Unrestricted Quantification

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Abstract: Semantic interpretations of both natural and formal languages are usually taken to involve the specification of a domain of entities with respect to which the sentences of the language are to be evaluated. A question that has received much attention of late is whether there is unrestricted quantification, quantification over a domain comprising absolutely everything there is. Is there a discourse or inquiry that has absolute generality? After framing the debate, this article provides an overview of the main arguments for and against the possibility of unrestricted quantification, highlighting some of the broader implications of the debate.

1   Introduction

Semantic theories of natural and formal languages usually appeal to the notion of domain of quantification in specifying the interpretations or the truth conditions of the sentences of the object language. In natural language, quantificational expressions such as 'every', 'some', 'most', are routinely evaluated with respect to a salient and typically restricted range of entities. Consider the following sentence uttered in an ordinary context:

(1)   The student knew everything.

The sentence can of course be true despite the fact that the student under discussion is not omniscient. This is to say that the sentence is to be evaluated only with respect to a restricted range of entities, perhaps the facts or propositions relevant to the exam. These entities constitute the domain of quantification of the sentence.[1] Our semantic understanding of natural language sentences thus seems to demand a notion of domain of quantification, and the appropriate truth conditions of quantified sentences

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[1]Multiple occurrences of a quantifier might call for multiple domains, one for each occurrence of the quantifier. For ease of exposition, this complication will be put aside. For an overview of the main issues surrounding the notion of domain, see among others Westerstahl 1985, Recanati 1996, Stanley and Szabó forthcoming, and Stanley and Szabó 2000.

are to be specified with reference to it. So we say that (1) is true if and only if the relevant student knew everything in the domain. As this example illustrates, the domain of quantification is not always determined by the constituents of the sentence, or at least by the constituents that are phonetically realized.

Familiar model-theoretic semantics specify the interpretations of the object language by fixing a domain of quantification and then assigning semantic values constructed from that domain to non-logical expressions of the language. Quantified sentences are again evaluated with respect to a domain of quantification. In the case of first-order logic, for example, interpretations or models are given by specifying a domain of quantification d and an interpretation function i assigning entities constructed from d to the non-logical terminology of the language. A quantified sentence such as 'everything Fs' (in symbols: $\forall x \, Fx$) is true in the interpretation determined by d and i if and only if every entity in d is also in the extension assigned to F by i.

So far the notion of domain of quantification has been given a functional characterization based on the potential to play a certain semantic role. No substantive assumption about the nature of domains has been made. However, according to a widespread view enshrined for example in standard model theory, a domain is an object—a set or a set-like object—whose members are the objects in the domain. This is known as the All-in-One Principle (Cartwright 1994, 7).

There are theoretical alternatives to the All-in-One Principle which play an important role in the debate about unrestricted quantification. One option is to employ plural talk, which can be nicely regimented in plural logic.[2] Thus, rather than describing a domain as an object whose members constitute the range of quantification, one may describe it as *some objects* without assuming that there must be a single set-like object to which they all belong as members. Another option arises in the presence of a type distinction between objects and properties (or concepts), where properties are construed as higher-order entities. In this context one may employ the resources of second-order logic and characterize a domain as a property (or concept). Since properties are construed as higher-order entities, domains no longer objects and therefore this approach avoids the All-in-One Principle. Whenever possible, we will remain neutral with respect to these alternative construals of the notion of domain. But we will not always be able to do that. Some of the main arguments below will turn on whether particular alternatives are assumed.

Once the notion of domain of quantification has been introduced, two general theoretical questions suggest themselves: which entities can form a domain? And which domains can be salient for the interpretations of our linguistic practice?

Concerning the first question, an issue that has received much attention in the last two decades is whether there is an all-inclusive domain of quantification. Is there a domain encompassing absolutely everything there is? If so, can it ever be salient for

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[2]For surveys of plural logic, see Rayo 2007 and Linnebo 2013a.

the interpretation of our linguistic practice? Following Rayo and Uzquiano (2006, 2), we may call these questions the *metaphysical question* and the *availability question*, respectively.

There is no shortage of *prima facie* examples of sentences that quantify over an all-inclusive domain. When we say that

(2)   the empty set contains no element,

we take ourselves to mean that it contains absolutely no element. We do not take ourselves to be denying only that it contains the elements of some specific, restricted range of entities. Likewise, if a nominalist asserts

(3)   everything is concrete,

she means to assert that every object whatsoever is concrete. Any exception seems inconsistent with the truth of what has been asserted.

