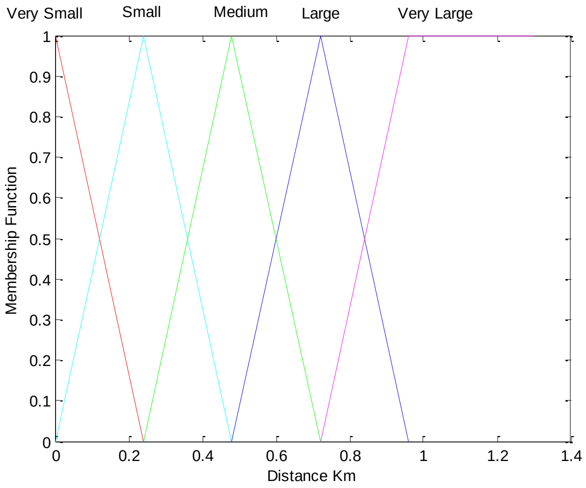
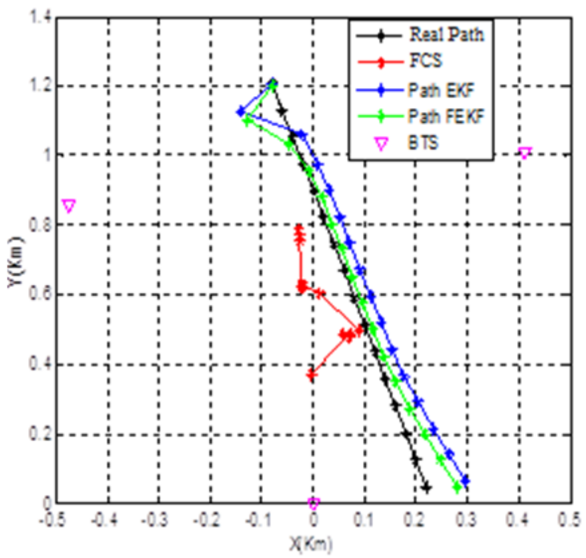
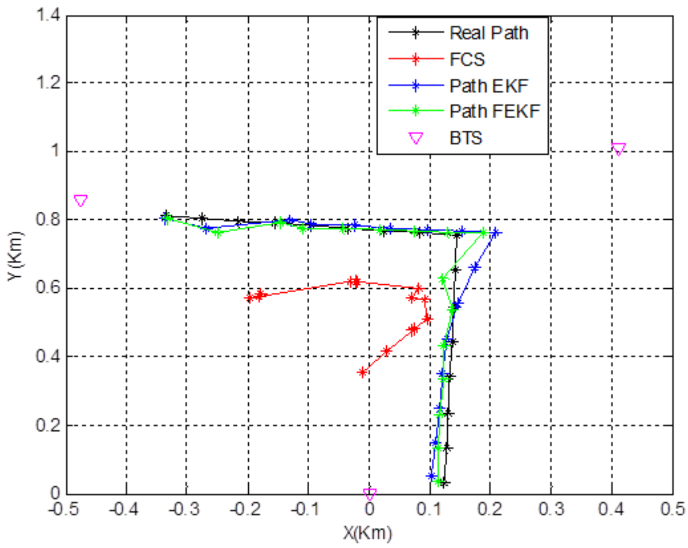


Figure 7. Membership function of input variable "distance" of FCS.

Figure 8. Tools and example of drive test image of TEMS software.

