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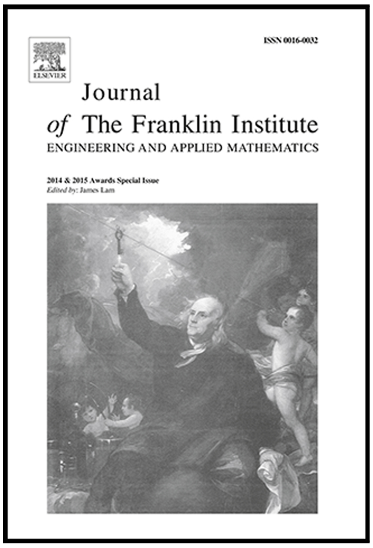

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Distributed Filtering for a Class of Discrete-time Systems Over Wireless Sensor Networks

Tao Wen, Chuanbo Wen, Clive Roberts and Baigen Cai

***Abstract*—This paper addresses the distributed filter design problem for a class of dynamic systems over wireless sensor networks. The missing measurements and the correlation among state noises and measurement noises are considered, where a set of mutually uncorrelated random variables is employed to describe the missing phenomena. Firstly, the construction of the designed filter is proposed and the prediction of the state at each node is given. Then, the filtering error covariance is presented and the filter parameters are determined to minimize the trace of such a covariance, where the network topology data are used to simplified the singular matrix. Subsequently, the relationship between the filter performance and missing probability of the measurement is discussed. Finally, a numerical simulation is presented to illustrate the effectiveness and capability of the proposed distributed filters.**

***Index Terms*—Distributed filters; wireless sensor network; missing measurements; state estimation**

I. Introduction

Over the past decades, filter design problems over wireless sensor networks (WSNs) have drawn particular research attention because of their wide-scope applications in many aspects such as multiple autonomous robot, objective tracking, air pollution detection integrated patient monitoring and so on [1]–[5]. Generally, a WSN contains a large number of intelligent sensing nodes. These nodes are distributed over a certain geographic area and have the capability of measuring the targets, processing received data, and sending information to their neighbors [6]–[8]. A fundamental issue over WSNs is the distributed filter design, where each node shares the local data over the network to help its neighbors complete the tasks. The key for the distributed filter design with a high precision is how to make rational use of the measurement information from each node in a cooperative manner.

For the distributed filtering problems, early work mainly focus on filter design for the network without information communication among nodes. To be specific, in [12], with the assumption that the system state satisfies Gaussian distribution, the optimal distributed filter design has been developed, which is furtherly indicated to be equal to the traditional least square estimate algorithm. In [13], by using the scalar weighting fusion structure, a distributed optimal fixed-lag Kalman smoother has been proposed for a class of discrete-time systems. The optimal sequential fusion problem with correlated measurement noises issue has been investigated in [9], where a successive orthogonalization of the correlated noises is performed. Furthermore, the work [14] has considered the weight design problem for the system with unknown noises statistics, where several different fusion schemes have been proposed by using the upper bounds of both the system noise variances and measurement noise variances. It should be pointed out that the aforementioned distributed filter design approaches were presented under the condition that each local estimate is calculated only based on its own measurements, not considering the helpful information from its neighbours [16]–[18]. Thus, these algorithms are not applicable directly to the state estimation problem over the WSNs. This is because each node in a WSN can communicate with its neighbors and obtain the estimate of the state by using the data not only

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from itself but also from other nodes in the light of the topology of networks.

In the WSNs environment, by assuming the directed graph representing the network topology is strongly connected, a consist distributed state estimator for the systems with global observability has been developed in [19]. In [22], a distributed Kalman filter over WSNs has been proposed without requiring directed graph being strongly connected. In such an algorithm, each node receives measurements from its neighbors. Different from [22], a different distributed filter has been constructed in [7] by communicating the innovation data between neighbors. The filter construction makes it being able to use the information from all the nodes directly or indirectly. By using a set of matrix inequalities, a distributed set-membership filtering method has been proposed for the systems with event-based communication mechanism in [23], [24]. Following [10], [11], the recursive distributed estimators for a class of state-saturated systems have been proposed over the WSNs. Their limitation is that only the upper bounds of the estimation error covariances are guaranteed, which limits their applications. In [25], for the WSNs systems with sensor gain degradation, solving a set of Riccati-like matrix equations, the exact expressions of the estimation error covariances has been given and the distributed minimum-variance filter has been designed, where the boundedness and monotonicity of the estimation error covariance has been also discussed. However, the filter gain, though flexible, is not accurate since the pseudo-inverse used in the computation process. Furthermore, the disturbances of the system model and measurements of each node under consideration are assumed to be uncorrelated with each other, which is not always satisfied in practical applications. The above discussion stimulates the main motivation for the work in this paper.

On the other hand, the filter performance is often affected by fading or missing phenomenon of the measurements. In fact, these phenomena are usually inevitable for a great deal of reasons, such as unreliable communication channels, power failures and potential sensor failures. Generally speaking, the fading phenomenon and missing phenomenon are often described by uniform distributed variables and Bernoulli distributed variables, respectively. To obtain accurate estimation of the state, when designing a filter to handle the observed targets, it is necessary to consider the influence from this incomplete information. The distributed filtering method for the system subject to fading measurements has been presented in [25], where the relationship between fading probability and filter performance also has been analyzed. Recently, the filter design problems for the systems with such phenomena have been investigated and a great deal of research results has been reported [19], [21]. Note that above works concern with dealing with the missing phenomenon and designing the corresponding distributed filter, but they do not consider neither the noises correlation among noises nor discussing how the missing probability affects the filter performance. Therefore, there is an essential need to investigate the distributed filter design problem for the system over WSNs subject to correlated noises and missing measurements.

Summarizing the above discussion, it can be concluded that the distributed filter design problems over WSNs have obtained a few initial results. When it comes to distributed filter design for the systems subject to correlated noises, despite their practical significance, the available results have been scattered. Note that such perturbations add substantial difficulties to the filter design and performance analysis such as the computation of estimation error covariance, not to mention the challenges offered from the missing measurements. As such, we intend to present the distributed filters over WSNs for a class of discrete-time systems with correlated noises and missing measurements. *The main contributions of the article are summarized as follows: 1) the distributed filter for the system that covers the missing measurements, random parameter matrices and the correlated noises is constructed over the WSNs; 2)the exact expression of the estimation error covariance is obtained, where a new matrix simplification method is employed to overcome the difficulties coming from the connection situation of the WSN nodes; and 3) the monotonicity of the arrival probability of the measurements and the precision of the designed distributed filter is studied.*

II. Problem Formulation and Preliminaries

A. Problem Formulation

Consider the following discrete time-varying system (target plant):

$$x_{k+1} = A_k x_k + \sum_{l=1}^{M} \xi_{l,k} G_{l,k} x_k + \Gamma_k w_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state to be estimated, $\xi_{l,k} \in \mathbb{R}$ $(l = 1, 2, \cdots, M)$ is the multiplicative noise with zero mean and variance $\Xi_{l,k}$, and $w_k \in \mathbb{R}^p$ is the system noise. A_k, $G_{l,k}$ and Γ_k are known system matrices of appropriate dimensions. The initial values of x_k are $\mathrm{E}\{x_0\} = \mu_0$ and $\mathrm{E}\{(x_0 - \mu_0)(x_0 - \mu_0)^T = P_0$.