Against the natural inclination to view some of our assertions as absolutely general, a number of philosophical arguments have been offered in support of the thesis that absolutely unrestricted quantification cannot be achieved. The aim of this article is to survey the current debate about absolutely unrestricted quantification. I begin by framing the debate. Then I discuss the main arguments for and against the thesis that there is an all-inclusive domain and that it can play a role in the interpretation of our linguistic practice. Finally, I illustrate some of the broader implications of the debate in semantics, metaphysics, and logic.

2   Framing the debate

Let us call *generality absolutism* (or simply *absolutism*) the view that it is possible to quantify over absolutely everything and *generality relativism* (or simply *relativism*) the denial of that view. A peculiar but not unique feature of the debate between generality absolutists and generality relativists is that it is riddled with expressibility problems. For instance, how can the relativist capture generalizations to the effect that no donkey talks or everything is self-identical? If the domain of quantification is not all-inclusive, these generalizations do not succeed in ruling out all counterexamples.

An especially pressing problem for relativists concerns the proper formulation of their view (see, e.g., Lewis 1991, Williamson 2003, and Fine 2006). It seems that any attempt to state relativism either undermine itself or fails to capture its intended content. To begin with, a proper expression of generality relativism cannot presuppose the truth of absolutism. So the warm-up formulation invoked above, i.e.

(4)   it is not possible to quantify over absolutely everything,

will be self-defeating if 'everything' ranges, in effect, over absolutely everything.

Suppose that the relativist attempts to express her view by asserting that no domain is absolutely unrestricted, i.e.

(5)   for every domain d, there is x such that x is not in d.

Let d^* be the domain of quantification of (5). If, on the one hand, (5) succeeds in quantifying over every domain (and hence its own domain), (5) implies (6):

(6)   there is x such x is not in d^*,

where the existential quantifier still ranges over d^*. From the truth of (6) it follows that there is something in d^* satisfying the formula 'x is not in d^*'. By a principle of disquotation, we then obtain the contradiction that there is something in d^* that is not in d^*. Therefore (5) leads to inconsistency. If, on the other hand, (5) does not succeed in quantifying over every domain, then it leaves open whether there is a domain containing absolutely everything. So it appears that the relativist's attempt to express her view is either self-undermining or it fails to rule out an all-inclusive domain. Alternative options invoking the notion of context or the distinction between object language and metalanguage are available to the relativist, though none appears particularly promising (for details, see Williamson 2003, 427-435).

A different relativistic approach pursues a schematic formulation of the view. The strategy is reminiscent of Russell's notion of *typical ambiguity*. Russell urges us not to confuse 'an ambiguous assertion with the definite assertion that the same thing holds in all cases" (Russell 1908, 227). Elaborating the view, he writes of the distinction between 'any' and 'all':

> In the case of such variables as propositions or properties, "any value" is legitimate, though "all values" is not. Thus we may say: "p is true or false, where p is any proposition", though we can not say "all propositions are true or false". (Russell 1908, 229-230)

Similarly, quantifiers can be thought to be 'typically ambiguous', liable to multiple interpretations (Parsons 1974, 2006, Glanzberg 2004), or to contain a schematic reference to a domain (Lavine 2006). While it is not asserted that absolutely every domain is restricted, one aims to reach the same effect by a suitably ambiguous assertion or schema that rules out counterexamples (e.g. 'there is something not in d' where d is arbitrary or can be reinterpreted as standing for any domain one may countenance).

Other attempts to provide a proper formulation of relativism resort to intensional notions (see Hellman 2006 and Fine 2006). According to Fine, for instance, there is a notion of possibility satisfying a necessitated version of the following condition: for any range of objects, there *could* be a set-like entity not among them whose members are exactly those objects. The suggestion is that any interpretation, no

matter which range of objects constitutes its domain, could be extended to one whose domain is constituted by a more inclusive range of objects. This view is taken to capture the relativist's position in a modal setting. The relevant modality is, for Fine, *postulational*, operating not a shift in circumstances or content, but in ontology (Fine 2005, 2006). The main question is whether this modal notion is legitimate and germane to the debate. One worry is that the expressive gain obtained by employing modal notions makes the correlative view a non-orthodox version of absolutism (see Linnebo 2010 and Linnebo 2013b; see also Santos 2013 for a parallel between this view and intuitionism in mathematics).

