Quantum Tests of the Einstein Equivalence Principle with the STE-QUEST Space Mission


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We present in detail the scientific objectives in fundamental physics of the Space-Time Explorer and QUantum Equivalence Space Test (STE-QUEST) space mission. STE-QUEST was pre-selected by the European Space Agency together with four other missions for the cosmic vision M3 launch opportunity planned around 2024. It carries out tests of different aspects of the Einstein Equivalence Principle using atomic clocks, matter wave interferometry and long distance time/frequency links, providing fascinating science at the interface between quantum mechanics and gravitation that cannot be achieved, at that level of precision, in ground experiments. We especially emphasize the specific strong interest of performing equivalence principle tests in the quantum regime, i.e. using quantum atomic wave interferometry. Although STE-QUEST was finally not selected in early 2014 because of budgetary and technological reasons, its science case was very highly rated. Our aim is to expose that science to a large audience in order to allow future projects and proposals to take advantage of the STE-QUEST experience.

I. INTRODUCTION

A. Scientific Motivations

Our best knowledge of the physical Universe, at the deepest fundamental level, is based on two theories: Quantum Mechanics (or, more precisely, Quantum Field Theory) and the classical theory of General Relativity. Quantum Field Theory has been extremely successful in providing an understanding of the observed phenomena of atomic, particle,
and high energy physics and has allowed a unified description of three of the four fundamental interactions that are known to us: electromagnetic, weak and strong interactions (the fourth one being gravitation). It has led to the Standard Model of particle physics that has been highly successful in interpreting all observed particle phenomena, and has been strongly confirmed with the recent discovery at the LHC of the Higgs (or, more precisely, Brout-Englert-Higgs) boson, which could in fact be viewed as the discovery of a fifth fundamental interaction. Although open questions remain within the Standard Model of particle physics, it is clearly the most compelling model for fundamental interactions at the microscopic level that we have at present.

On the other hand, Einstein’s theory of General Relativity (GR) is a cornerstone of our current description of the physical world at macroscopic scales. It is used to understand the flow of time in the presence of gravity, the motion of bodies from satellites to galaxy clusters, the propagation of electromagnetic waves in the vicinity of massive bodies, the evolution of stars, and the dynamics of the Universe as a whole. GR brilliantly accounts for all observed phenomena related to gravitation, in particular all observations in the Earth’s environment, the Solar system, in relativistic binary pulsars and, beyond that, on galactic and cosmological scales.

The assumed validity of GR at cosmological scales, and the fact that non-gravitational interactions are described by the Standard Model of particle physics, together with a hypothesis of homogeneity and isotropy of cosmological solutions of these theories, have led to the “concordance model” of cosmology, referred to as the Λ-CDM (Cold Dark Matter) model, which is in agreement with all present-day observations at large scales, notably the most recent observations of the anisotropies of the cosmic microwave background by the Planck satellite [2]. However, important difficulties remain, in particular the necessary introduction of dark energy, described by a cosmological constant Λ, and of cold dark matter, made of some unknown, yet to be discovered, stable particle.

There is a potential conflict on the problem of dark matter between the concordance model of cosmology and the Standard Model of particles. On the one hand, there is strong evidence [2] that 26.8 % of the mass-energy of the Universe is made of non-baryonic dark matter particles, which should certainly be predicted by some extension of the Standard Model of particles. On the other hand, there is no indication of new physics beyond the Standard Model which has been found at the LHC. For instance, the search of supersymmetry at LHC has for the moment failed.

Although very successful so far, GR as well as numerous other alternative or more general theories of gravitation are classical theories. As such, they are fundamentally incomplete, because they do not include quantum effects. A theory solving this problem would represent a crucial step towards the unification of all fundamental forces of Nature. Most physicists believe that GR and the Standard Model of particle physics are only low-energy approximations of a more fundamental theory that remains to be discovered. Several concepts have been proposed and are currently under investigation (e.g., string theory, loop quantum gravity, extra spatial dimensions) to bridge this gap and most of them lead to tiny violations of the basic principles of GR.

One of the most desirable attributes of that fundamental theory is the unification of the fundamental interactions of Nature, i.e. a unified description of gravity and the three other fundamental interactions. There are several attempts at formulating such a theory, but none of them is widely accepted and considered successful. Furthermore, they make very few precise quantitative predictions that could be verified experimentally. One of them is the Hawking radiation of black holes, which is however far from being testable experimentally for stellar-size black holes we observe in astrophysics.

Therefore, a fuller understanding of gravity will require observations or experiments able to determine the relationship of gravity with the quantum world. This topic is a prominent field of activity with repercussions covering the complete range of physical phenomena, from particle and nuclear physics to galaxies and the Universe as a whole, including dark matter and dark energy.

A central point in this field is that most unification theories have in common a violation at some (a priori unknown) level of one of the basic postulates of GR, which can be tested experimentally: the Einstein Equivalence Principle (EEP). Let us emphasize that the EEP is not a fundamental symmetry of physics, contrary to e.g. the principle of local gauge invariance in particle physics. An important challenge is therefore to test with the best possible accuracy the EEP. This is then the main motivation of many experiments in fundamental physics, both on Earth and in space.

Precise measurements are at the heart of the scientific method that, since Galileo’s time, is being used for unveiling Nature and understanding its fundamental laws. The assumptions and predictions of GR can be challenged by precision experiments on scales ranging from micrometers in the laboratory to the Solar System, in the latter case using spacecrafts or the orbiting Earth, Moon and planets. The implementation of tests with significantly improved sensitivity obviously requires the use of state-of-the-art technology, and in case of satellite-based experiments the challenge is to make such technology compatible with use in space, i.e. extremely robust, reliable, and automatized.
B. The Space Mission STE-QUEST

The satellite STE-QUEST (Space-Time Explorer and QUantum Equivalence Space Test) is specifically designed for testing different aspects of the EEP and searching for its violation with high precision. The weak equivalence principle has been verified with high precision using torsion balances on ground [118] and the Lunar laser ranging [136]. It will be tested in Earth orbit by the CNES satellite µ-SCOPE (Micro-Satellite à trainée Compensée pour l’Observation du Principe d’Équivalence) in 2016 [120]. On the other hand, the gravitational red-shift, a different aspect of the EEP, was first measured using gamma ray spectroscopy in the laboratory [110], and the most precise test so far was done in space with the GP-A experiment [28]. The ESA mission ACES (Atomic Clock Ensemble in Space) will test the gravitational red-shift with the highly accurate laser-cooled atomic clock PHARAO on the International Space Station (ISS) in 2016 [28].

Atomic clocks and high-performance time and frequency links, atom interferometers and classical accelerometers are today able to measure frequency, time, and distances, and furthermore to track the motion of massive bodies, quantum particles, and light to accuracy levels never reached before. These instruments achieve their ultimate performance in space, where the clean environment and the free-fall conditions become essential for identifying tiny deformations in space-time that might bring the signature of new physics or new fundamental constituents. From this point of view, it is not surprising that fundamental physics pervades all aspects of space science.

STE-QUEST was proposed in the fall of 2010 in response to ESA’s M3 call in the Cosmic Vision programme (with launch date in the 2022-24 time interval), by a science team under coordination by Schiller and Rasel with support from 67 colleagues from Europe and the USA. STE-QUEST is based on the earlier proposals EGE [117] and MWXG [55], submitted to ESA’s M2 call. ESA performed a “concurrent design facility” study of a mission concept similar to EGE, named STE, in 2010. Previously, ESA had also convened a Fundamental Physics Advisory Team which in 2009-10 developed a roadmap on fundamental physics in space.

STE-QUEST, together with three other mission proposals, was selected in early 2011 by ESA’s advisory structure as candidate mission. STE-QUEST went through an assessment phase study of the satellite and payload (see the Yellow Book of the mission [1]). As a result of the assessment phase and in agreement with the national space agencies, STE-QUEST was removed from the candidate pool in late 2013, before the final selection of a single mission for the M3 slot, because of budgetary and technological reasons. Nevertheless, ESA’s advisory committees evaluated the science aspects of STE-QUEST in early 2014 (together with the remaining M3 candidates) and ranked them highly. It is likely that STE-QUEST will recompete for the M4 launch slot.

The primary science objectives of STE-QUEST is testing the different aspects of the Einstein Equivalence Principle with quantum sensors. The payload consists of a differential atom interferometer comparing the free propagation of matter waves of different composition under the effect of gravity and a frequency comparison link in the microwave domain for comparing atomic clocks on ground. STE-QUEST performs a direct test of the Weak Equivalence Principle (WEP) by comparing the free fall of quantum objects of different composition. The Eötvös ratio between the matter waves of two isotopes of the Rubidium atom is measured in a differential atom interferometer down to the $2 \times 10^{-15}$ uncertainty level. While present limits on WEP tests involving classical objects reach an uncertainty of a few parts in $10^{18}$, measurements performed on quantum objects (matter waves in states which have no classical counterpart, e.g. spatio-temporal quantum superpositions) are still at the level of a few parts in $10^{13}$ [60, 113] [123]. From this point of view, STE-QUEST will explore the boundaries between gravitation and quantum mechanics, significantly improving existing measurements and complementing experiments such as µ-SCOPE, designed for a classical WEP test in space to the level $1 \times 10^{-15}$.

STE-QUEST also tests another complementary aspect of the Einstein Equivalence Principle, one of the most fascinating effects predicted by general relativity and other metric theories of gravity: the gravitational red-shift or gravitational time dilation effect. As direct consequence of the EEP, time runs (or clocks tick) more slowly near a massive body. This effect can be detected when comparing the time intervals measured by identical clocks placed at different depths in a gravitational field. The microwave link (MWL) of the STE-QUEST satellite allows comparing ground clocks down to the $1 \times 10^{-18}$ uncertainty level. Such measurements, far beyond the capabilities of existing long-distance time and frequency transfer systems, will perform clock red-shift tests in the field of the Sun and the Moon, respectively at the $2 \times 10^{-6}$ and $4 \times 10^{-4}$ uncertainty levels. For comparison, existing measurements of the Sun red-shift effect are at the few % uncertainty level while, to our knowledge, no such tests have ever been performed in the field of the Moon. An optional (depending on available funding) onboard clock allows additionally a red-shift measurement in the Earth field by taking advantage of the high apogee and high eccentricity of the orbit. The clock under consideration is derived from the PHARAO cold atom Cs clock to be flown on the ISS [28]. The version planned for STE-QUEST is designed to reach an uncertainty in the Earth field red-shift test of $2 \times 10^{-7}$, one order of magnitude better than the objective of ACES. The relativistic theory for time and frequency transfer needed for frequency links in space missions such as ACES and STE-QUEST is described in Ref. [21].

Clock red-shift measurements obtained in the field of the Earth, the Sun or the Moon test the Local Position
FIG. 1: The STE-QUEST spacecraft in orbit around the Earth. The mission is designed to test the Einstein Equivalence Principle by tracking the free-fall motion of quantum matter waves, by performing gravitational red-shift tests between ground clocks on intercontinental distances and with (optionally) a high-stability and high-accuracy onboard clock, and by performing tests of local Lorentz invariance and CPT symmetry. (OL: optical link; MWL: microwave link.)