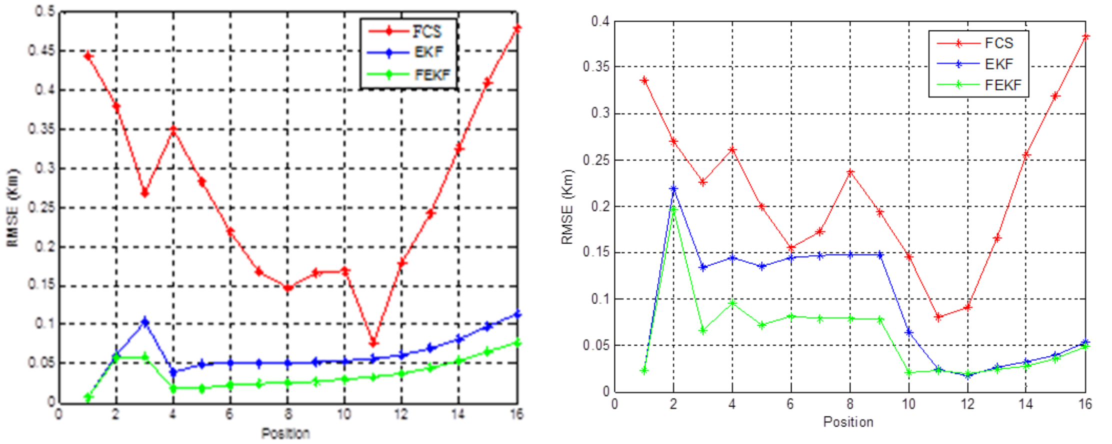


a) Linear Trajectory



b) Two linear Trajectories

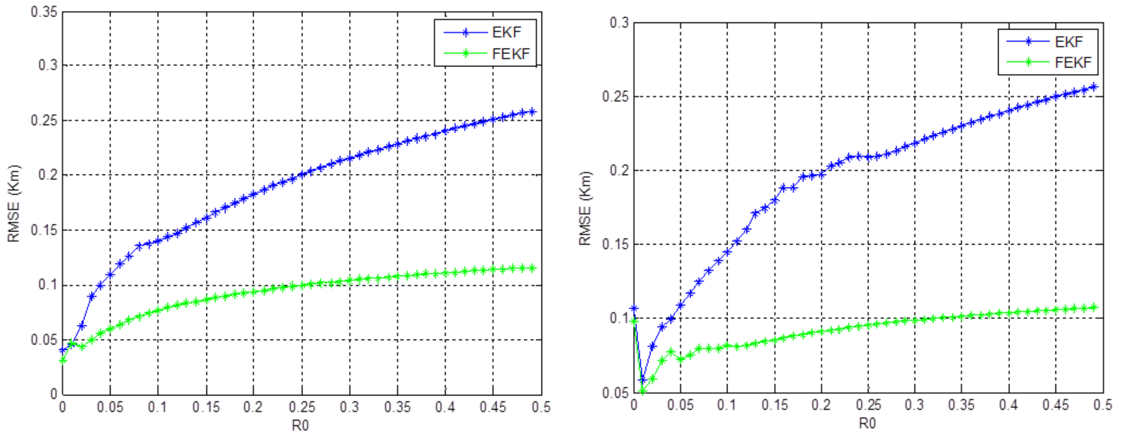
Figure 9. Real and estimated trajectories in case of linear itinerary



a) Linear Trajectory

b) Two-Linear Trajectory

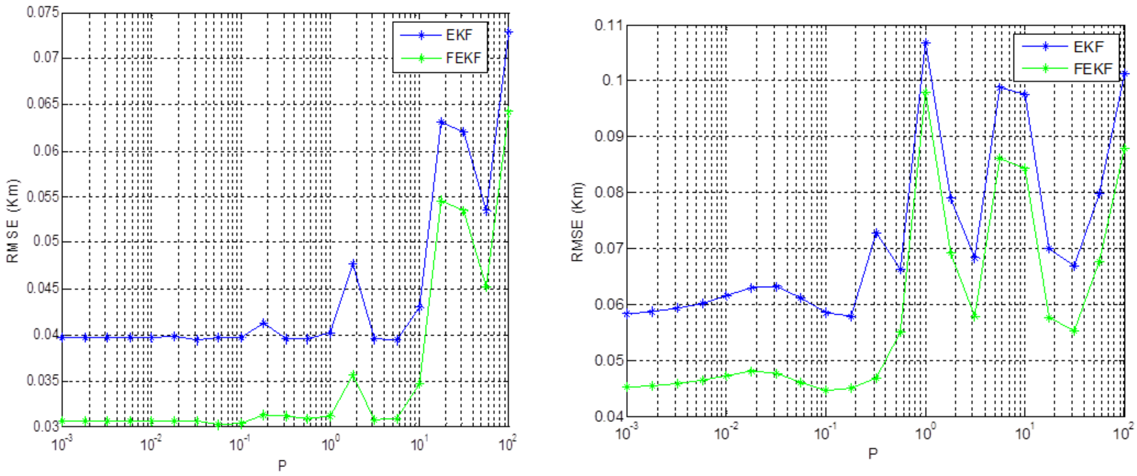
Figure 10. Performances in terms of RMSE in case of linear trajectory