Whether or not relativism may be properly expressed, the relativist at heart can still make trouble for the absolutist by attempting to reduce his position to absurdity or expose internal tensions in his view (see Button 2010). Arguments of this kind will be discussed below.

3   Arguments against absolutism

3.1   Paradox

The most powerful arguments against absolutism are taken to arise in connection with the paradoxes of Russell and Burali-Forti. Let us start with the latter.

Here is an informal rendering of the paradox. Given any well-ordered collection of objects, we may ask which position an object in the collection occupies. Indeed, since the collection is well-ordered, an object will come first (the least object in the ordering), another will come second (the least object in the ordering besides the first), yet another will come third (the least object in the ordering besides the first and the second), and so on. Other objects might come after those occupying the finite positions. So there might be the least object besides those occupying the finite positions, another object after that, and so on again. Ordinal numbers are objects representing the positions of all well-ordered collections of objects, i.e. they are types of well-orderings. The key observation is that the totality of all ordinal numbers, call it Ω, is itself well-ordered by the natural ordering of Ω. Now, if we can quantify over all ordinal numbers, we can define well-orderings that extend or 'prolong' the natural ordering of Ω. For example, if r is an object that is not an ordinal, we can characterize an order which places r immediately after all the ordinals. This defines a well-ordering of the collection obtained by adding r to Ω.[3] Since the ordinals represent the positions of all well-ordered collections of objects, each ordinal represents its own position in this well-ordering. However, there is a position occupied by r and thus there should an be ordinal representing it. But that cannot be: all the posi-

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[3]The order in question ($\prec$) can be characterized by the following condition. For distinct x and y in the collection, x $\prec y$ if and only if, either x and y are both ordinals and x comes before y, or $y = r$.

tions are already occupied by the ordinals themselves. This is, in informal terms, the paradox of Burali-Forti.

In other words, if we think of ordinals as types of well-orderings, the assumption that we can quantify over Ω affords us the resources to define well-orderings whose types are not in Ω, which implies paradoxically that some definable well-orderings have no order type (see Shapiro 2003, Shapiro and Wright 2006, Linnebo and Rayo 2012, Florio and Shapiro forthcoming, and Linnebo and Rayo forthcoming). Relativists conclude that we cannot quantify over all ordinals and often embrace the view, famously championed by Dummett, that the universe of mathematical objects is indefinitely extensible. According to Dummett (1993, 441) an 'indefinitely extensible concept' is 'one such that, if we can form a definite conception of a totality all of whose members fall under the concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it'. Given the tight connection between paradoxes and the view that the mathematical universe is indefinitely extensible, relativism and indefinite extensibility have tended to go hand in hand, though they may be divorced.

Another prominent argument against absolutism comes from versions of Russell's paradox. In the standard version, we start from the so-called naïve principle of set formation according to which every condition defines a set. The condition '$x \notin x$' yields the problematic instance sanctioning the existence of the Russell set (also denoted by $\{x : x \notin x\}$):

(R) $\exists y \, \forall x \, (x \in y \, \leftrightarrow \, x \notin x)$,

which leads to inconsistency on the assumption that the range of the universal quantifier in (R) includes all sets. For let r be a set witnessing the existential quantifier. Then we have:

(R*) $\forall x \, (x \in r \, \leftrightarrow \, x \notin x)$,

If the universal quantifier ranges over all sets, and hence over r, it follows that

(R**) $r \in r \, \leftrightarrow \, r \notin r$.

The relativist typically responds that the range of the universal quantifier is always restricted and restores consistency by postulating a domain expansion. The relevant instance of naïve comprehension now reads:

(R$^+$) $\exists^+ y \, \forall x \, (x \in y \, \leftrightarrow \, x \notin x)$,

where the domain of $\exists^+ y$ extends that of $\forall x$ (see Glanzberg 2006 for an account of how this shift might occur). By rejecting the possibility of unrestricted quantification over all sets, the relativist can regard the naïve principle of set formation as asserting, for any given domain, the existence of a set that is not in that domain.