<table>
<thead>
<tr>
<th>Science Investigation</th>
<th>Measurement Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weak Equivalence Principle Tests</strong></td>
<td></td>
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<tr>
<td><em>Universality of propagation of matter-waves</em></td>
<td>Test the universality of the free propagation of matter waves to an uncertainty in the Eötvös parameter better than $2 \times 10^{-15}$.</td>
</tr>
<tr>
<td><strong>Gravitational Red-shift Tests</strong></td>
<td></td>
</tr>
<tr>
<td><em>Sun gravitational red-shift</em></td>
<td>Test of the Sun gravitational red-shift effect to a fractional frequency uncertainty of $2 \times 10^{-6}$, with an ultimate goal of $5 \times 10^{-7}$.</td>
</tr>
<tr>
<td><em>Moon gravitational red-shift</em></td>
<td>Test of the Moon gravitational red-shift effect to a fractional frequency uncertainty of $4 \times 10^{-4}$, with an ultimate goal of $9 \times 10^{-5}$.</td>
</tr>
<tr>
<td><em>Earth gravitational red-shift (optional)</em></td>
<td>Measurement of the Earth gravitational red-shift effect to a fractional frequency uncertainty of $2 \times 10^{-7}$.</td>
</tr>
<tr>
<td><strong>Local Lorentz Invariance and CPT Tests</strong></td>
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</tr>
<tr>
<td><em>LLI and CPT</em></td>
<td>Provide significant improvements on the determination of several LLI and CPT parameters of the Lorentz and CPT symmetry violating Standard Model Extension.</td>
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*This scientific investigation can be performed only if the STE-QUEST payload is equipped with a high-stability and high-accuracy atomic clock.

TABLE I: Science investigations vs. measurement requirements for topics in fundamental physics that shall be investigated by STE-QUEST.

Invariance (LPI) principle and search for anomalous couplings depending on the composition of the source of the gravitational field. LPI is a constituent of EEP together with WEP and the Local Lorentz Invariance (LLI) principle, see Sec. II A. As we shall discuss in Sec. VA, in generic frameworks modelling a possible violation of EEP, WEP and clock red-shift tests are complementary and need to be pursued with equal vigor as, depending on the model used, either one of the tests can prove significantly more sensitive than the other. Improving the accuracy of these tests will bring significant progress in restricting the parameters space and discriminating between theories seeking to unify quantum mechanics with gravity. The eventual detection of an EEP violation would carry the signature of new fundamental constituents or interactions in the Universe (e.g. scalar fields for dark energy, particles for dark matter, fundamental strings, etc.). In this case, STE-QUEST tests would have a significant impact not only for fundamental physics research, but also for cosmology and particle physics. The ensemble of fundamental physics science objectives of STE-QUEST is summarized in Table I.
STE-QUEST has also important applications in domains other than fundamental physics, in particular in the fields of time and frequency metrology and for geodesy studies. As mentioned, the STE-QUEST high-performance MWL provides the means for connecting atomic clocks on ground in a global network, enabling comparisons down to the $1 \times 10^{-18}$ fractional frequency uncertainty level. Clock comparisons via STE-QUEST will contribute to the realization of international atomic time scales (UTC, TAI, etc.) and to the improvement of their stability and accuracy. Synchronization of clocks, space-to-ground and ground-to-ground, to better than 50 ps can be achieved through STE-QUEST for distributing time scales to unprecedented performance levels. Common-view comparisons of ground clocks, primarily used for gravitational red-shift tests in the field of Sun or Moon, also provide direct information on the geopotential differences at the locations of the two ground clocks. STE-QUEST will therefore contribute to establishing a global reference frame for the Earth gravitational potential at the sub-cm level through local measurements. This method is complementary to current and future satellite gravimetry missions such as CHAMP, GRACE and GOCE as well as to altimetry missions like JASON and Envisat in defining the Global Geodetic Observing System (GGOS). The Table IV (relegated in the conclusion section VI) summarizes the list of topics other than fundamental physics that shall be investigated by STE-QUEST.

The present paper is an adapted version of the fundamental physics science objectives of STE-QUEST extracted from the Yellow Book of STE-QUEST which is available in Ref. [1] (see also the mission paper [4]). The Yellow Book also gives an overview of science objectives in other fields (geodesy, time/frequency metrology, reference frames) and details on the mission and payload, which are however beyond the scope of this paper that focuses on the fundamental physics objectives. In Sec. II we shall review in more detail the EEP and its different facets. In Sec. III we shall discuss the status of EEP in Physics today and particularly in the contexts of cosmology and particle physics. Quantum mechanics and the EEP and the potential interest of quantum tests of the EEP will be analyzed in Sec. IV. The specific tests of the EEP which will be achieved by STE-QUEST will be presented in Sec. V. The paper ends with the main conclusions in Sec. VI.

II. THE EINSTEIN EQUIVALENCE PRINCIPLE

A. The Different Aspects of the EEP

The foundations of gravitational theories and the equivalence principle have been clarified by many authors, including Schiff [116], Dicke [48], Thorne, Lee & Lightman [125], and others. Following the book of Will [134] the EEP is generally divided into three sub-principles: the Weak Equivalence Principle (WEP) also known as the Universality of Free Fall (UFF), Local Lorentz Invariance (LLI), and Local Position Invariance (LPI). The EEP is satisfied if and only if all three sub-principles are satisfied. Below we describe these three sub-principles:

1. WEP (or UFF) states that if any uncharged test body is placed at an initial event in space-time and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition. The most common test of WEP consists in measuring the relative acceleration of two test bodies of different internal structure and composition freely falling in the same gravitational field. If WEP is satisfied, that relative acceleration is zero;

2. LLI states that the outcome of any local non-gravitational test experiment is independent of the velocity and orientation of the (freely falling) apparatus. Tests of LLI usually involve a local experiment (e.g. the comparison of the frequency of two different types of clocks) whose velocity and/or orientation is varied in space-time. LLI is verified if the result of the experiment is unaltered by that variation;

3. LPI states that the outcome of any local non-gravitational test experiment is independent of where and when in the Universe it is performed. Tests of LPI usually involve a local experiment (e.g. the measurement of a fundamental constant, or the comparison of two clocks based on different physical processes) at different locations and/or times. In particular, varying the local gravitational potential allows for searches of some anomalous coupling between gravity and the fields involved in the local experiment. A particular version of such tests, known as test of the gravitational red-shift, uses the same type of clock, but at two different locations (different local gravitational potentials) and compares them via an electromagnetic signal. Then it can be shown

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1 By test body is meant an electrically neutral body whose size is small enough that the coupling to inhomogeneities in the gravitational field can be neglected.
(see Sec. 2.4c in Ref. [134]) that the measured relative frequency difference is equal to $\Delta U/c^2$ (where $\Delta U$ is the difference in gravitational potential) if and only if LPI is satisfied.

One of the unique strengths of STE-QUEST is that it will test all three aspects of the EEP, using a combination of measurements in space and on the ground (relative acceleration of different atomic isotopes, comparison of distant clocks). Additionally, the explored domain of the possible violation of the LLI and LPI is maximized by the large variation of velocity and gravitational potential using a highly elliptic orbit of the spacecraft.

Since the three sub-principles described above are very different in their empirical consequences, it is tempting to regard them as independent. However, it was realized quite early that any self-consistent gravitational theory is very likely to contain connections between the three sub-principles. This has become known as Schiff’s conjecture [116], formulated around 1960. Loosely stated, the Schiff conjecture implies that if one of the three sub-principles is violated, then so are the other two. This conjecture can be understood heuristically by the following example. Suppose that WEP/UFF is violated; then two different clocks (with different internal compositions) will acquire different accelerations in a gravitational field. In the freely falling frame of one of the clocks, the other one will be accelerated (even though being located at the same position), and there will be an abnormal red-shift between the two clocks depending on their difference of internal composition, hence a violation of LPI. The Schiff conjecture has been proved within very general theoretical frameworks such as the Lagrangian formalism we shall review in Sec. VA. Alternative theories which do not satisfy the conjecture suffer from serious pathologies and are non-viable.

Schiff’s conjecture has given rise to much debate, in particular concerning its empirical consequences and the relative merit of tests of the different sub-principles. Whilst it is true that any theory respecting energy conservation (e.g. based on an invariant action principle) must satisfy Schiff’s conjecture, the actual quantitative relationship between violation of the sub-principles is model dependent and varies as a function of the mechanism used for the violation (see e.g. Sec. VA for a phenomenological example). As a consequence, it is not known a priori which test (WEP/UFF, LLI, or LPI) is more likely to first detect a violation and the most reasonable approach is to perform the tests of the three sub-principles. This is the philosophy of STE-QUEST.

For completeness, and to avoid possible confusion, we will say a few words about the Strong Equivalence Principle (SEP), although it is not directly related to, and will not be tested by STE-QUEST. The SEP is a generalization of EEP to include “test” bodies with non-negligible self-gravitation, together with experiments involving gravitational forces (e.g. Cavendish-type experiments). Obviously, SEP includes EEP as a special case in which gravitational forces can be ignored. Typical tests of SEP involve moons, planets, stars or local gravitational experiments, the best known example being lunar laser ranging that tests the universality of free fall, with the two test bodies being the Moon and the Earth falling in the field of the Sun. Clearly the two test bodies have non-negligible self-gravitation and thus provide a test of SEP. The empirical consequences of SEP and EEP are quite different; in general a violation of SEP does not necessarily imply a violation of EEP. Similarly the theoretical consequences are very different: a violation of EEP excludes not only GR as a possible theory of gravitation, but also all other metric theories (e.g. all PPN theories, Brans-Dicke theory, etc.). A violation of SEP on the other hand excludes GR, but allows for a host of other metric theories (e.g. PPN theories that satisfy a particular combination of PPN parameters). In that sense, SEP and EEP tests are complementary and should be carried out in parallel within experimental and observational possibilities. STE-QUEST focuses on EEP, but this does not preclude the interest of SEP tests like continued and improved lunar laser ranging.

### B. The Role of EEP in Theories of Gravitation

The EEP is the foundation of all curved space-time or “metric” theories of gravitation, including of course GR. It divides gravitational theories in two classes: metric theories, those that embody EEP and non-metric theories, those that do not. This distinction is fundamental, as metric theories describe gravitation as a geometric phenomenon, namely an effect of curvature of space-time itself rather than a field over space-time, quite unlike any of the other known interactions. It might thus appear unnatural to use a metric theory for gravitation, so different from the formalisms of the other interactions, and indeed most unification attempts cast doubt on precisely this hypothesis and thus on the validity of the EEP. Only experimental tests can settle the question and, in the light of the above, experimentally testing the EEP becomes truly fundamental. To be more precise (see e.g. Refs. 18, 125, 133), a metric theory of gravitation is one that satisfies the following postulates:

1. Space-time is endowed with a metric tensor $g_{\mu\nu}$, central to the metric equation that defines the infinitesimal line element, i.e. the space-time separation between two events

$$ds^2 = g_{\mu\nu}(x^\rho)dx^\mu dx^\nu,$$  

(2.1)

in some 4-dimensional space-time coordinate system $x^\rho$;
2. The trajectories of freely falling test bodies are geodesics of extremal length,

$$\delta \int ds = 0,$$

i.e. they depend only on the geometry of space-time, but are independent of the test body composition;

3. Clocks measure proper time $\tau$ along their trajectory, given by

$$d\tau^2 = -\frac{1}{c^2}ds^2,$$

independent of the type of clock used;

4. In local freely falling reference frames, the non-gravitational laws of physics (i.e. the other three fundamental interactions) satisfy the principles of special relativity.

Obviously the above postulates are a direct consequence of the EEP, for example LLI and LPI are the foundations of points 3 and 4 and WEP is the basis of point 2. It is important to note that GR is not the only possible metric theory that satisfies the above postulates. Indeed, there exist a large number of such theories like the scalar-tensor Jordan-Brans-Dicke theories \[25, 73\] and their generalizations. These theories differ from GR in the way that the metric tensor is related to the distribution of mass-energy through the existence of other fields associated with gravity (scalar field, vector field, etc.).