a) Linear Trajectory

b)Two_Linear Trajectory

Figure 11. Influence of R_0 values on RMSE performance in case of linear trajectory



a) Linear Trajectory

b)Two_Linear Trajectory

Figure 12. Influence of P_0 values on RMSE performance in case of linear trajectory

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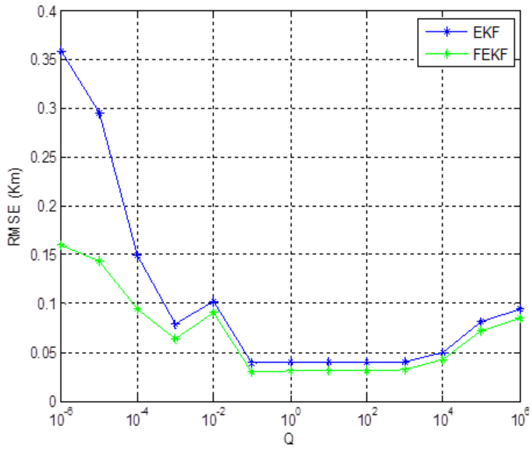
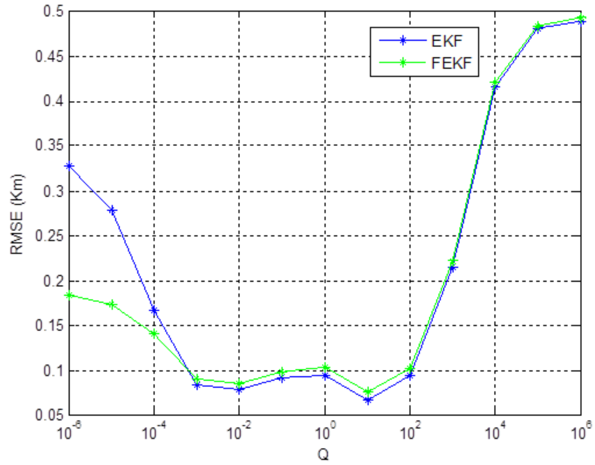
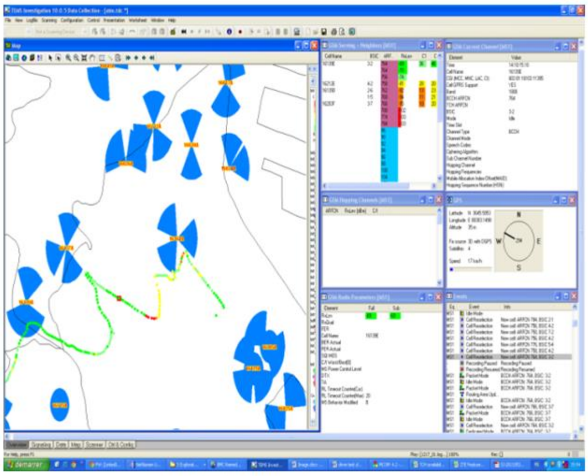


a) Linear Trajectory

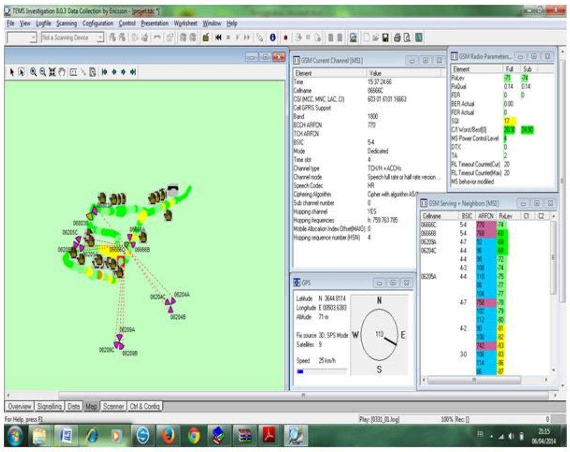


b)Two_Linear Trajectory

Figure 13. Influence of Q_0 values on RMSE performance in case of linear trajectory

