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The evolution of mathematics support: a literature review

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Abstract

Mathematics support, the provision of additional learning opportunities to, primarily, nonmathematics specialist undergraduates has grown significantly since the early 1990s, particularly in the UK, Ireland and Australia. Alongside the growth in volume of provision, there has been a marked increase in the amount of research and scholarship relating to mathematics support that has been carried out and disseminated. This paper reviews this literature and in doing so identifies areas in which mathematics support has evolved. This evolution has taken place in response to a range of crucial changes in the external policy and general environment and, in particular, in response to the changing nature of the so-called 'Mathematics Problem'. Key themes that emerge from the literature review, which are explored in detail, are the characteristics of students who engage with mathematics support and reasons why others do not; the role of the mathematics support tutor, who undertakes the tutoring task and how they are trained; the positioning of mathematics support within higher education structures; and the evaluation of the effectiveness of mathematics support.

Keywords: mathematics support, literature review, impact, evaluation.

1. Introduction

Mathematics support has been defined as 'a facility offered to students (not necessarily of mathematics) which is in addition to their regular programmes of teaching through lectures, tutorials, seminars, problems classes, personal tutorials, etc.' [1,p.9]. The two dominant forms of mathematics support are the drop-in centre and the bookable one-to-one appointment. Whilst the informal provision of extra support for students by committed mathematics lecturers has a very long history, what might be termed 'organised mathematics support' began in earnest in the UK less than thirty years ago in the early 1990s. The primary impetus for the development of mathematics support provision was what has become known as the 'Mathematics Problem'. This is discussed in more detail in the following section; for the present, the 'Mathematics Problem' can be summarised as the under-preparedness of new undergraduates (primarily in non-mathematical sciences disciplines) for the mathematical and/or statistical demands of their degree programme. *Making Mathematics Count* [2], the report of a Government Inquiry into post-14 mathematics education in Britain, includes the oft-quoted statement:

In the short-term, the Inquiry believes that Higher Education has little option but to accommodate to the students emerging from the current GCE process¹. [2, p.95]

¹ The GCE process refers to the qualifications taken by most students at that time immediately before entry to university, primarily Advanced levels also known as A-levels.

The provision of mathematics support was the response that many institutions had already put into place to make the required 'accommodation', in some institutions over a decade before the Smith Inquiry reached the above conclusion.

Mathematics support began as a response to circumstances that pertained in the late 1980s and early 1990s. A key motivation for the introduction of mathematics support was to attempt to tackle the problem of high failure rates on engineering degree courses. The engineering mathematics modules were seen as one of the major causes of these high failure rates [3]. Given that engineering courses typically recruited large numbers of students, high failure rates on these courses represented a major setback for many individual students and a significant loss of income to institutions. Mathematics support as originally established was therefore primarily remedial and often targeted at engineering students and, to a lesser extent, physical sciences students.

Over the last thirty years, the environment in which higher education in the UK operates has changed enormously and mathematics support provision has evolved in response to these changes. Key changes in both the policy and general external environment include:

- The election of the 1997 Labour Government whose leader, Tony Blair, set a target of 50% of young adults going into higher education in the next century [4]. To achieve this target, emphasis was placed on Widening Participation in Higher Education, as set out in the green paper *The Learning Age*: 'We cannot rely on a small elite, no matter how highly educated or how highly paid. Instead we need the creativity, enterprise and scholarship of all our people' [5,p.7]. The increased focus on widening participation resulted in student cohorts with even great inhomogeneity in their mathematical backgrounds and created new challenges for mathematics support.
- The introduction of university tuition fees in 1998 at the level of £1,000 per year and the subsequent 200% increases to £3,000 per year in 2006 and £9,000 per year in 2012. This created the notion of 'students as consumers' [6] and with it demands from students for increasing levels of support to ensure their success.
- Alongside the introduction of fees has been the general marketisation of higher education and competition between higher education providers has grown significantly. Competition for students has led universities to place more emphasis on their support services as a way of attracting potential recruits.
- A greater focus on student employment. The Destinations of Leavers from Higher Education (DLHE) survey was introduced in 2003 and its importance has grown since then. In 2017/18, the Teaching Excellence Framework relied on six core metrics (three of which were fully weighted and three of which were half weighted); of these, two of the fully weighted metrics were derived from the DLHE [7].
- Graduate employers have placed greater emphasis on applicants' quantitative skills and numerical reasoning tests are now routinely used as part of the selection process for a great many graduate jobs [8]. This has been challenging for many students, particularly those studying non-quantitative disciplines, and has created new demand for a different kind of mathematics support.