To observe the target, a WSN is used to measure the system, whose directed topology information is represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order N with the set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. The pair (i, j) stands for an edge of $\mathcal{G}$. The adjacency elements associated with the edges of the graph are nonnegative. When $a_{ij} > 0$, it means that node i can receive data from node j. Furthermore, a_{ii} $(i \in \mathcal{V})$ is assumed to be 1. The set of neighbors of node i plus the node itself are represented by $\mathcal{N}_i = \{j \in \mathcal{V} : a_{ij} > 0\}$. For the ith $(i = 1, 2, \cdots, N)$ node, the output of the sensor is modeled by:

$$z_{i,k} = \lambda_{i,k} C_{i,k} x_k + v_{i,k} \tag{2}$$

where $z_{i,k} \in \mathbb{R}^{m_i}$ is the measurement of sensor i, random variable $\lambda_{i,k}$ taking values of 0 and 1 describes the missing phenomenon of the measurement, and $v_{i,k}$ is the measurement noise. $C_{i,k}$ is a known matrix of appropriate dimension.

The following assumptions are made in this paper.

Assumption 1: w_k and $v_{i,k}$ are correlated Gaussian noises with $\mathrm{E}\{w_k\} = 0$, $\mathrm{E}\{v_{i,k}\} = 0$ and

$$\mathrm{E}\left\{ \begin{bmatrix} w_k \\ v_{i,k} \end{bmatrix} \begin{bmatrix} w_t^T & v_{j,t}^T \end{bmatrix} \right\} = \begin{bmatrix} Q_k, S_{j,k} \\ S_{i,k}^T, R_{ij,k} \end{bmatrix} \delta_{kt}. \tag{3}$$

Assumption 2: $\xi_{l,k}$ and $\lambda_{i,k}$ are independent of each other and of other random variables. The mean and variance of $\lambda_{i,k}$ are $\bar{\lambda}_{i,k}$ and $\bar{\lambda}_{i,k}(1 - \bar{\lambda}_{i,k})$, respectively.

Assumption 3: The initial value x_0 is uncorrelated with w_k, $v_{i,k}$ and $\lambda_{i,k}$.

Let $\hat{x}_{i,k|k-1}$ and $\hat{x}_{i,k|k}$ to be the prediction and the estimate of x_k at i-th sensor node, respectively. Next, we are aim to design the following distributed filter:

$$\hat{x}_{i,k|k-1} = A_{k-1}\hat{x}_{i,k-1|k-1} + \Gamma_{k-1}\hat{w}_{i,k-1|k-1}, \tag{4}$$

$$\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + \sum_{j \in \mathcal{N}_i} a_{ij} K_{ij,k} r_{j,k} \tag{5}$$

where $\hat{w}_{i,k-1|k-1}$ is the estimation of w_{k-1} at the ith node, $r_{j,k} = z_{j,k} - \bar{\lambda}_{j,k} C_{j,k} \hat{x}_{j,k|k-1}$ is the innovation of node j and $K_{ij,k}$ is the filter gain to be designed. The initial values of $\hat{x}_{i,k|k}$ is $\hat{x}_{i,0|0} = \mu_0$ and $P_{i,0|0} = P_0$ for each node.

Remark 1: From the construction of the above filter, it can be seen that $\hat{x}_{j,k|k-1}$ $(j \in \mathcal{N}_i)$ is employed to update $\hat{x}_{i,k|k-1}$ and obtain $\hat{x}_{i,k|k}$. Note that $\hat{x}_{j,k|k-1}$ is calculated by using the measurements of node j and its neighbors, which may not connect with node i. It implies that, at each node, the estimate $\hat{x}_{i,k|k}$ is obtained by using the information not only from itself and its neighbors, but also from the nodes that cannot connect with it. Compared with the traditional filter, the new designed filter indirectly employs more adequate information, and thus it may result better estimation results.

B. Preliminaries

For node i, estimate $\hat{x}_{i,k|k}$ is calculated by using its own measurement and the data sent from its neighbors. These measurements will be employed in section III to derive the estimation error of the filter, and the compact form of corresponding measurement equation can be written as

$$\bar{z}_{i,k} = \theta_{i,k} \check{C}_{i,k} x_k + \bar{v}_{i,k} \tag{6}$$

where

$$\bar{z}_{i,k} = \mathrm{col}_{\mathcal{N}_i}\{z_{j,k}\}, \quad \check{C}_{i,k} = \mathrm{col}_{\mathcal{N}_i}\{C_{j,k}\},$$
$$\bar{v}_{i,k} = \mathrm{col}_{\mathcal{N}_i}\{v_{j,k}\}, \quad \theta_{i,k} = \mathrm{diag}\{\lambda_{j,k} I\},$$
$$j \in \mathcal{N}_i \tag{7}$$

Furthermore, with Assumptions 1 and 2, it can be known that $\bar{v}_{i,k}$ $(i = 1, 2, \cdots, N)$ is a random vector with zero-mean and covariance $\bar{R}_{ij,k} = \mathbb{E}\left\{\bar{v}_{i,k}\bar{v}_{j,k}^T\right\} = [R_{tl,k}]$, $t \in \mathcal{N}_i$, $l \in \mathcal{N}_j$, and $\theta_{i,k}$ is a random vector with mean $\bar{\theta}_{i,k} = \mathrm{diag}\left\{\bar{\lambda}_{j,k} I\right\}$ and covariance $\check{\Theta}_{i,k} = \bar{\theta}_{i,k}(I - \bar{\theta}_{i,k})$.

Before establishing the main results, we would like to present some lemmas, which will be used in the following sections.

Lemma 1: [15] Let $R = [r_{ij}]_{p\times q}$ be a real-valued matrix and $M = \text{diag}\{m_1, m_2, \cdots, m_p\}$ and $N = \text{diag}\{n_1, n_2, \cdots, n_q\}$ be a diagonal random matrix. Then

$$\mathbb{E}\{MRN\} = \begin{bmatrix} \mathbb{E}\{m_1n_1\} & \mathbb{E}\{m_1n_2\} & \cdots & \mathbb{E}\{m_1n_q\} \\ \mathbb{E}\{m_2n_1\} & \mathbb{E}\{m_2n_2\} & \cdots & \mathbb{E}\{m_2n_q\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{m_pn_1\} & \mathbb{E}\{m_pn_2\} & \cdots & \mathbb{E}\{m_pn_q\} \end{bmatrix} \circ R$$

where $\circ$ is the Hadamard product.

