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Impure Sets May Be Located: A Reply to Cook

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Cook [2013] has argued that, given a Fregean view about sets and numbers, impure sets cannot be spatiotemporally located. §1 explains my understanding of his argument, whilst §2 identifies an ambiguity within it. In §§3-6 I explain how the argument fails under every resolution of that ambiguity.

1. Cook's Argument

Cook assumes a Fregean view of numbers such that "our account of the nature of numbers and sets needs to take into account the fact that numbers and sets are abstract objects corresponding, in a fundamental manner, to concepts" such that "the metaphysics of cardinal numbers should be relevantly similar to the metaphysics of sets." [2013: 2]. Given this, he argues that we should accept the *Fregean Spatial Uniformity Thesis (FUT)*: that the relations between the spatial location of the set of the concept *C* and the spatial locations of objects falling under *C* should mirror the spatial location of the number of the concept *C* and the spatial location of objects falling under *C*.

Assuming (as seems fair) that sets must be where their members are, he advances from FUT to impure sets not being located thus:

(1) {Leia} is spatially co-located with Leia.[1]

(2) {Han} is spatially co-located with Han.

As {Leia} is the set of the concept IDENTICAL TO LEIA (similarly for {Han}) and the number of both concepts is 1, it follows – given FUT – that:

(3) The number 1 is spatially co-located with Leia.

(4) The number 1 is spatially co-located with Han.

Then, from the substitutivity of identicals, we get the obviously false:

(5) Leia is spatially co-located with Han.

Given (5) is absurd, by *reductio* (1) and (2) are false (and sets cannot be located anywhere.)

[1] Cook uses many-membered sets whilst I use singletons. This is because it gets a little bit harder (exegetically, not philosophically) to talk about the 'exact location' of pluralities of things rather than one thing. Singletons will, however, make clear both Cook's argument and the problem it suffers from.

But when Cook says 'substitutivity of identical' I take it he means to be relying upon the transitivity of the co-location relation (for the number 1 isn't *identical* to either Han or Leia.) So he endorses:

> *Transitivity of Co-Location (TCL)*: If x is co-located with y and y is co-located with z then x is co-located with z.

TCL gets to the required conclusion.[2]

2. Ambiguous Location Relations

The ambiguity arises because '__ is located at __' is ambiguous, as are its cognate terms such as 'co-location'; there are, therefore, multiple ways to understand (1) – (5) and TCL. 'Location' can mean (at least) four different things: exact location; maximal filling; partial location; weak location. [*cf* Hudson 2005: 97-106; Parsons 2007]

To begin the disambiguation, we must get a grip on the primitive notion of 'exact location'. Examples of that relation include: me being exactly located at a 5'11" man-shaped region; a cube being exactly located at a region with six sides; a sphere of radius n units being exactly located at a region that has a volume of $4/3\pi n^3$. This relation might be many-one (it's contentious whether it is or not – Donnelly [2010: 203] lists both those who support its possibility and those who harbour doubts.) If it did hold many-one, some objects could be multi-located (where x is multi-located at the rs $=_{\mathrm{df}}$ x is exactly located at each r, and there are at least two rs.) Examples of multi-located things would be, say, immanent universals which exactly occupy every region an instance of it exactly occupies, or time travellers stood next to themselves (so time travelling Nikk would be exactly located at *two* man-shaped regions 5'11" in height.)

In cases of multi-location objects may, or may not, exactly occupy the union of those regions – that is, it's an open question whether the property *redness* exactly occupies the union of the regions that every instance is exactly located at (I suggest it doesn't, others [e.g. Barker and Dowe 2003] appear to disagree.) This open question is phrased by the helpful introduction of the next way of understanding location:

> x maximally fills region R $=_{\mathrm{df}}$ (i) there are some rs (possibly a single r) and x is exactly located at, and only at, each r (ii) R is the union of the rs. (Note that, by definition, singularly located objects exactly occupy the region they maximally fill.)

The open question, then, amounts to asking whether every object exactly occupies the region it maximally fills. The answer to that question doesn't concern us; it's only used to help key us into another possible disambiguation of what it is to be located at a region: does the object maximally fill it or not?

The remaining two ways of disambiguating location are less esoteric, and easier to define:

> x is partially located at region r $=_{\mathrm{df}}$ r is a sub-region of R and x is exactly located at R (e.g. I am partially located where my hand is, where my arm is, where my head is etc.)
>
> x is weakly located at region r $=_{\mathrm{df}}$ x partially occupies R and R is a sub-region of r (e.g. I am weakly located in England, Europe and the Milky Way, but not America or Andromeda.) [*cf* Parsons 2007: 203]

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[2] In private correspondence Cook has said he had something like this in mind.

