

## Introduction to Special Issue: Abstraction Principles

Florio, Salvatore

DOI:

[10.1093/philmat/nkw035](https://doi.org/10.1093/philmat/nkw035)

License:

None: All rights reserved

*Document Version*

Peer reviewed version

*Citation for published version (Harvard):*

Florio, S 2017, 'Introduction to Special Issue: Abstraction Principles', *Philosophia Mathematica*, vol. 25, no. 1, pp. 1-2. <https://doi.org/10.1093/philmat/nkw035>

[Link to publication on Research at Birmingham portal](#)

### **Publisher Rights Statement:**

This is a pre-copyedited, author-produced version of an article accepted for publication in [insert journal title] following peer review. The version of record is available online at: <https://doi.org/10.1093/philmat/nkw035>

Eligibility for repository: Checked on 9/2/2017

### **General rights**

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
- User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
- Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

### **Take down policy**

While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact [UBIRA@lists.bham.ac.uk](mailto:UBIRA@lists.bham.ac.uk) providing details and we will remove access to the work immediately and investigate.

## Introduction

Three decades after their contemporary rediscovery, abstraction principles continue to be at the center of one of the most active areas of research in philosophy of mathematics. The present special issue bears witness to this fact. It collects some of the work presented at two events held in 2014. The first was the workshop “Abstractionism / Neologicism” organized by Marcus Rossberg at the University of Connecticut (April 26-27). The second was the summer school and workshop “Abstraction: Philosophy and Mathematics” organized by Øystein Linnebo and me at the University of Oslo (May 22-24). The events were generously supported by the host universities. Additional funding for the summer school was provided by Kansas State University.

Given the thematic and temporal closeness of the two events, it felt natural to give all speakers an opportunity to publish their work side by side. We are very grateful to *Philosophia Mathematica* for accepting the proposal of a special issue and for providing double-blind peer review for the submissions.

The articles featured here make a number of novel contributions, both technical and philosophical, to the topic of abstraction. In the first article, Roy Cook examines the precise relation between four notions of invariance for abstraction principles. This sheds light on the special place occupied by Hume’s Principle among abstraction principles. For Hume’s Principle turns out to be the most fine-grained abstraction principle on concepts which is invariant according to the strongest of the four notions. Since the notion of invariance plays a crucial role in prominent accounts of logicality, the result may be viewed as supporting a neologist take on Hume’s Principle. But, as Cook points out that, the result also indicates a limit of neologicism. If Hume’s Principle is the most fine-grained abstraction principle that is invariant in a strong sense, no abstraction principle for sets can be invariant in the same sense.

The next three articles explore new abstraction principles, specifically abstraction principles for categories, structures, and points. Category theory is a well-established branch of mathematics with foundational significance. A difficult and so far neglected question is whether it can be recovered by abstraction. Shay Logan’s contribution addresses this question. After showing that a natural abstraction principle for categories is inconsistent, Logan proposes two consistent restrictions and outlines the challenges faced by the project of fully interpreting category theory in a theory of abstraction.

Like categories, structures can be obtained by abstraction. Graham Leach-Krouse discusses a class of principles that introduce structures as abstracts of systems satisfying certain axioms. Restricting attention to these structural principles yields a partial solution to the bad-company problem: it can be shown that abstraction principles based on first-order axioms and satisfiable in uncountable domains are consistent and compatible. Even from this perspective Hume’s Principle occupies a special place. It is the structural abstraction principle based on the empty set of axioms and, hence, the simplest of the structural abstraction principles. As it turns out, abstraction principles typically used to reconstruct set theory, such as Basic Law V and its restrictions, are not structural. There is an interesting analogy between this conclusion and that of Cook’s discussion.

Abstraction principles can be useful also in connection with theories of the continuum that take “gunky” regions, rather than points, as basic. Stewart Shapiro and Geoffrey Hellman show that, in

the context of those theories, points can be introduced by abstraction in multiple ways. The results are isomorphic, and there is a strong temptation to identify them. As tempting as this identification may be, influential criteria of identity for abstracts block it. They do not allow us to say that the points obtained by abstraction in one way are identical to the points obtained by abstraction in a different way. So we have another troublesome version of the Caesar problem.

The last article investigates abstraction from a nominalistic standpoint. Inspired by Russell's no-class theory, Kevin Klement outlines a general strategy for a nominalistic elimination of abstract objects using contextual definitions. A remarkable consequence of Klement's approach is that, by applying his strategy to cardinal numbers introduced by Hume's Principle, one can derive Peano arithmetic from an axiom of infinity relying only on purely logical means. This suggests an unexpected role for abstraction principles: they can be a key ingredient in nominalistic reinterpretations of talk of abstract objects.

A special issue is obviously the product of a collective effort. I would like to thank our authors, several anonymous reviewers, and all participants in the two events from which this special issue originates. Special thanks are owed to Øystein Linnebo and Marcus Rossberg.

Salvatore Florio  
University of Birmingham