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Catalytic performance of cobalt-silica catalyst for Fischer-Tropsch synthesis: effects of reaction rates on efficiency of liquid synthesis

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Abstract

A one-dimensional pseudo-homogeneous mathematical model of a fixed bed reactor for Fischer-Tropsch (FT) synthesis was developed for the flow of simulated N₂-rich syngas over an in-house cobalt-silica catalyst. This study aims at improving the efficiency of FT synthesis by maximizing the liquid productivities and selectivity, as well as maximizing the syngas conversion and minimizing the methane formation. The developed model predicts the fraction of the reactants and products along the reactor bed length. The rate of syngas conversion and the rate of CO₂, H₂O, CH₄, C₂H₆, C₃H₈, n-C₄H₁₀, i-C₄H₁₀ and C₆.₀₅H₁₂.₃₆ (C₅+) formation were calculated by developing advanced codes in MATLAB. The reaction equations were proposed as a number of lumped chemical reactions (8 reactions, including water gas shift reaction) by means of the molar coefficients of reaction molecules (11 reactive species). The kinetic parameters were estimated by global optimization in MATLAB using the global search method. Optimum values were achieved during the search process. The results predicted by the model were in very good agreement with those measured experimentally at different operating conditions, with respect to conversion and the FT products’ selectivity. The rates of production and consumption were derived from a modified power-law rate expression. This study shows that the adapted rate model can deliver a better prediction of final conversion and selectivity. The accuracy of the fitted model relative to the experimental data was determined by a quantitative analysis method using the mean absolute relative residual percentage (MARR %) for the total of 35 data points. It was found that the model based on the modified equation provided a better fit to the experimental data with a MARR of 6.57%, compared to the classic equation with a MARR of 12.24%.

The model predicts the influence of reaction rates on the performance of the fixed bed FT reactor using an unpromoted Co/SiO₂ catalyst at 503 K, 15 bar and 6 L g⁻¹ h⁻¹. As a result, the conversions of 91.57 and 97.26% were achieved for CO and H₂, respectively with the FT C₅+ synthesis reaction rate of 7.81×10⁻⁵ mol g⁻¹ s⁻¹. The higher rate of C₅+ formation was found by increasing FT reaction rates compared to the rate of lighter hydrocarbons’ formation. At the same condition, only 5.04 and 8.91% methane and C₂-C₄ selectivity respectively, were predicted; while the highest value of 86.05% liquid (C₅+) selectivity was obtained in this case. It was concluded that the higher rate of conversion of H₂ inside the pores filled with liquid products, compared to that of CO, caused an increase in the H₂/CO ratio in the catalyst pores; and thus, a shift towards the formation of lighter hydrocarbons.

Keywords: Fischer-Tropsch synthesis; fixed bed reactor; mathematical modelling; kinetic model
1. Introduction

In biomass to liquid technology where Fischer-Tropsch (FT) synthesis can be the final step there is great potential for the production of ultra-clean transportation fuels like diesel, jet fuel and other hydrocarbon (HC) products [1, 2]. FT synthesis has received considerable worldwide attention, both industrially and academically, due to an increased concern about the problem of environmental pollution and remote gas utilization; as well as the implementation of more stringent environmental legislation on liquid fuels [3, 4]. FT synthesis’ products are excellent high-performance, clean diesel fuels; due to the absence of sulphur, their aromatic compounds and their high cetane number. FT synthesis can be defined as the means of indirect coal liquefaction, in which synthesis gas containing CO and H\textsubscript{2} are catalytically converted to a mixture of linear gaseous, liquid and solid hydrocarbons [5]. A wide product spectrum of hydrocarbons is formed by the successive addition of C\textsubscript{1} units (-CH\textsubscript{2}- that represents a methylene group) to growing chains on the surface of the catalyst. Equation 1 represents the main reaction of the FT synthesis process [6]. Equation 2 is the methanation reaction which is often considered to be a separate reaction in this process. In addition, Equation 3 is the water gas shift (WGS) reaction. In principle, the WGS reaction is usually considered in the case of an Fe catalyst. With reference to the literature study [7], Co is not very active for the WGS reaction. In contrast, in the present study, the WGS reaction was taken into account since a considerable fraction of water is produced and subsequently CO is converted to CO\textsubscript{2}.

\[ \begin{align*}
\text{n CO} + 2\text{n H}_2 & \rightarrow (\text{CH}_2)_n + \text{n H}_2\text{O} & \Delta H_{R,298}^f = -165 \text{ kJ mol}^{-1} & \text{Equation 1} \\
\text{CO} + 3 \text{H}_2 & \rightarrow \text{CH}_4 + \text{H}_2\text{O} & \Delta H_{R,298}^f = -206 \text{ kJ mol}^{-1} & \text{Equation 2} \\
\text{CO} + \text{H}_2\text{O} & \leftrightarrow \text{CO}_2 + \text{H}_2 & \Delta H_{R,298}^f = -41 \text{ kJ mol}^{-1} & \text{Equation 3}
\end{align*} \]

Research studies have shown that only the four group VIII metals, iron (Fe), ruthenium (Ru), cobalt (Co) and nickel (Ni), have sufficiently high activities for FT synthesis. Among them, Fe and Co-based catalysts can be considered as the most practical FT catalysts. The Co-based catalyst has been widely investigated for FT synthesis due to its high selectivity to long chain hydrocarbons (HCs), high FT synthesis activity, high resistance to deactivation by water, low oxygenates selectivity, low water gas shift activity and better catalyst stability in hydrogen-rich environments. In the present work, an in-house developed cobalt catalyst on a silica support (37% Co/SiO\textsubscript{2}) was used.

To achieve an optimum in performance for the complete process, the reactor and the kinetic model should be developed mathematically. Also, the details of the products’ distribution, selectivity and reactants conversion must be obtained by the model. Generally, various types of reactors including a fixed bed reactor, slurry bed, trickle bed and fluidized bed reactors are used to run the FT synthesis process. The fixed bed reactor has several advantages such as the absence of the requirement to separate the catalyst from the product, the ease of the scaling up from a single tube to a pilot plant, and shutdown robustness compared with slurry bed reactor. However, the downside of fixed bed reactor includes high mass and heat transfer limitation, poor thermal stability, low catalyst reaction rate, offline-slow catalyst replacement, and high capital cost. Shell and Sasol are the pioneers and world leading companies for large scale FT liquids production using fixed bed reactor and slurry bed reactor, respectively. By modelling and optimizing the reactor’s operation, it is possible in many cases to achieve significantly enhanced throughput; better and more consistent product quality; rising conversion and selectivity; as well as a significant effect on the scaling up of the processes from the laboratory to production scale. Only a few studies [8-17] are available on the basis of the development of a mathematical model of a fixed bed reactor for FT synthesis.