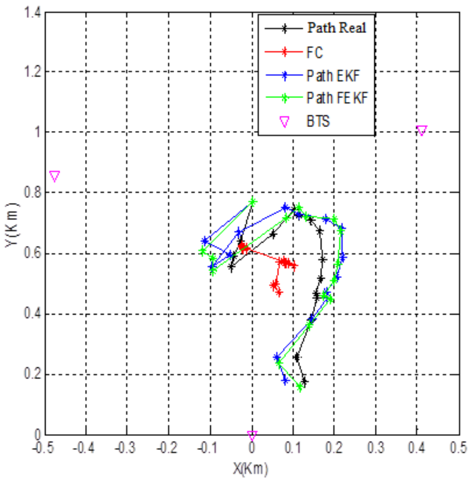


a) Urban environment

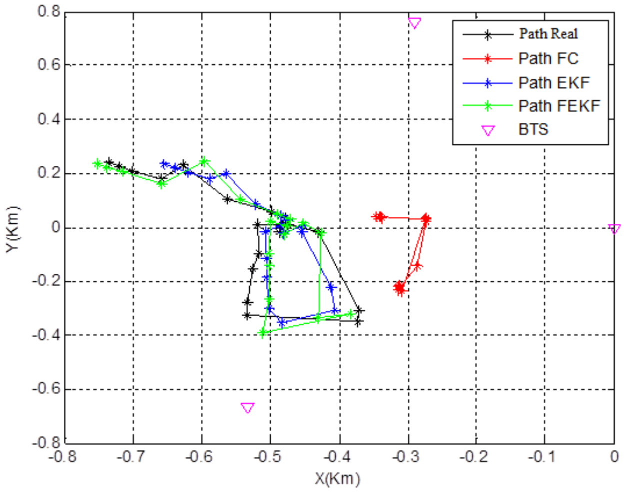


b) Semi-Urban environment

Figure 14. Example of drive test experience in Algiers' area



a) Urban environment



b) Semi-urban environment

Figure 15. Real and estimated trajectories in case of real-time experiment

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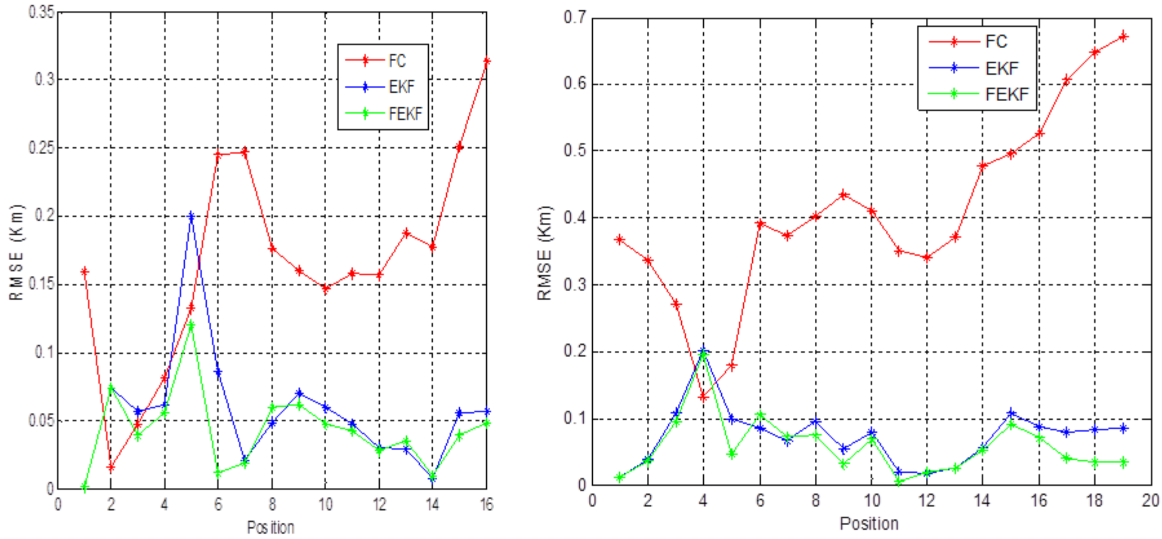
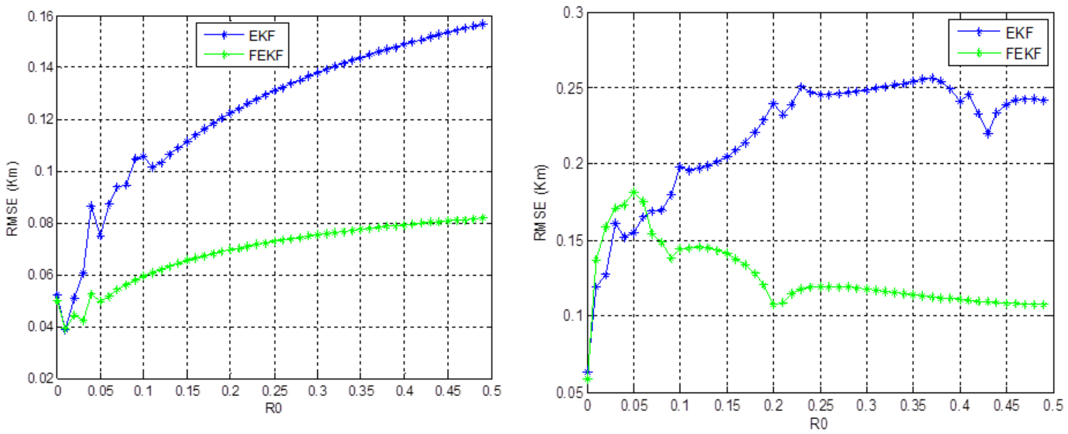