Lemma 2: [7] For matrices A, Z, E and L of appropriate dimensions, the following equalities hold:

$$\begin{aligned} &\frac{\partial}{\partial Z}\text{tr}(AZ^T) = A, \quad \frac{\partial}{\partial Z}\text{tr}(AZ^TE) = EA, \\ &\frac{\partial}{\partial Z}\text{tr}(AZEZ^TL) = F^TL^TZE^T + LAZE, \\ &\frac{\partial}{\partial Z}\text{tr}(AZE) = A^TE^T, \frac{\partial}{\partial Z}\text{tr}(ZA) = A^T \end{aligned} \tag{8}$$

III. Distributed Filter Design

The distributed filter is composed of the prediction (4) and update (5). In this section, the prediction (4) and the prediction error covariance are firstly presented. Subsequently, a new extended form of the error model for the whole sensor network is constructed and the corresponding estimation error covariance is constructed. Moreover, the filter gain $K_{ij,k}$ is designed to minimize such a covariance.

In order to derive the prediction error covariance and estimation error covariance, we first give the state second-order moment matrix. Denote the second-order moment matrix of x_k as $X_k = \mathbb{E}\{x_kx_k^T\}$, it follows from (1) and Assumption 3 that

$$X_{k+1} = A_kX_kA_k^T + \sum_{l=1}^{M}\Xi_{l,k}G_{l,k}X_kG_{l,k}^T + \Gamma_kQ_k\Gamma_k^T, \tag{9}$$

where the initial value $X_0 = \mu_0\mu_0^T + P_0$.

The calculation of prediction $\hat{x}_{i,k|k-1}$ and the corresponding prediction covariance is summarized as the following Theorem.

Theorem 1: For system (1)-(2) and the one step predictor (4), for the ith sensor, the prediction $\hat{x}_{i,k|k-1}$ is computed by

$$\hat{x}_{i,k|k-1} = A_{k-1}\hat{x}_{i,k-1|k-1} + \Gamma_{k-1}H_{i,k-1}\bar{z}_{i,k-1} \tag{10}$$

where

$$\begin{aligned} H_{i,k-1} &= \bar{S}_{i,k-1}\big(\check{\Theta}_{i,k-1}\circ(\check{C}_{i,k-1}X_{k-1}\check{C}_{i,k-1}^T) \\ &\quad +\bar{R}_{ii,k-1}\big)^{-1}, \\ \bar{S}_{i,k-1} &= \text{col}_N[S_{j,k-1}], \quad j\in\mathcal{N}_i. \end{aligned} \tag{11}$$

Denote $\tilde{x}_{i,k|k-1} = x_k - \hat{x}_{i,k|k-1}$. The corresponding prediction error covariance $P_{ii,k|k-1}$ and cross-covariance $P_{ij,k|k-1}$ are

$$\begin{aligned} P_{ii,k|k-1} &= \mathbb{E}\left\{\tilde{x}_{i,k|k-1}\tilde{x}_{i,k|k-1}^T\right\} \\ &= A_{k-1}P_{ii,k-1|k-1}A_{k-1}^T \\ &\quad +\Gamma_{k-1}\left[Q_{k-1} - H_{i,k-1}\bar{S}_{i,k-1}^T\right]\Gamma_{k-1}^T \\ &\quad +\sum_{l=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1}G_{l,k-1}^T, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &P_{ij,k|k-1} = \mathbb{E}\left\{\tilde{x}_{i,k|k-1}\tilde{x}_{j,k|k-1}^T\right\} \\ &= A_{k-1}P_{ij,k-1|k-1}A_{k-1}^T + \sum_{l=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1} \\ &\times G_{l,k-1}^T + \Gamma_{k-1}\big(Q_{k-1} - H_{i,k-1}\bar{S}_{i,k-1}^T - \bar{S}_{j,k-1} \\ &\times H_{j,k-1}^T + H_{i,k-1}\left[\check{\Theta}_{ij,k-1}\circ(\check{C}_{i,k-1}X_{k-1}\check{C}_{j,k-1}^T)\right. \\ &\left.+\bar{R}_{ij,k-1}\right]H_{j,k-1}^T\big)\Gamma_{k-1}^T, \end{aligned} \tag{13}$$

respectively, where $\check{\Theta}_{ij,k-1} = \mathbb{E}\left\{\theta_{i,k-1}\theta_{j,k-1}^T\right\}$ can be computed according to the statistical properties of $\theta_{i,k-1}$ $(i = 1, 2, \cdots, N)$ and the topology structure of the WSNs.

Proof: It can be seen from (4) that $\hat{x}_{i,k|k-1}$ will be obtained if $\hat{w}_{i,k-1|k-1}$ is known. Since w_{k-1} is correlated with $v_{i,k-1}$, $i = 1, 2, \cdots, N$, it is correlated with $z_{i,k-1}$. For node i, we can calculate $\hat{w}_{i,k-1|k-1}$ by using measurements $\bar{z}_{i,k-1}$ described in (6). It follows from the projection theory that

$$\begin{aligned} \hat{w}_{i,k-1|k-1} &= \mathbb{E}\{w_{k-1}|\bar{z}_{i,k-1}\} \\ &= H_{i,k-1}\bar{z}_{i,k-1} \end{aligned} \tag{14}$$

where $H_{i,k-1}$ is the gain to be calculated.

It is clear that $\widetilde{w}_{i,k-1|k-1} \perp \bar{z}_{i,k-1}$, namely,

$$\mathbb{E}\left\{\widetilde{w}_{i,k-1|k-1}\bar{z}_{i,k-1}^T\right\} = 0 \tag{15}$$

Inserting (14) into (15), $H_{i,k-1}$ can be expressed as

$$\begin{aligned} H_{i,k-1} &= \mathbb{E}\left\{w_{i,k-1}\bar{z}_{i,k-1}^T\right\}\mathbb{E}^{-1}\left\{\bar{z}_{i,k-1}\bar{z}_{i,k-1}^T\right\} \\ &= \bar{S}_{i,k-1}\mathbb{E}^{-1}\left\{\bar{z}_{i,k-1}\bar{z}_{i,k-1}^T\right\} \end{aligned} \tag{16}$$

It follows from Lemma 1 that

$$\begin{aligned} &\mathbb{E}\left\{\bar{z}_{i,k-1}\bar{z}_{i,k-1}^T\right\} \\ &= \mathbb{E}\left\{(\theta_{i,k-1}\check{C}_{i,k-1}x_{k-1} + \bar{v}_{i,k-1}) \right. \\ &\quad \left.\times (\theta_{i,k-1}\check{C}_{i,k-1}x_{k-1} + \bar{v}_{i,k-1})^T\right\} \\ &= \breve{\Theta}_{i,k-1} \circ \left(\check{C}_{i,k-1}X_{k-1}\check{C}_{i,k-1}^T\right) + \bar{R}_{ii,k-1} \end{aligned} \tag{17}$$

Combining (16) with (17), we have

$$\begin{aligned} H_{i,k-1} = \ &\bar{S}_{i,k-1}\left(\breve{\Theta}_{i,k-1} \circ \left(\check{C}_{i,k-1}X_{k-1}\check{C}_{i,k-1}^T\right)\right. \\ &\left.+\bar{R}_{ii,k-1}\right)^{-1} \end{aligned} \tag{18}$$

Inserting (18) and (14) into (4), we have (10).