In addition, the description of the FT kinetics is quite challenging due to the complexity of the reaction pathway and products involved in this process. The kinetics of cobalt-based FT catalysts have been a topic of investigations for decades. The principal governing factors of any FT synthesis kinetic mechanism are the temperature, total pressure, flow rate and the H\textsubscript{2}/CO ratio, which affect FT product distribution and conversion rates. There are a number of kinetic studies that were carried out empirically to fit the data based on the power-law rate mechanism. Yang et al. [18] and Wang [19] found empirical rate expressions for supported Co catalysts using a fixed bed reactor via regression of a power-law equation. Zennaro et al. [20] studied the kinetics on a
well-characterized 11.7% Co/TiO$_2$ catalyst in a differential fixed bed reactor. The experimental data of this study were fitted by a power-law expression. Das et al. [21] investigated the kinetics and effects of water for a 12.4 wt% Co/SiO$_2$ catalyst using a 1-L continuously stirred tank reactor (CSTR). The measured data of this study were also fitted by a power-law expression. Marvast et al. [22] modelled a two-dimensional fixed bed FT reactor packed with an Fe-HZSM5 catalyst using the same approach. According to this study, the C$_{5+}$ fraction was lumped in a single kinetic equation as representative FT products. However, their results (for rate of conversion and production) were not sufficiently accurate, with a relatively large error carried by the model. Jarosch et al. [23] developed a model using a similar approach for a Co:Re (21:1) catalyst on γ-Al$_2$O$_3$. Another relevant example is the work done by Ma et al. [24, 25] that was applied to a 25% Co/Al$_2$O$_3$ catalyst to fit the CH$_4$ kinetic data. Overall, the kinetic study based on the power-law rate model can be a powerful method to provide some insight into FT synthesis, without the need to represent a complex reaction network.

No-one has yet published the mathematical model of a fixed bed reactor over a Co/SiO$_2$ FT catalyst, including the reaction kinetics with the details of production rates of CO$_2$, H$_2$O, CH$_4$, C$_2$, C$_3$, C$_4$ and C$_5$, as well as the CO and H$_2$ conversion for this case. In the present study, a simple one-dimensional pseudo-homogeneous mathematical model of a fixed bed reactor is developed. The reaction kinetics were proposed on a 37% Co/SiO$_2$ catalyst using the modified power-law rate expression. The model predicts a mole fraction of the proposed species at five different sets of experimental conditions in which the data sets were adequate for this study. A parametric sensitivity analysis, with a detailed numerical simulation, is performed to illustrate the effects of reaction rates on the efficiency and performance of FT synthesis with respect to productivity, selectivity and conversion.

2. Experimental set-up

Figure 1 indicates the schematic diagram of the experimental setup (designed by co-worker [26]) that was conducted in a fixed bed reactor packed with 37 wt% Co catalysts on a silica support. The experiments were carried out at five different operating conditions with respect to the temperature, pressure and flow rate; they are tabulated in Table 1. The details of the reactor, as well as the in-house spherical Co catalyst and commercial silica support’s (provided by Fuji Silysia Chemical Ltd) specification, are tabulated in Table 2. A mini-structured vertical downdraft fixed bed reactor was used for the FT synthesis. The reactor was fixed in a tube furnace in order to provide the heat zone and a cast iron jacket was installed between the furnace and reactor to provide the uniform wall temperature. A simulated N$_2$-rich syngas bottle of 17% CO, 33% H$_2$ and 50% N$_2$ was fed into the reactor column at each experiment. Silicon carbide was used to dilute 2 g of the catalyst with a mass ratio of 1:12 (Co-SiO$_2$/SiC) for each experiment and then it was loaded into the reactor. A calibrated smart mass flow controller was employed to regulate the flow rate of the syngas (Bronkhorst Ltd).
Nitrogen gas was used to purge the reactor bed. Prior to the reaction, the catalyst was reduced in order to be activated. The catalyst was placed on stream for a 12 h reaction time to measure CO conversion, selectivity and productivity of hydrocarbon synthesis, as well as CH₄ and CO₂ product species. Two liquid/gas separators were used to separate liquid products from the gaseous stream. Changes in the concentration of CO and CO₂ production were monitored online by using a modified CO analyser (AVL Digas 440). A Gas Chromatogram Flame Ionization detector (GC-FID) (HP 5890) was employed to analyse the H₂-C₃-H₂C₄ online. The heavier liquid products were analysed offline by using Gas Chromatogram-Mass Spectrometry (GC-MS) (PerkinElmer ™). Utilization of nitrogen-rich syngas (with 50% of N₂ (volumetric percentage)) leads to reduction in production cost of diesel oil by eliminating the need for application of gas recycling loop after production of syngas by air partial oxidation. Waste generated heat of FT reaction could be removed effectively and temperature runaway probability can be minimized by nitrogen gas. Utilization of nitrogen as an inert in FT regime could result in an increase in tube diameter of reactor and decrease in number of tubes without temperature runaway problem [11].

![Diagram](image)

**Figure 1** Schematic diagram of the experimental setup [27].

**Table 1** Experimental conditions of the present study

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Temperature [K]</th>
<th>Pressure [bar]</th>
<th>WHSV [L_g_\text{cat}^{-1} h^{-1}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition 1</td>
<td>503</td>
<td>15</td>
<td>2.4</td>
</tr>
<tr>
<td>Condition 2</td>
<td>503</td>
<td>20</td>
<td>3.0</td>
</tr>
<tr>
<td>Condition 3</td>
<td>503</td>
<td>25</td>
<td>3.6</td>
</tr>
<tr>
<td>Condition 4</td>
<td>518</td>
<td>10</td>
<td>2.4</td>
</tr>
<tr>
<td>Condition 5</td>
<td>528</td>
<td>10</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Table 2** The details of the fixed bed reactor and catalyst/support used in the present study [27]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor length</td>
<td>cm</td>
<td>52.83</td>
</tr>
<tr>
<td>Reactor bed length</td>
<td>cm</td>
<td>11</td>
</tr>
<tr>
<td>Outer tube diameter</td>
<td>mm</td>
<td>19.05</td>
</tr>
<tr>
<td>Inner tube diameter</td>
<td>mm</td>
<td>15.7</td>
</tr>
<tr>
<td>Tube wall thickness</td>
<td>mm</td>
<td>1.65</td>
</tr>
<tr>
<td>Active site volume fraction</td>
<td>(-)</td>
<td>0.25</td>
</tr>
<tr>
<td>Void fraction</td>
<td>(-)</td>
<td>0.4</td>
</tr>
</tbody>
</table>
3. Development of the reactor mathematical model

3.1. Mathematical reactor model and kinetic description

A numerical one-dimensional mathematical model was developed to represent the process of FT synthesis using the mini-scale fixed bed reactor. The model was assumed to proceed isothermally for the proposed range of operating conditions. To develop the mass balance equation, a differential equation was used to determine the gradient of the change of concentration and partial pressure of the components against the axial dimension of the reactor bed length. Equations 4 and 5 describe the mole balance of the species $i$ with respect to concentration and partial pressure, respectively. The mole balance equations were first order ordinary differential equations (ODEs). The effects of axial dispersion, interphase and intraparticle mass transport were not taken into account.

