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The effects of tyre material and structure properties on relaxation length using Finite Element method

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Abstract:

This study investigates the influence of tyre structural layup and material properties on the relaxation length of a rolling tyre using finite element analysis. Relaxation length for rolling tyre under different operating conditions has been studied recently. However, the effects of tyre structural layup and material properties on relaxation length were ignored. In this present work, a finite element (FE) tyre model was built based on the material and geometry properties obtained from measurements of the tyre provided by a vehicle company. Rather than the common method (steady state rolling analysis) used for cornering behaviour simulations, ABAQUS/Explicit program was used for prediction of the cornering performance and relaxation length for a constant slip angle of the rolling tyre. Two different steer inputs were applied to the rolling tyre in terms of slip angle variation, namely step input and ramp input. The effects of various factors, including cross-section area, spacing, crown angle and strength of the tyre reinforcement cords, on relaxation length of the rolling tyre were investigated by numerical experiments using the design of experiment (DOE) method.

Key Words: relaxation length, finite element analysis, DOE, input function

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1. Introduction

Tyre characteristics during a dynamic change of wheel motion conditions have been concentrated on by engineers within the last two decades. It is well known that tyre lateral deformations do not occur instantaneously when a steering angle input is applied on it. The time delay of the lateral force response resulting from tyre lateral deformation is an important transient tyre property. Relaxation length is a property of a pneumatic tyre that describes the delay between when a slip angle is introduced and when the cornering force reaches its steady state value. Normally, relaxation length is defined as the rolling distance needed by the tyre to reach 63% of the steady state lateral force.

Relaxation length is an important factor for vehicle handling response. The shorter the relaxation length of a tyre, the more responsive its handling performance. Understanding the relaxation length behaviour of a rolling tyre can help to improve the simulation of vehicle handling performance. Tyre relaxation has been observed in the laboratory conditions through dynamic test methods [1-3].

Due to the influence of tyre behaviour on vehicle handling, it is necessary to implement accurate tyre dynamic models in vehicle handling simulation for predicting vehicle dynamic responses to different steering input functions. Therefore, accurate prediction of the tyre’s relaxation length and its implementation in the tyre dynamic model is of great importance in accurate simulations of vehicle response to steer inputs.

Maurice and Pacejka [2] determined the relaxation length from frequency response functions and step responses of a non-linear tyre model. In their simulations, the contact force and moment were generated by a tyre/road contact brush model. To improve the accuracy of the relaxation length, the lateral force responses of the model with an increment of the slip angle had to be fitted with an exponential function. Loeb et al. [4] used a first-order differential equation to describe the time varying lateral displacement of the tyre tread and, hence, to derive the relationship between the time and the lateral force since lateral force is directly proportional to the lateral displacement of the tread. However, this tyre model developed by them is only valid for small slip angles. Rill [5] also used first order differential equations to approximate the dynamic reactions of the tyre lateral forces and torques to disturbances, and the first order approximation was written as [5]

\[ F_y (v_y + \dot{y}_e) \approx F_y (v_y) + \frac{\partial F_y}{\partial v_y} \dot{y}_e \]  \hspace{1cm} (1)

where \( F_y \) represents the lateral force, \( v_y \) denotes the tyre lateral velocity, and the lateral tyre deflection \( y_e \) was also taken into account. Based on his derivation, the relaxation length for the lateral tyre deflection was expressed as a function of the wheel load and the slip...
angle. Mabrouka et al. [6] developed a steering system model to investigate the transient responses to steering torque input, in which the lateral flexible tyre model was built to predict lateral forces, but the prediction of transient responses was valid only for small slip angles.

There are very few studies reported in the literature which have been concentrated on the prediction of relaxation length using the finite element (FE) method although the FE method is now routinely used for various aspects of tyre static and dynamic analysis. Finite element analysis has the advantage of facilitating the investigation of the effect of tyre material and structural properties on tyre behaviour and is widely used by automotive engineers and tyre designers [7-10]. Some typical examples are: Yang et al. [11, 12] who investigated tyre durability properties based on the variations in carcass ply turn-up and bead reinforcement turn-up using FE method; Behroozi and Olatunbosun [13] who conducted a study on the influence of FE model complexity on aircraft tyre performance characteristics; Mohsenimanesh et al. [14] who developed a nonlinear and multi-laminated tractor tyre model to investigate the pressure distribution of an off-road tyre; Guo et al. [15] who developed a detailed aircraft finite element tyre model for dynamic simulations of tyre loading upon aircraft landing scenarios using rubber and fabric material properties which were characterized and correlated.