Denote $\widetilde{w}_{i,k-1|k-1} = w_k - \hat{w}_{i,k-1|k-1}$ $(i = 1, 2, \cdots, N)$. It follows from (1) and (4) that

$$\begin{aligned} \widetilde{x}_{i,k|k-1} = \ &A_{k-1}\widetilde{x}_{i,k-1|k-1} + \sum_{l=1}^{M}\xi_{l,k-1}G_{l,k-1}x_{k-1} \\ &+\Gamma_{k-1}\widetilde{w}_{i,k-1|k-1} \end{aligned} \tag{19}$$

and the corresponding error covariance and cross-covariance are

$$\begin{aligned} &P_{ii,k|k-1} = \mathbb{E}\left\{\widetilde{x}_{i,k|k-1}\widetilde{x}_{i,k|k-1}^T\right\} \\ &= A_{k-1}P_{ii,k-1|k-1}A_{k-1}^T + \sum_{j=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1} \\ &\times G_{l,k-1}^T + \Gamma_{k-1}\mathbb{E}\left\{\widetilde{w}_{i,k-1|k-1}\widetilde{w}_{i,k-1|k-1}^T\right\}\Gamma_{k-1}^T \\ &= A_{k-1}P_{i,k-1|k-1}A_{k-1}^T \\ &+\Gamma_{k-1}\left[Q_{k-1} - H_{i,k-1}\bar{S}_{i,k-1}^T\right]\Gamma_{k-1}^T \\ &+\sum_{l=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1}G_{l,k-1}^T \end{aligned} \tag{20}$$

and

$$\begin{aligned} &P_{ij,k|k-1} = \mathbb{E}\left\{\widetilde{x}_{i,k|k-1}\widetilde{x}_{j,k|k-1}^T\right\} \\ &= A_{k-1}P_{ij,k-1|k-1}A_{k-1}^T + \sum_{l=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1} \\ &\times G_{l,k-1}^T + \Gamma_{k-1}\mathbb{E}\left\{\widetilde{w}_{i,k-1|k-1}\widetilde{w}_{j,k-1|k-1}^T\right\}\Gamma_{k-1}^T \\ &= A_{k-1}P_{ij,k-1|k-1}A_{k-1}^T + \sum_{l=1}^{M}\Xi_{l,k-1}G_{l,k-1}X_{k-1} \\ &\times G_{l,k-1}^T + \Gamma_{k-1}\left(Q_{k-1} - H_{i,k-1}\bar{S}_{i,k-1}^T - \bar{S}_{j,k-1}\right. \\ &\times H_{j,k-1}^T + H_{i,k-1}\left[\breve{\Theta}_{ij,k-1} \circ \left(\check{C}_{i,k-1}X_{k-1}\check{C}_{j,k-1}^T\right)\right. \\ &\left.\left.+\bar{R}_{ij,k-1}\right]H_{j,k-1}^T\right)\Gamma_{k-1}^T, \end{aligned} \tag{21}$$

respectively. The proof is complete. ■

Based on $\hat{x}_{i,k|k-1}$, we are now in a position to calculate $\hat{x}_{i,k|k}$ by employing measurements from itself and its neighbors. Considering the missing phenomena and using Assumption 2, (2) can be rewritten as

$$\begin{aligned} z_{i,k} &= \bar{\lambda}_{i,k}C_{i,k}x_k + (\lambda_{i,k} - \bar{\lambda}_{i,k})C_{i,k}x_k + v_{i,k}, \\ &\quad i = 1, 2, \cdots, N. \end{aligned} \tag{22}$$

Denote $\widetilde{x}_{i,k|k} = x_k - \hat{x}_{i,k|k}$ $(i = 1, 2, \cdots, N)$. It follows from (1) and (5) that

$$\begin{aligned} &\widetilde{x}_{i,k|k} \\ &= \widetilde{x}_{i,k|k-1} - \sum_{j\in\mathcal{N}_i}a_{ij}K_{ij,k}(z_{j,k} - \bar{\lambda}_{j,k}C_{j,k}\hat{x}_{j,k|k-1}) \\ &= \widetilde{x}_{i,k|k-1} - \sum_{j\in\mathcal{N}_i}a_{ij}K_{ij,k}\bar{\lambda}_{j,k}C_{j,k}\widetilde{x}_{j,k|k-1} \\ &- \sum_{j\in\mathcal{N}_i}a_{ij}K_{ij,k}(\lambda_{j,k} - \bar{\lambda}_{j,k})C_{j,k}x_k \\ &- \sum_{j\in\mathcal{N}_i}a_{ij}K_{ij,k}v_{j,k} \end{aligned} \tag{23}$$

To facilitate the later research, (23) can be rewritten in a compact form for the whole sensor network.

$$\begin{aligned} \widetilde{x}_{k|k} = \ &\widetilde{x}_{k|k-1} - \sum_{i=1}^{N}E_iK_kF_i\bar{\Lambda}_k\bar{C}_k\widetilde{x}_{k|k-1} - \sum_{i=1}^{N}E_i \\ &\times K_kF_i\widetilde{\Lambda}_k\bar{C}_kx_k - \sum_{i=1}^{N}E_iK_kF_iv_k, \end{aligned} \tag{24}$$

where

$$\begin{aligned} &\widetilde{x}_{k|k} = \mathrm{col}_N\{\widetilde{x}_{i,k|k}\}, \quad \widetilde{x}_{k|k-1} = \mathrm{col}_N\{\widetilde{x}_{i,k|k-1}\}, \\ &v_k = \mathrm{col}_N\{v_{i,k}\}, E_i = \mathrm{diag}\{\underbrace{0,\cdots,0}_{i-1}, I, \underbrace{0,\cdots,0}_{N-i}\}, \\ &F_i = \mathrm{diag}\left\{a_{i1}I,\cdots,a_{iN}I\right\}, \bar{C}_k = \mathrm{col}_N\left\{C_{i,k}\right\}, \\ &\bar{\Lambda}_k = \mathrm{diag}\left\{\bar{\lambda}_{1,k}I,\cdots,\bar{\lambda}_{N,k}I\right\}, \quad \widetilde{\Lambda}_k = \Lambda_k - \bar{\Lambda}_k, \\ &\Lambda_k = \mathrm{diag}\left\{\lambda_{1,k}I,\cdots,\lambda_{N,k}I\right\}, K_k = [K_{ij,k}]_{N\times N}, \\ &\widetilde{C}_k = \mathrm{diag}\left\{C_{1,k}I,\cdots,C_{N,k}I\right\}. \end{aligned}$$

Furthermore, letting $\bar{L}_i = \operatorname{diag}\{\sqrt{a_{ij}}I\}$ and noting that $a_{ij}=0$ makes some columns of $\bar{L}_i$ to be zero, a simple matrix denoted as L_i will be obtained after deleting the corresponding zero columns of $\bar{L}_i$, which will be used in the following main results.