A series of eggshell cobalt catalyst supported with silica powder with dissimilar structure were used. The utilization of eggshell catalyst in mini scale biodiesel generator is an innovative intention which could overcome the mass transfer limitation in fixed-bed reactor system [26]. Eggshell catalysts have been proposed to overcome difficulties due to diffusion limitations in catalyst pellets in fixed bed reactors and this application (of eggshell catalysts) was also reported by literature [28]. Also, the catalyst was loaded in the reactor in powder form (2 g catalyst with particle size of 75-150 µm) in order to prevent internal mass transfer limitations. Similar to this, Visconti et al. [29] studied the detailed kinetics of the Fischer-Tropsch synthesis on cobalt catalyst in the form of powder to prevent the internal mass transfer limitations. The mixture velocity was calculated from the continuity equation (Equation 6). The density of the fluid was computed by applying the chain rule to the ideal gas law (Equation 7). The classic Ergun law for laminar flow was applied to calculate the overall pressure drop (Equation 8). Several parameterizations for the friction factor, $f$, have been given in the literature [30]. Equation 9 was used, which is valid for laminar flow for spheres over a relatively broad range of particle Reynolds numbers. Equation 10 is the size of the mesh in which the model was discretized in the dimension needed by the code (i.e. "$\Delta z$", "$i" and "$t" which are length, species and number of experimental conditions, respectively) [31]. Table 3 lists the important equations and correlations that were employed in the present model [27].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density of support catalyst</td>
<td>0.38 g ml$^{-1}$</td>
</tr>
<tr>
<td>Particle diameter</td>
<td>150 µm</td>
</tr>
<tr>
<td>Catalyst/support weight</td>
<td>2.0 g</td>
</tr>
<tr>
<td>Surface area</td>
<td>257 m$^2$ g$^{-1}$</td>
</tr>
<tr>
<td>Water content</td>
<td>0.6 wt %</td>
</tr>
<tr>
<td>Pore volume</td>
<td>1.20 ml g$^{-1}$</td>
</tr>
<tr>
<td>No. of species</td>
<td>12</td>
</tr>
<tr>
<td>No. of reactions</td>
<td>8</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{dC_i}{dz} &= \rho_f \beta \sum_{j=1}^{NR} v_{ij} R_j - C_i \frac{du_s}{dz} \\
\frac{u_s dp_i}{R_f dz} &= \rho_f \beta \sum_{j=1}^{NR} v_{ij} R_j - \left( \frac{p_T}{R_f} \frac{du_s}{dz} \right) \\
\rho_f \frac{du_s}{dz} &= -u_s \frac{d\rho_f}{dz} \\
\frac{d\rho_f}{dz} &= \frac{d}{dz} \left( \frac{p_T M_f}{R_f} \right) M_m \left[ \frac{1}{F} \frac{dp_T}{dz} \right] \\
\frac{dp_T}{dz} &= -f \frac{u_s^2 \rho_f}{d} \\
f &= \frac{(1 - e)^2 36 (25/6)}{\varepsilon Re} \\
\end{align*}
\]
Size of the mesh

\[ = [\text{Number of nodes in } z \text{ direction}] \times [\text{Number of species}] \times [\text{Number of experimental conditions}] \]

Table 3 Correlations and equations used in this study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Species concentration</td>
<td>( C_i = \frac{Y_i P_f}{R_g T} )</td>
<td>Equation 11 [32]</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>( Re = \frac{\rho u L_p d_p}{\mu_m} )</td>
<td>Equation 12 [33]</td>
</tr>
<tr>
<td>Mixture viscosity</td>
<td>( \mu_m = \sum_{i=1}^{N_S} \frac{Y_i \mu_i}{\sum_{j=1}^{N_S} \mu_j} )</td>
<td>Equation 13 [34]</td>
</tr>
<tr>
<td>Mixture molecular weight</td>
<td>( M_m = \sum_{i=1}^{N_S} Y_i M_i )</td>
<td>Equation 14 [34]</td>
</tr>
<tr>
<td>Species viscosity</td>
<td>( \mu_i = a + b T + c T^2 + d T^3 )</td>
<td>Equation 15 [35]</td>
</tr>
<tr>
<td>Initial fluid density</td>
<td>( \rho_f = \frac{P_f M_m}{R_g T} )</td>
<td>Equation 16</td>
</tr>
<tr>
<td>Initial superficial velocity</td>
<td>( u_{s,0} = \frac{Q}{A_r} )</td>
<td>Equation 17</td>
</tr>
<tr>
<td>Conversion of reactants</td>
<td>( x_i = \frac{C_{i,\text{in}} - C_{i,\text{out}}}{C_{i,\text{in}}} \times 100 )</td>
<td>Equation 18</td>
</tr>
<tr>
<td>CO(_2) selectivity</td>
<td>( S_{CO_2} = \frac{C_{CO_2,\text{out}}}{C_{CO_2,\text{in}} - C_{CO_2,\text{out}}} \times 100 )</td>
<td>Equation 19 [36]</td>
</tr>
<tr>
<td>Light hydrocarbons selectivity</td>
<td>( S_{Li} = \frac{C_{Li,\text{in}} - C_{Li,\text{out}}}{C_{CO_2,\text{out}} - C_{CO_2,\text{out}}} \times 100 ) for ( 1 \leq i \leq 4 )</td>
<td>Equation 20</td>
</tr>
<tr>
<td>Total liquid hydrocarbon selectivity</td>
<td>( S_{CS+} = 100 - (S_{C_1} + S_{C_2} + S_{C_3} + S_{C_4}) )</td>
<td>Equation 21</td>
</tr>
</tbody>
</table>

In order to complete the mathematical description of the reactor model, the details relating to the kinetics of the reactions, need to be provided. Kinetic models of reduced complexity are attractive for reactor analysis and design purposes. The FT synthesis kinetic mechanism is based on a complex reaction system with many chemical compounds. The reaction network can be classified as a number of lumped reactions by means of the kinetic characteristics of reaction molecules. These models are capable of capturing the essential features of the FT synthesis products’ distribution without the need for a parameter, such as chain growth probability (\( \alpha \)).