As exemplified by Boolos, the absolutist usually denies the naïve principle and thus has to provide a principled distinction between conditions that define a set and conditions that do not—it is common here to invoke the iterative conception of set. Similarly, the relativist has to give some reason to think that the naïve principle should be upheld rather than abandoned. Let us briefly survey some of the reasons that have been offered.

As noted just above, the absolutist is likely to hold that the condition '$x \notin x$' does not define a set, forsaking the naïve principle of set formation. This is often seen as a loss, but it can be mitigated by the introduction of proper classes, which enjoy an independent justification on the basis of their mathematical fruitfulness (see, e.g., Uzquiano 2003). If the absolutist cannot take the condition '$x \notin x$' to define a set but avails himself of proper classes, he can at least hold that this condition defines a proper class. As a result, he has a consistent surrogate for the Russell set. However, as argued by Parsons (1974, 10-11), one 'could view the addition of classes to a set theory [...] so that they are indistinguishable from another layer of sets'. The resulting picture is that of the open-ended universe of sets proposed by Zermelo (1930). Concerning set-theoretic paradoxes, Zermelo writes:

> [T]hese are only apparent 'contradictions', and depend solely on confusing *set theory itself*, which is not categorically determined by its axioms, with individual *models* representing it. What appears as an 'ultrafinite non- or super-set' in one model is, in the succeeding model, a perfectly good, valid set with both a cardinal number and an ordinal type, and is itself a foundation stone for the construction of a new domain. To the unbounded series of Cantor ordinals there corresponds a similar unbounded double-series of essentially different set-theoretic models, in each of which the whole classical theory is expressed. [...] This series reaches no true completion in its unrestricted advance [...]. (1233)

Any theory that purports to characterize all sets can be reinterpreted as describing only an initial segment of the universe of sets, with the proper classes representing next layer of sets.

The absolutist might decide to forego the benefits of class theory and argue that any such theory simply fails to capture the intended content of set theory. After all, as Boolos famously insisted, set theory is meant to be 'a theory about all, "absolutely" all, the collections that there are.' (Boolos 1998, 35) The relativist will stress the importance of proper classes and regard the absolutist's rejection of class theory as symptomatic of his 'lack of a more comprehensive conception of set' (Parsons 1974, 11).

Another line of defense of the relativist's position in this context appeals to some version of the view that there are 'lightweight' objects and that, for particular classes of entities such as sets, being is 'easy'. The defense of relativism put forth in Fine

2006 embodies an approach of this kind, as does Glanzberg 2004. According to Glanzberg, for instance, the 'logical notion of object' includes anything that can be referred to by a singular term. He writes:

> We are looking at an object like $\{x : x = x\}$ where 'x' ranges over elements of the domain specified by an interpretation. Insofar as we really have given a determinate specification, we have specified exactly what falls in this class. As classes are extensional, I see nothing else that could be required to convince ourselves that this object exists. [...] I must grant that denying its existence is coherent. But outside of the question-begging insistence that the original domain was absolutely everything, and hence there can be no such object, it is hard to see why a plausible case for its existence has not been made. (Glanzberg 2004, 553-554)

A parallel argument would show that $\{x : x \notin x\}$ has a determinate specification and thus the corresponding set exists.

Aiming to provide general considerations to adjudicate the acceptability of a comprehension principle, Linnebo (2010) suggests employing plural logic and regarding the naïve principle of set formation as the combination of two underlying principles. The first is Plural Comprehension, the principle according to which any condition satisfied by at least one object determines a plurality of objects:

(7)   If there is an x such that $\varphi(x)$, then there are some things such, for any x, x is among them if and only if $\varphi(x)$.

The second is Plural Collapse:

(8)   Any plurality of objects forms a set.

Plural Comprehension and Plural Collapse jointly entail the naïve principle. The relativist, as we have seen, will insist on the plausibility of the two principles and read the naïve principle as embodying an expansion of the domain. The absolutist is likely to uphold Plural Comprehension and reject Plural Collapse. Based in part on modal considerations, Linnebo defends the opposite view: he restricts plural comprehension, while upholding unrestricted Plural Collapse. He then uses this view to develop a notion of absolute but indefinite generality.

An important semantic variant of Russell's argument has been given by Williamson (2003; for discussion, see Glanzberg 2004, Glanzberg 2006, Linnebo 2006, McKay 2006, and Parsons 2006). Consider the general Russellian schema:

(RS) $\exists y \, \forall x \, (R(x,y) \leftrightarrow \varphi(x))$.

