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Orbital shadowing, ω -limit sets and minimality

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Abstract

Let X be a compact Hausdorff space, with uniformity \mathcal{U} , and let $f: X \rightarrow X$ be a continuous function. For $D \in \mathcal{U}$, a D -pseudo-orbit is a sequence (x_i) for which $(f(x_i), x_{i+1}) \in D$ for all indices i . In this paper we show that pseudo-orbits trap ω -limit sets in a neighbourhood of prescribed accuracy after a uniform time period. A consequence of this is a generalisation of a result of Pilyugin *et al*: every system has the second weak shadowing property. By way of further applications we give a characterisation of minimal systems in terms of pseudo-orbits and show that every minimal system exhibits the strong orbital shadowing property.

Keywords: Orbital shadowing, ω -limit set, Minimal, Pseudo-orbit, Dynamical system

2010 MSC: 37B99, 37C50, 54H20

Let $f: X \rightarrow X$ be a continuous map on a compact metric space X . We say (X, f) is a (discrete) *dynamical system*. A sequence (x_i) in X is called a δ -pseudo-orbit provided $d(f(x_i), x_{i+1}) < \delta$ for each i . Pseudo-orbits are clearly relevant when calculating an orbit numerically, as rounding errors mean a computed orbit will in fact be a pseudo-orbit. The (finite or infinite) sequence (y_i) in X is said to ε -shadow the (x_i) provided $d(y_i, x_i) < \varepsilon$ for all indices i . First used implicitly by Bowen [3], a system has *shadowing*, or the *pseudo-orbit tracing property*, if pseudo-orbits are shadowed by true orbits. Since then various other notions of shadowing have been studied, for example, ergodic, thick and Ramsey shadowing [4, 5, 8, 10, 21], limit shadowing [2, 16, 23], s -limit shadowing [2, 16, 19], orbital shadowing [13, 22, 23], and inverse shadowing [7, 14, 18].

The *orbital shadowing property* was introduced in [22] where the authors studied its relationship to classical stability properties, such as structural stability and Ω -stability. Informally, a system has orbital shadowing if the closure of the set of points in any pseudo-orbit is close to an orbit closure of a point (see below of precise definitions). Orbital shadowing has since been studied by various authors (e.g [13, 15, 23]). A stronger type of orbital shadowing was introduced in [13], aptly named *strong orbital shadowing*, as part of the authors' quest to characterise when the set of ω -limit sets of a system coincides with the set of closed internally chain transitive sets.

In this paper we prove (Theorem 2.3) that every compact metric dynamical system enjoys a weak form of shadowing, specifically: For any $\varepsilon > 0$ there exist $n \in \mathbb{N}$ and $\delta > 0$ such that given any δ -pseudo-orbit $(x_i)_{i \geq 0}$ there exists $z \in X$ such that

$$B_\varepsilon(\{x_i\}_{i=0}^n) \supseteq \omega(z).$$

Thus initial segments of pseudo-orbits trap ω -limit sets, and consequently full orbits, in their neighbourhood. As an application of this result we show that compact minimal systems have the strong orbital shadowing property as introduced in [13]. Our methodology allows us to give a characterisation of minimal systems in terms of pseudo-orbits (Theorem 2.5). Along the way we generalise a result of Pilyugin *et al* [22] by showing that every compact Hausdorff system has the second weak shadowing property.

In order to keep our results as general as possible we take the phase space throughout to be compact Hausdorff but not necessarily metric; this is a setting which has attracted an increasing amount of attention in topological dynamics (e.g. [1, 6, 11, 12, 15, 17, 20, 25]). In particular this means all of our results hold in a compact metric setting.

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1. Preliminaries

1.1. Dynamical systems

A *dynamical system* is a pair (X, f) consisting of a compact Hausdorff space X and a continuous function $f: X \rightarrow X$. We say that the *orbit* of x under f is the set of points $\{x, f(x), f^2(x), \dots\}$; we denote this set by $\text{Orb}(x)$. For a point $x \in X$, we define the ω -*limit set* of x under f , denoted $\omega(x)$, to be the set of limit points of its orbit sequence. Formally

$$\omega(x) = \bigcap_{N \in \mathbb{N}} \overline{\{f^n(x) \mid n > N\}}.$$

It follows that $\overline{\text{Orb}(x)} = \text{Orb}(x) \cup \omega(x)$. Note also that since X is compact $\omega(x) \neq \emptyset$ for any $x \in X$ by Cantor's intersection theorem.