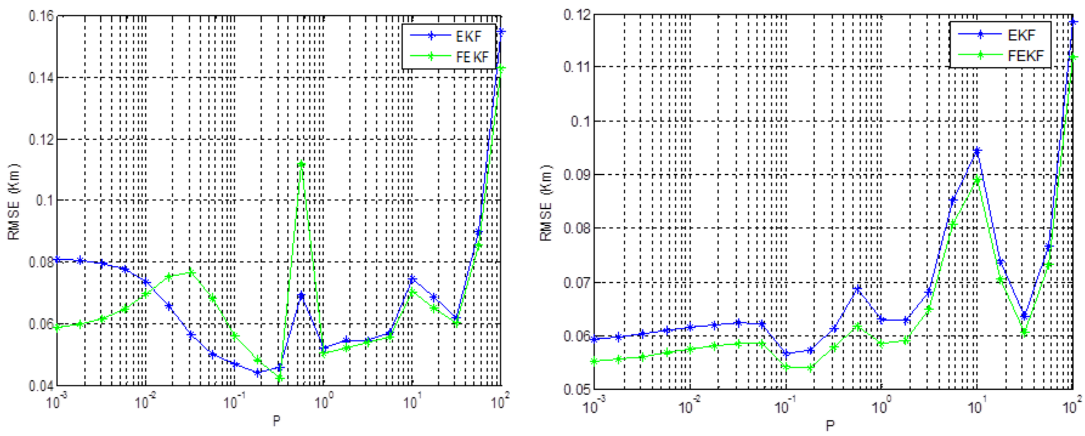
a) Urban environment b) Semi-urban environment

Figure 16. Performances in terms of RMSE in case of real time experiment



a) Urban environment b) Semi-urban environment

Figure 17. Influence of R_0 values on RMSE performance in case of real time experiment



a) Urban environment b) Semi-urban environment

Figure 18. Influence of P_0 values on RMSE performance in case of real time experiment.