With four different ways of interpreting what one might mean when one says that an object 'is located at' a region there are four corresponding ways of understanding 'co-located':

x is co-located$_E$ with $y =_{df} x$ and y are both exactly located at some region.

x is co-located$_F$ with $y =_{df} x$ and y both maximally fill some region.

x is co-located$_P$ with $y =_{df} x$ and y are both partially located at some region.

x is co-located$_W$ with $y =_{df} x$ and y are both weakly located at some region.[3]

So there are four ways of understanding Cook's argument depending upon what co-location relation he has in mind. Given the first, TCL ends up being false because of nuances to do with multi-location; given the second, (3) and (4) are false; given the third, TCL admits of straightforward counterexamples; given the fourth, (5) is no longer absurd.

3. Co-location$_E$

Here are some examples to get a grip on being co-located$_E$: statues, and the lumps of clay that constitute them, are co-located$_E$; if bosons interpenetrated they'd be co-located$_E$; if a ghost of the right shape and size passed through me, we'd be co-located$_E$; and, of course, an immanent universal is co-located$_E$ with its instance. It seems natural to think numbers, if located, are located like immanent universals i.e. exactly located where the things that fall under their associated concept are exactly located (so the number 1 is exactly located where every singular entity is.) Whilst that might be false (alternatives will be considered below) let's imagine what happens if this is the case.

If Cook meant co-location$_E$ throughout, then TCL is now false. This isn't obvious when we think about singularly located things, but it becomes apparent when we think about multi-located things. If, for instance, you believe immanent realism then a property like *redness* is exactly located at multiple places, being co-located$_E$ with every red instance, but it doesn't thereby follow that every red instance is co-located$_E$ with every other red instance (my red pen isn't where your red shirt is!) Or imagine time travel is possible and Marty time travels back in time to a region in Europe whilst his much younger self is at a region in America. If two ghosts simultaneously walk through both versions of Marty, they are each co-located$_E$ with Marty (one with his younger version, one with his older version.) But they are not co-located$_E$ with one another, for the American ghost is in America, and the European ghost is in Europe. The lesson is: when things become multiply located, weird things happen with intuitively transitive relations (not just those concerning co-location: I argue elsewhere that similar oddities arise with mereological relations [Effingham 2010].)

This isn't that counterintuitive, especially as we can rescue a similar transitivity principle (one that means that TCL holds when only considering singularly located things.) When things are multi-located we have multiple versions of them (e.g. Marty's earlier and later versions, or *redness*'s

[3] More options arise if we add in an 'all and only' clause e.g. x and y are co-located$_{E*}$ iff they are exactly located at <u>all and only</u> the same regions. But this doesn't help Cook. For instance the number 1 won't be co-located$_{E*}$ with Han or Leia and so (3) and (4) will be false. To demonstrate: if the number 1 is multi-located where, say, Leia is (see §2), then it has at least one more exact location than Leia (for it also exactly occupies where Han is); if it is singularly located (as described in §2) then it isn't exactly located where Leia is at all; so either ways the number 1 fails to be co-located$_{E*}$ with Leia and (3) is false; similarly for Han and (4). The same problem arises for similar variations of co-location$_p$ and co-location$_w$. And the variation of co-location$_F$ is just co-extensive with co-location$_F$.

many different instantiations.)[4] We should apply this to all multi-located things: when *redness* is instantiated by many red things, it is exactly located where every red thing is, and we can say that there's a different version of *redness* present in each place; when 1 is exactly located where Leia is and exactly located where Han is, there are two different 'versions' of the number 1. We then get a principle like:

> TCL*: If version v_x of x is co-located$_E$ with version v_y of y, and v_y is co-located$_E$ with version v_z of z then v_x is co-located$_E$ with v_z.

For singularly located things TCL* is the same as TCL, but not so for multi-located things e.g. as different versions of the number 1 are where Leia and Han are, TCL* doesn't lead us to believe that Leia and Han are co-located$_E$ (and so doesn't cause the problem TCL does.)

Cook might remain unmoved. TCL is clearly true, he may say, ergo – if numbers are multi-located – then all the worse for numbers (*a fortiori* immanent universals, time travellers etc.) Cast like this, the crucial part of Cook's argument is that Fregeanism entails FUT, which entails that numbers are multi-located. But we can avoid even that. Consider the current state of play: Cook believes we should endorse the following principle about the location of sets (the mirror principle of which entails that numbers are multi-located):

> (6) Set *s*, associated with concept *C*, exactly occupies the region that those things which fall under *C* exactly occupy (e.g. Han exactly occupies r_H, Leia exactly occupies r_L, so Han and Leia collectively exactly occupy $r_L \cup r_H$; given FUT {Leia, Han} exactly occupies $r_L \cup r_H$.)[5]

Cook believes this principle because he assumes, fairly, that sets should be where their members are. But there is an alternative principle that both meets Fregean sensibilities and meets the demands of that fair assumption, whilst nevertheless allowing sets and numbers to be singularly located:

> (7) Set *s*, associated with some concepts – the *C*s – exactly occupies the region that is maximally filled by those things which fall under each *C*. (e.g. Han exactly occupies r_H, Leia exactly occupies r_L, so they maximally fill $r_L \cup r_H$, ergo {Leia, Han} exactly occupies $r_L \cup r_H$.)

