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Parameters and Boundary Conditions in Modelling the Track Deterioration in a Railway System

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Abstract. The main function of the railway track is to support the loads of the railway vehicles and to guide their movements. To investigate the effect of a specific load on the track, the evaluation of the different function of the elements is necessary. Each track component has its own mechanical parameters, which in most case cannot be restored without parts replacements. The development of the track structure has some component properties more important than others. With Finite Element Model (FEM) software packages available, the use of them for simulation and analysis of track components has become an accurate tool when supported by a hybrid approach. In FEM, special attention needs to be given to the boundary conditions and linear or nonlinear interactions between track components. The aim of this paper is to present, in applying to model the track deterioration in a railway system, the mapping of both parameters and boundary conditions, their descriptions, properties, interfaces and how these occur in a railway track in-service.

1. Introduction
The main function of the railway track is to support the loads of the railway vehicles [1] and to guide their movements [2]. To achieve this role each component must perform its specific functions satisfactorily in response to the traffic loads and environmental factors imposed on the system [3]. If the permanent way is not perfectly levelled and aligned, irregularities cause oscillations or vibrations of the train, which can cause discomfort to the passenger and damage to the freight [4].

Track condition has a significant influence on the railway system behaviour in terms of ride safety, maintenance and passenger comfort [5], and must be assessed by measurable degradation parameters [6]. It is defined by the condition of its components and its geometry. These two groups of elements are closely interrelated within the complex process of track deterioration; if one is in poor condition this will contribute to deteriorate the other, and if the components are in poor condition it will not be possible to correct the geometry efficiently [7].

An accurate knowledge of the track mechanical behaviour (stress, strain, moments, etc.) is, [as a first step], essential for a rational dimensioning [and studying] of the various components of the track system,

which should satisfy safety and economy requirements [8]. There is a variety of methods which can be adjusted to the nature of the problem under study.

Some modern methods related to mechanical behaviour use finite element analysis, which permits to take into account the real geometry and the real stress-strain relationship [8], and load dynamic effects. In this case, knowledge regarding both parameters and boundary conditions are essential to investigate the track deterioration phenomenon, and the aim of this paper is to present, in applying to model, the mapping of these elements, their descriptions, properties, interfaces and how these occur in a railway track in-service

Therefore, this paper is structured as follows. Section 2 presents a background of track components, loads, mechanistic model, and FEM. Following this, in Section 3, it presents the mapping of the track loads, and curves and gradients elements. In Section 3 is also shared the results related to the track component characteristic parameters and proposed boundary conditions to deal with the complex track deterioration process. Finally, Section 4 concludes the paper and outlines the challenges to apply the mapped elements in modelling.

2. Background

A modern conventional track can be subdivided into 7 components (rails, pads, sleepers, ballast, sub ballast, geosynthetics, and subgrade), each having a specific function in the train load support [9]. The track has been termed "permanent way" [1]. It is a fundamental part of railway infrastructure and its components can be classified into two main categories: superstructure and substructure. Both of them are mutually important to ensure the passenger safety and comfort, the travel quality [10], and also the freight preservation.

The first one (superstructure) supports and distributes the train loads and is subjected to periodic maintenance and replacement. The second (substructure) is the one in which the train loads, after proper distribution in the superstructure, are transferred and in which, in principle, it should not be subjected to interventions during periodic maintenance [8].

There is no definite structural coordination in the permanent way. Rails, track joints, sleepers, ballast and subgrade experience differential movements between all components, with consequences on wear and deterioration. The lateral stability and the resistance to arch depend on the fit of the ballasted sleepers, the rail weight and the axle load [4], and the speed. Rail support varies in stiffness and local deformations in the roadbed cause (and are caused by) impacts due to loads applied dynamically at a wide variety of intensities. The drainage instabilities and deep settlement require occasional corrections; surface deformations, alignment and calibrations should be a continuous process.