This paper presents an approach for prediction of relaxation length using a developed FE tyre model. Detailed description of the rubber material property definition and tyre structural layup definition are presented in the FE tyre model development. In this study, the relaxation length derivation is based on prediction of the transient dynamic behaviour in the time domain using Abaqus/Explicit program. The transient dynamic analysis has been applied by Wei and Olatunbosun [16] in investigating the tyre performance when impacting large obstacles. Cho et al. [17] also analysed the transient dynamic responses of 3D patterned tyre rolling over a small cleat fixed on a drum. Koishi et al. [18] used the explicit FE analysis code PAM-SHOCK to conduct cornering simulations, in which the fiber-reinforced rubber composites were modelled with multi-layered shell elements. Rao et al. [19] discussed the simulation of combined cornering-cum-braking behaviour of a pneumatic tyre by use of the explicit finite element code. Different from the transient dynamic analysis in the literature [16-22], two different inputs are applied to steer the rolling tyre.

2. Finite Element tyre model

The 2D tyre model was built based on a 235/60 R18 tyre product, in which the rubber materials are composed of tread component, sidewall component and apex component, and the reinforcements are embedded in these rubber materials. The 2D FE tyre model is illustrated in Figure 1. The definition of the structural layup of the reinforcements, together with rubber material properties are described in the following sections. In order to achieve an accurate geometry model, the 2D tyre cross-section was extracted from a real tyre product and the geometric shape was captured using a digital camera.
2.1 Structural layup of reinforcements

As is well known, the tyre consists of different reinforcements which are embedded in rubber component in the form of layers. Different reinforcement components are positioned in rubber material which have different characteristics. Rebar elements in ABAQUS are able to define the structural layup for different layers in membrane and surface elements. Cord spacing, cord cross-section area, cord orientation inside a ply and cord material property are all necessary parameters for definition of the structural layup of reinforcements. A schematic representation of reinforcement components is shown in Figure 2, where the orientations of the cords are given.
The area of cord cross-section can be obtained by measuring the diameter of the reinforcement using micrometer gauge as shown in Figure 3. The spacing between the centre of the two neighbouring cords and orientation can be easily obtained using image processing techniques (Figure 4).
Measurements of the structural characteristics for different reinforcements were carried out and the test data are shown in Table 1.

<table>
<thead>
<tr>
<th>Component</th>
<th>Area per Bar (mm$^2$)</th>
<th>Spacing (mm)</th>
<th>Orientation Angle ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap ply</td>
<td>0.1521</td>
<td>0.5128</td>
<td>90.0</td>
</tr>
<tr>
<td>Steel belt 1</td>
<td>0.3165</td>
<td>1.2983</td>
<td>110.0</td>
</tr>
<tr>
<td>Steel belt 2</td>
<td>0.3165</td>
<td>1.2983</td>
<td>70.0</td>
</tr>
<tr>
<td>Carcass</td>
<td>0.2917</td>
<td>0.5928</td>
<td>0.0</td>
</tr>
<tr>
<td>Reinforced strip</td>
<td>0.1898</td>
<td>0.8055</td>
<td>80.0</td>
</tr>
</tbody>
</table>

2.2 Material property definition

Material properties of the rubber material were defined by combining the tests and evaluation using existing material models. Because of the limited support from tyre manufacturers in supplying material samples, rubber samples were separately extracted from the tread, the sidewall, and the apex sections of a tyre product. Normally, the tensile test sample should be better extracted as a dumbbell or ring shaped specimen. However, because of the narrow sections of rubber in a tyre product, it is not realistic to acquire either a dumbbell or a ring specimen from it. In this study, some straight narrow strip rubber specimens were prepared for testing. These specimens satisfy the ASTM-D412 requirements [23] for test specimens. In this case, the length of the test sample needs to be more than 10
times longer than its width and thickness so it can produce the same reliable test data as the other two shaped samples in hyperelastic property test of rubber.

The uniaxial extension method was applied to carry out the hyperelastic property test of the rubber components [12]. The temperature for the test was set as the common room temperature (about 23°C) according to the standard in ASTM-D412 [23], and the rubber samples were stretched for more than ten pre-conditioning cycles until the stress/strain relationship becomes stable prior to data collection. For the formal tests, the uniaxial procedure is repeated at least three times in order to obtain a realistic average test data.