A contradiction ensues if we take $\varphi(x)$ to be $\neg R(x,x)$ and take the quantifiers to be unrestricted, or at least take the range of the existential quantifier to be included in

that of the universal one. Williamson's version of the argument exploits the schema to obtain a naïve principle for the existence of semantic interpretations. Let L^1 be the usual countable language of first-order logic and let P be a predicate of L^1. Say that P applies to an object x according to a semantic interpretation i if and only if x is in the extension assigned to P by the interpretation i. In symbols:

$$i \vDash P(v) \; [a(v/x)],$$

where a is any variable assignment suitable for i, and $a(v/x)$ is just like a except possibly for the fact that the singular variable v denotes x. The naïve principle of interpretations is obtained by instantiating $R(x, y)$ with 'P applies to x according to y'. The principle tells us that for any formula $\varphi(x)$, there is an interpretation according to which the extension of P is given by $\varphi(x)$:

(I) $\exists i \, \forall x \; (P$ applies to x according to $i \; \leftrightarrow \; \varphi(x))$.

The problematic instance results from taking $\varphi(x)$ to be 'P does not apply to x according to x':

(I*) $\exists i \, \forall x \; (P$ applies to x according to $i \; \leftrightarrow \; P$ does not apply to x according to $x)$.

If the range of the universal quantifier is all-inclusive, or at least contains every interpretation, then we obtain a contradiction:

(I**) P applies to i^* according to $i^* \; \leftrightarrow \; P$ does not apply to i^* according to i^*,

for some interpretation i^*.

   Relativists have taken the argument to show that we cannot quantify over all interpretations. As in the case of the set-theoretic version of Russell's argument, a relativist can restore consistency by postulating an expansion of the domain and by reading the principle accordingly:

(I⁺) $\exists^+ i \, \forall x \; (P$ applies to x according to $i \; \leftrightarrow \; \varphi(x))$.

It is easy to see how one could construct even more versions of the Russellian argument based on notions such as proposition, property, or even domain of quantification.

   The absolutist, on the one hand, might abandon the naïve principles driving these Russellian arguments and reject the relativist's reasons for accepting them. For example, in the case of Williamson's version of the argument, the absolutist might deny the problematic instances of the naïve principle, which is consistent with the recognition that quantification over all interpretations is essential to theorize about logical consequence. The absolutist can hold that a sentence is logically true if and only if it is true under *every* interpretation but deny that the there is an interpretation corresponding to the problematic condition. (See section 4.1, for an alternative response

available to the absolutist.) The relativist, on the other hand, will complain that the absolutist's rejection of the naïve principle leads to arbitrariness. If not all conditions determine an instance of the relevant notion (set, interpretation, etc.), which conditions do and which ones do not? It is often said that at this point the two parties have reached a standoff.

3.2   Semantic Indeterminacy

An alternative line of attack against absolutism is based on traditional arguments for semantic indeterminacy arising from the Löwenheim-Skolem theorem (e.g. Skolem 1923, Quine 1968, Putnam 1980). A suitable version of the theorem in the current setting implies that for any theory T in the language of L^1, if T is satisfied in an interpretation with an all-inclusive domain, then T is satisfied in an interpretation whose domain is not all-inclusive. This means that no theory can discriminate between interpretations with an all-inclusive domain and interpretations with a restricted domain. Anything the absolutist may say, if true by his lights, can be so interpreted as to hold true of a restricted domain.

   This raises an epistemic problem for the absolutist. How does he know that he is actually managing to quantifying over absolutely everything in his purported examples of absolutely unrestricted quantification? Absolutists may appeal to independent considerations to single out, among all the candidate interpretations, the one with an all-inclusive domain as the intended interpretation (see Rayo 2002a and Sider 2009). However, a stronger reading of the theorem has been given. According to it, the theorem shows that

> there simply cannot be any fact of the matter concerning which [interpretation] is intended. The universe of discourse of quantification is intrinsically and essentially ambiguous when there is an attempt to quantify in an unrestricted fashion. Thus [...] it is not merely impossible to communicate the intention to quantify over absolutely everything, it is impossible to form such an intention. (Lavine 2006, 106)

   McGee (2000, 2006) has argued that if the rules of inference associated with L^1 are understood as open-ended, we can single out interpretations with an all-inclusive domain as the only ones that satisfy this understanding of the rules. McGee's line of thought would provide the absolutist with a response to the argument from semantic indeterminacy, but it has been subject to criticism (Field 2001, 275-277, and Lavine 2006; see Williamson 2006 for further discussion).