Theorem 2: For system (1)-(2) and filter (5), for the ith sensor, filtering gain matrix $K_{ij,k}$ is computed by

$$K_{ij,k} = \begin{cases} \bar{K}_{ij,k}\sqrt{a_{ij}}^{-1}, & \text{if} j \in \mathcal{N}_i \\ 0, & \text{if} j \notin \mathcal{N}_i. \end{cases} \tag{25}$$

where $\bar{K}_{ij,k}$ ($j \in \mathcal{N}_i$) is the j-th element of $\bar{K}_k^{(i)}$ and $\bar{K}_k^{(i)} = D_k^{(i)} L_i \left(L_i^T B_k L_i\right)^{-1}$, $D_k^{(i)}$ is the i-th line submatrix of D_k, $D_k = P_{k|k-1}\widetilde{C}_k^T \bar{\Lambda}_k^T$, $B_k = \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1} \widetilde{C}_k^T \bar{\Lambda}_k^T + \check{\Lambda}_k \circ \left(\bar{C}_k X_k \bar{C}_k^T\right) + R_k$, $\check{\Lambda}_k = \bar{\Lambda}_k \tilde{\Lambda}_k$ and $R_k = [R_{ij,k}]_{N\times N}$.

The filtering error covariance $P_{ii,k|k}$ is

$$\begin{aligned} P_{ii,k|k} &= \mathbb{E}\left\{\widetilde{x}_{i,k|k}\widetilde{x}_{i,k|k}^T\right\} \\ &= P_{ii,k|k-1} + \sum_{t\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i} a_{it}a_{il}\bar{\lambda}_{t,k}\bar{\lambda}_{l,k}K_{it,k}C_{t,k} \\ &\times P_{tl,k|k-1}C_{l,k}^T K_{il,k}^T + \sum_{t\in\mathcal{N}_i} a_{it}^2\bar{\lambda}_{t,k}(1-\bar{\lambda}_{t,k})K_{it,k} \\ &\times C_{t,k}X_kC_{t,k}^T K_{it,k}^T + \sum_{t\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i} a_{it}a_{il}K_{it,k}R_{tl,k}K_{il,k}^T \\ &- \sum_{t\in\mathcal{N}_i} a_{it}\bar{\lambda}_{t,k}K_{it,k}C_{t,k}P_{ti,k|k-1} \\ &- \sum_{t\in\mathcal{N}_i} a_{it}\bar{\lambda}_{t,k}P_{it,k|k-1}C_{t,k}^T K_{it,k}^T \end{aligned} \tag{26}$$

and

$$\begin{aligned} P_{ij,k|k} &= \mathbb{E}\left\{\widetilde{x}_{i,k|k}\widetilde{x}_{j,k|k}^T\right\} \\ &= P_{ij,k|k-1} + \sum_{t\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_j} a_{it}a_{jl}\bar{\lambda}_{t,k}\bar{\lambda}_{l,k}K_{it,k}C_{t,k} \\ &\times P_{tl,k|k-1}C_{l,k}^T K_{jl,k}^T + \sum_{t\in\mathcal{N}_i\bigcap\mathcal{N}_j} a_{it}a_{jt}\bar{\lambda}_{t,k}(1-\bar{\lambda}_{t,k}) \\ &\times K_{it,k}C_{t,k}X_{k-1}C_{t,k}^T K_{jt,k}^T + \sum_{t\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_j} a_{it}a_{jl}K_{it,k} \\ &\times R_{tl,k}K_{tl,k}^T - \sum_{t\in\mathcal{N}_i} a_{it}\bar{\lambda}_{t,k}K_{it,k}C_{t,k}P_{tj,k|k-1} \\ &- \sum_{l\in\mathcal{N}_j} a_{jl}\bar{\lambda}_{l,k}P_{il,k|k-1}C_{l,k}^T K_{jl,k}^T \end{aligned} \tag{27}$$

Proof: Let $P_{k|k-1}$ and $P_{k|k}$ denote the prediction error covariance and estimation error covariance of the extended state, respectively. From (24), we have

$$\begin{aligned} P_{k|k} &= \left(I - \sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)P_{k|k-1} \\ &\times \left(I - \sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)^T + \left(\sum_{i=1}^N E_iK_kF_i\right) \\ &\times\mathbb{E}\left\{\tilde{\Lambda}_k\bar{C}_kx_kx_k^T\bar{C}_k^T\tilde{\Lambda}_k^T\right\}\left(\sum_{i=1}^N E_iK_kF_i\right)^T \\ &+ \left(\sum_{i=1}^N E_iK_kF_i\right)R_k\left(\sum_{i=1}^N E_iK_kF_i\right)^T \end{aligned} \tag{28}$$

where $P_{k|k-1}$ can be obtained from (12), (13) and

$$P_{k|k-1} = [P_{ij,k|k-1}]_{N\times N}. \tag{29}$$

After some algebraic manipulation, (28) becomes

$$\begin{aligned} &P_{k|k} \\ &= P_{k|k-1} - \left(\sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)P_{k|k-1} \\ &-P_{k|k-1}\left(\sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)^T + \left(\sum_{i=1}^N E_iK_k\right. \\ &\left.\times F_i\bar{\Lambda}_k\widetilde{C}_k\right)P_{k|k-1}\left(\sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)^T \\ &+\left(\sum_{i=1}^N E_iK_kF_i\right)\check{\Lambda}_k\circ\left(\bar{C}_kX_k\bar{C}_k^T\right) \\ &\times\left(\sum_{i=1}^N E_iK_kF_i\right)^T + \left(\sum_{i=1}^N E_iK_kF_i\right)R_k \\ &\times\left(\sum_{i=1}^N E_iK_kF_i\right)^T. \end{aligned} \tag{30}$$

Note that, for arbitrary matrix J of appropriate dimension and $i \neq j$, we have

$$\operatorname{tr}\{E_iJE_j^T\} = \operatorname{tr}\{E_j^TE_iJ\} = 0. \tag{31}$$