For a dynamical system (X, f) , a subset $A \subseteq X$ is said to be *positively invariant* (under f) if $f(A) \subseteq A$. The system is *minimal* if there are no proper, nonempty, closed, positively-invariant subsets of X . Equivalently, a system is minimal if $\omega(x) = X$ for all $x \in X$.

Definition 1.1. Let X be a metric space. The system (X, f) has the *orbital shadowing property* if for all $\varepsilon > 0$, there exists $\delta > 0$ such that for any δ -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that

$$d_H \left(\overline{\{x_i\}_{i \geq 0}}, \overline{\{f^i(z)\}_{i \geq 0}} \right) < \varepsilon.$$

Here d_H denotes the Hausdorff metric, defined on the compact subsets of X , which is given by:

$$d_H(A, A') = \inf\{\varepsilon > 0 : A \subseteq B_\varepsilon(A') \text{ and } A' \subseteq B_\varepsilon(A)\}.$$

The following weakening of orbital shadowing was introduced in [22].

Definition 1.2. Let X be a metric space. The system (X, f) has the *second weak shadowing property* if for all $\varepsilon > 0$, there exists $\delta > 0$ such that for any δ -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that

$$\text{Orb}(z) \subseteq B_\varepsilon(\{x_i\}_{i \geq 0}).$$

The following strengthening of orbital shadowing was introduced in [13]. The authors demonstrate it to be distinct.

Definition 1.3. Let X be a metric space. The system (X, f) has the *strong orbital shadowing property* if for all $\varepsilon > 0$, there exists $\delta > 0$ such that for any δ -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that, for all $N \in \mathbb{N}_0$,

$$d_H \left(\overline{\{x_{N+i}\}_{i \geq 0}}, \overline{\{f^{N+i}(z)\}_{i \geq 0}} \right) < \varepsilon.$$

1.2. Uniform spaces

In this subsection we give a brief description of a uniform space. A more thorough introduction to the topic can be found in [9, Chapter 8].

Let X be a nonempty set and $A \subseteq X \times X$. Let $A^{-1} = \{(y, x) \mid (x, y) \in A\}$. The set A is said to be *symmetric* if $A = A^{-1}$. For any $A_1, A_2 \subseteq X \times X$ we define the composite $A_1 \circ A_2$ as

$$A_1 \circ A_2 = \{(x, z) \mid \exists y \in X : (x, y) \in A_2, (y, z) \in A_1\}.$$

For any $n \in \mathbb{N}$ and $A \subseteq X \times X$ we denote by nA the n -fold composition of A with itself, i.e.

$$nA = \underbrace{A \circ A \circ A \cdots A}_{n \text{ times}}.$$

The *diagonal* of $X \times X$ is the set $\Delta = \{(x, x) \mid x \in X\}$. A subset $A \subseteq X \times X$ is called an *entourage* if $A \supseteq \Delta$.

A *uniformity* \mathcal{U} on X is a collection of entourages such that

- a. $E_1, E_2 \in \mathcal{U} \implies E_1 \cap E_2 \in \mathcal{U}$.
- b. $E \in \mathcal{U}, E \subseteq D \implies D \in \mathcal{U}$.
- c. $E \in \mathcal{U} \implies D \circ D \subseteq E$ for some $D \in \mathcal{U}$.
- d. $E \in \mathcal{U} \implies D^{-1} \subseteq E$ for some $D \in \mathcal{U}$.

We call the pair (X, \mathcal{U}) a *uniform space*. We say \mathcal{U} is *separating* if $\bigcap_{E \in \mathcal{U}} E = \Delta$. A subcollection \mathcal{V} of \mathcal{U} is said to be a *base* for \mathcal{U} if for any $E \in \mathcal{U}$ there exists $D \in \mathcal{V}$ such that $D \subseteq E$. Note that the collection of symmetric entourages of a uniformity form a base for said uniformity.