Theories in which varying non-gravitational coupling constants are associated with dynamical fields that couple to matter directly are not metric theories. In such theories, the fine structure constant $\alpha$ for instance would vary with space and time. Neither, in this narrow sense, are theories in which one introduces additional fields (dilatons, moduli) that couple differently to different types of mass-energy, e.g. some versions of Superstring theory. The fundamental ingredient of all such non-metric theories is non-universal coupling to gravity of all non-gravitational fields, i.e. the fields of the Standard Model of particle physics. In metric theories, coupling to the gravitational field is universal, and as a consequence the metric of space-time can be studied by a variety of devices made up of different non-gravitational fields and particles, and, because of universality, the results will be independent of the device. For instance, the proper time between two events is a characteristic of space-time and of the location of the events, not of the clocks used to measure it \[134\].

Thus experimental tests of the EEP are often viewed as tests of the universal coupling of gravity (through the metric of space-time $g_{\mu\nu}$) to all non-gravitational fields of the Standard Model of particle physics \[44\]. Violations occur when the coupling is dependent on some attribute of the non-gravitational fields at hand that may be different for different test bodies, e.g. electromagnetic charge, nuclear charge, total spin, nuclear spin, quark flavor, lepton number, etc. Exploring all possibilities of such anomalous couplings is the fundamental aim of experimental tests of the EEP. Note also that in any particular experimental situation, symmetry requires that such anomalous couplings be not only a function of the composition of the test body, but also of the mass which is the source of the gravitational field. As a consequence, the widest possible range of source and test body configurations needs to be explored when testing the different aspects of EEP, and this is one of the aims of STE-QUEST, which will test for EEP violation in the gravitational fields of the Sun and the Moon. Furthermore, although not discussed further here, the STE-QUEST data can also be analyzed to search for violation of EEP in other source fields, e.g. that of galactic dark matter as in Ref. \[118\]. Such future searches will be part of the legacy of STE-QUEST.

C. Why Would the EEP be Violated?

It has already been pointed out that the EEP is in fact rather unnatural in the sense that it renders gravity so different from other interactions, because the corresponding universal coupling implies that gravitation is a geometrical attribute of space-time itself rather than a field over space-time like all other known interactions. Einstein himself initially called it the hypothesis of equivalence before elevating it to a principle once it became clear how central it was in the generalization of special relativity to include gravitation. This shows how surprising it is in fact that such an hypothesis should be satisfied at all, let alone down to the uncertainties of present-day tests. Therefore, rather than asking why the EEP should be violated, the more natural question to ask is why no violation has been observed yet. Indeed most attempts at quantum gravity and unification theories lead to a violation of the EEP \[9, 42, 49, 97, 115, 124\], which in general have to be handled by some tuning mechanism in order to make the theory compatible with existing limits on EEP violation. For example, in string theory moduli fields need to be rendered massive (short range) \[124\] or stabilized by e.g. cosmological considerations \[42\] in order to avoid the stringent limits already imposed by EEP tests.
Similarly M-theory and Brane-world scenarios using large or compactified extra dimensions need some mechanism to avoid existing experimental limits from EEP tests or tests of the inverse square law \[3, 9, 10, 97, 115\]. The latter tests explore a modification of the gravitational inverse square law (e.g. in the form of a Yukawa potential) and are in many respects complementary to EEP tests. However, violations of the inverse square law will also be detected by certain EEP tests (e.g. red-shift tests), allowing for a much richer phenomenology with different distance dependences and anomalous couplings. Therefore, not only do we expect a violation of EEP at some level, but the non-observation of such a violation with improving uncertainty is already one of the major experimental constraints for the development of new theories in the quest for quantum gravity and unification (see Fig. 2). This makes experimental tests of EEP in all its aspects one of the most essential enterprises of fundamental physics today.

It is interesting to note that experimental constraints for EEP violations at low energy are rather closely related to present-day physics at the very small scale (particle physics) and the very large scale (cosmology). These connections are discussed in more detail in Sec. III A and III B. Notably, the recent experimental confirmation of the Higgs boson has thus lent strong credibility to the existence of scalar fields, as the Higgs is the first fundamental scalar field observed in Nature. It is thus likely that additional long and/or short range scalar fields exist, as postulated by many unification theories, and EEP tests are one of the most promising experimental means for their observation.

At the other extreme, in cosmology, most models for Dark Energy (DE) are also based on long-range scalar fields that, when considered in the context of particle physics, are non-universally coupled to the fields of the Standard Model \[79, 80\]. As a consequence, one would expect EEP violations from such fields at some level, which might be detectable by experiments like STE-QUEST thus shedding light on the dark energy content of the Universe from a completely different angle. Similarly, long-range scalar fields coupled to Dark Matter (DM) have been investigated as a possible source of EEP violations \[34\], which again provides a very appealing route towards independent confirmation of DM, making it more tangible than only a hypothesis for otherwise unexplained astronomical observations.
III. EEP IN THE CONTEXT OF PHYSICS TODAY

A. Cosmology Context

One of the most important discoveries of the past decade has been that the present Universe is not only expanding, but it is also accelerating [2,108,112]. Such a scenario is problematic within the standard cosmological model, based on GR and the Standard Model of particle physics. Together, these two models provide a set of predictions well in agreement with observations: the formation of light elements in the early Universe — the big bang nucleosynthesis (BBN), the existence of the cosmic microwave background (CMB), and the expansion of the Universe. However, now the big challenge of modern cosmology and particle physics is to understand the observed acceleration of the Universe. Observations indicate that the content of matter and energy in our Universe is about 68.3% dark energy (DE), 26.8% dark matter (DM), and 4.9% baryonic matter [2].

These values are obtained assuming the Λ-CDM model. There are independent measurements of the DE component of the Universe from observations of high red-shift Type Ia Supernovae [52,108,112]. The evidence for DM comes essentially from the analysis of galactic rotation curves [56], acoustic oscillations in the CMB [72,74,139], large scale structure formation [53,54], and gravitational lensing [37,140]. Nevertheless, although there is such a strong evidence for the existence of DE and DM, almost nothing is known about their nature and properties.

The simplest explanation of DE is the existence of a small, but non-zero, cosmological constant Λ. The latter does not undergo a dynamical evolution, and is conventionally associated to the energy of the vacuum in a quantum field theory. In other words, the cosmological constant is a constant energy density filling space homogeneously and isotropically, and is equivalent physically to vacuum energy. As a consequence, it should store the energy density of the present day Universe and its value should be of the order of the critical density. In fact, from the observations it follows that Λ ≈ H_0^2, where H_0 = 2 × 10^{-42} GeV is the present value of the Hubble parameter and is related to the dimension of the Universe. The vacuum energy density associated to the cosmological constant is therefore ρ_A = Λ/8πG ≈ 10^{-47} GeV^4 (∼ ρ_{critical}).

On the other hand, arguments from quantum field theory imply that the vacuum energy density is the sum of zero point energy of quantum fields with a cutoff determined by the Planck scale (m_P ∼ 1.22 × 10^{19} GeV) giving ρ_{vacuum} ≈ 10^{74} GeV^4, which is about 121 orders of magnitude larger than the observed value. A lower scale, fixed for example at the QCD scale, would give ρ_{vacuum} ≈ 10^{-3} GeV^4 which is still much too large with respect to ρ_A. From a theoretical point of view, at the moment, there is no explanation as to why the cosmological constant should assume the correct value at the scale of the observed Universe. The only argument we can give is based on the anthropic principle, i.e. the idea that much larger values would not have lead to the formation of stars, planets and ultimately humans.

Rather than dealing directly with the cosmological constant to explain the accelerating phase of the present Universe, a number of alternative approaches and models have been proposed in the last years. Some of these models are briefly summarized below:

1. Quintessence models [32,111,133] — These models invoke a time evolving scalar field with an effective potential that provides the observed inflation;

2. Chameleon fields [29,72,80] — In this model the scalar field couples to the baryon energy density and is homogeneous, varying across space from solar system to cosmological scales;

3. K-essence [11,12,36] — Here the scalar field sector does contain a non-canonical kinetic term;

4. Modified gravity arising out of string theory [52] — In this model the feedback of non-linearities into the evolution equations can significantly change the background evolution leading to acceleration at late times without introducing DE;

5. Chaplygin gases [10,20,76] — This model attempts to unify DE and DM in a unique setting, by allowing for a fluid with an equation of state which evolves between the two;

6. f(R)-gravity [30,104] — In this model one considers instead of the Einstein-Hilbert action a generic function of the scalar curvature R, not necessarily linear in R as in the conventional GR. f(R)-gravity contains many features which make these models very attractive, as for example: (i) they provide a natural unification of the early-time inflation and the later-time acceleration of the Universe owing to the different role of the gravitational terms relevant at small and large scales; (ii) they allow to unify DM and DE; (iii) they provide a framework for the explanation of the hierarchy problem and unification of Grand Unified Theories (GUT) with gravity. However, some f(R)-models of gravity are strongly constrained (or ruled out) by solar system tests restricting the possible models;
7. Phantom Dark Energy [29].

Many of the models proposed in the literature are characterized by the fact that a scalar field (or more than one scalar field) coupled or not to gravity and ordinary matter is included in the action of gravity. On a fundamental ground, there are several reasons to introduce a scalar field in the action describing gravity. A scalar field coupled to gravity is an unavoidable aspect of all theories aimed at unifying gravity with the other fundamental forces. These theories include Superstring, Supergravity (SUGRA), M-theories. Moreover, scalar fields appear both in particle physics and cosmology: the Higgs boson in the Standard Model, the string dilaton entering the supermultiplet of the higher dimensional graviton, the super-partner of spin-$\frac{1}{2}$ in SUGRA. It also plays a non-trivial role in models based on composite boson condensates. The introduction of a scalar field gives rise typically to a violation of the EEP depending on its coupling to the Lagrangian describing ordinary matter.

The above considerations apply to most extended theories of gravity (scalar-tensor theories, f(R)-gravity, etc.), which leaves the foundation of relativistic gravity on a rather shaky ground. That becomes a problem especially when trying to isolate the fundamental properties of classical gravity which should be preserved in approaches to quantum or emergent gravity [90]. The STE-QUEST experiment will therefore play a crucial role not only for searching possible violation of the EEP, but will shed light also on what effective theories of gravity among those above mentioned is the true theory for describing gravity.

A violation of the EEP in the dark sector (DM and DE), comes also from a possible coupling of DM to a scalar field. More precisely, a (light) scalar field coupled to DM could mediate a long-range force of strength comparable to gravity [15, 34, 35]. This kind of investigation is also motivated by the fact that such interactions could account for features related to the DM distribution as well as to DM-quintessence interactions [15, 34, 35, 41, 67]. Limits on such a force have been derived from observations of DM dynamics in the tidal stream of the Sagittarius dwarf galaxy which yields a force with strength less that 20% of gravity for a range of 20 kpc [77, 78]. Moreover, as noted in Refs. [15, 34, 35], if a new long-range force will be detected in future, then it would be a signal of the presence of a new mass hierarchy between the light scalar mass $m_\phi \lesssim 10^{-20}$ eV and the weak scale $m_W \approx 10^2$ GeV, in addition to the one between the weak scale and the Planck scale. The possibility that a scalar field couples to Standard Model particles implies that the force acting on ordinary matter could be composition-dependent [43]. As a consequence, such forces are tightly constrained by Eötvös experiments looking for violations of the weak EEP [118]. On the other hand, even if $\phi$ has only an elementary (i.e., renormalizable) coupling to DM, interactions between DM and ordinary matter will still induce a coupling of $\phi$ to ordinary matter [15, 34, 35]. This can be thought of as arising from the scalar coupling to virtual DM particles in ordinary atomic nuclei. Hence, a fifth force coupled to the Standard Model is naturally expected in the case in which a light scalar couples to a field having Standard Model interactions.