The railway wheel transmits vertical and horizontal forces onto the track. Furthermore, the long welded railway track is subject to the influence of longitudinal forces arising because of changes in temperature. The track is stressed by quasi-static (low-frequency) and dynamic force components of higher frequency [11].

According to Lichtberger [11], the entire track components [sub-system] consist not only of ballast structures with sleepers "swimming" in them, and of the rails supported on them, but also of the track formation with its protective layers or improved structure, as well as of the subsoil itself. They are separated by the sleeper-ballast interface [3]. Each track component has its own mechanical parameters which in most case cannot be restored without part replacements [8].

In order to unify the concepts, this paper proposes that the track components in a ballasted track may be divided into 2 main groups: superstructure (rail, rail pad, fastening, sleeper, and sleeper pad), and substructure (ballast, sub-ballast, and sub-grade – geosynthetic, protective layer, and subsoil). Figure 2 illustrates a typical ballasted track component. Descriptions and function regarding each one of these elements are explained exhaustively in [1], [3], [8], [12], [11], [4], and [13].

When considering the track structure as a set of structural components, the mechanical properties must be characterized. The track structure development – not to complicate – has some properties of the components more important than others. The fundamental distinction is made between components with

mass and inertia properties and those with elastic properties. There are, however, some components which cannot be classified in these two groups [14].

The track component mechanical properties are an essential input to their behaviour. They are a valuable outlet after grappling the permanent way behaviour. Mechanical properties and track design define the relationship between the forces acting on the track and the forces, stresses and displacements occurring in and below the line [14].

The combination of rail vehicle and track should be regarded as one system. This applies to the function as a transport system, but also with respect to the technical point of view. A strong integration exists between the permanent way and the vehicles. The separation between both subsystems and the place where the interaction manifests itself is the contact between wheel and rail throughout the bearing and guidance of the vehicles [12].

The rail deflection magnitude at the wheel-rail interface is the key to promoting a stable track system capable of safely supporting trains at the circulation operating speeds, and collectively all track system components (and their action / reaction properties) contributing to this. When a rail deflects less than a certain amount under loading, damage will occur to the wheel and suspension of a train vehicle as well as to the track; however, excessive deflection also results in damage to both track and vehicle. Track design requires consideration of load responsive behaviour of all components of your system for acceptable response, maintenance regime optimization, and overall lifetime performance [9]. Figure 1 illustrates various parts of ballasted track components, and a simple view of the pressure distribution of the wheel force (Q) in the individual track components.
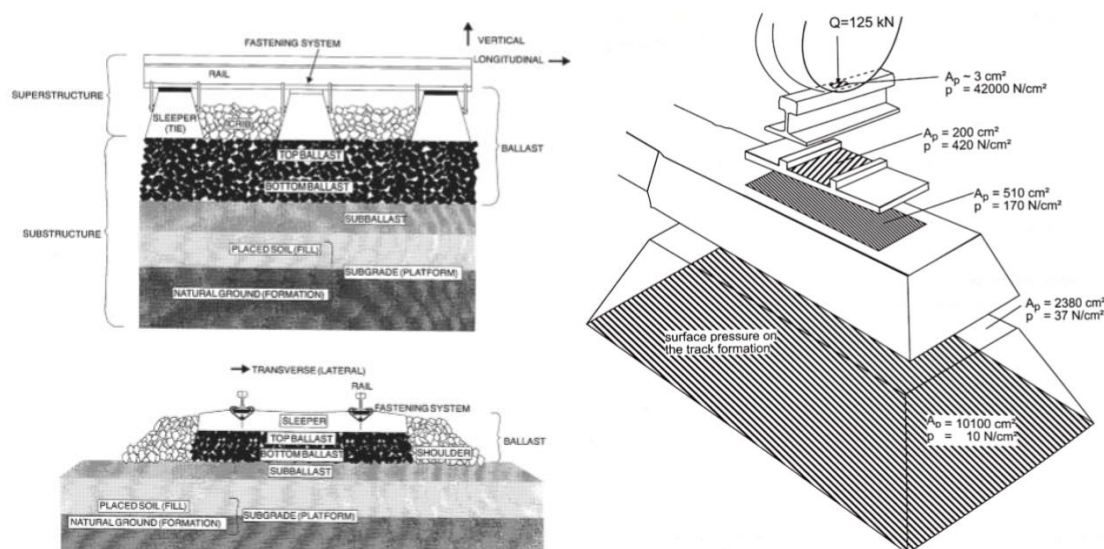