3.3   Further arguments

More arguments have been leveled against absolutism (see also Rayo and Uzquiano 2006). However, they have been thought to be carry less weight than the arguments

surveyed above. If, for instance following Carnap (1950), ontology is taken to be relative to a conceptual framework, then quantification too is relative to a conceptual framework and fails in some sense to be absolutely unrestricted (see, e.g., Hellman 2006 and Rayo 2012, which also examines different ways of understanding absolutism in light of the possibility of reconceptualization). Another argument develops the idea, endorsed for example by Dummett (1981, chapter 15), that quantification is sortal-relative in the sense that domains of quantification correspond to extensions of general terms associated with well-defined criteria of identity. Whether absolutely unrestricted quantification is possible depends on whether there is a term (e.g.'thing','object', or 'entity') that can perform the required function. This opens up a new line of attack for the relativist, who may deny that there is a general term that has an all-inclusive extension and is associated with well-defined criteria of identity. Finally, an objection to absolutism has arisen out of substitutional interpretations of quantifiers (see McGee 2000 and Lavine 2006).

4   Broader implications

The debate on absolute generality has ramifications in areas such as semantics, metaphysics, and logic. In this section, I provide a brief overview of some of these ramifications, focusing primarily on semantics.

4.1   Semantics

If unrestricted quantification is possible and our discourse is at least sometimes absolutely general, we might expect to be able construct a semantic theory which captures this possibility (for a dissenting view, see Peters and Westerstahl 2006). As noted above, standard model-theoretic semantics, based on Zermelo-Fraenkel set theory, does not do justice to it. Since sets serve as domains of quantification and a universal set cannot be admitted, absolutely general quantification cannot be represented in this framework.

The situation generalizes to every broadly model-theoretic semantics upholding a version of the All-in-One Principle and a construal of the notion of domain subject to constraints parallel to those which govern the notion of set (e.g. separation). Some technical results associated with Williamson's semantic version of Russell's pardox illuminate the situation.

As a preliminary, let us introduce a hierarchy of languages, beginning with L^1, the standard language of first-order logic. We countenance two sorts of expansion of this language. First, one can introduce second-order quantifiers and variables. Let us call the resulting language L^2. In this context, there are two main options for the interpretation of of the second-order quantifiers. The first is Fregean and reads them as quantifying into predicate position over concepts. The second option reads them

as plural quantifiers. The other sort of expansion consists in augmenting L^2 with predicates taking the second-order variables as arguments. The resulting language, call it L^2+, corresponds to the language of full second-order logic or plural logic with plural predicates.[4] The axioms governing the second-order quantifiers are the usual ones: introduction and elimination rules plus a principle of full comprehension.

This process of expansion may be repeated. Each language L in the hierarchy may be augmented with quantifiers and variables of the next higher order. Let L^* denote the resulting system. We say that L is of order n if the highest order of quantifiers of L is n. Moreover, we say that a predicate is of level n if the highest order of variables it takes as arguments is n. If L is of order n, we may also augment L with n-level predicates obtaining $L+$. (Note that, as defined, this operation is redundant if L already contains such predicates.) Finally, if L is of order n, let us denote by $L_{\mathbf{Sat}}$ the addition to L of a new n-level predicate functioning as a satisfaction predicate.

Before stating the results, let us record the main assumptions on which they rest:

  (i) absolutism for the quantifiers of every order;

 (ii) semantic optimism, i.e. the view that proper semantic theorizing is possible at every stage of the hierarchy;

(iii) a liberalized version of model theory in which every expression of the object language is assigned any possible semantic value of the appropriate type (this is a 'generalized semantics' in the terminology of Rayo 2006 and Linnebo and Rayo 2012);

(iv) each stage of the hierarchy may be treated as an interpreted language so that it can be legitimately used in theorizing (as indicated above, two main interpretations are available, one in terms of Fregean concepts, the other in terms of plurals and higher-order plurals).