Based on (31), we can obtain

$$\begin{aligned} &\operatorname{tr}(P_{k|k}) \\ &= P_{k|k-1} - \left(\sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\bar{C}_k\right)P_{k|k-1} \\ &-P_{k|k-1}\left(\sum_{i=1}^N E_iK_{k+1}F_i\bar{\Lambda}_{k+1}\widetilde{C}_k\right)^T + \sum_{i=1}^N E_iK_kF_i \\ &\times\bar{\Lambda}_k\bar{C}_kP_{k|k-1}\left(\sum_{i=1}^N E_iK_kF_i\bar{\Lambda}_k\widetilde{C}_k\right)^T \end{aligned}$$

$$+\left(\sum_{i=1}^{N} E_i K_k F_i\right) \check{\Lambda}_k \circ \left(\bar{C}_k X_k \bar{C}_k^T\right) \times \left(\sum_{i=1}^{N} E_i K_k F_i\right)^T + \left(\sum_{i=1}^{N} E_i K_k F_i\right) R_k \times \left(\sum_{i=1}^{N} E_i K_k F_i\right)^T$$

$$= \mathrm{tr}(P_{k|k-1}) - \mathrm{tr}\left(\sum_{i=1}^{N} E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1}\right) - \mathrm{tr}\left(P_{k|k-1}(\sum_{i=1}^{N} E_i K_{k+1} F_i \bar{\Lambda}_{k+1} \widetilde{C}_k)^T\right) + \mathrm{tr}\left(\sum_{i=1}^{N} E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1} (E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k)^T\right) + \mathrm{tr}\left(\sum_{i=1}^{N} E_i K_k F_i R_k (E_i K_k F_i)^T\right) + \mathrm{tr}\left(\sum_{i=1}^{N} E_i K_k \times F_i \bar{\Lambda}_k \bar{C}_k \, \check{\Lambda}_k \circ \left(\bar{C}_k X_k \bar{C}_k^T\right) (E_i K_k F_i \bar{\Lambda}_k \bar{C}_k)^T\right). \quad (32)$$

Setting the partial derivation of $\mathrm{tr}(P_{k|k})$ with respect to K_k to be zero and using Lemma 2, we obtain

$$\begin{aligned} \frac{\partial \mathrm{tr}(P_{k|k})}{K_k} &= 2\sum_{i=1}^{N} E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1} (F_i \bar{\Lambda}_k \widetilde{C}_k)^T \\ &\quad -2\sum_{i=1}^{N} E_i P_{k|k-1} (F_i \bar{\Lambda}_k \widetilde{C}_k)^T \\ &= 2\sum_{i=1}^{N} E_i K_k F_i B_k F_i^T - 2\sum_{i=1}^{N} E_i D_k F_i^T \\ &= 0 \end{aligned} \quad (33)$$

Recalling the definition of E_i, we can rewrite (33) as

$$K_k^{(i)} F_i B_k F_i^T = D_k^{(i)} F_i^T, \quad i = 1, 2, \cdots, N. \quad (34)$$

where $D_k^{(i)}$ is the i-th line submatrix of D_k. From the fact that F_i can be expressed as $F_i = L_i L_i^T$, (34) becomes

$$K_k^{(i)} L_i L_i^T B_k L_i L_i^T = D_k^{(i)} L_i L_i^T \quad (35)$$

It can be easily verified that L_i is a matrix with full row rank, thus we have

$$K_k^{(i)} L_i L_i^T B_k L_i = D_k^{(i)} L_i \quad (36)$$

and thus

$$K_k^{(i)} L_i = D_k^{(i)} L_i \left(L_i^T B_k L_i\right)^{-1} \quad (37)$$

where we have used the fact that matrix $L_i^T B_k L_i$ is invertible. Noting the construction of L_i^T, we can obtain the filter gain $K_{ij,k}$ as described in (25).

Note that $P_{ii,k|k} = \mathbb{E}\left\{\widetilde{x}_{i,k|k}\widetilde{x}_{i,k|k}^T\right\}$ and $P_{ij,k|k} = \mathbb{E}\left\{\widetilde{x}_{i,k|k}\widetilde{x}_{j,k|k}^T\right\}$. By using the expression of $\widetilde{x}_{i,k|k}$ in (23) and filter gain (25), we have (26) and (27), which completes the proof. ■

Remark 2: In Theorems 1 and 2, a distributed filtering algorithm over the wireless sensor network is presented for the addressed discrete-time systems. To minimize the estimation error covariance $P_{k|k}$, we take the partial derivation of $tr\{P_{k|k}\}$ and obtain (34). Unfortunately, since each node only transmits data to its neighbors, F_i ($i = 1, 2 \cdots, N$) in (34) may be singular, and it makes difficulty in calculating the inverse of matrix $F_i B_k F_i^T$. To solve this problem, a new method is used to design the gain by using the topology information of the WSN. Specially, the simplified matrix L_i is adjusted in light of the adjacency elements of each node, which implies that the topology information is also employed.

Finally, let us summarize the algorithm of designing distributed filter (4)-(5) as follows,

Algorithm 1 Distributed filtering algorithm

Step 1. Calculate $\hat{x}_{i,k|k-1}$ by (10) and (11). Calculate $P_{ii,k|k-1}$ and $P_{ij,k|k-1}$ by use (12) and (13), respectively. Obtain $P_{k|k-1}$ by (29).

Step 2. Calculate $K_{ij,k}$ by (25).

Step 3. Calculate $P_{ii,k|k}$ and $P_{ij,k|k}$ by use (26) and (27), respectively.

Remark 3: In [25], the filter gain is is obtained by using the Moore-Penrose pseudo inverse of a singular matrix. In the current paper, to avoid using the Moore-Penrose pseudo inverse, matrix F_i is simplified. Specifically, a novel matrix simplification technique is used to handle F_i by using $F_i = L_i L_i^T$, based on which, an new invertible matrix $L_i^T B_k L_i$ is obtained. The filter gain $K_{ij,k}$ is computed without using Moore-Penrose pseudo inverse.

IV. Property Analysis

Above section has given the recursive distributed filter. It can be seen from the filtering process that the precision of the proposed filter is related with the missing probability of the measurements. In this section, we will study how the arrival probabilities of the measurements affect the estimation error covariance of the proposed filter. Before analyzing the performance, we need first introduce the following assumption.

Assumption 4: At time instant k, the arrival probabilities of all the measurements are the same, i.e. $\bar{\lambda}_{i,k} = \bar{\lambda}_k$ $(i = 1, 2, \cdots, N)$.

Theorem 3: For system (1)-(2) and filter (4)-(5). Given the estimation error covariance $P_{k-1|k-1}$, at time instant k, $\mathrm{tr}\{P_{k|k}\}$ is non-increasing as $\bar{\lambda}_k$ increases.

Proof: From (32), we can obtain

$$\begin{aligned} &\mathrm{tr}\{P_{k|k}\} \\ &= \mathrm{tr}\{P_{k|k-1}\} + \mathrm{tr}\left\{\sum_{i=1}^{N} E_i K_k F_i B_k F_i^T K_k^T E_i^T\right\} \\ &-\mathrm{tr}\left\{\sum_{i=1}^{N} E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1} - P_{k|k-1}\right. \\ &\left.\times \sum_{i=1}^{N}\left(E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k\right)^T\right\}. \end{aligned} \tag{38}$$

Note that the second item on the right-hand side of (38) becomes

$$\begin{aligned} &\mathrm{tr}\left\{\sum_{i=1}^{N} E_i K_k F_i B_k F_i^T K_k E_i^T\right\} \\ &= \mathrm{tr}\left\{\sum_{i=1}^{N} E_i D_k F_i K_k^T E_i^T\right\} \\ &= \mathrm{tr}\left\{P_{k|k-1}\widetilde{C}_k^T \bar{\Lambda}_k^T \sum_{i=1}^{N} (E_i K_k F_i)^T\right\}. \end{aligned} \tag{39}$$

and we have

$$\mathrm{tr}\{P_{k|k}\} = \mathrm{tr}\{P_{k|k-1}\} - \mathrm{tr}\left\{\sum_{i=1}^{N} E_i K_k F_i \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1}\right\}. \tag{40}$$