Figure 1. Various parts of a conventional ballasted railway track structure (left) [3] and pressure distribution of the wheel force (Q) in the individual track components (right) [11]

Many factors can affect track condition and suitable modelling techniques must be employed. As an alternative, there is the mechanistic model which involves establishing, by theory or by testing, the track component mechanical properties [15]. Mechanistic or physical models are based on a priori physical information [16]. In another definition, a mechanistic model is described as a model of mechanical reactions in the track, which results in its degradation. Track structure analysis models based on these properties are used to calculate, for individual track components, the forces, stresses and likelihood of defect development in the components.

The advantage of this method is that the track response to traffic parameters can be incorporated, although the response of some track components is hard to quantify. For instance, a single defective fastening may not cause any noticeable effects, but several adjacent defective fastenings will affect the deterioration of other components and the track as a whole.

The track structure analysis models based on the mechanical properties are successful in calculating forces, tensions and probability of failure development in the individual components. An advantage for the solution may be incorporated, in addition to a response of some components from the difficulty of quantification [15]. In fact, these models cannot handle ranges of operating, environmental, and maintenance conditions that demonstrate a different degradation behaviour [17]. Another major problem of the mechanistic model is to quantify the track and vehicle properties. Understanding the interactions between the components and the properties of the track can also be difficult in some cases [17].

The FEM permits to study the actual physical system without extreme simplifications, to take into account accurate limit conditions (i.e. the conditions imparting to the stresses or strains specific values at limit positions, for instance, in supports displacements is zero) and the accurate constitutional law of behaviour (i.e. the relationship between stress and strain for every material) [8] [20].

According to Skoglund [18], the FEM is inextricably tied to constitutive modelling as this method makes it possible to numerically solve boundary value problems where a constitutive model is an ingredient. It is a piecewise approximation to the actual stress and strain fields. To be more accurate, the structure is divided into a number of pieces of finite size, not necessarily of equal size, called finite elements (hence the technique name). When the structure is divided into elements the individual elements are assigned a set of nodes in-between which the internal element deformation is interpolated. This suggests that the deformations are known at the nodes, which in fact is true when the analysis is completed.

There are some specific questions regarding the use of FEM in modelling railway sections related to the large differences between the size and mechanical properties of the elements, and the continuity or otherwise of the layer to be analysed. However, a FEM becomes useful for determining the overall structure behaviour in terms of deflection, making the interaction between elements and layers easy to treat, and understanding and enabling the application of different boundary conditions, as well as introducing different geometries within the same model [19].

3. Parameters and boundary conditions

In modelling the track deterioration based on mechanistic model and numerical approach (FEM), it is fundamental to define, in successive steps, whether the analysis will be static or dynamic, the material mechanical behaviour will be elastic or elastoplastic or viscoelastic, the external force values, and the material mechanical property values [8]. Gathering these technical information, it is possible to proceed with the numerical calculation, and comparing its results with measurements or with results of other methods.

3.1. Track curves and gradients

Longitudinal track alignment consists of the straight flat track sections, in a straight direction, the curves, cant, transition curves, and transition gradients, in the horizontal direction, and last, the gradients and vertical curves. The physical appearance of these elements is determined by the vehicle behaviour characteristics, translated into simple instruction for use and formulae based on safety, comfort, and cost-effectiveness criteria [12]. According to elementary physics, a vehicle running at a speed V on a curve of radius R develops a centrifugal acceleration $\gamma = V^2 / R$ and a centrifugal force $F = m * V^2 / R$ with reduction in passenger comfort, important transverse forces, increase transverse loading, and increase vibrations [8]. In order to reduce these unfavourable effects, it is used large radius of curvature R, cant, and reduction in train speed – the last resort solution.