In the present work, the rate of CO and H\(_2\) disappearance and the rate of CO\(_2\), H\(_2\)O, CH\(_4\), C\(_2\)H\(_4\), C\(_3\)H\(_6\), C\(_4\)H\(_8\), n-C\(_{10}\)H\(_{22}\), i-C\(_{10}\)H\(_{22}\) and C\(_{6.05}\)H\(_{12.36}\) (C\(_{6.05}\)) formation were taken into account. The reaction equations were proposed as a number of lumped reactions (Equations 22–29) by means of the kinetic characteristics of reaction molecules for FT synthesis over Co/SiO\(_2\) (Table 4). Equation 29 is the representative single reaction equation that corresponds the lumped rate of C\(_{6.05}\) formation by setting C\(_{6.05}\)H\(_{12.36}\) as the average molecular value of liquid HC components.

Table 4 Postulated lumped FT synthesis kinetic model over Co/SiO\(_2\) and estimated kinetic parameters

<table>
<thead>
<tr>
<th>Assumed reaction pathway</th>
<th>( n_j )</th>
<th>( m_j )</th>
<th>( \delta_j )</th>
<th>( \alpha_j )</th>
<th>( E_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO + 3H(_2) ( \xrightarrow{R_1} ) CH(_4) + H(_2)O</td>
<td>Equation 22</td>
<td>0.38</td>
<td>0.15</td>
<td>1.99</td>
<td>65.56</td>
</tr>
<tr>
<td>2CO + 4H(_2) ( \xrightarrow{R_2} ) C(_2)H(_4) + 2H(_2)O</td>
<td>Equation 23</td>
<td>-1.17</td>
<td>1.80</td>
<td>-2.05</td>
<td>4.54×10(^2)</td>
</tr>
<tr>
<td>2CO + 5H(_2) ( \xrightarrow{R_3} ) C(_3)H(_6) + 2H(_2)O</td>
<td>Equation 24</td>
<td>0.81</td>
<td>-2.00</td>
<td>1.73</td>
<td>1.14×10(^3)</td>
</tr>
</tbody>
</table>
\[ 3\text{CO} + 7\text{H}_2 \rightarrow \text{C}_3\text{H}_8 + 3\text{H}_2\text{O} \]  
Equation 25 -1.03 1.70 -0.41 1.19\times10^{-6} 31.03

\[ 4\text{CO} + 9\text{H}_2 \rightarrow \text{n-C}_4\text{H}_{10} + 4\text{H}_2\text{O} \]  
Equation 26 -0.86 1.38 0.68 1.84\times10^{-10} 27.59

\[ 4\text{CO} + 9\text{H}_2 \rightarrow \text{i-C}_4\text{H}_{10} + 4\text{H}_2\text{O} \]  
Equation 27 -0.69 1.25 2.21 6.59\times10^{-9} 10.14

\[ 6.05\text{CO} + 12.23\text{H}_2 \rightarrow \text{C}_6\text{H}_{12}\text{(C}_5\text{)} + 6.05\text{H}_2\text{O} \]  
Equation 28 1.71 -1.00 -2.57 1.98\times10^{-8} 12.44

\[ \text{CO} + \text{H}_2\text{O} \rightarrow \text{CO}_2 + \text{H}_2 \]  
Equation 29 -1.71 1.57 2.98 3.78\times10^{-5} 10.00

The kinetic parameters in Table 4 were estimated by global optimization in MATLAB using a global search method. This method was applied as an alternative to the traditional (gradient-based) optimization methods to avoid convergence to the local minima during the search process. Optimum values were achieved during the search process and they were not significantly different. These values were also determined within the acceptable range compared to the literature values. The modified power-law rate expression (Equation 30) was derived to calculate the rate of consumption and production of the species mentioned above.

\[ r_j = \alpha_j \left( \frac{T}{T_p} \right)^{\delta_j \exp} \left( - \frac{E_j}{R_j T} \right)^{n_j \exp} \frac{m_j}{p_{\text{CO}} p_{\text{H}_2}} \]  
Equation 30

In this equation, \( p_{\text{CO}} \) and \( p_{\text{H}_2} \) stand for partial pressure of \( \text{CO} \) and \( \text{H}_2 \), respectively; \( \alpha_j \) and \( \delta_j \) stand for the pre-exponential factor and the explicit temperature dependence factor of the pre-exponential factor of rate constant, respectively. In this equation, \( E_j \) denotes the activation energy of the \( j^{th} \) reaction equation; \( n_j \) and \( m_j \) indicate the order of reaction with respect to \( \text{CO} \) and \( \text{H}_2 \), respectively.

### 4. Results and discussion

#### 4.1. Model prediction

Optimal values of kinetic parameters were calculated by minimizing a multi-criteria objective function, Equation 31. This function is based on the sum of the absolute relative deviation percentage of seven species at five proposed experimental conditions.

\[ F_{\text{obj}} = \sum_{j=1}^{n_{exp}} \sum_{i=1}^{n_{exp}} w_i \left( \frac{|y_{i,j}^{\exp} - y_{i,j}^{\text{cal}}|}{y_{i,j}^{\exp}} \times 100 \right) \]  
Equation 31

In this equation, \( n_{exp} \) denotes the total number of species (\( \text{CO}, \text{CO}_2, \text{CH}_4, \text{C}_2, \text{C}_3, \text{C}_4 \) and \( \text{C}_5\)\)) and \( n_{exp} \) shows the total number of experimental conditions; \( y_{i,j}^{\exp} \) and \( y_{i,j}^{\text{cal}} \) terms are the experimental and calculated values, respectively and take reactants conversion and products selectivity into account, for the \( i^{th} \) component and \( j^{th} \) experiment; and \( w_i \) corresponds to the different weights taken to normalize the contributions of each species.

A qualitative analysis of the predicted results by the model was illustrated in Figure 2. The model’s results were compared against measured data using a parity diagram. The majority of the data points were between -10% and +10% error. This comparison was carried out with respect to the selectivity of \( \text{CO}_2, \text{CH}_4, \text{C}_2, \text{C}_3, \text{C}_4 \) and \( \text{C}_5\), as well as \( \text{CO} \) conversion at five different experimental conditions. Therefore, a total number of 35 data points were investigated, which were enough for the present analysis.
Figure 2 The parity of the calculated CO₂, CH₄, C₂, C₃, C₄ and C₅⁺ selectivity as well as CO conversion against experimental results, using the modified power-law rate expression, with the total of 35 data points.

