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Kripke semantics for full ground references (work in progress)

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“Full ground references” means references to integers and to other references, but not functions or thunks. Game semantics for full ground references was given in [1]. This work aims to give Kripke semantics.

**Language** We fix a set $S$ of *sorts*. We extend the call-by-push-value types with a reference type $\text{Ref}_s$ for each sort $s$. We then define

$$D ::= 0 \mid D + D \mid \sum_{i \in \mathbb{N}} D_i \mid 1 \mid D \times D \mid \text{Ref}_s \ (s \in S)$$

and fix, for each sort $s$, a full ground type $D_s$, intended to be the type of values stored in a reference cell of sort $s$.

A *world* is a finite set over $S$. (Alternatively: a finite sequence of sorts.) The judgements are $w, \Gamma \vdash v : A$ for values and $w, \Gamma \vdash c : B$ for computations, where $w$ is a world and $\Gamma$ a typing context (finite set of identifiers with an associated value type). Term syntax and operational semantics is defined as usual. Evaluation terminates because of the restriction to full ground references.

**Denotational semantics** A value type $A$ will denote a functor from the category $\text{Inj}$ of worlds and injections to $\text{Set}$. Intuitively, $[A]w$ is a semantic domain for closed values in world $w$, and such values can be renamed along an injection $w \to w'$. In particular $[\text{Ref}_s]w$ is the set of $s$-sorted cells in $w$.

A $w$-*store* associates to each cell $l$ in $w$ a value $w \vdash v_{s_l} : D_{\text{sort}(l)}$. The semantic domain for these is

$$Sw \overset{\text{def}}{=} \prod_{l \in w}[D_{\text{sort}(l)}]w$$

Although for ground references $S$ is a functor $\text{Inj}^\text{op} \to \text{Set}$, that is not so for full ground references.

Computation types have more subtle semantics. To understand it, say that a $w$-*store* $s$ associates to each cell $l$ in $w$ a value $w \vdash v_{s_l} : D_{\text{sort}(l)}$. An SC-configuration $\Gamma \vdash^{sc} x, s, M : B$ consists of

- a world $x$—we think of cells in $x$ as local/private, whereas those in $w$ are global/public
- a $w + x$-store $s$
- and a computation $w + x, \Gamma \vdash c M : B$.

These arise in the operational semantics of a language that has both $w$-many global cells and generation of local cells.

We want $[B]w$ to be a semantic domain for closed SC-configurations in world $w$. What category should $[B]$ be a functor from?

Let’s start with the ground ref setting. An *initialization* $(i, p) : w \to w'$ consists of

- an injection $i : w \to w'$—we write $\text{new}(i)$ for the cells in $w'$ not in the range of $i$
- and an element $p \in \prod_{l \in \text{new}(i)}[D_{\text{sort}(l)}]$.

These form a category $\text{Init}$. A *partial initialization* is defined similarly, except that $i$ is a partial injection. The latter form a category $\text{PInit}$ that contains both $\text{Inj}^\text{op}$ and $\text{Init}$, and indeed is freely generated by these subcategories modulo two equations.

A partial initialization $i : w \to w'$ converts an SC-configuration in world $w$ to one in world $w'$, by
• hiding the cells in $w$ that are not in the domain of $i$
• renaming each cell in the domain of $i$ to one in the range
• creating each cell $l \in \text{new}(i)$ with value $p_l$.

So a computation type $B$ should denote a functor $\text{Plinit} \to \text{Set}$.

Turning to full ground references, an initialization $(i, p) : w \to w'$ consists of
• an injection $i : w \to w'$
• and an element $p \in \prod_{l \in \text{new}(i)}[D_{\text{sort}(i)}](w + \text{new}(i))$

These form a category $\text{Init}$, and $S$ is a functor $\text{Init} \to \text{Set}$. But partial initializations are more subtle. Let’s first say that a stateful value $\Gamma \vdash^s V, s : A$ consists of
• a world $x$—again, cells in $x$ are local/private, whilst those in $x$ are global/public
• for each local cell $l$ in $x$, a value $w + x \vdash^v s_l : D_{\text{sort}(l)}$
• and a value $w + x \vdash^v V : A$.

For any functor $A : \text{Inj} \to \text{Set}$, define
\[
(\Psi A) w \overset{\text{def}}{=} \int_{x \in \text{Init}} \prod_{l \in x} [D_{\text{sort}(l)}](w + x) \times [A](w + x)
\]
so that $(\Psi[A])w$ is a semantic domain for closed stateful values of type $A$ in world $w$. If $A$ is a constant functor, then $(\Psi A)w \cong Aw$. A partial initialization $(i, p) : w \to w'$ consists of
• a partial injection $i : w \to w'$
• and an element $p \in (\Psi \prod_{l \in \text{new}(i)}[D_{\text{sort}(l)}])(w + \text{new}(i))$.

These form a category $\text{Plinit}$ that contains both $\text{Inj}_{\text{ap}}$ and $\text{Init}$ and indeed is freely generated by these subcategories modulo two equations. A computation type $B$ should denote a functor $\text{Plinit} \to \text{Set}$. Also, if $A : \text{Inj} \to \text{Set}$ then $\Psi A : \text{Plinit}_{\text{ap}} \to \text{Set}$.

Semantics of judgements:
• A value $w, \Gamma \vdash^v V : A$ denotes a family of functions $[\Gamma](w + x) \to [A](w + x)$, natural in $x \in \text{Inj}$.
• A computation $w, \Gamma \vdash^c M : B$ denotes a family of functions $S(w + x) \times [\Gamma](w + x) \to [B](w + x)$, natural in $x \in \text{Init}$. Here overline represents the forgetful functor $\text{Init} \to \text{Inj}$.

Semantics of types:
\[
\begin{align*}
[FA]w & \overset{\text{def}}{=} \int_{x \in \text{Init}} S(w + x) \times [A](w + x) \\
[UB]w & \overset{\text{def}}{=} \int_{x \in \text{Init}} S(w + x) \to [B](w + x) \\
\prod_{i \in I} B_i w & \overset{\text{def}}{=} \prod_{i \in I} [B_i]w \\
[A \to B]w & \overset{\text{def}}{=} \int_{x \in \text{Init}} [A](w + x) \to [B](w + x) \\
[U \prod_{i \in I}(A_i \to B_i)]w & \overset{\text{def}}{=} \int_{x \in \text{Init}} S(w + x) \to \prod_{i \in I}([A_i](w + x) \to [B_i](w + x))
\end{align*}
\]

References