On a straight line, curvature is zero, while on a curve of radius R curvature is 1 / R. Therefore, between a straight and a curved track, the curvature changes abruptly from zero to 1 / R. Passengers feel this sudden change of curvature as a jolt. Basically, to reduce this effect, transition curves are used between straight track and curves or between two adjacent curves to allow gradual change in lateral acceleration [12].

Wherever possible, the longitudinal profile of a line follows the ground profile [8]. If differences in track level cannot be avoided, gradients are used, and they cannot be too steep in connection with the maximum available adhesion force between the driven wheels and the rails. When there are changes in gradient, vertical transition curves of suitable radius must be used [12]. Track curves and gradients have a role in the track deterioration process.

3.2. Track loads

Both the vehicle and the track have irregularities. They may be irregularities of track geometry (long wavelength) or discrete irregularities (short wavelength) on the surface of the wheel or on the rolling surface of the track. These two classifications of track irregularities (long / short wavelengths) produce different force magnitudes due to the resonances they create within the permanent way structure. To understand the track damage mechanism and its geometry deterioration, it is necessary to recognize all forces on the rail of a train traveling above and the responses made to these forces [4].

The requirements for bearing strength and track quality depend to a large extent on the load parameters: axle load – static vertical load per axle, tonnage borne – sum of the axle loads, and running speed. The first one, to which the dynamic increment is added, in principle determines the track required strength. The second (tonnage borne) is a measure of when maintenance and renewal are necessary. The dynamic load component, which depends on speed, and horizontal and vertical track geometry, also plays an essential part here [12].

According to Esveld [12], the forces acting on the track as a result of train loads are considerable and sudden, characterized by rapid fluctuations. The loads can be considered from 3 main angles: vertical, horizontal – transversal to the track, and horizontal – parallel to the track. Generally, the loads are unevenly distributed over the two rails and are often difficult to quantify. Depending on the load nature, they can be divided as: quasi-static loads – result of the gross rate, centrifugal force and the centering force in curves and switches, and cross winds, and the last, the dynamic loads caused by track irregularities (horizontal and vertical) and irregular track stiffness due to variable characteristics and settlements of ballasted bed and formation, discontinuities at welds, joints, switches etc., irregular rail running surface (corrugations), and vehicle defects such as wheel flats, natural vibrations, hunting. Additionally, the temperature effects on continue weld rail track can cause considerable longitudinal tensile and compressive forces, which in the latter case can result in track instability (risk of buckling).

3.3. Track components

The deterioration modes could be associated with the track component degradation. The service life of rails, sleepers, and fastener systems plays an essential role in the railway infrastructure since their failure could cause train derailment as well as important maintenance costs. Granular layers also present a further progressive deformation with the passage of trains, causing an accumulative track geometry deterioration. The failure mode is due to the granular layer settlements as consequence of the contact loss between particles or the breakage of them caused by the repeated dynamic loads [21]. The main track components to be defined in modelling the track deterioration are: rail, fastening, rail pad, sleeper, sleeper pad, ballast, sub-ballast, and sub-grade.

3.3.1. Rail. The rail is running surface, carrying and guiding element at the same time. It is subject to equal static and dynamic stress. In heavy haul traffic, axle loads up to 40 t are applied. Nowadays in regular high-speed traffics speeds of up to 350 km/h are reached. Depending on the topography, rails are laid with radius as low as 300 m, therefore, they are subject to the very high lateral forces exerted by the wheel flange striking against the gauge corner of the outer rail [11]. Steel rail sections may be connected either by bolted joints or by welding. The bolted rail joints have been one the major locations of maintenance problems. Discontinuity of the track running surface produces dynamic impact loads battering the rail surface and the joint ends (see figure 2). The combination of the impact load and the reduced rail stiffness at the joints causes greater stress on the ballast and subgrade. Although much