The increasing quantification of many disciplines. In the early 1990s, the users of mathematics support were mainly students of engineering and the physical sciences. However, technological and scientific advances over the last 30 years have resulted in many subjects such as the biosciences and social sciences becoming much more reliant on mathematical modelling and statistical analysis (see, for example, [9,10]). These changes in the nature of practice in several disciplines have produced demand for support, particularly in statistics, from groups of students who previously would not have sought such support.

Mathematics support provision has responded to these and other changes in the higher education environment to establish itself as a permanent feature of the higher education landscape. Whilst once mathematics support may have been regarded as a 'Cinderella service' [11], according to Youdan [12,p.49] it is now viewed as an essential part of the provision of a university:

Mathematics and statistics support has now attained a critical mass and overcome the significant hurdle where universities worry whether offering such support is an indication of modest aspirations. The accepted position is now that it is a student's right to receive support with the mathematical content of their degree.

This paper reflects on the evolution in the nature of mathematics support and the way that it is viewed through exploring a number of key themes. In the next section, the changing nature of the 'Mathematics Problem' is described in some detail. This is followed by sections addressing who the users of mathematics support are; who the tutors providing mathematics support are and how they are trained for this role; how mathematics support is positioned within the structures of higher education; and, evaluating the effectiveness of mathematics support.

Particularly in this century, those who deliver mathematics support have engaged in research and scholarship to underpin and develop their practice [13,14]. This has resulted in an everincreasing literature base. The themes outlined in the previous paragraphs have been identified through a systematic review of the mathematics support literature dating back to 2000, covering the majority of English language mathematics education research journals as well as some broader higher education research journals. A range of the grey literature, particularly practitioner association and professional body publications, has also been examined; although, due to the nature of grey literature, this review has not been as systematic as the review of journal publications. In addition to this literature, there is a substantial body of conference publications, notably from the CETL-MSOR (Continuing Excellence in Teaching and Learning in Mathematics, Statistics and Operational Research) series of annual conferences. We have chosen to not systematically review conference presentations, since significant work in mathematics support that has been presented at conferences usually leads on to a journal or other publication. The few conference papers that are cited in this paper have no relevant follow-on publication. We have also drawn on some publications before 2000 where these have been cited in articles we have reviewed

and/or where the publication provides important baseline information relating to the evolutionary development of mathematics support provision.

2. The Mathematics Problem

In the late 1980s, higher education institutions (HEIs) in the UK identified problems with high drop out and failure rates in mathematically based courses and low enrolment in courses with A-level mathematics entry requirements [15]. Several HEIs responded to this phenomenon by introducing some kind of mathematics support, such as drop-in workshops and bridging courses. Such responses were typically bespoke to the local situation and introduced by staff 'at the chalk-face' as an attempt to improve the outcomes for their students. Whilst most academic staff aware of these issues may have complained internally to their colleagues, a few decided to try to raise the profile of this issue nationally. In 1994, several articles appeared in the national press highlighting some of the issues under headlines such as '*Where* x = inadequate teaching' [16], '*Engineers unable to bridge the maths gap*' [17], '*Superior sums that don't add up to much*' [18].

The following year, two major reports were published by professional bodies and learned societies: *The changing mathematical background of undergraduate engineers* [19] and *Tackling the mathematics problem* [20]. These reports highlighted that new undergraduates, particularly those in the engineering and physical sciences disciplines, on entry to university, did not have the same range of mathematical skills as their counterparts from previous years. Sutherland and Pozzi (1995), Commissioned by the Engineering Council, [19] surveyed academic staff involved in teaching mathematics to engineering undergraduates. One of their most notable findings was that 83% of those surveyed "expressed considerable concern about students' facility with algebraic manipulation" [19,p.5]. The joint report [20] from the London Mathematical Society, the Institute of Mathematics and Its Applications and the Royal Statistical Society (the three leading learned and professional societies in the mathematical sciences in the UK), highlighted that:

The serious problems perceived by those in higher education are:

- 1. A serious lack of essential technical facility the ability to undertake numerical and algebraic calculation with fluency and accuracy;
- 2. A marked decline in analytical powers when faced with simple problems requiring more than one step;
- 3. A changed perception of what mathematics is in particular of the essential place within it of precision and proof [20,p.2].