Table 5 shows the results obtained by the model (the modified power-law rate model) and those measured by experiments with respect to CO conversion and products’ selectivity. The modelling results were in good agreement with the experimental data. The accuracy of the fitted model relative to the experimental data was determined by a quantitative analysis method using the mean absolute relative residual percentage (MARR %). It was found that the model based on the modified equation provided a better fit to the experimental data with a MARR of 6.57% compared to the classic equation with MARR of 12.24%.

\[
MARR(\%) = \frac{1}{n_{\text{exp}} n_{\text{exp,cal}}^{\text{cal}}} \sum_{i=1}^{n_{\text{exp}}} \left( \sum_{j=1}^{n_{\text{exp,cal}}} \left( \frac{y_{i,j}^{\text{exp}} - y_{i,j}^{\text{cal}}}{y_{i,j}^{\text{cal}}} \right) \times 100 \right)
\]

Equation 32

<table>
<thead>
<tr>
<th>Component (%)</th>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
<th>Condition 4</th>
<th>Condition 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO Conversion</td>
<td>79.34</td>
<td>79.76</td>
<td>66.55</td>
<td>65.80</td>
<td>54.34</td>
</tr>
<tr>
<td>CO₂ Selectivity</td>
<td>4.46</td>
<td>4.38</td>
<td>2.63</td>
<td>2.66</td>
<td>1.72</td>
</tr>
<tr>
<td>CH₄ Selectivity</td>
<td>16.58</td>
<td>17.03</td>
<td>19.20</td>
<td>15.85</td>
<td>15.32</td>
</tr>
<tr>
<td>C₂ Selectivity</td>
<td>1.47</td>
<td>1.47</td>
<td>1.52</td>
<td>1.35</td>
<td>1.27</td>
</tr>
<tr>
<td>C₃ Selectivity</td>
<td>2.79</td>
<td>2.80</td>
<td>2.90</td>
<td>2.63</td>
<td>2.51</td>
</tr>
<tr>
<td>C₄ Selectivity</td>
<td>2.93</td>
<td>2.85</td>
<td>2.53</td>
<td>2.55</td>
<td>2.18</td>
</tr>
<tr>
<td>C₅⁺ Selectivity</td>
<td>76.24</td>
<td>75.85</td>
<td>73.85</td>
<td>77.62</td>
<td>78.71</td>
</tr>
</tbody>
</table>

Table 5 A comparison of the results obtained by the model (Mod.) with experimental data (Exp.) with respect to CO₂, CH₄, C₂, C₃, C₄ and C₅⁺ selectivity as well as CO conversion at operating conditions 1 to 5.
A good agreement was achieved based on the CO$_2$ production rate in all the experimental data set. This confirms the validity of the model and reaction equations to predict the rate of CO$_2$ formation. Similarly, CO conversion was predicted very well except in a higher temperature condition (condition 5). The main difference was achieved with respect to the CH$_4$ formation, rate especially at conditions 2 and 4. Overall, the predicted values were in good agreement with the experimental data.

According to the results, 90.78% CO conversion was achieved at condition 5. At the same condition, the undesirable CH$_4$ selectivity was 40.29%, which is very high compared to other operating conditions. Also, the results indicate that the methane rate of formation was remarkably higher compared to higher paraffinic hydrocarbons. This can be due to the lower energy barrier of methane for formation, compared to other paraffins [37, 38]; however, in contrast the $E_{\text{CH}_4}$ value was higher. Mathematically, the higher the value of the activation energy, the lower the value of the exponential term in the reaction equation. Regardless of the value of the pre-exponential factor, it can be concluded that the higher value of formation rate corresponds to the lower value of the energy barrier. This contradiction was explained by the fact that the pre-exponential value of methane was far greater than other paraffins that result in a higher methane formation rate and subsequently higher selectivity value, compared to C$_2$-C$_4$. In Figure 3, the axial conversion profiles of CO and H$_2$ species are depicted along the reactor bed length. As it can be seen, the gradients of the conversion’s profile of the syngas components are slowed down in all experimental data sets as they proceeded to the reactor outlet. The decreasing of CO and H$_2$ mole fractions results in a reduction in concentration of syngas components and an increase in the conversion.

4.2. Effects of FT reaction rate

Literature studies showed [39, 40] that the promoted Co-based catalyst had higher CO hydrogenation rates than an unprompted Co catalyst. The catalyst activity strongly depended on its weight fraction and the addition of a promoter in the solid catalyst. The addition of a promoter to a Co-based catalyst tripled the activity of the catalyst and increased the C$_{\text{+}}$ selectivity from 84% to 91% [39]. The results revealed by the present study, indicate that the cobalt particle size was controlled by utilizing mesoporous support SiO$_2$ to achieve the highly reducible and dispersed active phase; as well as preventing the sintering of particles. Although the Co catalyst used in this study was an unprompted catalyst, it was very active due to the high percentage of cobalt content (37%). A higher amount of Co catalyst in the reactor bed is associated with its activity, which causes an increase in the rates of the FT reaction and therefore the CO and H$_2$ conversion.

The effects of the FT reaction rates on CO and H$_2$ conversion as well as selectivity of CO$_2$, CH$_4$, C$_2$, C$_3$, C$_4$, and C$_5$ were investigated at 503 K, 15 bar and 6 L g$_{\text{cat}}$^{-1} h$^{-1}$. In order to minimize the effect of the highly exothermic methanation reaction, the high value of the flow rate was taken into account. Figure 5 shows CO and
H$_2$ conversion as well as CO$_2$, CH$_4$, C$_2$, C$_3$, C$_4$, and C$_5$ selectivity as the function of the FT reaction rates. Light HC products (i.e. C$_2$-C$_4$) increased slowly with an increase in FT reaction rates. The results showed that the rate of conversion of CO and H$_2$ strongly depended on the FT reaction rates, increasing along with their increases. It was found that the rate of conversion of H$_2$ is slightly greater than CO (Figure 4). By increasing the FT reaction rates from 1.95×10$^{-5}$ to 7.81×10$^{-5}$ mol g$_{\text{cat}}^{-1}$ s$^{-1}$ the conversion of CO and H$_2$ increased by 46.76% and 47.79%, respectively (Table 6). Figure 5 shows higher methane selectivity was predicted at smaller FT reaction rates. Methane selectivity drops from 15.86 to 5.04% as the FT reaction rates increase from 1.95×10$^{-5}$ to 7.81×10$^{-5}$ mol g$_{\text{cat}}^{-1}$ s$^{-1}$. Based on literature studies, FT reaction rates increase as pore size increases. Higher reducibility of large cobalt particles is likely to be one of the reasons for the higher FT reaction rates and lower methane selectivity on wider pore cobalt catalysts. The higher rate of C$_5$ formation was found with increasing the FT reaction rates (from $R_{\text{FT}}$ (1) to (6)), compared to the rate of lighter HCs formation. It was suggested that this can be due to a greater rate of conversion of H$_2$ ($R_{\text{FT}}$ (6)) inside pores filled with liquid products, compared to that of CO, caused an increase in the H$_2$/CO ratio in the catalyst’s pores; and thus there is a shift towards the formation of lighter HCs. The FT reaction rates increase with an increase in pore size. The catalyst with a small pore diameter exhibits low activity; a significant increase in FT reaction rates can be observed by increasing the pore size. The higher methane formation rate and lower C$_5$ selectivity can be expected with a small pore size. Catalyst characterization carried out by Khodakov et al. [28] revealed that a thin pore catalyst contained smaller cobalt particles. Thus, the activity of the catalysts seems to be affected by the size of the cobalt particles. The FT reaction rates were found to be lower for smaller particles than for larger particles. The lower FT reaction rates, the higher the selectivity of CH$_4$ and the lower the C$_5$ selectivity.