On these assumptions, the following results can be established for an arbitrary language L in the hierarchy.

THEOREM 1
Neither a truth theory nor a model theory for L can be given in L. (Tarski 1935)

THEOREM 2
A truth theory for L can be given in $L_{\mathbf{Sat}}$. It can also be given in L^*. (Tarski 1935, Boolos 1985, Rayo and Uzquiano 1999, Rayo 2002b)

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[4]Terminological conventions vary among authors: see the distinction between meager (or basic) and full languages in Yi 2005, Yi 2006, and Rayo and Uzquiano 2006, or the use of the plus sign in Rayo 2002b and Linnebo 2003.

THEOREM 3

A model theory for L can be given in L^*. Moreover, if L is of order n but does not contain n-level predicates, a model theory for L can also be given in $L_{\mathbf{Sat}}$. (Rayo and Uzquiano 1999, Rayo and Williamson 2003, McGee 2004, Rayo 2006)

An extension of these results to languages of transfinite orders is given in Linnebo and Rayo forthcoming.[5]

These results bring out an important consequence of absolutism, namely that it naturally leads to adopt a type-theoretic hierarchy (see Studd forthcoming for an alternative argument to this effect based on generalized quantifiers). Note that the type-theoretic framework provides the absolutist with a solution to the problem raised by Williamson's semantic version of Russell's paradox. In particular, it enables the absolutist to block the argument while still retaining a version of the naïve principle of interpretations. The key point is that interpretations can now be construed not as objects, but as higher-order entities (concepts, properties, or pluralities) encoding the relevant semantic information. For example, the absolutist can invoke all-inclusive domains by means of a universal property (or concept), such as that of being self-identical, without committing to the All-in-One Principle. Indeed, the All-in-One Principle requires a domain to be an object, whereas in the type-theoretic framework properties (concepts) are not objects but higher-order entities. Thus, insofar as one aims to capture domains containing absolutely every *object*, this approach may be said to succeed.

Although some absolutists have embraced the type-theoretic framework (e.g. Williamson 2003, Yi 2005, 2006, McKay 2006, and Rayo 2006), this position has been heavily criticized. Common objections are based on learnability and expressibility. The learnability objection (Glanzberg 2004) holds that, past perhaps the first few levels, the languages in the type-theoretic hierarchy cannot be learned and thus they cannot be deployed by us in semantic theorizing. The expressibility objection

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[5]It might be helpful to provide more precise references to these results in the literature. THEOREM 1 is essentially Tarski's (see also Rayo and Williamson 2003, footnote 7 and Theorem 2 in the Appendix of Linnebo and Rayo 2012). For a second-order language without predicates of level 2, the first part of THEOREM 2 is found in Boolos 1985, 335-337, while the second part is found in Rayo and Uzquiano 1999, 322-323. The first part of THEOREM 2 relative to second-order languages with predicates of level 2 is covered in Rayo 2002b, 459-460. The second part of THEOREM 2 and its generalizations to languages of higher order can be extracted from these results (but see Linnebo and Rayo 2012).

For first-order languages, the first part of THEOREM 3 is sketched in McGee 2004, 362-363. For second-order languages, the second part of THEOREM 3 is found in Rayo and Uzquiano 1999. Generalizations of the first part of THEOREM 3 are discussed in Rayo and Williamson 2003, 353, and characterized in Rayo 2006, 243-245 and 248-252. Theorem 3 in the Appendix of Linnebo and Rayo 2012 presents the first part of THEOREM 3 for languages of finite and transfinite orders.

points out that the absolutist who embraces the type-theoretic hierarchy faces expressive limitations. He can quantify over specific levels of the hierarchy but never over all levels at once. This means that he cannot express certain semantic insights concerning the entire hierarchy. For instance, he has no way to capture the intuitive content of 'no domain is empty' or 'every interpretation has a domain' (Linnebo 2006 and Linnebo and Rayo forthcoming; see Krämer forthcoming for a response on behalf of the absolutist). This is because the type of the entities playing the role of domain changes as we move up the hierarchy. Some proposals that bear on the expressibility objection involve changing the logic (Weir 2006) or developing a suitably constrained notion of property (Linnebo 2006).