Without any doubts, the equivalence of gravitational mass and inertial mass represents one of the most fundamental postulates in Nature. Theoretical attempts to connect general relativity to the Standard Model of particles are affected by a violation of the EEP [45]. Therefore, tests of the EEP turn out to be important tests of unification scale physics far beyond the reach of traditional particle physics experiments. The discoveries of DM and DE have provided strong motivation to extend tests of the EEP to the highest precision possible. In this respect, the STE-QUEST experiment will play a significant role.

B. Particle Physics Context

In the previous section, it already became clear that the difficulties of GR in cosmology are closely related to those in particle physics. In particular, in a quantum field theory (like the Standard Model of particle physics), one would expect that the vacuum energy of the fundamental fields should be observed in its gravitational consequences, especially on the large scale of the Universe. However, there is a huge discrepancy (121 orders of magnitude, or at least 44 orders of magnitude if one assumes the QCD scale, see Sec. IIA) between the observed vacuum energy density of the Universe (dark energy) and the one expected from the Standard Model of particle physics. This has been considered a major problem in modern physics, even before the discovery of dark energy when the “observed” value of the cosmological constant (or vacuum energy) was compatible with zero [152]. And one might argue that this problem has become even worse since the discovery of the accelerated expansion of the Universe, and the associated small but non-zero value of $\Lambda$, as now one requires a mechanism that does not completely “block” the gravitational effect of vacuum energy, but suppresses it by a huge factor, i.e. some extreme fine tuning mechanism is required that is difficult to imagine.

Another conceptual problem is that the Standard Model of particle physics requires a number of dimensionless coupling constants to be put in by hand, which seems somewhat arbitrary and is not very satisfactory [46]. One of the aims of theoretical developments is then to replace these constants by some dynamical fields that provide the coupling constants (e.g. moduli fields in string theory, dilaton, etc.), similarly to the Higgs field giving rise to the
mass of fundamental particles. As a consequence the coupling constants become dynamical quantities that vary in 
space-time (e.g. space-time variation of the fine structure constant $\alpha$), which necessarily leads to violations of the 
EEP (violation of LPI, but also of WEP/UFF and LLI). However, the resulting phenomenological consequences are 
such that in most approaches one requires some mechanism to stabilize these fields in order to be compatible with 
present-day constraints from EEP tests [24, 124]. Although no firm predictions exist, this makes the discovery of the 
effect of such fields (e.g. EEP violation) a distinct possibility [10].

The recent discovery of the Higgs particle at LHC confirms the existence of the first fundamental scalar field, at 
least fundamental down to the scale probed by the Standard Model. As discussed in the previous section, scalar fields 
are ubiquitous in cosmology because they easily provide a diffuse background: they play a central role in most models 
of inflation or dark energy. It is thus important to have identified at least one fundamental scalar field. There has 
been attempts to make the Higgs field itself play a role in cosmology, by coupling it to the curvature of space-time. 
This is for example the model of Higgs inflation [45]. At first glance, this might seem to lead to violations of the 
equivalence principle but, going to an Einstein frame, this gives rise to nonlinear interactions of the Higgs field, which 
ares from the powers of the Planck mass (or, more precisely, $M_P/\xi$ if $\xi$ is the coupling of the Higgs to curvature) [19].

Even if one disregards gravity, the Standard Model of particle physics still does not address all the fundamental 
questions: in particular, whereas it attributes the origin of mass to the Higgs non-vanishing vacuum value, it does 
not explain the diversity of the masses of the fundamental particles, i.e. it does not explain the diversity of the couplings 
of the matter to the Higgs field. One thus has to go to theories beyond the Standard Model in order to answer these 
questions. Most of these theories make heavy use of scalar fields, the most notable examples being supersymmetry, 
which associates a scalar field to any spin-$1/2$ matter field, string theory and higher-dimensional theories. Some of 
these scalar fields may be extremely light, or even massless, which leads to new types of long range forces, and thus 
potential EEP violations, unless these fields are universally coupled, a difficult property to achieve.

Moreover, the values of these scalar fields often have a predictive role in setting the value of fundamental constants 
or ratios of mass scales. Because they are weakly coupled to ordinary matter, they may not have reached their 
actual state, in which case they are still evolving with time. This leads to a time dependence of the corresponding 
constants or mass scales, and thus again to a potential violation of the equivalence principle.

IV. QUANTUM MECHANICS AND THE EEP

Quantum tests of the Equivalence Principle differ from classical ones because classical and quantum descriptions of 
motion are fundamentally different. In particular, the Universality of Free Fall (or WEP) has a clear significance in 
the classical context where it means that space-time trajectories of test particles do not depend on the composition 
of these particles. How UFF/WEP is to be understood in Quantum Mechanics is a much more delicate point. The 
subtlety of discussions of the EEP in a quantum context is also apparent in the debate about the comparison of various 
facets of the EEP, in particular the UFF and the LPI [65, 100, 135]. More generally, considering quantum phenomena 
in the context of gravity poses many conceptual and fundamental difficulties as discussed below. Although not all 
of these are directly explored by STE-QUEST, they provide a broad picture of the limits of our knowledge in this 
domain and thus the interest of experiments like STE-QUEST that have the discovery potential for expected and 
unexpected results that might shed light on this frontier of physics.

Let us first discuss the case where no distinction is made between classical and quantum tests, by evaluating 
different UFF tests with respect to non-standard theories, as was done for example by Damour & Donoghue [45] for 
the specific case of couplings to a light dilaton. The same type of argument is also valid for a vector field like the $U$ 
boson [57, 58]. In these cases a similar value for the fractional inaccuracy (e.g. $10^{-15}$) leads to a larger sensitivity for 
the free fall of titanium ($^{48}$Ti) and platinum ($^{195}$Pt) test masses (e.g. the $\mu$-SCOPE mission) than for STE-QUEST 
where two isotopes of rubidium ($^{85}$Rb and $^{87}$Rb) are compared. To be more quantitative, the sensitivity depends 
on the difference in $E_i/M$ ratios between the two test masses, with $E_i$ being a particular type of nuclear binding energy. 
The difference in sensitivities then simply stems from the locations of these atoms along the sequence of stable heavy 
elements in the $(N, Z)$ plane as shown in Fig. 3.

Let us be more quantitative on the comparison STE-QUEST versus $\mu$-SCOPE based on Fig. 3. In a model such as 
in Ref. [45], the nuclear binding energy of nuclei is the sum of several contributions ($A = N + Z$):

$$E = -16.0 A + 17.0 A^{2/3} + 23.0 \frac{(N-Z)^2}{A} + 0.7 \frac{Z(Z-1)}{A^{1/3}} - 6.0 \frac{(-1)^N + (-1)^Z}{A^{1/2}} \text{ MeV}.$$ \hspace{1cm} (4.1)

These are, respectively, the volume energy $\propto A$, the surface energy $\propto A^{2/3}$, the asymmetry energy, the (repulsive) 
Coulomb energy and the pairing energy. In a model for the coupling of the dilaton to fundamental matter fields 
(quarks and gluons), one expects that all the terms in the formula (4.1) will contribute separately to the violation of
the EEP. For two atoms A and B and a given type of energy \( i \) in Eq. (4.1) we pose

\[
\delta_i(A, B) = \frac{E_i(A)}{M(A)} - \frac{E_i(B)}{M(B)}.
\]

Then an indicator of the uncertainty with which STE-QUEST and \( \mu \)-SCOPE (both having fractional inaccuracy of the order of \( 10^{-15} \)) can test each of the separate terms in Eq. (4.1) is provided by the ratio

\[
s_i(A, B) = \frac{10^{-15}}{\delta_i(A, B)}.
\]

The results are tabulated in Tables II and III. As we see the pair \( ^{48}\text{Ti}–^{196}\text{Pt} \) is between a factor \( \sim 5 \) and a factor \( \sim 60 \) times more sensitive (depending on which \( E_i \) is considered) than the pair \( ^{85}\text{Rb}–^{87}\text{Rb} \). In addition the pair \( ^{85}\text{Rb}–^{87}\text{Rb} \) has no sensitivity to the pairing energy because the difference of the number of neutrons in the two isotopes is even.

However, from a wider phenomenological point of view, notice that the \( ^{85}\text{Rb}–^{87}\text{Rb} \) test has to be considered as exploring a variation in the table of nuclei which is orthogonal and complementary to that along the main sequence, mainly tested by the pair \( ^{48}\text{Ti}–^{196}\text{Pt} \). In this context, the precision of STE-QUEST has to be compared to existing tests between two Rb isotopes rather than to \( \mu \)-SCOPE. This is similar to methods used in the context of particle physics where the CERN Scientific Council has accepted experiments [51], [107] for testing the free fall of cold antihydrogen atoms at a level of the order of \( 10^{-2} \). With most non-standard theories used to compare the interest of experiments, the targeted precision is far from what is already known from classical tests of the EEP. The fact that the experiments have been judged to be worthy shows the peculiar interest of tests of UFF performed with non-classical objects like antimatter or quantum objects.

Let us now discuss a number of physical hypotheses one is implicitly making when doing the above classical comparison, simply looking at the different locations of the tested materials in Fig. 3, i.e. assuming that there is nothing special about quantum tests.
The first implicit assumption is that Quantum Mechanics is valid in the freely falling frame associated with classical test bodies in the definition of WEP. Indeed, the usual definition of the EEP states that special relativity holds in the freely falling frame of WEP without reference to quantum mechanics.² Of course, this extension of the notion of freely falling frame to quantum mechanics is always implicit and “obvious”. It is used when one computes the phase shift of a matter wave interferometer in a gravity field, using the full machinery of quantum mechanics, for instance Feynman’s path integral formalism [122].

Another important implicit assumption is that any possible violation of the EEP must be due to a new fundamental interaction (which superposes to the gravitational force), and that fundamental interactions are described in the framework of quantum field theory (QFT) by bosonic fields. In particular, the formalism of QFT must be true for that field, e.g. the procedures of second quantization and of renormalization. The consequence is that the violation of the EEP is either due to a scalar spin-0 field (dilaton) or a vector field (for instance the U boson [57, 58]). Indeed, recall that higher-order spin fields (s > 2) yield difficulties, for instance the coupling of an additional spin-2 field to the metric spin-2 field of GR is problematic. Of course there are theorems that fundamental interactions in the framework of relativistic quantum mechanics necessarily involve the notion of fields, but as physicists we also want to prove our theorems experimentally.

The previous statements represent the state-of-the-art of Physics that we have today; if one of these would turn out to be wrong this would provoke a major crisis in Physics. Nevertheless, because they are so fundamental, these statements are worth being experimentally verified wherever possible. It is true that they are tested every day in particle accelerators, but in a regime where the gravitational field plays essentially no role. Testing the EEP for quantum waves in the presence of gravity represents a new way of testing some of our deepest beliefs in Physics at the interplay between QM and GR.

Additionally, there are a number of other concerns regarding the quantum to classical comparison in general, which illustrate the difficulties in this region of Physics and thus the interest of any experimental guidance.

The variety of quantum states is much larger than that of classical ones and it seems therefore plausible that quantum tests may ultimately be able to see deeper details of couplings between matter and gravity than classical ones. When considering non-standard couplings of matter to gravity, there might be a difference between how the wave packet centre is moving and how it is deforming [23, 75, 127]. As an illustration in a concrete example, let us consider the free fall in a gravitational field of a particle in QM described by the wave function \( \Psi \). We assume that

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² Recall that relativistic quantum mechanics did not exist at the time of the earliest formulation of the equivalence principle by Einstein.
the wave function is initially Gaussian. Schrödinger’s equation with Hamilton operator

$$\dot{H} = \frac{\partial^2}{2m} + mg \dot{z}$$  \hspace{1cm} (4.4)$$

is satisfied, where the second term is the usual Newtonian gravitational potential. We compute the time of flight of this particle from some initial position $z_0$ up to $z = 0$, the initial position being determined by the expectation value $z_0 = \langle \dot{z} \rangle_{\Psi_0}$ of the position in the Gaussian initial state $\Psi_0$. The time of flight is statistically distributed with the mean value agreeing with the classical universal value,

$$T = \sqrt{\frac{2z_0}{g}}.$$  \hspace{1cm} (4.5)$$

However, the standard deviation of the measured values of the time of flight around $T$ depends on the mass of the particle

$$\sigma = \frac{\hbar}{\Delta_0 mg},$$  \hspace{1cm} (4.6)$$

where $\Delta_0$ is the width of the initial Gaussian wave packet. In this sense the quantum motion of the particle is non-universal, as it depends on the value of its mass [47, 95, 129].