progress has been made in improving joints, a better solution has been to eliminate the joint entirely by the use of continuous welded rail (CWR) [3]. Despite of extended rail life resulting, reduced damage to the substructure, and improved riding quality, the CWR may present rail breakage or track buckling from induced temperature changes. The rail characteristic parameter is its Elastic Modulus (E), for instance, E = 209,800 MPa [22].
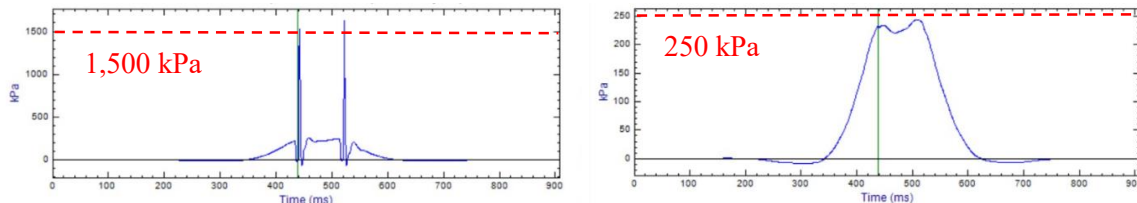

Figure 2. Ballast pressure under sleeper with deep joint (right), and without irregularities (right) from simulation on D-Track Software by mechanistic model.

3.3.2. Fastenings. It is the purpose of the fastenings to maintain the track gauge and to transmit forces acting on and in the rails to sleepers. As the fastening system must be damped in a vertical direction for movements upwards as well as downwards, the properties of the fastenings have to be considered when the rail is lifted off under traffic load. It is important that the fastening system offers sufficient resistance in a vertical direction [11]. They should also provide resilience and adequate deflection, and avoid abrasion between components and of over-stressing. Fastenings are distinguished in rigid and elastic fastenings. Rigid fastenings are used only with timber and steels sleepers, and the rail is connected to sleeper with bolts and nails. During trains passage the rail compresses the sleeper and part of the strain is plastic. In turn, elastic fastenings – mandatory with concrete sleepers, may be distinguished in two types: screw-type – great fastening strength, and spring-type. Fastenings should retain sufficient clamping force over the years [8]. The rail fastening characteristic parameter is its Elastic Modulus (E), for instance, E = 210,000 MPa [23].

3.3.3. Rail pads. The function of rail pads is to transfer the rail load to the sleeper while filtering out the high frequency force components. Modern rails pads vary considerably in appearance and their material properties [12]. They are normally made out of rubber, high-density polyethylene (HPDE), thermoplastic polyester elastomer (TPE), and ethylene vinyl acetate (EVA) [24]. They serve as an elastic damping element, which decouple vibration. Due to the rail settlement connected with elasticity, the load is distributed over several sleepers. This means that the rail pad also has a load distribution effect [11]. Specially designed softer rail pads are used with degrees of stiffness around 100 kN/mm or even less, especially if combined with concrete sleepers [12]. As general rule, these pads come in various designs in order to better adapt to the railway system, and can thus range in thickness from 4.5 to 15.0 mm. The rail pad characteristic parameters of the are their Stiffness (k) and Damping (c), for example, k = 100 kN/mm and c = 32 kN s/m [25].

3.3.4. Sleepers. Sleepers are the track components positioned between rails and ballast. They must ensure appropriate load transfer and distribution from the rails to the ballast, constant rail spacing, mounting of the rails on the sleepers at 1/20 or 1/40 inclination, and adequate mechanical strength both in the vertical and in the horizontal direction [8]. The tradition materials used to manufacture railway sleepers are timber, concrete and in some case steel, which are generally designed for 20, 50 and 50 years. Additionally, there are the composite sleepers as a potential alternative to timber sleepers. The sleeper characteristic parameter is its Elastic Modulus (E), for instance, in the case of a Softwood AREMA (timber sleeper), E = 7,400 MPa [26].