Due to the accuracy and ease of application of the Yeoh material model, it was chosen to define the hyperelastic property of the rubber components by fitting the uniaxial extension test data. The expression of the Yeoh model is shown as

\[
U = C_{i0}(I_1 - 3) + C_{i1}(I_1 - 3)^2 + C_{i2}(I_1 - 3)^3 + \frac{1}{D_1}(J^{el} - 3)^2 + \frac{1}{D_2}(J^{el} - 3)^4 + \frac{1}{D_3}(J^{el} - 3)^6
\]

where \( U \) represents the strain energy density; \( C_{i0}, C_{i1}, C_{i2}, D_1, D_2, D_3 \) are material constants which describe the shear behaviour and material compressibility respectively, and are to be determined by testing and test data fitting in ABAQUS; \( J^{el} \) is the elastic volume ratio, while \( I_1 \) is the first deviatoric strain invariant. The fitting of the test data for the tread components is illustrated in Figure 5, and the calculated parameters for all the rubber components can be found in Table 2.

![Figure 5 Hyperelastic Property fitting for the tread](image-url)

Figure 5  Hyperelastic Property fitting for the tread
Table 2 Hyperelastic property constants for rubber materials

<table>
<thead>
<tr>
<th>Rubber Material</th>
<th>Yeoh strain energy potentials constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
<td>C_{10}</td>
</tr>
<tr>
<td>Tread</td>
<td>0.73</td>
</tr>
<tr>
<td>Sidewall</td>
<td>0.71</td>
</tr>
<tr>
<td>Apex</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Viscoelastic property is also considered in this study because of its importance in dynamic simulations. Actually, the viscoelastic property of rubber material provides a more accurate representation of the real world rubber behaviour, and the finite element model should include consideration of this characteristic, especially for transient dynamic analysis.

The viscoelastic property test of rubber material was carried out using the method of stress relaxation, and the test data was collected and analysed in the time domain. In the measurements, the extracted rubber samples were each stretched up to 50% of their original length respectively, and they were held for more than 900 seconds whilst the stress levels were recorded.

The normalized time domain viscoelastic property test data obtained from stress relaxation testing was implemented for evaluation in ABAQUS. The relaxation function $g_R(t)$ in terms of a series of exponentials known as Prony series is used to fit viscoelastic property data [24]

$$
g_R(t) = G_0 \left(1 - \sum_{i=1}^{N} \overline{g}_i^p \left(1 - e^{-t/\tau_i^G}\right)\right) \tag{3}
$$

where $\overline{g}_i^p$ represents the shear relaxation modulus ratio, $\tau_i^G$ represents relaxation time; these material constants are determined by modelling the physical test in ABAQUS. The evolution results of the tread component are illustrated in Figure 6 and the Prony series parameters for all the rubber materials can be found in Table 3.
Figure 6  Viscoelastic property evolution of the tread

Table 3  Viscoelastic property constants for rubber materials

<table>
<thead>
<tr>
<th>Rubber Material</th>
<th>Component</th>
<th>Prony series parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g₁</td>
<td>k₁ τ₁</td>
</tr>
<tr>
<td>Tread</td>
<td>0.08</td>
<td>0  2.39E-5</td>
</tr>
<tr>
<td>Sidewall</td>
<td>0.10</td>
<td>0  2.07E-6</td>
</tr>
<tr>
<td>Apex</td>
<td>0.15</td>
<td>0  5.76</td>
</tr>
</tbody>
</table>

2.3 Three-dimensional tyre model and cornering simulation

Figure 7 illustrates the 3D FE tyre model, which was generated by revolving the 2D tyre model about the rotational symmetric axis, and the 2D axisymmetric elements (CGAX4H and CGAX3H) were transformed into 3D solid elements (C3D8H and C3D6H). In order to constrain the bead nodes of the tyre model, a rigid body between rim node (tyre centre) and the tyre-rim assembly nodes was defined using the tie function in ABAQUS. In the cornering simulation of the tyre model, only the rotational degree of freedom of the rim about the lateral axis was allowed to be free, and the other five degrees (three translational degrees and two rotational degrees) were constrained. The road was modelled using an analytical surface which can move in the S direction at a given speed, and in this way the tyre can rotate reversely because of the interaction of the tyre and road, the friction coefficient between the tyre and the road is set as 1.0. The lateral forces of the tyre in the time domain can be recorded for a slip angle of \( \alpha \).
3. Relaxation length derivation

In the cornering simulation, the lateral force normally increases from 0 to a steady-state value over a period of time, and the relaxation length is considered as the distance needed to reach 63% of the steady-state tyre lateral force. In this study, the tyre rolling velocity applied here is 10 km/h, together with a tyre inflation pressure of 200 kPa and radial load of 3000 N.