This report also introduced the phrase the 'Mathematics Problem' as a shorthand for the inadequate preparation of many new undergraduates for the mathematical demands of their course.

Both of these reports drew primarily on the opinions of academic staff. LMS et al. acknowledge that the analysis in their report 'is based on *judgement*, and is not susceptible of absolute "proof" [20,p.2]. In 2000, the Engineering Council published a further report *Measuring the Mathematics Problem* [21]. This report sought to use quantitative evidence alongside the judgement of academic staff to highlight the mathematics problem. Significant quantitative evidence was provided by Lawson [22]. This analysis explored results from diagnostic testing of new students on entry to university. The same test had been used every year since 1991. The results showed year on year decline in performance of students with the same A-level mathematics grade and also showed that students entering university in 1997 with a grade C in A-level mathematics performed at the same level on the diagnostic test as students entering university in 1991 with a grade N² ('narrow fail'). Later work [23] showed that the 2001 A-level mathematics grade B cohort had remarkably similar performance to the 1991 grade N cohort.

Alongside the discourse that pre-university mathematics qualifications no longer adequately prepared new undergraduates for the mathematical demands of their courses, there was a second theme relating to the national policy of widening participation in higher education. According to Kent et al. [24,p.10], writing about the situation at Imperial College, London³:

the expansion in student numbers means that departments which used to insist on a good A-level in maths, in addition to good grades in the 'core' topics, no longer do so. For example, last year in the Chemistry Department about 20% of students had A-level grade D or E or GCSE⁴.

If an institution as prestigious as Imperial College had been forced to drop the entry requirement of a good grade in mathematics A-level for some of its engineering and science courses in order to recruit the required number of students then this would have become the practice across the sector. This problem was further exacerbated following the introduction of Curriculum 2000 (a reform of the entire A-level system) which a later Government inquiry bluntly stated was 'a disaster for mathematics' [2,p.8]. One manifestation of this 'disaster' was that the number of entries for A-level Mathematics and Further Mathematics fell from 66,247 in 2001 to 53,940 in 2002, a drop of almost 20%. For universities already struggling to find sufficient applicants with A-level mathematics to fill places on engineering and science courses, this massive reduction in numbers was a major blow.

The impact of Curriculum 2000 led to further intensive lobbying from HEIs and professional bodies and the Government finally responded by setting up a national inquiry into post-14 mathematics education. In 2004, the report of this Inquiry, *Making Mathematics Count*, was

³ Imperial College, London is one of the most prestigious higher education institutions in the UK for the study of science and technology and, as such, able to recruit from amongst the best qualified students.

² In the early 1990s, the majority of new undergraduates in England entered university having taken A-levels as their final qualification in school. A-levels were graded A-E (pass grades), N (narrow fail), U (unclassified).

⁴ GCSE is the qualification taken at age 16 i.e. at the end of compulsory study of mathematics in England and is considerably below the level of the A-level qualification.

published [2]. Several of its recommendations were acted upon reasonably quickly (such as the creation of a high-level post in the Department for Education and Science with dedicated subject specific responsibility for mathematics [recommendation 1.1] and the establishment of a National Centre for Excellence in the Teaching of Mathematics [recommendation 6.12]). However, it was clear that it would take several years for the majority of recommendations to be implemented (including those relating to addressing the shortage of well-qualified mathematics teachers) and have an effect on new undergraduates; hence the already quoted conclusion of the Inquiry that, in the short-term, higher education would have to 'accommodate to the students emerging from the current GCE process' 2,p.95].