3.3.5. Sleeper pads. Sleeper pads or under sleepers pads (USPs) are resilient special pads which are placed between the sleepers and the ballast. They are about 10mm to 20mm thick and have been used

for about 20 years in special applications [27]. According to Johansson et al. [28], the USPs often consist of a polyurethane elastomeric material, which has a foam structure. The materials must be well chosen in order to get good damping and stiffness values. For the installation in the railway track, the USPs are often glued onto the underside surface of the sleepers. The main reason for using USPs is to reduce the damage of the ballast. By using USPs, the force of the train is expected to be distributed to more sleepers at the same time. This also reduces the contact pressure and the wear of the ballast and therefore the track settlement. Furthermore, it is expected that vibrations in the ballast and in the ground are reduced by using USPs [27]. The sleeper pad characteristic parameters are their Stiffness (k) and Damping (c), for example, $k = 0.15$ N/mm^2 [29].

3.3.6. Ballast. Railway ballast is the crushed stone that forms the top layer of the substructure, in which the tie (sleeper) is embedded and supported. Mainline ballast material is usually large, uniformly graded crushed stone. Although crushed stone is used for a variety of engineering purposes, as railway ballast it is subjected to a uniquely severe combination of loading stresses and environmental exposure. In particular, the upper portion of ballast that is directly below the tie is the zone that must endure the highest stresses from traffic loads and from the surfacing operation, leading to more rapid ballast deterioration in this area [13]. The ballast characteristic parameters are its Stiffness (k) and Damping (c), for instance, $k = 17$ MN/m [30] and $c = 25$ kN s/m [31].

3.3.7. Sub-ballast. It is the layer between the ballast and the subgrade [3], which has been either placed as a specific layer or evolved in-place from the particle wear, densification, and settlement of old ballast layers due to decades of loading and track maintenance. The latter condition is very typical railway lines that have been active for a long time and where the old roadbed acts essentially as a sub-ballast layer. As a structural layer, it reduces stress to the subgrade, similar to ballast, by an amount dependent on its resilient modulus (stiffness) and thickness. Sub-ballast stiffness generally controls the load-spreading ability of the sub-ballast layer and depends largely on its compacted density, which in turn is controlled by its gradation. It must not plastically deform over many load cycles [13]. Some of the sub-ballast functions may be provided by cement, lime, or asphalt stabilized local soils, asphalt concrete layers, and geosynthetic materials like membranes, grids, and filters fabrics (geotextiles) [3]. The sub-ballast characteristic parameters will depend on the provided material and technology.

3.3.8. Sub-grade. It is the platform upon which the track structure is constructed. Its main function is to provide a stable foundation for the sub-ballast and ballast layers [3]. It is known as the foundation on which everything above depends for support often the most variable, and potentially the weakest of track components. The subgrade inaccessibility makes it challenging to assess its condition, diagnose a problem, and prescribe or implement a remedy with confidence. To function as a stable foundation layer, the subgrade must be structurally sound and not sensitive to environmental damage. Structural considerations for the subgrade include ensuring the subgrade is stable under self-weight, stable under train loading, and does not strain plastically to form a progressive shear failure. The primary governing issue for subgrade is the soil type. This is followed by determination of the soil physical state, including the density and moisture content, and physical properties and engineering characteristics [13]. The subgrade may be divided in two categories: natural ground (formation), and placed soil (fill). The sub-grade characteristic parameters are its Stiffness (k) and Damping (c), for instance, $k = 80 \times 10^3$ kN/m and $c = 45$ kN s/m [31].

3.4. Boundary conditions

The term "boundary conditions" is used to cover all possible additional conditions that may be necessary to fully describe a particular problem [32]. According to Li et al [13], in general, a FEM would include a track segment and may only include half of the track cross section assuming symmetry along its longitudinal centerline. Depending on the degree of accuracy and the computer running time, the size and shape of the FEM mesh (element) can vary, and can include various sized elements of triangular,

rectangular, brick, or other shapes. In FEM, special attention needs to be given to the boundary conditions and linear or nonlinear interactions between track components such as from sleeper to ballast, and sub-ballast to subgrade. The main boundary conditions to be defined in modelling the track deterioration are: loading conditions, and interactions.