Another example is the role of intrinsic spin of quantum probes, that has no classical equivalent. For classical particles, the EEP is implemented by the rule of the minimal coupling (see e.g. Ref. [131]): in the presence of the gravitational field we replace the Minkowski metric $\eta_{\alpha\beta}$ in the Lagrangian of special relativity by the curved space-time metric $g_{\mu\nu}$ of GR. Suppose that a classical body is made of $N$ particles with positions $x_a$ and velocities $v_a$ interacting through the classical electromagnetic field $A_\alpha$, with dynamics resulting from the Lagrangian

$$L_{SR} = L[x_a, v_a, A_\alpha, \eta_{\alpha\beta}]$$  \hspace{1cm} (4.7)$$
in special relativity. Then the Lagrangian describing the dynamics of this body in GR will simply be

$$L_{GR} = L[x_a, v_a, A_\mu, g_{\mu\nu}].$$  \hspace{1cm} (4.8)$$

The procedure to couple a quantum field to gravity is much more complex and, we argue, more fundamental than for the coupling of classical fields. The Lagrangian of the quantum field (e.g. the Dirac field) depends on the derivative of the field because of the intrinsic spin, and requires additional formalisms like tetrads and the spinorial representations of the Lorentz group and the associated spinorial derivative. So, while classical matter is coupled to gravity by using only the metric ($\eta_{\alpha\beta}$ replaced by $g_{\mu\nu}$), quantum fields associated with electrons and other fermions are coupled to gravity through tetrads, which may be considered as a deeper representation of space-time (the metric is immediately deduced from the tetrad, but the inverse is not true), with a more complicated formalism.

Of course, atom interferometry tests of the EEP are usually performed with unpolarized spin states ($m_F = 0$) because the latter are insensitive to magnetic fields at first order. They can also be performed with other Zeeman sublevels (spin polarized states with $m_F \neq 0$) with a somewhat reduced precision due to the first-order coupling with magnetic fields. This possibility of performing spin-dependent tests is an obvious advantage of quantum tests over classical versions of EEP tests, though the latter ones may of course perform tests with spin-polarized matter [70]. Comparison of these various tests could be done by following the line already opened for spin-dependent clock measurements [137], using the Standard Model Extension (SME) framework [91].

Our main concern about the frontier between QM and GR is of course the absence of a consistent quantum theory of gravity. Its non-renormalisability within standard perturbative methods has led to a variety of suggestions and alternative approaches, the most pragmatic being to incorporate the gravitational field into the effective-field-theory (EFT) program, which results in definite prescriptions for the computations of low-energy quantum corrections at scales well above the Planck scale [27]. In particular, it allows for computations of metric fluctuations in inflationary cosmology and subsequent applications to explain CMB anisotropies. On the other hand, taking the geometric interpretation of gravity as the central paradigm, it has been suggested that gravity will eventually defy standard quantization approaches and that the sought-for reconciliation will impose at least as much change on our present notion of Quantum(=Field) Theory as it does on classical GR [106]. In view of these diverging attitudes it is important to note that they will already differ in their respective answers to the most mundane physical questions, such as: what is the gravitational field sourced by a quantum system in a state represented by a wave function $\Psi$? Here the so-called semi-classical theory comes into play, which states that the gravitational field obeys the Einstein field equation

$$G^{\mu\nu} = \frac{8\pi G}{c^4} (\dot{T}^{\mu\nu}) \phi,$$  \hspace{1cm} (4.9)
where the expectation value of the stress-energy quantum operator $\hat{T}_{\mu\nu}$ in the given quantum state $\Psi$ replaces the classical stress-energy tensor $T_{\mu\nu}$.

The obvious and orthodox interpretation of this equation is that of an approximation to quantum gravity for states $\Psi$ producing negligible fluctuations in the sourcing stress-energy. If we assume quantum gravity to obey the usual rules of measurement and quantum-state reduction, then it is easy to see that (4.9) cannot be a valid description of such processes [130]. Indeed, suppose that we have a state of matter made of the superposition of two states (each with probability $1/2$) in which the matter is localized, respectively, into two different disjoint regions. Then, according to Eq. (4.9), the gravitational field will be generated by half the matter in the first region and by half the matter in the other distinct region. This is clearly incompatible with a measurement of the location of the matter (and the associated collapse of the wave function), since after measurement all the matter will be either located entirely in the first, or entirely in the second region.

On the other hand, if we allow for the possibility that quantum gravity will imply changes to the usual rules of quantum-state reduction, e.g., along the lines of Ref. [106], then Eq. (4.9) might acquire a more fundamental status. In fact, the first suggestion of Eq. (4.9) was made in connection with the logical possibility that gravity is not to be quantized at all [114]. Based on this, it has further been suggested to experimentally scrutinize the alleged necessity of quantum gravity, e.g., by looking at situations in which the classical gravitational self-field computed according to (4.9) affects the quantum-mechanical dispersion as a result of the non-linearities that the dependence of the spacetime metric on the state $\Psi$ [resulting from (4.9)] introduces into the quantum dynamical equations for $\Psi$ (which in turn depend on the metric) [31, 64].

Back to the orthodox viewpoint, where Eq. (4.9) is only of approximate validity, we maintain that it can be an effective description of quantum systems under gravity, including their own field, and as such it makes sense and has interesting physical consequences, like the back-reaction of Hawking’s radiation on black holes, or that of quantum fields in the early Universe. Although the semi-classical theory will not be checked directly by quantum tests of the EEP, the above issues and paradoxes remind us that we do not dispose of a consistent quantum theory of gravity and that experimental evidence exploring the relationship between QM and GR is direly needed.

In summary, although there is no established theory that favors quantum tests of the EEP, there are nonetheless a number of difficulties in the frontier between QM and GR due to the absence of a quantum theory of gravitation, that call for experiments lying at that frontier, like quantum tests of the EEP proposed by STE-QUEST.

On the experimental side, quantum mechanical tests have to be considered as opening a new technological avenue based on quantum sensors, which is probably the best solution for future much improved tests. Whereas macroscopic tests approach their ultimate limits after years of scientific research and technical development, this is not the case for atomic tests. In particular, the accuracy of the atom interferometry test in STE-QUEST ($2 \times 10^{-15}$) promises an improvement by 8 orders of magnitude over the best existing test ($10^{-7}$) between two Rb isotopes. With this improvement, it already would reach the level of accuracy of $\mu$-SCOPE [126], while still having possibilities for future improvements.

V. STE-QUEST TESTS OF THE EINSTEIN EQUIVALENCE PRINCIPLE

In this section, we discuss specifically the tests of the EEP carried out by STE-QUEST. We first describe a general theoretical framework for the WEP/UFF and LPI tests that allows a classification and comparison of the different experiments and clarifies the complementarity between the different types of tests. We then use that framework to compare each of the planned STE-QUEST experiments to existing and expected measurements in the same domain and point out the improvements expected from STE-QUEST. Finally we address the possible STE-QUEST tests of Lorentz Invariance and CPT symmetry using another theoretical framework particularly adapted for that purpose.

A. Different Tests of the EEP

Tests of the different aspects of EEP (i.e. WEP, LLI, and LPI), and the relations between them, are best discussed within the “modified Lagrangian framework”, which is a powerful formalism allowing deviations from GR and metric theories of gravity, but at the same time permitting a coherent analysis of various experiments [69, 105, 134, 138]. The formalism describes a large class of non-metric theories in a way consistent with Schiff’s conjecture and energy conservation. This class of theories is defined by a single Lagrangian, in which the coupling between gravitation and different types of mass-energies is generically not universal. In a simplified variant of the formalism, we consider a composite body of mass $m$ (e.g. an atom in a STE-QUEST experiment) in the Newtonian gravitational potential...
\[ U(\mathbf{x}) = \frac{GM}{r} \] of the Earth, where \( r = |\mathbf{x}| \), thus obeying the Lagrangian
\[ L = -mc^2 + mU + \frac{1}{2}mv^2. \]  

(5.1)

We postulate that the mass \( m = m(\mathbf{x}) \) of this body depends on the position \( \mathbf{x} \) through a violation of the LPI. This is modelled by assuming that a particular internal energy of the body, \( E_X = E_X(\mathbf{x}) \), behaves anomalously in the presence of the gravitational field, where \( \mathbf{x} \) refers to the type of interaction involved (electromagnetic, nuclear, spin-spin, spin-orbit, etc.). For simplicity, because we have in mind the discussion of the red-shift test versus the test of the UFF, both being performed by STE-QUEST, we consider only a dependence on \( \mathbf{x} \) to model the violation of the LPI. It could be possible to include also a dependence on the velocity \( \mathbf{v} \) to model a violation of LLI. Separating \( E_X(\mathbf{x}) \) from the other forms of energies \( E_Y \) composing the body and which are supposed to behave normally, we write
\[ m(\mathbf{x}) = \overline{m} + \frac{1}{c^2} \left[ E_X(\mathbf{x}) + \sum_{Y \neq X} E_Y \right]. \]

(5.2)

Here \( \overline{m} \) denotes the sum of the rest masses of the particles constituting the body; \( \overline{m} \) and all \( E_Y \)'s are constant. The violation of LPI is modeled in the simplest way by assuming that at the leading order
\[ E_X(\mathbf{x}) = \overline{E}_X + \beta^{(a)}_X \overline{m} \Delta U(\mathbf{x}), \]

(5.3)

where \( \Delta U = U_\oplus - U \) is the potential difference with respect to some reference point, e.g. the surface of the Earth. The parameter \( \beta^{(a)}_X \) is dimensionless and characterizes the violation of LPI. It depends on the particular type of mass-energy or interaction under consideration, e.g. \( \beta^{(a)}_X \) would be different for the electromagnetic or the nuclear interactions, with possible variations as a function of spin or the other internal properties of the body, here labelled by the superscript \((a)\). Thus \( \beta^{(a)}_X \) would depend not only on the type of internal energy \( X \) but also on the type of body \((a)\). Defining now the “normal” contribution to the total mass,
\[ m_0 = \overline{m} + \sum_{Y} E_Y/c^2, \]

(5.4)

and replacing \( m(\mathbf{x}) \) by its explicit expression into the Lagrangian (5.1) we obtain
\[ L = -m_0 c^2 + m_0 \left( U - \beta^{(a)}_X \overline{m} \Delta U \right) + \frac{1}{2}m_0v^2, \]

(5.5)

where we have neglected higher-order terms, which are of no relevance for the discussion.

We can now analyze the traditional free-fall and red-shift experiments. By varying Eq. (5.5), we obtain the equation of motion of the body as
\[ \frac{dv}{dt} = (1 + \beta^{(a)}_X) \nabla U, \]

(5.6)

which shows that the trajectory is affected by the violation of LPI and is not universal. In fact, we see that \( \beta^{(a)}_X \) measures the non-universality of the ratio between the body’s passive gravitational mass and inertial mass. Thus, in this framework, the violation of LPI implies a violation of UFF (and WEP), and \( \beta^{(a)}_X \) is the WEP-violating parameter. This is a classic proof of the validity of Schiff’s conjecture \[116\].