The technological advances of the 21st century, particularly in the field of computing led to a rapid increase in the amount of data available and this has affected virtually all disciplines in higher education. Consequently, other bodies (in addition to traditional ones from engineering and the physical sciences) have raised concerns in relation to the mathematical and quantitative skills of the students studying their disciplines. For example, in the biosciences 'UK graduates lack the quantitative skills necessary to analyse and interpret data and to have confidence in their analysis. Some candidates show a lack of even basic mathematical skills' [25,p.14]. Concerns were not restricted to the sciences. In 2012, the British Academy, the UK's national body for the humanities and social sciences, issued a position statement *Society Counts* [10], the opening words of which are:

The British Academy is deeply concerned that the UK is weak in quantitative skills, in particular but not exclusively in the social sciences and humanities [10,p.1].

This position statement not only speaks of the lack of quantitative skills of students, it extends the Mathematics Problem to academic staff: 'Another reason for the poor skills of undergraduates is the dearth of academic staff able to teach quantitative methods' [10,p.4].

Echoing the titles of earlier reports (but with greater ambition than 'measuring' or 'tackling'), the Royal Society of Arts (RSA) published a report *Solving the maths problem* [26]. One of its conclusions is that

English universities are side-lining quantitative and mathematical content because students and staff lack the requisite confidence and ability. This has the potential to damage standards in English universities [26,p.11].

Like the British Academy position statement, the RSA report drew attention to the lack of mathematical skills amongst academic staff and the danger of a self-perpetuating situation – students receiving undergraduate education in which mathematical content is side-lined in the fullness of time come to make up the bulk of the academic staff and do not have the confidence to make the curriculum more mathematical.

A key factor contributing to the issues raised in these reports in relation to students is the English secondary education system, where mathematics is compulsory only to age 16

(GCSE-level). Most learners study no mathematics after the age of 16. Hodgen et al. [27] compared upper secondary mathematics education internationally and found that the UK is an outlier. Their work reviewed educational jurisdictions within 24 developed countries and found that in only six of these jurisdictions is mathematics not compulsory after the age of 16 – four of these six were England, Scotland, Wales and Northern Ireland (the other two were Ireland and Australia (NSW)). England, Wales and Northern Ireland were the only jurisdictions where fewer than 20% of upper secondary students studied mathematics.

The impact of this low participation rate in mathematics post-16 was investigated by the Advisory Committee on Mathematics Education (ACME). In their report *Mathematical Needs: Mathematics in the workplace and in higher education* [28,p.1] they state that

We estimate that of those entering higher education each year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 have done so.

This gap of 205,000 represents students who are likely to need mathematics support once they enter higher education.

In response to these concerns, Michael Gove, the then Minister of Education, announced in 2011 that he was setting a new goal for the education system that 'within a decade the vast majority of pupils are studying maths right through to the age of 18' [29]. Such goals are easy to set but harder to deliver; in particular, given that the shortage of specialist mathematics teachers highlighted by Smith [2] had not been remedied, it was not clear where the teachers required to implement this policy goal would be found. One key part of the strategy to achieve this has been the introduction of a new qualification, called Core Mathematics, that may be taken post-16 alongside A-levels by students who are not taking A-level mathematics. In 2018, there were 6,849 entries⁵ for this qualification, some way short of the 205,000 new undergraduates ACME had identified as needing to study some mathematics beyond GCSE level.

This focus on subjects such as the biosciences, social sciences and humanities should not be taken as a sign that the original mathematics problem raised by engineering and the physical sciences had been solved. In 2011, the Institute of Physics published a report *Mind the Gap: mathematics and the transition from A-levels to physics and engineering degrees* [30]. This report returned to the theme of earlier reports, mentioned above, that mathematics A-level was not preparing students adequately for the demands of engineering and physics degree courses. A House of Lords Select Committee report confirmed this stating:

⁵ Source: Mathematics Education Innovation (MEI) http://mei.org.uk/files/pdf/MEI-media-release-level-3-qualifications-August-2018.pdf

In 2006, the Royal Society argued that the gap between the mathematical skills of students when they entered HE and the mathematical skills needed for STEM⁶ first degrees was a problem which had become acute ... The evidence we received suggested that the problem remains [31,p.15].