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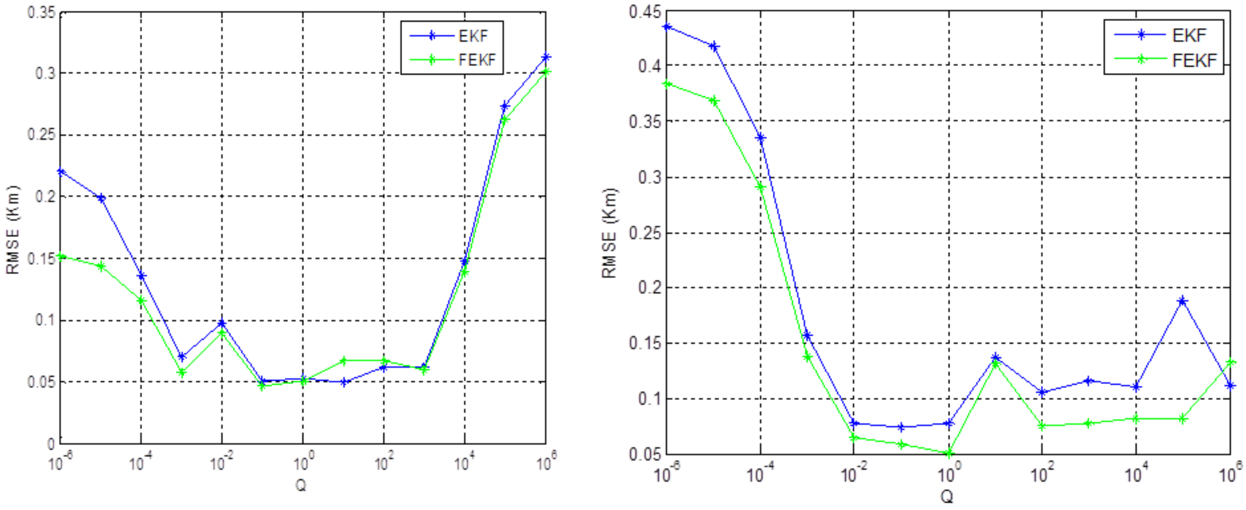

a) Urban environment b) Semi-urban environment

Figure 19. Influence of Q_0 values on RMSE performance in case of real time experiment.

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TABLE I: Fuzzy Rules for FEKF

S_k^i \ Inn_k^i	Very Large	Large	Neutral	Small	Very Small
Large	**0.25**	**0.5**	**0.5**	**0.5**	**0.75**
Small	**0**	**0.25**	**0.5**	**0.75**	**1**

For instance,
If Inn_k^i is Small AND S_k^i is Large THEN $y_i = 0.75$
IF Inn_k^i is Large AND S_k^i is Small THEN $y_i = 0.25$
The above table shows an optimal five partitions on the input variable Inn_k^i and two partitions for input variable S_k^i, as well as five distinct discrete values of c_i.

TABLE II. FEKFS pseudo-code

Input: Output:	B_k^i, H_k^i, d_k^i (i=1, N), $\hat{X}_k^-$, P_k^-, R, P_0, Q_0. $X_k = X_k^{est}, P_k = P_k^{est}, \hat{X}_{k+1}^-, P_{k+1}^-$.

FOR i = 1 to N
- Calculate Filter gain

$$K_k^i = P_k^- {H_k^i}^T \left(H_k^i P_k^- {H_k^i}^T + R_k \right)^{-1}$$

- Calculate predicted distance $\hat{D}_k^i = H_k^i \hat{X}_k^-$
- Calculate innovation $Inn_k^i = d_k^i - \hat{D}_k^i$
- Calculate innovation covariance

$$S_k^i = H_k^i P_k^- {H_k^i}^T + R \,.$$

- Calculate local solution

$$\hat{X}_k^i = \hat{X}_{k-1} + K_k^i (Inn_k^i)$$

- y_i = *Fuzzy_Inferrence_System1* (Inn_k^i, S_k^i).

END

Calculate target estimate X_k^{est} and P_k^{est} using (15-16)
Calculate next prediction $\hat{X}_{k+1}^-$, P_{k+1}^- using (2-3).