Here the threat of a standoff appears again. If the absolutist's adoption of a type-theoretic hierarchy saddles him with expressive limitations, then there is a strong analogy between his predicament and that of the relativist. While the absolutist embraces a type-theoretic hierarchy, the relativist postulates a hierarchy of classes or domains. As a result, they both face serious obstacles when attempting to express generalizations about all the items in their respective hierarchies.

4.2 Generality of theories

Accepting the possibility of unrestricted quantification means acknowledging that *some* theory is absolutely general. But which theories enjoy this feature? In order to establish that a particular theory is absolutely general, it is not enough to show that the theory is true in an interpretation with an all-inclusive domain, even if this interpretation can be regarded as intended. Indeed, since some theories impose incompatible constraints on the domain, there might be theories that have a reasonable claim to absolute generality when taken in isolation, but not when taken together.

Uzquiano (2006a, 2006b; see also Hawthorne and Uzquiano 2011) has presented some interesting cases of this phenomenon. He has shown that the absolute generality of a second-order version of Zermelo-Fraenkel set theory (i.e. second-order ZFC with the axiom that the urelements form a set) is at odds with the absolute generality of a second-order version of atomistic extensional mereology. Likewise, the absolute generality of the former theory is also at odds with the absolute generality of an attractive theory of abstraction. This puts the absolutist in the uncomfortable position of having to adjudicate incompatible claims to absolute generality.

4.3 Logic

Absolutists will be interested in the logical properties of a system whose quantifiers range over absolutely everything. The adoption of the higher-order semantics mentioned in section 4.1 may be seen as one step in this direction. However, a well-known argument by Kreisel (1967) vindicates standard set-theoretic model theory by

showing that interpretations with set-sized domains yield an extensionally adequate relation of logical consequence for first-order languages. So nothing is lost from an extensional point of view if we ignore interpretations with domains that are too big to form a set. With the help of suitable reflection principles in set theory, versions of Kreisel's result can be obtained for second-order languages (Shapiro 1987) and beyond.

However, the absolutist may be interested in giving a syncategorematic treatment of the quantifiers, i.e. he may want to give a semantics where 'everything' in the object language always means 'everything' as understood in the metalanguage. In this semantics, the quantifiers would always be interpreted as ranging over an all-inclusive domain and would not be subject to reinterpretation (for discussion, see Rayo and Williamson 2003). Kreisel's argument is not applicable here, as it presupposes that sets correspond to legitimate domains of quantification. So what are the logical properties of a system capturing unrestricted quantification?

Working within a basic theory of predication—a theory equivalent to full second-order arithmetic—Friedman (1999) has proved the following completeness result for L^1. Let Infinity be the set of sentences 'there are at least n objects', one for each natural number n. Then, on the assumption that the universe of the metatheory is linearly ordered, every set of sentences of L^1 consistent with Infinity is satisfiable (see Florio 2010 for a detailed exposition of the result). Working within a stronger system—second-order set theory with urelements—Rayo and Williamson (2003) have shown that, by assuming that there is a well-ordering of the universe, one can extend the completeness result from a countable language to an arbitrary one. It is an open question whether these results are optimal.

5   Conclusion

As we have seen, there is controversy over whether the denial of the thesis that we can quantify over absolutely everything can be coherently articulated. However, the relativist may still attempt to reduce the absolutist's position to absurdity. We have surveyed a number of arguments against absolutism. The most common objections are based on the paradox of Russell and Burali-Forti and are associated with the idea that the mathematical universe is indefinitely extensible. These objections rely essentially on some principle of comprehension for the relevant notions (set, interpretation, etc.). The issue is whether there are compelling reasons for the absolutist to accept any such principle. Another main line of objection against absolutism relies on the alleged semantic indeterminacy of quantification and deploys a suitable version of the Löwenheim-Skolem theorem.

If the absolutist can resist these objections, he still faces challenges associated with the ramifications of his view. In semantics, a potential cost is that of having to embrace a hierarchy of higher-order languages. Moreover, the absolutist has to

exercise care in distinguishing theories that have a legitimate claim to absolute generality from those that must be regarded as having more limited ambitions. Finally, he is likely to have to make non-trivial assumptions on the structure of the universe in order to prove completeness results for a logical system that captures his absolutely unrestricted understanding of the quantifiers. Thus, even if absolutism is ultimately tenable, it carries with it substantive philosophical commitments which might be avoided on the rival view.[6]

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[6][Acknowledgements]

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