(7) strays from the exact letter of FUT because it talks about a set's relation to those things falling under *every* concept associated with it – but I don't think this deviation is substantial or that it drives us away from Fregeanism.

Both (6) and (7) exactly locate sets in the same place (e.g. either ways {Leia, Han} exactly occupies $r_L \cup r_H$) so either principle meets Cook's fair assumption that sets are where their members are – and both are compatible with his Fregean scruples. But whilst the mirror principle of (6) left numbers being multi-located, the same does not go for the mirror principle of (7), which is:

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[4] By this I don't mean to *reify* versions (or instantiations) anymore than a nominalist reifies properties by asserting that red is their favourite colour. So whilst some people might reify versions (e.g. a perdurantist might think temporal parts are versions) I make no such claim here, but do help myself to the version-talk.

[5] Why not think that {Leia, Han} exactly occupies both r_L and r_H rather than the region they collectively exactly occupy? This doesn't sound like the natural treatment, and further {Leia, Han} would then be multi-located which is what we're meant to be avoiding.

(8) A number n, associated with some concepts – the Cs – exactly occupies the region that is maximally filled by those things which fall under each C.

Given (8) a number ends up exactly occupying just one region – the union of all of the regions that the things that fall under the associated concept exactly occupy (for that union is the region that they maximally fill together.) So as every entity falls under a concept associated with 1, the region that is maximally filled by every entity – that is, the largest occupied region that there is – is where the number 1 is exactly located. (Indeed, every number n, where $n \leq$ the number of things in the cosmos, will end up exactly occupying that same region.) So, if Cook thinks TCL is inviolable (and, thus, that the multi-location of *anything* is impossible) he can't complain when the locator of numbers agrees that they must be singularly located, and takes that as a reason to endorse (7) and (8). But given (8) the, singularly located, number 1 is exactly located at the largest occupied region (which is much bigger than either Han or Leia) and, thus, (3) and (4) are false. So even if Cook takes for granted anti-multi location sentiments, and holdfasts TCL, his argument isn't sound.

In short: if Cook has in mind 'co-location$_E$' then either multi-location is possible, and TCL is false, or multi-location is impossible and we have every reason to endorse (7) and (8), thereby denying (3) and (4).

4. Co-location$_F$

Alternatively, Cook may acknowledge that numbers are multi-located entities, but believe that the relevant relation is co-location$_F$ not co-location$_E$ (this alternate disambiguation won't help if everything must be singularly located since 'co-location$_E$' and 'co-location$_F$' are co-extensive when it comes to singularly located things). Now the argument isn't sound as (3) and (4) are, again, false. Imagine the universe consisted solely of Leia and Han. The number 1 would be exactly located where Leia is (r_L) and Han is (r_H) (and where each of their parts are and fusion is, but ignore these things for simplicity's sake.) The number 1 thereby maximally fills $r_L \cup r_H$. But Leia doesn't maximally fill *that* region; Leia is exactly located at r_L and (by definition) maximally fills only r_L. Similarly for Han. So Leia is not co-located$_F$ with the number 1, nor is Han; so (3) and (4) are false and the *reductio* can't work.

5. Co-location$_P$

Perhaps we are meant to concentrate on partial location, and co-location$_P$, throughout. But read in this fashion TCL is straightforwardly false for it then reads:

TCL$_P$: If x is co-located$_P$ with y and y is co-located$_P$ with z then x is co-located$_P$ with z.

A counterexample is easy to find. Imagine two semi-detached houses with kitchens bordering each house's outer wall and not its inner wall. So the houses themselves share a wall, but the kitchens do not. Each house partially occupies where the inner wall is, so the houses are co-located$_P$. Each house is partially located where its respective kitchen is, so each house is co-located$_P$ with its respective kitchen. Clearly, though, the kitchens – which exactly occupy disjoint regions from one another – do not partially occupy one and the same region i.e. they are *not* co-located$_P$. TCL$_P$ is, therefore, false.

6. Co-location$_W$

Finally, we might disambiguate 'co-location' as 'co-location$_W$'. But now we can't carry out the *reductio* for we are an absurdity short. Let TCL_W be the principle we get when we read 'co-location' in TCL as 'co-location$_W$'. TCL_W is true, but trivially so for everything is co-located$_W$ with everything else, since everything is weakly located at the largest region that there is! However (5) is not the absurdity that Cook is looking for, for all (5) now amounts to is the claim that Leia partially occupies one region, Han partially occupies another, and there's a larger region with both as parts. Few claims are more innocuous than that!

In conclusion, no matter how Cook disambiguates 'co-location' it won't be possible to motivate the *reductio*.

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