3.1 Different input functions

To identify tyre relaxation properties when different steering inputs are applied, two different input functions were defined and applied to effect the road rotation and then to control the tyre motion as shown in Figure 8. These functions are used to prescribe the variation of the road’s horizontal rotation angle, which is expressed in terms of time history of amplitude variation. One input function is the instantaneous application of steering input i.e. application of the road rotation angle (from 0 to the specified slip angle) with a step function, while the other input function is defined with a ramp input definition, which is expressed as [24]

\[
a(t) = \begin{cases} 
A_0 & \text{for } t \leq t_0 \\
A_0 + (A_1 - A_0) \xi^2(10 - 15\xi + 6\xi^2) & \text{for } t_0 < t < t_1 \\
A_1 & \text{for } t \geq t_1 
\end{cases}
\]  (4)
in which $\xi = \frac{t - t_0}{t_1 - t_0}$, $t_0 = 0$, $A_0 = 0$, $t_1 = 0.1$, $A_1$ is the amplitude of the angle displacement. It is noted that the road motion velocity is kept constant, say 10km/h, while the slip angle is varied from 0 to 7 degrees with the input function.

Figure 8  Tyre steering input functions applied on the road rotation angle (a) instantaneous step input function and, (b) ramp input function
3.1 Relaxation length for different slip angles

Cornering force (lateral force) variation for different slip angles was predicted, and Figure 9 shows the lateral force variation for different target slip angles in the step input scenario. For the validation of the cornering properties reference can be to the authors’ previous paper [25], in which satisfactory results were obtained for the comparison between the simulation and measurement. It can be seen from Figure 9 that increasing the slip angle results in a higher steady-state lateral force. With two different input functions (step input and ramp input), the relaxation length as a function of slip angle is plotted and illustrated in Figure 10. It can be observed that reducing the slip angle for the rolling tyre results in higher relaxation length for both the two input functions. This can be explained by the increase of the shear stress in the tread rubber associated with the tyre tread deformation for higher slip angle due to tyre-road interaction. For the tyre rolling on the road with a larger slip angle, the tread section suffers more deformation because of the adhesion between tyre and road, resulting in the steady-state lateral force being established more quickly.

![Figure 9](image)

*Figure 9  Cornering force variation for different slip angles*
In comparison with the step input scenario, the ramp input scenario leads to a higher relaxation length for the same target slip angle. For example, for the target slip angle of 7°, the relaxation length for step input is 0.07 m while that for ramp input is 0.20 m. Due to the fact that the steady state slip angle value for the step input function is applied instantaneously, the relaxation length for the rolling tyre is shorter compared to that of the ramp input function. It is evident that the quicker the application of slip angle, the shorter the relaxation length.

4. Parametric studies

DOE matrix method has been applied for the parametric studies because of its convenience and importance in identifying the effect of tyre properties on tyre dynamic performance such as steady-state rolling properties and transient dynamic behaviour. In the literature [11, 26], the DOE matrix method was used for investigating the effect of yield strength, cross-sectional area, and spacing of the carcass rebars on the tyre burst pressure and the effect of tyre reinforcement turn-up together with the bead reinforcement size and distribution on the tyre durability respectively. This present work investigates the effects of the reinforcement properties on the relaxation length using DOE orthogonal array scheme. Seven factors, including the layup structures and material properties, were established as shown in Table 4. Different levels of these factors were set up to identify the relaxation length properties for different input parameters. Based on the layup structure and material property parameters, an orthogonal matrix of fifteen experiments was generated and summarised in Table 5.
The aim of DOE matrix method is to provide an approach for assessing the effects of the tyre material and layup structure properties on the relaxation length, whereby the potential factors in the seven parameters which significantly affect the relaxation length can be evaluated. Taking into account the driver’s ability to apply the required steer input in practice, two different input functions were applied in the simulation in terms of road rotation (tyre slip) angle. For the scenarios with a target slip angle of 7 degree, the values of relaxation length for different run cases have been calculated and plotted against tyre parameters, as shown in Figure 10 and Figure 11. Figure 10 summarises the variations in relaxation length at different levels of tyre material and structural properties with an instantaneous step input function, while Figure 11 presents the relaxation length variation for a ramp input function.