The violation of LPI is best reflected in classical red-shift experiments, which can be analysed using a cyclic Gedanken experiment based on energy conservation. This was done in Ref. \[105\], extending a famous argument by Einstein himself. The result for the fractional frequency shift \( z \) in a Pound & Rebka-type experiment \[110\] is then
\[ z = \left( 1 + \alpha^{(a)}_X \right) \Delta U/c^2, \]

(5.7)

where the LPI-violating parameter \( \alpha^{(a)}_X \) is again non-universal. The important point is that, within the framework of the modified Lagrangian (5.5), the LPI-violating parameter \( \alpha^{(a)}_X \) is related in a precise way to the WEP-violating parameter \( \beta^{(a)}_X \) (see Ref. \[105\]):
\[ \beta^{(a)}_X = \alpha^{(a)}_X \frac{E_X}{m c^2}. \]

(5.8)
Therefore tests of LPI and WEP are not independent, and we can compare their different qualitative meanings. Since for typical energies involved we shall have $E_X \ll m c^2$, this means that $\beta_X \ll \alpha_X$, where $\beta_X$ and $\alpha_X$ denote some typical values of the parameters. For a given set of LPI and WEP tests, their relative merit is given by Eq. (5.8) and it is dependent on the model used, i.e. the type of anomalous energy $E_X$ and the employed materials or bodies.

For example, let us assume a model in which all types of electromagnetic energy are coupled in a non-universal way, i.e. $\beta_{EM} \neq 0$ (with all other forms of energies behaving normally), and where the clock transition is purely electromagnetic. The UFF test between two materials $(a)$ and $(b)$, both containing electromagnetic energy (e.g. binding energy), is carried out with an uncertainty of $|\beta_{EM}^{(a)} - \beta_{EM}^{(b)}| \approx |\beta_{EM}| \lesssim 10^{-13}$ in best current experiments [118]. On the other hand the LPI test for a clock of type $(c)$ based on an electromagnetic transition, is carried out with an uncertainty of $|\alpha_{EM}^{(c)}| \approx |\alpha_{EM}| \lesssim 10^{-4}$ in the GP-A experiment [128]. For macroscopic test bodies, the nuclear electromagnetic binding energy contributes typically $E_{EM}/(m c^2) \approx 10^{-13}$ of the total mass, so from Eq. (5.8) we have $|\beta_{EM}| \approx 10^{-3} |\alpha_{EM}|$, which means that the WEP test yields $|\alpha_{EM}| \lesssim 10^{-10}$, a much more stringent limit than the red-shift test ($|\alpha_{EM}| \lesssim 10^{-4}$).

However, that result depends on the particular model used. If we assume another model in which the nuclear spin plays a role leading to a non-universal coupling of atomic hyperfine energies, i.e. $\beta_{HF} \neq 0$ (with other forms of energies and properties of the body behaving normally), the result is different. Atomic hyperfine energies are of order $10^{-24}$ J (corresponding to GHz transition frequencies), which for typical atomic masses leads to $E_{HF}/(m c^2) \approx 10^{-16}$. As a consequence, WEP tests set a limit of only $|\alpha_{HF}| \lesssim 10^3$, while LPI tests using hyperfine transitions (e.g. H-masers) set a limit of about $|\alpha_{HF}| \lesssim 10^{-4}$. The conclusion is therefore radically different in this model where LPI tests perform orders of magnitude better than WEP tests.

To summarize, the two types of tests, WEP (or UFF) and LPI (red-shift), are complementary, and need to be all pursued, because depending on the model used either one of the tests can prove significantly more sensitive than the other. The main goal of STE-QUEST is to perform at once the different types of tests of the EEP, with good and in some case unprecedented precision: the WEP/UFF test, which will be done by mean of atom interferometry, the red-shift/LPI test through clock comparisons with optical and microwave links, and also a test of LLI, whose comparison with the WEP and LPI tests could be discussed in a way similar to what was presented above.

B. STE-QUEST Test of the Weak Equivalence Principle

The atom interferometer (ATI) of STE-QUEST is described in detail in the STE-QUEST Yellow Book [1] and in the article [4]. Here we only recall the main principle and some key numbers in the operation and measurements. The STE-QUEST ATI is a dual-species atom interferometer using the two isotopes of Rb (namely $^{85}$Rb and $^{87}$Rb) which are simultaneously trapped and cooled by a sequence involving atoms manipulation by lasers and magnetic fields. Atoms are cooled to temperatures below the critical temperature (a few nK) for Bose-Einstein Condensation (BEC), which allows operation of the interferometer with degenerate quantum gases (BECs). The complete trapping and cooling process lasts about 10 s. The two isotopes are then released into free fall and subject simultaneously to a Mach-Zender interferometer. Each atom undergoes three laser pulses that coherently split, reverse, and recombine the wave packets during a time interval of 10 s. The actual separation of the coherent wave packet parts during the interferometer sequence is up to 10 cm and larger than their respective size by more than two orders of magnitude. During each of the pulses, the laser phase is “imprinted” onto the matter-wave phase, so that on recombination the interference of the two waves (read out via the populations of internal states) provides the information on the acceleration of the freely falling matter waves with respect to the laser source. The final observable is then the difference between the measured accelerations of the two isotopes, i.e. the differential acceleration of the $^{85}$Rb and $^{87}$Rb matter waves.

The STE-QUEST ATI thus provides a test of the universality of free fall or weak equivalence principle (UFF/WEP). Such tests are generally quantified by the E"otv"os ratio $\eta_{AB}$ for two test objects $A$ and $B$ and a specified source mass of the gravitational field:

$$\eta_{AB} = 2 \frac{a_A - a_B}{a_A + a_B} = \beta_A - \beta_B ,$$

(5.9)

where $a_i$ $(i = A, B)$ is the acceleration of object $i$ with respect to the source mass and $\beta_i$ is the parameter introduced in Eq. (5.6). Note that for a given experiment the data can be interpreted with respect to different source masses (see

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3 We assume in this example that electromagnetism plays the same role in the nuclear binding energy and the hydrogen hyperfine transition.
FIG. 4: Present and upcoming tests of WEP/UFF in the Earth field (except for LLR, which is in the Sun field) adapted from Ref. [135]. Experiments using macroscopic test masses are represented by red arrows on a yellow background; experiments involving at least one quantum object by black arrows on a blue background. COW stands for the Collela-Overhauser-Werner experiment [40] using neutron interferometry compared to macroscopic test masses. Cs/Rb-macro [98, 109] are the similar atom interferometry experiments. For completeness the violet entry is the hydrogen vs. anti-hydrogen test under construction at CERN.

The useful ATI measurements are performed during about 0.5 h around perigee, when the sensitivity is largest. When taking into account all perturbing effects (gravity gradients, vibrations, magnetic fields, etc.) the single-shot (20 s cycle time) sensitivity is about $2.9 \times 10^{-12} \text{m/s}^2$ in differential acceleration of the two isotopes, and is dominated by the atomic shot noise. As a consequence, with the STE-QUEST baseline orbit, the goal of $2 \times 10^{-15}$ sensitivity (statistical uncertainty) in the Eötvös ratio can be reached in less than 1.5 years (see [1, 4] for details) with good prospects for reaching $1 \times 10^{-15}$ in the mission lifetime. Systematic effects are estimated to be below the $2 \times 10^{-15}$ level once calibrated, with the possibility of carrying out some of the calibrations during the rest of the orbit (away from perigee) thus not impacting the useful measurement time (see Ref. [121] and the STE-QUEST Yellow Book [1].

The final uncertainty of the UFF/WEP test can be compared to present and upcoming tests, by considering the corresponding Eötvös ratios. Figure 4 presents such a comparison for different tests in the Earth field (except LLR which is in the Sun field).

When examining Fig. 4 one should bear in mind that the compared experiments all use different test masses, and that thus a direct comparison can be misleading, as discussed in Sec. IV and V A. The only experiment that the STE-QUEST UFF/WEP test can be compared to directly is the ground measurement of the differential free fall of the two Rb isotopes [23, 60] with respect to which STE-QUEST represents an impressive improvement by eight orders of magnitude. The same is true when comparing to other purely quantum matter wave experiments [119, 123]. Even when comparing to macroscopic tests, with best present ground tests from the Eötv-Wash group [118] or LLR [136].
FIG. 5: Common-view comparison between Turin and Boulder for the test of LPI in the field of the Sun. The two panels show the different locations of the clocks in the field of the Sun as the Earth rotates, and their common-view comparison by STE-QUEST.

both at the $2 \times 10^{-13}$ level, STE-QUEST still represents an improvement by two orders of magnitude. However, it is important to stress here that STE-QUEST measurement is truly quantum in nature (see Sec. IV), in particular:

1. The observable is the phase difference of interfering matter waves in a coherent superposition;

2. The coherent superposition is well separated spatially by $\sim 10$ cm, more than two orders of magnitude larger than the size of the individual wave packet parts;

3. The atoms are condensed to a quantum degenerate state (Bose-Einstein Condensate);

4. The coherence length of the atoms is of the order of a micrometer, many orders of magnitude larger than the de Broglie wavelength of the macroscopic test masses ($10^{-27}$ m or less).

Ground [50], parabolic flights [63], drop tower [101] and sounding rocket tests using coherent matter waves are also likely to improve within the STE-QUEST time frame. However, STE-QUEST does not suffer from inherent limits of the ground laboratory environment (short free-fall times, gravity gradients, perturbed laboratory environment, etc.), which will ultimately limit tests on ground. Moreover, unique advantages of a satellite operation (see Sec. 2.5 in the Yellow Book [1]) are the chief assets towards the performance announced. This is somewhat akin to classical tests where the next significant improvement is expected from going into space with the $\mu$-SCOPE mission.

Finally, we note that so far no analysis in the field of other sources (e.g., galactic dark matter [118]) has been carried out for STE-QUEST. This might lead to further interesting limits and experimental possibilities, e.g., by considering parts of the orbit that are not useful for UFF/WEP in the Earth field, but are useful in the field of more distant bodies. Such analysis and corresponding optimization of the measurement scenario will be carried out as the mission progresses and will further enhance the scientific discovery potential of STE-QUEST.

C. STE-QUEST Test of Local Position Invariance

In the baseline configuration (without clock on-board), STE-QUEST will be able to compare distant ground clocks using the microwave link (MWL) in common-view mode. In the common-view technique, two ground clocks are simultaneously compared. The difference of simultaneous measurements provides then a direct comparison of the two clocks on the ground. This measurement does not require a high-performance frequency reference on-board the STE-QUEST spacecraft. Indeed, the noise of the space clock, which appears as common mode in the two simultaneous link measurements, is rejected to high degree when the difference of the two space-to-ground comparisons is evaluated. According to the STE-QUEST reference orbit, common-view contacts between USA and Europe, Europe and Japan, Japan and USA have uninterrupted durations longer than 10 hours with each of them repeated every two days. The concept of the LPI test in the gravitational field of the Sun is shown in Fig. 5. In this example the frequency ratio $\nu_T/\nu_B$ between two ground clocks in Turin and Boulder is measured.

In the framework discussed in Sec. V A, we can consider a generalization in which the Sun acts as the source of the anomalous gravitational coupling. The measured frequency ratio of the two clocks can be written as

$$\frac{\nu_T}{\nu_B} = 1 - \frac{1}{c^2} \left[ U_B^\odot - U_T^\odot + \frac{v_B^2 - v_T^2}{2} + a_B^\odot U_B^\odot - a_T^\odot U_T^\odot \right] + \Delta,$$

(5.10)
where $U^\odot_B$ and $U^\odot_T$ are the solar Newtonian gravitational potentials at the locations of the ground clocks and $v_B$ and $v_T$ are the corresponding velocities in a solar-system barycentric reference frame. The LPI violating parameters $\alpha_B^\odot$ and $\alpha_T^\odot$ depend on the type of transition used in the respective clocks and possibly on the source of the gravitational field (here the Sun); $\Delta$ represents all corrections due to the other solar system bodies (including the Earth) assumed to behave normally, as well as higher-order correction terms.