On the other hand, similar to (37), from (33), we can also have

$$K_k L_i = D_k L_i \left(L_i^T B_k L_i\right)^{-1}, \tag{41}$$

inserting which into (40), we have

$$\begin{aligned} \mathrm{tr}\{P_{k|k}\} &= \mathrm{tr}\{P_{k|k-1}\} - \mathrm{tr}\left\{\sum_{i=1}^{N} E_i D_k L_i\right. \\ &\left.\times \left(L_i^T B_k L_i\right)^{-1} L_i^T \bar{\Lambda}_k \bar{C}_k P_{k|k-1}\right\} \end{aligned} \tag{42}$$

Moreover, from Assumption 4, the mean and variance of $\lambda_{i,k}$ can be calculated as $\bar{\lambda}_k$ and $\bar{\lambda}_k(1-\bar{\lambda}_k)$, respectively. Correspondingly, we have $\bar{\Lambda}_k = \bar{\lambda}_k I$ and $\check{\Lambda}_k = \bar{\lambda}_k(1-\bar{\lambda}_k)I$, combining which with (42), we obtain

$$\begin{aligned} &\mathrm{tr}\{P_{k|k}\} \\ &= \mathrm{tr}\{P_{k|k-1}\} - \mathrm{tr}\left\{\sum_{i=1}^{N} E_i D_k L_i \left(L_i^T B_k L_i\right)^{-1}\right. \\ &\left.\times L_i^T \bar{\Lambda}_k \widetilde{C}_k P_{k|k-1}\right\}. \\ &= \mathrm{tr}\{P_{k|k-1}\} - \mathrm{tr}\left\{\sum_{i=1}^{N} \bar{\lambda}_k^2 E_i P_{k|k-1} \widetilde{C}_k^T L_i\right. \\ &\left.\times \left(L_i^T \bar{B}_k L_i\right)^{-1} L_i^T \widetilde{C}_k P_{k|k-1}\right\}. \end{aligned} \tag{43}$$

where

$$\bar{B}_k = \bar{\lambda}_k^2 \widetilde{C}_k P_{k|k-1} \widetilde{C}_k^T + \bar{\lambda}_k(1-\bar{\lambda}_k)\left(\bar{C}_k \bar{X}_k \bar{C}_k^T\right) + R_k \tag{44}$$

Following the expressions of (12), (13) and (29), it can be seen that, for a given $P_{k-1|k-1}$, the value of $P_{k|k-1}$ will also be determined. $P_{k|k}$ and $\bar{B}_k$ are affected by the arrival probability $\bar{\lambda}_k$. Taking the derivative of $P_{k|k}$ with respect to parameter $\bar{\lambda}_k$, we have

$$\begin{aligned} &\frac{\partial}{\partial \bar{\lambda}_k}\mathrm{tr}\{P_{k|k}\} \\ &= -\mathrm{tr}\left\{\sum_{i=1}^{N} 2\bar{\lambda}_k E_i P_{k|k-1}\widetilde{C}_k^T L_i \left(L_i^T \bar{B}_k L_i\right)^{-1} L_i^T\right. \\ &\left.\times \widetilde{C}_k P_{k|k-1}\right\} + \mathrm{tr}\left\{\bar{\lambda}_k^2 \sum_{i=1}^{N} E_i P_{k|k-1}\widetilde{C}_k^T L_i \left(L_i^T \bar{B}_k\right.\right. \\ &\left.\times L_i)^{-1} L_i^T \widetilde{B}_k L_i \left(L_i^T \bar{B}_k L_i\right)^{-1} L_i^T \widetilde{C}_k P_{k|k-1}\right\} \\ &= \mathrm{tr}\left\{\bar{\lambda}_k \sum_{i=1}^{N} E_i P_{k|k-1}\widetilde{C}_k^T L_i \left(L_i^T \bar{B}_k L_i\right)^{-1} L_i^T\right. \\ &\left.\times [\bar{\lambda}_k \widetilde{B}_k - 2\bar{B}_k] L_i \left(L_i^T \bar{B}_k L_i\right)^{-1} L_i^T \widetilde{C}_k P_{k|k-1}\right\} \end{aligned} \tag{45}$$

where

$$\begin{aligned} \widetilde{B}_k &= \frac{\partial \bar{B}_k}{\partial \bar{\lambda}_k} \\ &= 2\bar{\lambda}_k \widetilde{C}_k P_{k|k-1}\widetilde{C}_k^T + (1-2\bar{\lambda}_k)\bar{C}_k X_k \bar{C}_k^T \end{aligned} \tag{46}$$

Note that

$$\begin{aligned} \bar{\lambda}_k \widetilde{B}_k - 2\bar{B}_k &= -\bar{\lambda}_k \widetilde{C}_k P_{k|k-1}\widetilde{C}_k^T - 2R_k \\ &< 0 \end{aligned} \tag{47}$$

Then combining (45) and (47), we have

$$\frac{\partial}{\partial\bar{\lambda}_k}\mathrm{tr}\left\{P_{k|k}\right\} \leq 0.$$

Thus, we can conclude that $\mathrm{tr}\{P_{k|k}\}$ is non-increasing as $\bar{\lambda}_k$ increases, which completes the proof. ■

Above theorem discusses the relationship between arrival probability of the measurements and the filter performance, which shows that the estimation error covariance is non-increasing as the arrival probability increases.

Remark 4: Theorem 3 investigates the monotonicity of the filter performance with respect to the missing probability. In such a case, the systems noises and measurement noises are allowed to be correlated with each other. It is different with [25], where the monotonicity is discussed for the systems with uncorrelated noises.

V. A Numerical Example

To illustrate the effectiveness and the potential applicability of the proposed filtering method, a second-order discrete-time system is investigated in this section. Assume the dynamic of the targeted follows the system (1) with parameters:

$$\begin{aligned} A_k &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \Gamma_k = \begin{bmatrix} 0.7 \\ 0.25 \end{bmatrix}, \\ G_{l,k} &= \begin{bmatrix} 0.46\sin(lk) & 0.05 \\ 0 & 0.14\cos(lk) \end{bmatrix}, l = 1, 2, 3. \end{aligned}$$

The state $x_k = [x_1, x_2]^T$, whose first and second elements are the position and velocity, respectively. Let sample time $T = 1s$, $\Xi_{l,k} = 0.1, l = 1, 2, 3$ and w_k is a Gaussian white noise with zero-mean and variance 0.3.

The mean and the covariance of the initial state x_0 are $\mu_0 = [0, 1]^T$ and $P_0 = \mathrm{diag}\{2, 1\}$, respectively. The WSNs used to track the target is consisting of 4 nodes, where the set of edges $\mathcal{E} = \{(1, 1), (1, 4), (2, 2), (2, 4), (3, 2), (3, 3), (4, 2), (4, 4)\}$, and all the adjacency elements are $a_{ij} = 1$.