The trend of changes of CO$_2$ selectivity was found to be slightly different from other observations. Table 6 shows the CO$_2$ selectivity was reduced first with the increasing of the FT reaction rates at lower rates (from 1.95×10$^{-5}$ to 4.88×10$^{-5}$ mol g$_{\text{cat}}^{-1}$ s$^{-1}$) and then its value was increased with the further raising of FT reaction rates; probably because CO$_2$ was produced via water gas shift reaction from the reaction between CO and water. An increase of FT reaction rates increased the rate of CO conversion, which is related to a decrease of CO mole fraction. The water was produced as the reaction occurred at the surface of the catalyst. It was attributed to the observed increase in the rate of growing of the H$_2$O fraction with the increasing of the FT reaction rates; however, the rate of CO mole fraction reduction (i.e. the slope of CO consumption rate) reduces by the increasing of the FT reaction rates. At some point, the rate of the H$_2$O formation is more than the rate of reduction of the CO mole fraction and therefore could result in increasing CO$_2$ selectivity.

Figure 4 The conversion profiles of (a) CO and (b) H$_2$ species along the reactor bed length (normalized axial dimension) at different FT reaction rate values. The results are based on the developed mathematical model.
Figure 5 The Effects of FT reaction rate on CO and H\textsubscript{2} conversion as well as selectivity of CO\textsubscript{2}, CH\textsubscript{4}, C\textsubscript{2}, C\textsubscript{3}, C\textsubscript{4}, and C\textsubscript{5+}. Reaction conditions: 503 K, 15 bar and 6 L g\textsubscript{cat}\textsuperscript{-1} h\textsuperscript{-1}.

Table 6 The values of final syngas conversion and products’ selectivity obtained by the model at different FT reaction rate values

<table>
<thead>
<tr>
<th>Components (%)</th>
<th>R\textsubscript{FT} (1)</th>
<th>R\textsubscript{FT} (2)</th>
<th>R\textsubscript{FT} (3)</th>
<th>R\textsubscript{FT} (4)</th>
<th>R\textsubscript{FT} (5)</th>
<th>R\textsubscript{FT} (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO conversion</td>
<td>44.81</td>
<td>57.86</td>
<td>68.22</td>
<td>76.29</td>
<td>82.52</td>
<td>91.57</td>
</tr>
<tr>
<td>H\textsubscript{2} conversion</td>
<td>49.47</td>
<td>63.15</td>
<td>73.97</td>
<td>82.35</td>
<td>88.72</td>
<td>97.26</td>
</tr>
<tr>
<td>CO\textsubscript{2} selectivity</td>
<td>2.40</td>
<td>1.95</td>
<td>1.75</td>
<td>1.69</td>
<td>1.73</td>
<td>2.33</td>
</tr>
<tr>
<td>CH\textsubscript{4} selectivity</td>
<td>15.86</td>
<td>11.42</td>
<td>9.01</td>
<td>7.50</td>
<td>6.45</td>
<td>5.04</td>
</tr>
<tr>
<td>C\textsubscript{2} selectivity</td>
<td>1.23</td>
<td>1.34</td>
<td>1.43</td>
<td>1.52</td>
<td>1.62</td>
<td>1.88</td>
</tr>
<tr>
<td>C\textsubscript{3} selectivity</td>
<td>2.40</td>
<td>2.60</td>
<td>2.76</td>
<td>2.92</td>
<td>3.08</td>
<td>3.50</td>
</tr>
<tr>
<td>C\textsubscript{4} selectivity</td>
<td>2.38</td>
<td>2.60</td>
<td>2.78</td>
<td>2.95</td>
<td>3.13</td>
<td>3.53</td>
</tr>
<tr>
<td>C\textsubscript{5+} selectivity</td>
<td>78.13</td>
<td>82.04</td>
<td>84.02</td>
<td>85.11</td>
<td>85.72</td>
<td>86.05</td>
</tr>
</tbody>
</table>

4.3. Effect of water gas shift reaction rate

In principle, WGS reaction is usually considered in the case of Fe catalysts. With reference to a literature study [7], Co is not very active for WGS reaction. It means that the effects of water were normally not considered in the kinetics of FT synthesis with cobalt catalysts. Some studies showed that the WGS reaction occurred to a small extent, acting as a one-way reaction and producing a small amount of carbon dioxide. In this case, carbon dioxide is typically treated as a carbon-containing product. In the present model, the CO\textsubscript{2} formation was non-negligible and H\textsubscript{2}O partial pressure was identified as one of the key factors influencing the reaction rate; this leads to the conclusion that the WGS reaction is responsible for this result. As explained above, this reaction is a very well-known reversible process; however, the best fit of the experimental data corresponded to a model postulating the irreversibility of CO\textsubscript{2} formation.

The amount of catalyst active sites available for the FT reaction changes with the partial pressures of water and hydrogen. The presence of hydrogen prevents the catalyst from oxidation when the water’s partial pressure
is not too high to cause permanent deactivation. In contrast, water functions by reoxidizing the cobalt catalysts which turns into an activity loss for the FT reaction [41].