3.4.1. Loading conditions. In order to study the global response of the railway track system due to the passing train load, a conventional train model may be proposed in according to the Authority's Code and the technical specifications relating to rolling stock. Figure 3 illustrates an example of conventional train and its static load.
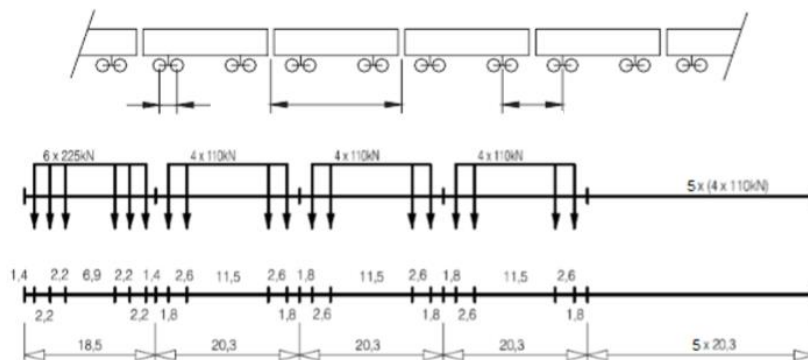

Figure 3. Conventional train and static load of passing train [33]

3.4.1. Interactions. The track model may be a linear finite-element model (see Figure 4). The rail is modelled as a continuous beam with shear deformation, supported by the pads, which are linear spring and damper elements. The sleepers are regarded as a concentrated mass. The ballast layer is represented in a simplified manner by discretely linear spring and damper elements. The shear continuity of the interlocking ballast particles and the sub-ballast are not considered in this example. The subgrade is modelled as a viscoelastic element without mass. The vehicle is considered as independent 1/8 vehicle models, and spring and damper elements represent the secondary and primary suspension connecting the car body with the bogie, and the bogie with the wheel set, respectively. The wheel set has contact with the rail by means of a non-linear Hertzian spring element, which provides stiffness in the vertical direction only [31]. It is possible to include rail joints (spring), fastening system (spring), sleeper pad (spring and damper), and other interfaces. Additionally, depending on the length of the train, may be included straight and curves, and gradients, as weel, in order to increase the modelling accuracy. On the other hand, it is known that as much more complex is the modelling, much more time and resources are necessary to process it.

4. Conclusions
It is very important to understand the mechanism that underpins track deterioration to predict better the development of track irregularities. An accurate knowledge of the track mechanical behaviour is an essential step for a rational studying of its components. There is a variety of methods, which can be adjusted to the nature of the problem, and the FEM is one of them that permits to study the actual physical system without extreme simplifications, taking into account accurate limit conditions and behaviour constitutional law. Therefore, it is fundamental mapping the track parameters and the boundary conditions, recognizing when it is adequate their simplification – or not, once they will be an essential input in modelling the track deterioration in a railway in-service.
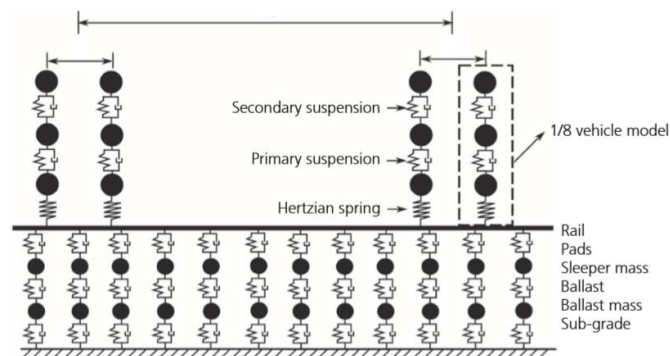
Figure 3. Simplified 2D model for the analysis of a couple vehicle-track system [31]

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