An essential point to note is that, in the absence of an LPI violation ($\alpha_B^\odot = \alpha_T^\odot = 0$), the leading part in Eq. (5.10) is equal to zero (up to small tidal correction terms in $\Delta$ and constant terms from the Earth field). This is a direct consequence of the EEP, as the Earth is freely falling in the Sun field [71]. The LPI test in the Sun field is thus a null test, verifying whether the measured frequency ratio is equal to the expected value, i.e. $1 + \Delta$ in this example.

In general, the types of clocks used at the different ground stations may be of different type so $\alpha_B \neq \alpha_T$. In the following, we will assume for simplicity clocks of the same type which simplifies the LPI violating term in (5.10) to $\alpha^\odot(U^\odot_B - U^\odot_T)$, with the aim of the experiment being the measurement of $\alpha^\odot$. More precisely the experiment will measure the time evolution of the ratio $\nu_T/\nu_B$, which again should be one in GR (up to correction terms), but will evolve in time if the LPI violating parameter is non-vanishing because of the time evolution of $U^\odot_B - U^\odot_T$, mainly related to the rotation of the Earth. The time evolution of $(U^\odot_B - U^\odot_T)/c^2$ will be predominantly periodic with a daily period and peak-to-peak amplitude of about $1 \times 10^{-12}$.

Then, the determination of the LPI parameters boils down to a search for a periodic signal with known frequency and phase in the clock comparison data. As detailed in Ref. [1], in the baseline configuration the measurement uncertainties of the MWL and the ground clocks should allow a detection of any non-zero value of the LPI violating parameter $\alpha$ in the Sun field that exceeds $2 \times 10^{-6}$ after four years of integration. In the case that the optional optical link is included in the payload, that goal can be reached in 72 days of integration with the ultimate performance of $5 \times 10^{-7}$ reached in 4 years. Note however, that these results are based on only frequency measurements without making use of the phase cycle continuity provided by the STE-QUEST MWL. When phase cycle continuity is maintained by the link, the measurement duration is not affected by the dead-time between one common-view comparison and the next, resulting in a reduction of the integration time needed to reach the ultimate accuracy. Such a data analysis approach is presently being implemented in the numerical simulations [1].

The procedure for the LPI test in the Moon field is identical to the Sun field test described above. The difference is that the frequency and phase of the signal that one searches for are different and that the sensitivity is decreased by a factor $\sim 175$, see Eq. (5.12).

In the case where the onboard clock option of STE-QUEST is realized, it will be possible to perform also an LPI test in the field of the Earth. Given that this is only an option, we will discuss the test and the results that can be achieved only briefly. Some more details can be found in Ref. [1]. In this case the MWL (or optical) link is used to compare the onboard clock to ground clocks. In the formalism of Sec. [V.A] the frequency ratio can be written as

$$\frac{\nu_{\text{STE}}}{\nu_B} = 1 - \frac{1}{c^2} \left[ U^\odot_B - U^\odot_{\text{STE}} + \frac{\nu_B^2 - \nu_{\text{STE}}^2}{2} + \alpha_B^\odot U^\odot_B - \alpha_{\text{STE}}^\odot U^\odot_{\text{STE}} \right] + \Delta', \quad (5.11)$$

where $U^\odot_B$ and $U^\odot_{\text{STE}}$ are the Earth’s Newtonian gravitational potentials at the locations of the ground clock and the onboard clock, and $\nu_B$ and $\nu_{\text{STE}}$ are the corresponding velocities in a geocentric reference frame. The LPI violating parameters $\alpha_B^\odot$ and $\alpha_{\text{STE}}^\odot$ depend on the type of transition used in the respective clocks. As in Eq. (5.10), $\Delta'$ represents all corrections due to the other solar system bodies, as well as higher-order or numerically smaller correction terms (see Ref. [24] for more details on relativistic time and frequency transfer).

The main difference with respect to the Sun LPI test above is that the ground clocks are not freely falling in the field of the Earth, so even in the absence of an LPI violation the frequency ratio is not one and is varying in time with the eccentric orbit of STE-QUEST. The test then compares the theoretically calculated frequency ratio (from the knowledge of the STE-QUEST orbit and the ground station locations) to the actually measured one. This leads to two methods for the test, one based on the accuracy of the clocks (so-called DC measurement) that searches for an offset with respect to the expected value, and one based on the periodic variation due to the orbit eccentricity (so-called AC measurement) that searches for the time varying signature and thus relies on the clock stability. The former is carried out mainly when the satellite is at apogee, when the LPI violating term in (5.11) is largest; the latter uses measurements over the full orbit. As detailed in Ref. [1], simulations taking into account the MWL and clock noise and accuracy show that with both methods an uncertainty of $2 \times 10^{-7}$ on the LPI violating parameter $\alpha^\odot$ can be reached after 4 days (DC measurement) and 840 days (AC measurement) of integration. In the case of the DC measurement, the limit is imposed by the clock accuracy rather than the measurement duration. In the case of the AC measurement the uncertainty can be decreased to $1.5 \times 10^{-7}$ when integrating over the whole mission duration.

The sensitivities of STE-QUEST estimated above can be compared to present and upcoming LPI tests by looking directly at the limits on the corresponding parameters (see Sec. [V.A]). Such a comparison is presented in Fig. [6] adapted and updated from Ref. [135].
FIG. 6: Present and upcoming limits on LPI violation adapted from Ref. [135]. Red arrows (after 1990) represent “null red-shift measurements” that can only provide the difference \( \alpha_i - \alpha_j \) for two different types of clocks \( i \) and \( j \). Inside the green circles are the best direct LPI tests in the Sun field providing directly \( \alpha_{\odot} \) for the respective transition. Inside the blue circles are the best direct LPI tests in the Earth field for \( \alpha_{\oplus} \) (optional in the case of STE-QUEST).

Figure 6 shows a number of experiments, including null tests and direct tests. The latter set limits directly on the parameter \( \alpha_i \) for the relevant transition, e.g. the H-maser experiment of 1979 [128] sets a limit on \( \alpha_{\odot}^H \) for the hydrogen hyperfine transition in the Earth gravitational field. The “Null Red-shift” experiments in Fig. 6 consist of two co-located clocks of different type in the same laboratory whose relative frequency is monitored as the local gravitational potential varies in time. Thus one measures \( (\alpha_i - \alpha_j)U/c^2 \) for two clocks of type \( i \) and \( j \) and sets a limit on the difference \( \alpha_i - \alpha_j \). The most precise such test at present sets a limit of \( \alpha_{\odot}^{Rb} - \alpha_{\odot}^{Cs} = (0.11 \pm 1.0) \times 10^{-6} \) for the Rb vs. Cs hyperfine transitions [68], using the annual variation of the solar potential in the laboratory due to the eccentricity of the Earth’s orbit. Depending on the underlying model, the difference \( \alpha_{\odot} - \alpha_{\odot} \) might be much smaller than the individual values, especially when similar transitions are used (both hyperfine or both electronic, i.e. optical), so direct tests that measure \( \alpha_i \) individually rather than differences \( \alpha_i - \alpha_j \) are necessary and complementary to co-located tests, which is one of the main drivers for experiments like ACES or STE-QUEST. In the STE-QUEST LPI test, a non-zero signal will be observed no matter what the actual values of \( \alpha_T^B \) and \( \alpha_T^T \) in Eq. (5.10) are, provided at least one of them is non-zero, because of the different temporal variations of \( U_T^B \) and \( U_T^T \). This is not the case in null-tests with co-located clocks, where one necessarily has \( U_i = U_j \) and thus a signal can only be detected if \( \alpha_i \neq \alpha_j \), which is not the case for STE-QUEST.

Finally, all Sun LPI science objectives also apply to a test with the Moon as the source mass. STE-QUEST will carry out a direct LPI test in the Moon field using the same methods (and data) as the test in the Sun field described above. Note that the two putative signals can be easily de-correlated in the data due to the different frequency and phase. The sensitivity of STE-QUEST to a possible violation of LPI sourced by the Moon is then simply given by a
reduction factor with respect to the Sun effect of

\[
\left( \frac{M_{\text{Sun}}}{d_{\text{Sun}}} \right) / \left( \frac{M_{\text{Moon}}}{d_{\text{Moon}}} \right) = 175.
\]  

(5.12)

In the baseline configuration the measurement uncertainties of the MWL and the ground clocks should allow a detection of any non-zero value of the LPI violating parameter \( \alpha \) sourced by the Moon that exceeds \( 4 \times 10^{-4} \) after four years of integration. In the case that the optional optical link is included in the payload, that goal can be reached in 72 days of integration with the ultimate performance of \( 9 \times 10^{-5} \) reached in 4 years. Like for the Sun test, note that these results are based on only frequency measurements without making use of the phase cycle continuity requested for the MWL.

Clock tests as described above are sometimes interpreted as searches for a space-time variation of fundamental constants, in particular those of the Standard Model (fine structure constant, electron, proton and quark masses, QCD mass scale, etc.). Such an interpretation is model-dependent (one assumes the validity of the Standard Model of particle physics to describe atomic transitions) so we do not use it here as our aim is to remain as general as possible. In order to best constrain all possible variations of constants the comparison of as many different transitions as possible is essential. Comparisons of ground clocks based on different types of transitions repeated during the STE-QUEST mission (see the Yellow Book [1]) will provide a wealth of data to search for temporal variations of fundamental constants, the fine structure constant \( \alpha \) and the electron-to-proton mass ratio \( \mu \) in particular. Different clock transitions have different dependency on fundamental constants. Therefore, the results of crossed frequency comparisons repeated in time provides a clear interpretation of any observed drift over time and imposes unambiguous limits on time variations of fundamental constants. Current best limits on time variations of fundamental constants from laboratory experiments are consistent with zero: the \( \alpha \) drift was recently measured to \( \dot{\alpha}/\alpha = (5.8 \pm 6.9) \times 10^{-17} \) yr\(^{-1} \) in the dysprosium experiment [96]; the \( \alpha \) and \( \mu \) drifts were determined to \( \dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \) yr\(^{-1} \) and \( \dot{\mu}/\mu = (1.9 \pm 4.0) \times 10^{-16} \) yr\(^{-1} \) in the frequency comparison of a Hg\(^+ \) and an Al\(^{17} \) clock [113]. At the same time, data obtained from astronomical observation of quasar absorption spectra are providing complementary information exploring completely different measurement systematics: \( \Delta \alpha/\alpha = (-0.57 \pm 0.11) \times 10^{-5} \) [102] and \( \Delta \mu/\mu < 1.8 \times 10^{-6} \) (95% confidence level) [103], at approximately half the Universe's current age. Interestingly enough, even if their interpretation is still controversial, these data seem to indicate a time variation of \( \alpha \). Additional and more precise measurements are clearly needed to better understand and resolve the puzzle. These limits are expected to improve by at least one order of magnitude thanks to STE-QUEST.

D. STE-QUEST Tests of Lorentz Invariance and CPT Symmetry

Lorentz Invariance is the third sub-principle of the EEP as described in Sec. II A. Currently, there is a great deal of interest in Lorentz Invariance and the combined charge conjugation, parity, time reversal (CPT) symmetry — and, in particular, the question of whether these related symmetries are truly exact in Nature. Both the Standard Model of particle physics and GR are precisely invariant under (local) Lorentz and CPT symmetries, which makes these symmetries particularly fundamental. Whilst Lorentz and CPT symmetries have been discussed frequently in frameworks similar to the one introduced in Sec. V A, a more general, broad, and complete framework for tests of Lorentz Invariance and CPT symmetry has been developed over the last decade: the Standard Model Extension (SME) [38, 39]. We will use this framework to analyze Lorentz Invariance tests that will be carried out with STE-QUEST. The first estimates presented here give a general idea of the potential of STE-QUEST in this field.