The effects of WGS reaction rate on conversion, rate of formation and selectivity were studied at a temperature of 503 K, a total pressure of 15 bar and a flow rate of 6 L g\(^{-1}\) h\(^{-1}\). Figure 6 illustrates the trend of the changes in CO and H\(_2\) conversion at different WGS reaction rates along the normalized axial dimension (reactor bed length). Figure 7 shows that the effects of WGS reaction rate on HCs’ selectivity and rate of each formation was almost negligible. The conversion rate of CO was slightly changed with WGS reaction rates, increasing with increases in the WGS reaction rate. In contrast, the increases in the WGS reaction rates did not change the H\(_2\) conversion rate (Figure 6). The WGS reaction rates were found to be effective on the rate of unwanted CO\(_2\) formation, increasing from 2.40 to 8.37% with increases in WGS reaction rates from 1.04×10\(^{-6}\) to 4.16×10\(^{-6}\) mol g\(^{-1}\) s\(^{-1}\) (Table 7).

Figure 6 (a) CO and (b) H\(_2\) conversion profiles along the reactor bed length (normalized axial dimension) at different water gas shift reaction rate values. The results are based on the developed mathematical model.

Figure 7 Effects of WGS reaction rate on CO and H\(_2\) conversion as well as selectivity of CO\(_2\), CH\(_4\), C\(_2\), C\(_3\), C\(_4\), and C\(_{5+}\). Reaction conditions: 503 K, 15 bar and 6 L g\(^{-1}\) h\(^{-1}\).
Table 7 The values of final syngas conversion and products’ selectivity obtained by the model at different water gas shift reaction rate values.

<table>
<thead>
<tr>
<th>Components (%)</th>
<th>( R_{WGS} ) (1) 1.04E-06</th>
<th>( R_{WGS} ) (2) 1.56E-06</th>
<th>( R_{WGS} ) (3) 2.08E-06</th>
<th>( R_{WGS} ) (4) 2.60E-06</th>
<th>( R_{WGS} ) (5) 3.12E-06</th>
<th>( R_{WGS} ) (6) 4.16E-06</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO conversion</td>
<td>44.81</td>
<td>45.44</td>
<td>46.06</td>
<td>46.68</td>
<td>47.28</td>
<td>48.47</td>
</tr>
<tr>
<td>( H_2 ) conversion</td>
<td>49.47</td>
<td>49.31</td>
<td>49.15</td>
<td>49.01</td>
<td>48.86</td>
<td>48.59</td>
</tr>
<tr>
<td>( CO_2 ) selectivity</td>
<td>2.40</td>
<td>3.51</td>
<td>4.57</td>
<td>5.59</td>
<td>6.56</td>
<td>8.37</td>
</tr>
<tr>
<td>( CH_4 ) selectivity</td>
<td>15.86</td>
<td>15.82</td>
<td>15.77</td>
<td>15.73</td>
<td>15.69</td>
<td>15.60</td>
</tr>
<tr>
<td>( C_2 ) selectivity</td>
<td>1.23</td>
<td>1.22</td>
<td>1.20</td>
<td>1.19</td>
<td>1.17</td>
<td>1.15</td>
</tr>
<tr>
<td>( C_3 ) selectivity</td>
<td>2.40</td>
<td>2.37</td>
<td>2.35</td>
<td>2.32</td>
<td>2.30</td>
<td>2.25</td>
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<tr>
<td>( C_4 ) selectivity</td>
<td>2.38</td>
<td>2.36</td>
<td>2.34</td>
<td>2.32</td>
<td>2.30</td>
<td>2.26</td>
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<tr>
<td>( C_5^+ ) selectivity</td>
<td>78.13</td>
<td>78.23</td>
<td>78.34</td>
<td>78.44</td>
<td>78.53</td>
<td>78.73</td>
</tr>
</tbody>
</table>

4.4. Effect of methane formation reaction rate

Methane formation via methanation reaction is one of the primary variables affecting any FT synthesis process. Methane formation controls the most valuable FT products’ selectivity, which are liquid HCs in this case [42]. The results showed that a higher \( CH_4 \) selectivity of the Co catalyst is primarily correspondent to a higher \( CH_4 \) rate constant, compared with other lighter HCs’ formation (\( C_2 - C_4 \)). This conclusion is consistent with the previous report from the literature [25].

Figure 8 illustrates the trend of changes in CO and \( H_2 \) conversion at different methanation reaction rates along the normalized axial dimension. The rate of conversion of CO and \( H_2 \) strongly depend on the methanation reaction rate, increasing with increases in the rate values. It was found that the rate of conversion of \( H_2 \) is greater than CO (Figure 8). By increasing the methanation reaction rate from \( 4.40 \times 10^{-6} \) to \( 1.76 \times 10^{-5} \) mol g\(^{-1}\) cat s\(^{-1}\) the conversion of CO and \( H_2 \) increased by 12.62% and 21.85%, respectively (Table 8). Figure 9 shows CO and \( H_2 \) conversion as well as \( CO_2 \), \( CH_4 \), \( C_2 \), \( C_3 \), \( C_4 \), and \( C_5^+ \) selectivity as functions of the methanation reaction rate. The effects of the methanation reaction rate on \( CO_2 \) formation and lighter HCs’ selectivity, excluding \( CH_4 \), were almost negligible. The results showed that the rate of \( CH_4 \) formation and \( C_5^+ \) strongly depend on the methanation reaction rate. It was apparently found by increasing the methanation reaction rate that the selectivity of \( CH_4 \) increased, while liquid formation dropped significantly. By increasing the methanation reaction rate from \( 4.40 \times 10^{-6} \) to \( 1.76 \times 10^{-5} \) mol g\(^{-1}\) cat s\(^{-1}\) the \( CH_4 \) selectivity increased by 28.82% and the \( C_5^+ \) selectivity decreased by 27.63% (Table 8).

Figure 8 The conversion profiles of (a) CO and (b) \( H_2 \) species along the reactor bed length (normalized axial dimension) at different methanation reaction rate values. The results are based on the developed mathematical model.
5. Conclusion