For an entourage $E \in \mathcal{U}$ and a point $x \in X$ we define the set $B_E(x) = \{y \in X \mid (x, y) \in E\}$; we refer to this set as the *E-ball about x*. This naturally extends to a subset $A \subseteq X$; $B_E(A) = \bigcup_{x \in A} B_E(x)$; in this case we refer to the set $B_E(A)$ as the *E-ball about A*. We emphasise that (see [24, Section 35.6]):

- For all $x \in X$, the collection $\mathcal{B}_x := \{B_E(x) \mid E \in \mathcal{U}\}$ is a neighbourhood base at x , making X a topological space.
- The topology is Hausdorff if and only if \mathcal{U} is separating.

For a compact Hausdorff space X there is a unique uniformity \mathcal{U} which induces the topology (see [9, Chapter 8]).

We may use uniformities to give appropriate definitions of orbital shadowing, second weak shadowing and strong orbital shadowing in the more general setting of uniform spaces. First of all, given an entourage $D \in \mathcal{U}$, a sequence $(x_i)_{i \geq 0}$ in X is called a *D-pseudo-orbit* if $(f(x_i), x_{i+1}) \in D$ for all $i \geq 0$.

Definition 1.4. Let X be a uniform space. The system (X, f) has the *orbital shadowing property* if for all $E \in \mathcal{U}$, there exists $D \in \mathcal{U}$ such that for any D -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that

$$\text{Orb}(z) \subseteq B_E(\{x_i\}_{i \geq 0})$$

and

$$\{x_i\}_{i \geq 0} \subseteq B_E(\text{Orb}(z)).$$

Definition 1.5. Let X be a uniform space. The system (X, f) has the *second weak shadowing property* if for all $E \in \mathcal{U}$, there exists $D \in \mathcal{U}$ such that for any D -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that

$$\text{Orb}(z) \subseteq B_E(\{x_i\}_{i \geq 0}).$$

Definition 1.6. Let X be a uniform space. The system (X, f) has the *strong orbital shadowing property* if for all $E \in \mathcal{U}$, there exists $D \in \mathcal{U}$ such that for any D -pseudo-orbit $(x_i)_{i \geq 0}$, there exists a point z such that, for all $N \in \mathbb{N}_0$,

$$\{f^{N+i}(z)\}_{i \geq 0} \subseteq B_E(\{x_{N+i}\}_{i \geq 0})$$

and

$$\{x_{N+i}\}_{i \geq 0} \subseteq B_E(\{f^{N+i}(z)\}_{i \geq 0}).$$

When X is a compact metric space these definitions coincide with the previously given metric versions.

2. Main results

Henceforth X is a compact Hausdorff space and the unique uniformity associated with X will be denoted by \mathcal{U} . Since the collection of symmetric entourages of \mathcal{U} forms a base for \mathcal{U} , we can assume, without loss of generality, that all entourages we refer to are symmetric. This will be a standing assumption throughout.

Lemma 2.1. *Let (X, f) be a dynamical system where X is a compact Hausdorff space. Then (X, f) satisfies the following:*

$$\forall E \in \mathcal{U} \forall x \in X \exists n \in \mathbb{N} \exists z \in X \text{ s.t.} \\ \bigcup_{i=1}^n B_E(f^i(x)) \supseteq \omega(z).$$

Proof. Take $E \in \mathcal{U}$ and pick $x \in X$. Let $E_0 \in \mathcal{U}$ be such that $2E_0 \subseteq E$. Take a finite subcover of the open cover $\{\text{int}(B_{E_0}(y)) \mid y \in \omega(x)\}$ of $\omega(x)$. For each element of this subcover there exists m such that $f^m(x)$ lies inside it. Pick one such m for each element and then let n be the largest. The result follows by taking $z = x$. \square

Lemma 2.2. *Let (X, f) be a dynamical system where X is a compact Hausdorff space. Then (X, f) satisfies the following:*

$$\forall E \in \mathcal{U} \exists n \in \mathbb{N} \text{ s.t. } \forall x \in X \exists z \in X \text{ s.t.} \\ \bigcup_{i=1}^n B_E(f^i(x)) \supseteq \omega(z).$$