TABLE III: Fuzzy Rules associated to distance input fuzzy inference system.

distance	Very Small	Small	Medium	Large	Very Large
Weight c	1	0.75	0.5	0.25	0

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TABLE IV FCS pseudo-code

Input:	B_k^i , H_k^i , d_k^i (i=1, N), $\hat{X}_k^-$, P_k^-, R, P_0, Q_0.
Output:	Target estimate $X_k = X_k^{est}$

FOR i=1 to N

- y_i = *Fuzzy_Inferrence_System2* (d_k^i.).
- Calculate Filter gain

$$K_k^i = \mathrm{P}_\mathrm{k}^- {H_k^i}^T \left(H_k^i \mathrm{P}_\mathrm{k}^- {H_k^i}^T + R_k\right)^{-1}$$

- Calculate the innovation $Inn_k^i = d_k^i - H_k^i \hat{X}_k^-$
- Calculate the local solution
- $\hat{X}_k^i = \hat{X}_{\mathrm{k}-1} + K_k^i (Inn_k^i)$

END

Calculate target estimate X_k^{est} and P_k^{est} using (15-16)
Calculate next prediction $\hat{X}_{k+1}^-$, P_{k+1}^- using (2-3).

TABLE V Average RMSE along the whole trajectory.

	FCS	EKF	FEKF
Average RMSE (km) (one-Linear Traj.)	0.1642	0.0518	0.0360
Standard deviation σ	0.1148	0.0250	0.0184
Average RMSE (km) (two-Linear Traj.)	0.2181	0.0937	0.0605
Standard deviation σ	0.0820	0.0824	0.0434

TABLE VI Comparaison test for RMSE along the linear trajectories

	FEKF Vs EKF	FEKF Vs FCS
t-test p-value (Single Linear Traj.)	0.0185	<0.00001
t-test p-value(Two-Linear Traj.)	0.0250	<0.00001

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TABLE VII Average RMSE along the whole trajectory for various noise intensities.

Noise Intensity σ	F CS	EKF	FEKF
0.001 (Linear Traj.)	0.1640	0.0520	0.0360
0.001 (Two-Linear Traj.)	0.2181	0.0937	0.0605
0.002 (Linear Traj.)	0.1648	0.0565	0.0364
0.002 (Two-Linear Traj.)	0.2188	0.0939	0.0612
0.004 (Linear Traj.)	0.2114	0.0681	0.0565
0.004 (Two_LinearTraj.)	0.2213	0.0988	0.0646
0.006 (Linear Traj.)	0.2224	0.0762	0.0682
0.006 (Two-Linear Traj.)	0.2281	0.1037	0.0690
0.008 (Linear Traj.)	0.2442	0.0942	0.1126
0.008 (Two-Linear Traj.)	0.2321	0.1532	0.0914
0.1 (Linear Traj)	0.258	0.180	0.133
0.1 (Two-Linear Traj.)	0.2367	0.1794	0.1002
0.2 (Linear Traj.)	0.2561	0.1880	0.1500
0.2 (Two-Linear Traj.)	0.2454	0.2113	0.1186
0.3 (Linear Traj.)	0.266	0.203	0.156
0.3 (Two-Linear Traj.)	0.2446	0.2303	0.1263
0.4 (Linear Traj)	0.2532	0.3027	0.1899
0.4 (Two-Linear Traj.)	0.2533	0.2362	0.1433
0.5 (Linear Traj.)	0.251	0.438	0.326
0.5 (Two_LinearTraj.)	0.3028	0.2390	0.1611
0.6 (Linear Traj.)	0.3264	0.6549	0.5382
0.6 (Two_LinearTraj.)	0.5878	0.2572	0.3268

TABLE VIII. Example of distance calculation using Walfish-Ikigami model

Cell ID	Hb	Tx	Rxlev	ARFCN	Walfish distances (Km)
16139E	52	43	-69	764	0.455334
16212F	36	45	-74	756	0.537776
16203F	15	45	-95	766	0.443333

TABLE IX Average RMSE along the whole trajectory.

	FCS	EKF	FEKF
Average RMSE (km) Urban environment	0.1642	0.0518	0.0360
Average RMSE (km) Semi-urban environment	0.4102	0.0736	0.0583

TABLE X. Statistical t-test results along whole trajectory

	FEKF Vs EKF	FEKF Vs FCS
t-test p-value (Urban environment.)	0.0211	<0.00001
t-test p-value(Semi-urban environment.)	0.0427	<0.00001