Many candidate theories of quantum gravity suggest the possibility of Lorentz and CPT symmetry breaking in certain regimes. For example, the symmetries could be broken spontaneously, either in string theory [52, 53] or in quantum field theories with fundamental tensor fields [6, 8]. There could also be Lorentz-violating physics in loop quantum gravity [5, 61] and non-commutative geometry theories [33, 69]; Lorentz violation through spacetime-varying couplings [50, 80]; or breaking of Lorentz and CPT symmetries by quantum anomalies in certain space-times with non-trivial topologies [81]. Moreover, since CPT violation in a well-behaved low-energy effective quantum theory automatically requires Lorentz violation as well [66], any predictive theory that entails violations of CPT will also include violations of Lorentz Invariance.

So far, there is no compelling evidence that Lorentz and CPT symmetries are not actually exact in Nature. In fact, there have been numerous experimental tests of these theories, using a very wide variety of techniques. Recent experimental tests have included studies of matter-antimatter asymmetries for trapped charged particles and bound state systems, measurements of muon properties, analyses of the behavior of spin-polarized matter, frequency standard comparisons, Michelson & Morley-type experiments with resonators, Doppler effect measurements, measurements of neutral meson oscillations, polarization measurements on the light from cosmological sources, high-energy
A general effective field theory that describes Lorentz violation for elementary particles is the SME [38, 39]. As a quantum field theory, the SME contains all Lorentz-violating operators that can be written down using Standard Model fields, along with coefficients for Lorentz violation that parameterize the Lorentz-violating effects. These coefficients vanish in a perfectly Lorentz-invariant theory. It has also been expanded (as a classical field theory) to give a systematic way of studying Lorentz-violating and CPT-violating gravitational interactions [87]. The SME gravitational action includes both space-time curvature effects and space-time torsion phenomena; some of the torsion effects turn out to be equivalent, at least locally, to spin-dependent Lorentz-violating operators in the particle physics sectors of the SME [88]. Although many theories describing new physics suggest the possibility of Lorentz violation, none of them are understood well enough to make firm predictions. The greatest utility of the SME is the theory’s generality. The SME provides a framework for placing constraints on Lorentz and CPT-violating effects, without worrying about the underlying mechanism by which the symmetry violation arises.

In the presence of Lorentz violation, experimental results will depend on the orientation of the apparatus (for violations of spatial isotropy) and on the velocity of the apparatus (for violations of Lorentz boost invariance). For Earthbound experiments the changes of orientation and velocity are limited to the Earth rotation and orbital motion or to slow modulations imposed in the laboratory (e.g. turntables). For a satellite experiment, there are new forms of motion, and this enhances the sensitivity to Lorentz violation. The highly eccentric and time-varying orbit of the STE-QUEST satellite will be extremely advantageous for several reasons. Tests of Lorentz boost symmetry require comparisons of data collected in different Lorentz frames. It is necessary to physically boost the experiment into different frames and compare the results observed under the different conditions. The sensitivity to boost invariance violations is then determined by the velocity differences \( v \) between different observation frames. The direction of \( v \) determines the specific linear combination of violation coefficients that can be constrained by a single comparison. Simultaneously, the speed determines the strength of the constraint for nonrelativistic relative speeds, \( v/c \ll 1 \) is a direct suppression factor. For these reasons, it is advantageous to sample as many frames, moving as rapidly in relation to one-another as possible.

When coupled to gravity, one finds that WEP/UFF tests can provide the best available sensitivity to certain types of Lorentz violation in the SME [92]. In fact, several Lorentz-violating possibilities can only be tested using such precision gravitational experiments [89]. Hence, the impressive WEP tests of the STE-QUEST mission would provide the best sensitivities to date on an additional set of coefficients for Lorentz violation.

Effective WEP violation in the SME originates from its generality in allowing the possibility of coefficients for Lorentz violation that differ among fermions of different flavors. That is, the degree of Lorentz violation may differ from protons, to neutrons, to electrons, for example. When gravitational couplings are considered for fermions, this species dependence leads to a differing gravitational response. Since the effect is due to Lorentz violation, variation in the size of the effective WEP violation with the orientation and boost direction of the experiment typically results, as do modifications in the direction of the gravitational acceleration. Thus, WEP tests such as those on the STE-QUEST mission typically have the ability to distinguish a signal due to Lorentz violation from other sources of WEP violation via the dependence of the signal on orientation and velocity as well as the unique direction dependence of the acceleration.

The proposed WEP/UFF experiment is of the class that was analyzed extensively by Kostelecký [92]. Explicit predictions obtained for experiments on Earth extend naturally to STE-QUEST through replacement of appropriate boost and gravitational factors. Performing such an experiment in space provides the benefits of variable boost orientations and longer free-fall times, but there is no fundamental change in the existing analysis. The results of that analysis along with the WEP sensitivity goals of STE-QUEST imply that sensitivities ranging from the \( 10^{-11} \) to \( 10^{-7} \) levels per measurement cycle will be possible for up to 8 combinations of SME coefficients. After incorporating data from the large number of orbits throughout the mission, constraints ranging from \( 10^{-14} \) to \( 10^{-10} \) levels are expected. These sensitivities would provide improvements of up to 5 orders of magnitude over existing constraints.

Another way in which the STE-QUEST mission as currently proposed could attain sensitivity to Lorentz violation is through red-shift shifts. Coefficients for Lorentz violation, which couple to gravitational fields in the SME lead to modified space-time curvature [13, 92] as well as additional modifications to clock frequencies, and specific predictions for red-shift experiments have been made [13, 92]. These predictions include that of a variation in the red-shift signal as the clock explores the gravitational potential that is qualitatively different from the conventional red-shift signal [13]. This effect arises due to the impact of rotation-invariance violation on the gravitational field. While the sensitivity to the relevant SME coefficients available via red-shift tests will not exceed the maximum reach currently available via other types of experiments, such tests are still interesting from a SME perspective for two reasons. The sensitivities to Lorentz violation achieved in a given experiment often constrain or measure a large combination of coefficients from the theoretical framework, hence additional tests can provide the necessary information to disentangle these combinations. Secondly, the present analysis of such experiments considers implications of the minimal gravitationally coupled SME.
**Science Investigation** | **Measurement Requirement**
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**Clock Comparisons and International Atomic Time Scales** |  
**Common-view comparisons of ground clocks** | Common-view comparison of ground clocks at the 1 $\times 10^{-18}$ fractional frequency uncertainty level after a few days of integration time with the STE-QUEST microwave link and a few hours by using the optical link.  
**Space-to-ground time transfer** | Space-to-ground time transfer with accuracy better than 50 ps.  
**Synchronization of ground clocks** | Synchronization of clocks on ground to better than 50 ps.  
**Atomic time scales** | Contribution to the generation of atomic time scales to fractional frequency inaccuracy lower than 1 $\times 10^{-16}$.  
**GNSS clocks and time scales (optional)** | Monitoring of the stability of on-board GPS, GALILEO, and GLONASS clocks.  
**Geodesy** |  
**On-site differential geopotential measurements** | Differential geopotential measurements between two points on the Earth’s surface with resolution in the gravitational potential at the level of 0.15 m$^2$/s$^2$ (equivalent to 1.5 cm on the geoid height difference).  
**Reference Frames** |  
**Earth terrestrial and celestial reference frame** | Realization and unification of the terrestrial and the celestial reference frame of the Earth.

*This scientific investigation can be performed only if the STE-QUEST payload is equipped with a high-stability and high-accuracy atomic clock.

**TABLE IV: Science investigations vs. measurement requirements for topics other than fundamental physics that shall be investigated by STE-QUEST.**

Only. Higher dimension operators [90, 93] for which specific predictions for experiments of this type have not yet been made may result in additional effects that can be measured in this way.

Additional tests of Lorentz Invariance are possible if the optional onboard clock is flown. The dependences of the atomic clock frequencies on the minimal SME parameters is already known [22, 84]. These dependences were determined using the effective Hamiltonian that may be derived from the SME Lagrangian [85]. The algorithm for calculating the frequency shifts is quite general and can be applied to virtually any atomic clock transition. In the context of the Schmidt nuclear model [120], all the angular momentum $I$ of odd-even nucleus is carried by a single unpaired nucleon. The principal sensitivities (given by the Schmidt model and the atomic shell structure) are to Lorentz violation coefficients in the proton sector, with secondary sensitivities in the electron sector. Searching for modulations in the transition frequency with the characteristic satellite orbital frequency will make it possible to place constraints on up to 25 coefficients in the proton sector, with sensitivity levels ranging from $10^{-21}$ down to $10^{-28}$. A further 18 electron-sector coefficients may be constrained with potentially $10^{-19}$ to $10^{-27}$ level sensitivities. For most of these coefficients, these are unprecedented levels of sensitivity. There are also dependences on additional SME coefficients, which are not captured by the Schmidt model. These include dependences on neutron coefficients and dependences that exist because of the relatively rapid movement of the nucleons inside an atomic nucleus [7]. The extremely sensitive data provided by STE-QUEST would allow extracting these additional dependences.

In conclusion of this section, STE-QUEST offers the possibility to explore a large parameter space of the SME and to thereby constrain, or uncover, violations of Lorentz and CPT symmetry. In particular, coefficients in the proton and electron sector will be constrained from the clock measurements, while the $^{85}$Rb–$^{87}$Rb atom interferometer will provide new constraints in the gravitational sector, with expected improvements of up to five orders of magnitude on present limits.

VI. CONCLUSIONS

We have presented the fundamental physics science objectives of STE-QUEST, which are centered on tests of the three different aspects of the Einstein Equivalence Principle (EEP): the Weak Equivalence Principle (WEP) or Universality of Free Fall, Local Position Invariance (LPI) or Universality of Clock Rates, and Local Lorentz Invariance (LLI) coupled to CPT symmetry. One of the unique strengths of STE-QUEST is that it will test all three aspects of the EEP, using a combination of measurements in space and on the ground (relative acceleration of different atomic isotopes, comparison of distant clocks). Although the three sub-principles are connected by Schiff’s conjecture, the
actual quantitative merit of the different experiments is model-dependent (see Sec. VA). As a consequence, it is not known a priori which test (WEP, LPI, or LLI) is more likely to first detect a violation and the most reasonable approach is to pursue tests of the three sub-principles with equal vigor. This is the baseline of STE-QUEST, which carries out, and improves on, state-of-the-art tests of all three sub-principles.

Another unique feature of STE-QUEST is its capability to carry out the WEP test using quantum matter waves in superpositions that have no classical counterpart. Although, we know of no viable model that predicts an EEP violation specific to such quantum matter waves, one should be prepared for surprises as there are numerous open questions at the interface between gravitation and quantum mechanics reviewed in Sec. IV. Those issues provide good reason for exploring the foundations of general relativity, i.e. the EEP, with as diverse objects as possible, like antimatter or quantum degenerate gases and superposition states.

Finally let us mention that although the primary science objectives of STE-QUEST are in fundamental physics, the mission will also provide a wealth of legacy science for other fields like time/frequency metrology, reference frames and geodesy. These are summarized in Table IV with more details available in the STE-QUEST Yellow Book [1].

To conclude, all of this gives a unique science case for STE-QUEST, making it the first space mission to carry out such a complete test of the foundations of gravitation theory using quantum sensors, and providing additionally some legacy science for other applications. If selected in an upcoming call, it will follow in the wake of precursor missions like ACES and µ-SCOPE (to be launched by ESA and CNES in 2016) and firmly establish Europe’s lead in fundamental physics as a space science discipline.

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