Proof. Fix $E \in \mathcal{U}$. Let $E_0 \in \mathcal{U}$ be such that $2E_0 \subseteq E$. For each $x \in X$ let $n_x \in \mathbb{N}$ be as in the condition in Lemma 2.1 for E_0 and let $D_x \in \mathcal{U}$ be such that, for any $y \in X$, if $(x, y) \in D_x$ then, for each $i \in \{0, \dots, n_x\}$, $(f^i(x), f^i(y)) \in E_0$. Without loss of generality, the collection $\{B_{D_x}(x) \mid x \in X\}$ forms an open cover. Let

$$\{B_{D_{x_i}}(x_i) \mid i \in \{1, \dots, k\}\},$$

be a finite subcover. Take $n = \max_{i \in \{1, \dots, k\}} n_{x_i}$. Pick $x \in X$ arbitrarily. There exists $l \in \{1, \dots, k\}$ such that $x \in B_{D_{x_l}}(x_l)$, which in turn implies $(f^i(x), f^i(x_l)) \in E_0$ for each $i \in \{0, \dots, n_{x_l}\}$. By Lemma 2.1 there exists $z \in X$ such that

$$\bigcup_{i=1}^{n_l} B_{E_0}(f^i(x_l)) \supseteq \omega(z).$$

Since $2E_0 \subseteq E$, by entourage composition combined with the fact that $n_l \leq n$, it follows that

$$\bigcup_{i=1}^n B_E(f^i(x)) \supseteq \omega(z).$$

\square

Theorem 2.3. *Let (X, f) be a dynamical system where X is a compact Hausdorff space. Then for any $E \in \mathcal{U}$ there exist $n \in \mathbb{N}$ and $D \in \mathcal{U}$ such that given any D -pseudo-orbit $(x_i)_{i \geq 0}$ there exists $z \in X$ such that*

$$B_E(\{x_i\}_{i=0}^n) \supseteq \overline{\text{Orb}(z)}.$$

In particular,

$$B_E(\{x_i\}_{i \geq 0}) \supseteq \overline{\text{Orb}(z)}.$$

Proof. Let $E \in \mathcal{U}$ be given and let $E_0 \in \mathcal{U}$ be such that $2E_0 \subseteq E$. Take $n \in \mathbb{N}$ as in the condition in Lemma 2.2 with respect to E_0 . By uniform continuity we can choose $D \in \mathcal{U}$ such that every D -pseudo-orbit E_0 -shadows the first n iterates of its origin. Explicitly: Let $D_1 \subseteq E_0$ be an entourage such that, for any $y, z \in X$, if $(y, z) \in D_1$ then $(f(y), f(z)) \in E_0$. For each $i \in \{2, \dots, n\}$ let $D_i \in \mathcal{U}$ be such that $2D_i \subseteq f^{-1}(D_{i-1}) \cap D_{i-1}$.

Now take $D := D_n$. Suppose $(x_i)_{i \geq 0}$ is a D -pseudo-orbit. Then $(f^i(x_0), x_i) \in E_0$ for all $i \in \{0, \dots, n\}$. By Lemma 2.2 there exists $y \in X$ such that

$$\bigcup_{i=1}^n B_{E_0}(f^i(x_0)) \supseteq \omega(y).$$

Since $2E_0 \subseteq E$ and, for each $i \in \{0, \dots, n\}$, $(f^i(x_0), x_i) \in E_0$ it follows by entourage composition that

$$B_E(\{x_i\}_{i=0}^n) \supseteq \omega(y).$$

Since X is compact, $\omega(y) \neq \emptyset$. Pick $z \in \omega(y)$. Because ω -limit sets are closed and positively invariant $\text{Orb}(z) \subseteq \omega(y)$. The result follows. \square

Notice that we could replace $\overline{\text{Orb}(z)}$ with either $\omega(z)$ or $\text{Orb}(z)$ in the statement of Theorem 2.3. Our methodology in the proof means that each of these would be equivalent statements.

The fact that all compact Hausdorff systems exhibit second weak shadowing now follows as an immediate corollary to Theorem 2.3. Note that Corollary 2.4 is a generalisation of [22, Theorem 3.1].