In Figure 10, it can be observed that the relaxation length values are only slightly affected by different levels of steel belt cross-section area, steel belt spacing, and steel belt elasticity, whereas the effects of different levels of cross-section area of carcass cord, carcass cord
spacing, carcass elasticity and crown angle of steel belts on relaxation length are significant. It can be seen that increasing the carcass cord spacing leads to increase in the relaxation length for both the two input functions. This can be explained by the variation in contact patch when increasing the carcass cord spacing. As increasing the carcass cord spacing reduces the carcass stiffness, the tyre deforms more and the area of the tyre-road contact patch is enlarged, which leads to a higher relaxation length.

In terms of the cross-section area of the carcass cord, carcass elasticity and crown angle of the steel belts, it is found that increasing the level of these parameters results in a lower relaxation length. Since increasing these parameters increase the tyre stiffness and therefore reduces the tyre-road contact area, the result is a quicker build-up of the lateral force in turning the tyre to a specified slip angle with steering inputs and hence a reduction in the relaxation length. It is also interesting to note that carcass properties including structural and material properties have significant impact on the relaxation length, whereas the steel belt properties, excluding steel belt crown angle have only very slight influence on the relaxation length.

A similar trend can also be found in Figure 11: the steel belt cross-section area, steel belt spacing and steel belt elasticity have very little impact on the relaxation length, whereas the other four parameters (cross-section area of carcass cord, carcass cord spacing, carcass elasticity, and crown angle of steel belts) have significant influence on the relaxation length variations. It is noted that the relaxation length value has only a slight difference for the carcass cord spacing of 1.04 mm and 1.34 mm, but increases considerably when the spacing is increased to 1.64 mm. Consistent with the instantaneous step condition, increasing of cross-section area of carcass cord, carcass elasticity and crown angle of steel belts lead to the decrease of relaxation length.

![Step Input](image)

**Figure 10** relaxation length for instantaneous step input function
5. Conclusions

Explicit finite element program has been used in predicting the relaxation length of the rolling tyre. The detailed information of the tyre layup structure and material properties has been defined by a combination of measurements and mathematical evaluation using existing material model. Rebar element in ABAQUS™ was used to define the reinforcement properties, in which the adjacent cords’ spacing, cross-section area of the cords and elasticity of the cords are specified. The Yeoh model and Prony series model have been used to evaluate the hyperelastic and viscoelastic measurement data respectively.

The relaxation length is derived by calculating the distance needed to reach 63% of the steady-state tyre lateral force. The tyre steering simulations were carried out in the time domain, and two different input functions (step and ramp) were applied to effect the road’s horizontal rotation angle, relative to the tyre’s rolling direction.

DOE orthogonal array scheme was used in the parametric analysis in investigating the influence of the layup structure and material properties on the variation in relaxation length. Seven factors including the carcass and the steel belt properties at three different levels were considered and in this way fifteen experimental runs were established. With the analysis for the instantaneous step and ramp input conditions, it can be observed that the carcass properties including the spacing of neighbouring carcass cords, cross-section of the carcass cord and carcass elasticity together with the crown angle have significant influence on the relaxation length for both of the two steering input functions, whereas the variation in cross section area of the steel belt, the spacing of neighbouring steel belts and the...
elasticity of steel belt have very little effect on the relaxation length. It is also noted that increasing the spacing of neighbouring carcass cords leads to the increase of relaxation length, while increase in the elasticity and cross-section area of the carcass cord and the crown angle of steel belts leads to lower relaxation length.
References

Graphical Abstract

Tyre Structure Design Factors
A: Cross-section area of carcass cord
B: Carcass cords spacing
C: Carcass elasticity modulus
D: Crown angle of steel belts
E: Steel belt cross-section area
F: Steel belts spacing
G: Steel belt elasticity modulus

Effects of Design factors on Relaxation Length
Highlights

- The structural layup of the tyre has been characterised using image processing techniques to identify reinforcement cord orientation and spacing.
- The material properties of rubber have been obtained using a combination of material tests and Material modelling in Abaqus to characterise the hyperelastic and viscoelastic properties.
- Relaxation length for application of two types of steer input (step and ramp) has been obtained for different target slip angles using time domain simulation in Abaqus/Explicit.
- The effect of various tyre structure design factors on relaxation length were characterised by numerical experiment using design of experiment (DOE) method.
- The tyre carcass reinforcement properties i.e. cord diameter, spacing and elasticity modulus as well as the belt reinforcement crown angle are the factors which control the relaxation length.