A mathematical model of a fixed bed reactor for Fischer-Tropsch (FT) synthesis was developed by a one-dimensional pseudo-homogeneous flow over simulated N$_2$-rich syngas on an in-house Co/SiO$_2$ catalyst. The reaction equations were proposed as a number of lumped reactions by means of the molar coefficient of the reaction molecules. The rates of production and consumption were derived from a modified power-law rate expression. According to the results, the adapted rate model can deliver a better prediction of final conversion and selectivity. Therefore, the temperature dependence of the pre-exponential factor was considered explicitly due to its considerable effects on the rate of reactions. The kinetic parameters were estimated by global optimization in MATLAB using a global search method. This method was applied as an alternative to traditional (gradient-based) optimization methods to avoid convergence to the local minima during the search process. Optimum values were achieved during the search process and they were not significantly different. These values were determined within the acceptable range compared to the literature values. The modelling results were in good agreement with the experimental data. It was found that the model based on the modified equation provided a better fit to the experimental data with a mean absolute relative residual (MARR) of 6.57%, compared to the classic equation with a MARR of 12.24%.
The effects of Fischer-Tropsch, water gas shift and methanation reaction rates on CO and H\textsubscript{2} conversion as well as selectivity of CO\textsubscript{2}, CH\textsubscript{4}, C\textsubscript{2}, C\textsubscript{3}, C\textsubscript{4}, and C\textsubscript{5+}, were investigated at 503 K, 15 bar and 6 L g\textsubscript{cat}\textsuperscript{-1} h\textsuperscript{-1}. It was concluded that the conversion rates of CO and H\textsubscript{2} are strongly dependent on the FT reaction rates in which the conversion of 91.57 and 97.26\% respectively, were achieved with the FT reaction rate of 7.81\times10\textsuperscript{-5} mol g\textsubscript{cat}\textsuperscript{-1} s\textsuperscript{-1}. The higher rate of C\textsubscript{5+} formation was found with the increasing of the FT reaction rates compared to the rate of lighter hydrocarbons’ formation. At the same condition, only 5.04 and 8.91\% methane and C\textsubscript{2}-C\textsubscript{4} selectivity were predicted while obtaining the highest value of 86.05\% liquid (C\textsubscript{5+}) selectivity. It was concluded that the greater rate of conversion of H\textsubscript{2} inside pores filled with liquid products, compared to that of CO, caused an increase in the H\textsubscript{2}/CO ratio in the catalyst’s pores; and thus, a shift towards the formation of lighter HCs. Also, the 37\% Co/SiO\textsubscript{2} catalyst was very active due to the high percentage of cobalt content; this was one of the contributing factors to the increasing of the FT reaction rates. The FT reaction rates increase with an increase in the pore size. The catalyst with a small pore diameter exhibits low activity; a significant increase in FT reaction rates can be observed by increasing the pore size. Higher reducibility of large cobalt particles is likely to be one of the reasons for the higher FT reaction rates and lower methane selectivity on wider pore cobalt catalysts. In the present model, the CO\textsubscript{2} formation was non-negligible and H\textsubscript{2}O partial pressure was identified as one of the key factors influencing the reaction rate; this led to the conclusion that the WGS reaction is responsible for this result. The best fit of the experimental data corresponded to a model postulating the irreversibility of CO\textsubscript{2} formation. Water was produced as the reaction occurred at the surface of the catalyst. It was shown that the production of H\textsubscript{2}O increased by increasing the FT reaction rates; however, the rate of CO mole fraction reduction reduced by the increasing of the FT reaction rates. At some point, the rate of the H\textsubscript{2}O formation is larger than the rate of reduction of CO mole fraction and therefore could result in a slight increase of CO\textsubscript{2} selectivity.

Acknowledgments

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Nomenclature

Symbols

\begin{itemize}
  \item \(a_j\) pre-exponential factor of rate constant in reaction \(j\), \([\text{mol Pa}^{(m+n)} g_{\text{cat}}^{-1} s^{-1}]\)
  \item \(A_r\) reactor surface area, \([\text{m}^2]\)
  \item \(C_{i'}\) concentration of species \(i'\), \([\text{mol m}^{-3}]\)
  \item \(d_p\) average particle diameter, \([\text{m}]\)
  \item \(E_j\) activation energy of reaction \(j\), \([\text{J mol}^{-1}]\)
  \item \(f\) friction factor, \([-]\)
  \item \(k_{j'}\) rate constants, \([\text{mol Pa}^{(m+n)} g_{\text{cat}}^{-1} s^{-1}]\)
  \item \(M_i\) molecular weight of species \(i\), \([\text{g mol}^{-1}]\)
  \item \(M_m\) molecular weight of mixture, \([\text{g mol}^{-1}]\)
  \item \(m\) partial order of the reactant with respect to hydrogen, \([-]\)
  \item \(n\) partial order of the reactant with respect to carbon monoxide, \([-]\)
  \item \(p_i\) partial pressure of species \(i\), \([\text{bar}]\)
  \item \(p_0\) total pressure, \([\text{bar}]\)
  \item \(Q\) volumetric flow rate of the fluid, \([\text{m}^3 \text{s}^{-1}]\)
  \item \(R_j\) rate of reaction \(j\), \([\text{mol g}_{\text{cat}}^{-1} s^{-1}]\)
  \item \(Re\) Reynolds number, \([-]\)
  \item \(R_g\) universal gas constant, 8.314 \([\text{J mol}^{-1} \text{K}^{-1}]\)
  \item \(S_i\) selectivity of species \(i\), \([\%]\)
  \item \(T\) reaction temperature, \([\text{K}]\)
  \item \(T_{r}\) reference temperature, \([\text{K}]\)
  \item \(u_{p}\) superficial fluid velocity, \([\text{m s}^{-1}]\)
  \item \(x\) conversion, \([\%]\)
\end{itemize}
\( Y_i \) mole fraction of species 'i', [-]

**Greek letters**
- \( \beta \) volume fraction of active site of the solid particles, [-]
- \( \delta \) explicit temperature dependence factor of the pre-exponential constant, [-]
- \( \epsilon \) void fraction, [-]
- \( \mu_i \) dynamic viscosity of species 'i', [kg m^{-1} s^{-1}]
- \( \mu_m \) dynamic viscosity of the mixture, [kg m^{-1} s^{-1}]
- \( \rho_f \) density of the fluid, [kg m^{-3}]
- \( \rho_B \) density of the bulk, [kg m^{-3}]

**Subscripts**
- 0 initial value
- \( i \) species number
- \( j \) reaction number
- \( m \) mixture
- \( z \) axial dimension

**Abbreviations**
- CSTR Continuously stirred tank reactor
- Co Cobalt
- FT Fischer-Tropsch
- FID Flame Ionization detector
- Fe Iron
- WHSV Weight Hourly Space Velocity
- GC Gas Chromatogram
- HC Hydrocarbon
- HCs Hydrocarbons
- MS Mass Spectrometry
- Ni Nickel
- ODEs Ordinary differential equations
- Re Rhenium
- Ru Ruthenium
- WGS Water gas shift

**References**


Highlights

- Fischer–Tropsch synthesis over cobalt-silica was conducted in a fixed bed reactor.
- Conversion and production rates were correlated by modified power-law rate model.
- Reactor model and reaction kinetics were developed and fitted against experiments.
- The performance of process was examined for its potential in liquid production.
- The influence of reaction rates on syngas conversion was studied for more efficient synthesis.