Corollary 2.4. *Let (X, f) be a dynamical system where X is a compact Hausdorff space. Then the system has second weak shadowing.*

Proof. Let $E \in \mathcal{U}$ be given and let $D \in \mathcal{U}$ correspond to this as in Theorem 2.3. Take a D -pseudo-orbit $(x_i)_{i \geq 0}$. By Theorem 2.3 there exists $z \in X$ such that

$$B_E(\{x_i\}_{i \geq 0}) \supseteq \text{Orb}(z).$$

\square

Theorem 2.5. *Let X be a compact Hausdorff space and $f: X \rightarrow X$ be a continuous function. The system (X, f) is minimal if and only if for any $E \in \mathcal{U}$ there exist $n \in \mathbb{N}$ and $D \in \mathcal{U}$ such that for any two D -pseudo-orbits $(x_i)_{i \geq 0}$ and $(y_i)_{i \geq 0}$*

$$\{y_i\}_{i=0}^n \subseteq B_E(\{x_i\}_{i=0}^n)$$

and

$$\{x_i\}_{i=0}^n \subseteq B_E(\{y_i\}_{i=0}^n).$$

Proof. First suppose the system is minimal. Let $E \in \mathcal{U}$ be given. Take $n \in \mathbb{N}$ and $D \in \mathcal{U}$ corresponding to E as in Theorem 2.3. Now let $(x_i)_{i \geq 0}$ and $(y_i)_{i \geq 0}$ be two D -pseudo-orbits. By Theorem 2.3 there exist $z_1, z_2 \in X$ such that $B_E(\{x_i\}_{i=0}^n) \supseteq \omega(z_1)$ and $B_E(\{y_i\}_{i=0}^n) \supseteq \omega(z_2)$. As (X, f) is minimal $\omega(z_1) = \omega(z_2) = X$. It follows that $B_E(\{x_i\}_{i=0}^n) = B_E(\{y_i\}_{i=0}^n) = X$. Hence

$$\{y_i\}_{i=0}^n \subseteq B_E(\{x_i\}_{i=0}^n)$$

and

$$\{x_i\}_{i=0}^n \subseteq B_E(\{y_i\}_{i=0}^n).$$

Now suppose the system is not minimal. Then there exists $x \in X$ such that $\omega(x) \neq X$. Pick $y \in \omega(x)$ and let $z \in X \setminus \omega(x)$. Take $E \in \mathcal{U}$ such that $B_E(z) \cap \omega(x) = \emptyset$. Consider the pseudo-orbits given by the orbit sequences of y and z : these are D -pseudo-orbits for any $D \in \mathcal{U}$. As ω -limit sets are positively invariant, $\text{Orb}(y) \subseteq \omega(x)$. Since $z \notin B_E(\omega(x))$ it also follows that $z \notin B_E(\text{Orb}(y))$. In particular, for any $n \in \mathbb{N}$, $\{f^i(z)\}_{i=0}^n \not\subseteq B_E(\{f^i(y)\}_{i=0}^n)$. \square

For the case when X is a compact metric space Theorem 2.5 may be formulated as follows: A dynamical system (X, f) is minimal precisely when for any $\varepsilon > 0$ there exist $\delta > 0$ and $n \in \mathbb{N}$ such that for any two δ -pseudo-orbits $(x_i)_{i \geq 0}$ and $(y_i)_{i \geq 0}$

$$d_H(\{x_i\}_{i=0}^n, \{y_i\}_{i=0}^n) < \varepsilon.$$

Corollary 2.6. *Let X be a compact Hausdorff space and $f: X \rightarrow X$ be a continuous function. The system (X, f) is minimal if and only if for any $E \in \mathcal{U}$ there exist $n \in \mathbb{N}$ and $D \in \mathcal{U}$ such that for any D -pseudo-orbit $(x_i)_{i \geq 0}$ we have $B_E(\{x_i\}_{i=0}^n) = X$.*

Proof. Immediate from the proof of Theorem 2.5. □

Corollary 2.7. *Let X be a compact Hausdorff space. If (X, f) is a minimal dynamical system then it exhibits the strong orbital shadowing property.*

Proof. Let $E \in \mathcal{U}$ be given. Take $n \in \mathbb{N}$ and $D \in \mathcal{U}$ corresponding to E as in Theorem 2.5. Now let $(x_i)_{i \geq 0}$ be a D -pseudo-orbit and pick any $z \in X$. Since $(x_{N+i})_{i \geq 0}$ and $(f^{N+i}(z))_{i \geq 0}$ are D -pseudo-orbits for all $N \in \mathbb{N}_0$, by Corollary 2.6,

$$B_E(\{x_{N+i}\}_{i \geq 0}) = X = B_E(\{f^{N+i}(z)\}_{i \geq 0}).$$

The result follows. □

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