The measurement matrices are:

$$\begin{aligned} C_{1,k} &= [0.8 + 0.2\sin(k), 0.1\cos(k)], \\ C_{2,k} &= [0.6, 0.2 + \sin(k)], \\ C_{3,k} &= [0.4, 0], \\ C_{4,k} &= [0.9\mathrm{e}^{-k}, 0.3]. \end{aligned}$$

The variances of the measurement noises are $R_{ii,k} = 0.3$ $(i = 1, 2)$ and $R_{ii,k} = 0.4$ $(i = 3, 4)$. The correlation parameters among state noises and measurement noises are $R_{ij,k} = 0.05$, $(i, j = 1, 2, 3, 4, i \neq j)$ and $S_{j,k} = 0.03$ $(j = 1, 2, 3, 4)$. The means of $\lambda_{i,k}$ are $0.4 + 0.1i$, respectively.

According to (25), (4), (5), (26) and (27), the estimate and estimation error covariance of x_k are computed recursively. Simulation curves are presented in Figs.1-5. Specifically, Figs.1-2 show the true state and respective estimates at each node with $\bar{\lambda}_{i,k} = 0.8$ $(i = 1, 2, 3, 4)$. It is verified that the performance of the proposed filter is favorable. To compare the filter precision, the curves of the estimation error covariance at node 1 based on the new method and the traditional method are given in Figs.3-4, from which we can see that the former has higher estimation precision. This is because the traditional method can only use the measurements from the neighbors of this node, while the construction of new filter (4)-(5) makes it able to use the information from all nodes directly or indirectly. To show the relationship between the filter performance and the arrival probability of the measurements, the curves of the traces of the estimation error covariances of the extended state are presented with $\bar{\lambda}_k = 0.3, 0.6, 0.8, 0.9$, respectively, where the arrival probabilities of all the sensors are assumed to be the same. It can be seen that the trace becomes smaller as the arrival probability increases.

VI. Conclusion

Some mutually independent Bernoulli random variables are employed to describe the missing measurements where all sensors are allowed to be with individual arrival probabilities. By using projection theory and the correlation information of the noises, at each node, the system noise as well as the system state has been predicted. Subsequently, the estimation error covariance has been presented and the filter gain has been properly designed. Moreover, the monotonicity property with respect to the arrival probability has been investigated in the case that all the sensors have the same means.

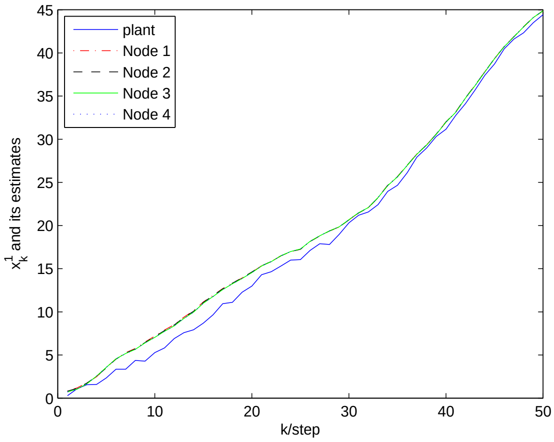

Fig. 1. State $x_{1,k}$ and its estimation.

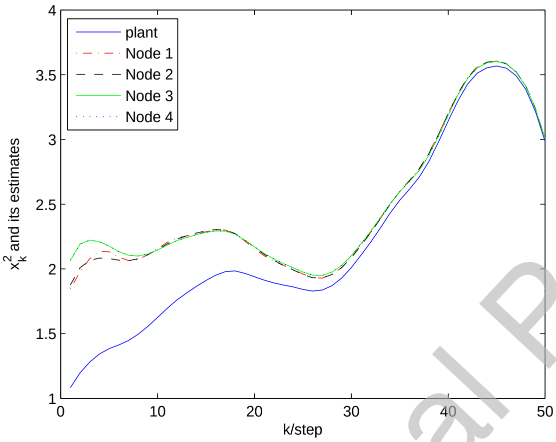

Fig. 2. State $x_{2,k}$ and its estimation.

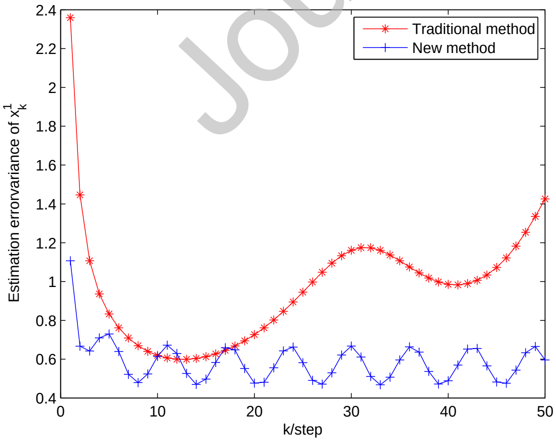

Fig. 3. Estimation error variance of x_k^1.

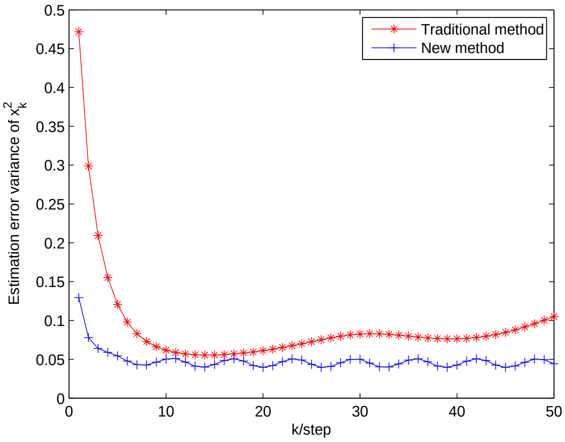

Fig. 4. Estimation error variance of x_k^2.

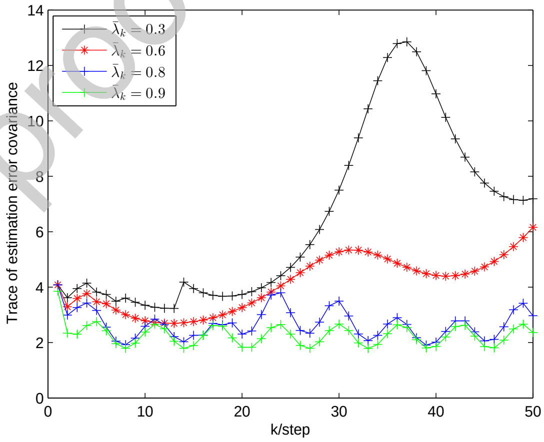

Fig. 5. Trace of the estimation error covariance with $\bar{\lambda}_k = 0.3, 0.6, 0.8, 0.9$, respectively.

Finally, a numerical example is employed to show the usefulness of the new distributed filter.

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