The data explosion and the impending demands of the fourth industrial revolution means that employers are increasingly valuing analytical skills in their graduate recruits [32]. As part of the employment selection process for graduates, many employers now use numerical reasoning tests to assess applicants' analytical skills. Similar numerical reasoning tests had been introduced in 2000 by the Department of Education for all students seeking to gain Qualified Teacher Status. Consequently, even where students have studied one of the few remaining disciplines in higher education with no quantitative skills demand beyond GCSE level, their mathematical abilities will often still be tested as part of the employment process. For students who have not studied any mathematics since the age of 16, this is often a daunting prospect.

Whilst this paper focuses its attention on the position in the United Kingdom, it should be pointed out that the phenomena described above are not unique to the United Kingdom. These issues, or similar ones have been encountered in many countries throughout the world. For example, the opening words of the foreword to a report for the Australian Council of Deans of Science, *The State of Quantitative Skills in Undergraduate Science Education* [33, p.i] are 'The *QS in Science* project raises alarm bells for the higher education sector'. The report itself states

The lack of quantitative confidence and preparedness among secondary school students is presenting significant challenges to the tertiary sector ... Students entering science programs have weaker foundations in, and stronger negative beliefs towards, mathematics but at the same time advances in science and technology require more complex quantitative knowledge and skills [33,p.3].

In Ireland, long-term evaluation of new student preparedness at the University of Limerick, using results from diagnostic testing in a manner similar to previously cited work in England [23], provided evidence of a decline, over a 12 year period, in the mathematical competencies of students entering science and technology courses [34]. The National Council for Curriculum and Assessment instigated a review of mathematics education in secondary education in Ireland stating that the context of the review was a 'fundamental evaluation of the appropriateness of the mathematics that students engage with in school and its relevance to their needs' [35,p.3]. The outcome of this review was the introduction of a new curriculum called Project Maths which was rolled out in schools from 2010.

⁶ STEM = Science, Technology, Engineering and Mathematics.

A discussion group at the European Society for Engineering Education (SEFI) Mathematics Working Group addressed the question 'What are the major problems facing Engineering Maths Education in Europe?' [36]. This group concluded that

The lack of basic skills of university freshmen is well known and seems to be Europewide. Participants saw as reasons for this state the expansion of higher education, the lack of training at schools, the abuse of the calculator and also missing qualifications of school teachers [36,p.1].

In summary, the Mathematics Problem has evolved over the last twenty five years. It began as a concern, primarily in engineering and the physical sciences, that good A-level mathematics grades did not signify adequate preparation of students for higher education in the way that they did previously. With the focus on widening participation, the problem grew to include students not being as well qualified (in terms of A-level grades or even not having A-level mathematics at all). The increasing quantification of many disciplines extended the problem still further to include biosciences, social sciences, the humanities and other disciplines where the key element of the Mathematics Problem was that compulsory mathematics post-16. The latest development of the Mathematics Problem (as it exists at present) is the demands of employers for graduates from every discipline to display analytical skills (particularly in relation to numerical data); such skills are viewed as essential for employability in the light of the fourth industrial revolution.

3. Users and non-users of mathematics support

3.1 User characteristics

As has been explained, mathematics support had its origins predominantly in responding to the challenges engineering and physical sciences students were facing as they embarked on their university courses. Although the term 'remedial' was never explicitly used, there was a strong motivation to improve the retention rates on these courses, so the primary target audience for mathematics support was students who were at risk of failing their mathematics modules. In the early days of mathematics support therefore, there were three common characteristics of users of mathematics support (at least, in the intentions of those providing the support):

- 1. They were students of engineering and the physical sciences;
- 2. They were students newly enrolled in higher education;
- 3. They were weaker ('at risk') students.

Lawson, Halpin and Croft [37], reporting on a national survey of mathematics support provision, indicated that engineering students were usually the major users. Further reports [38-40] reinforce the focus of mathematics support on engineering and science students.

However, as is explained in Section 2, the nature of the mathematics problem has diversified and expanded since the 1990s and many more disciplines are now requiring students to use quantitative methods to some extent. Consequently, the demand for mathematics support from students of other disciplines has grown. This is reflected in the literature with several articles relating to discipline-specific mathematics support being published covering disciplines such as pharmacy [41], teacher education [42], biosciences [43], business [44] and nursing [45]. In addition, it has also been observed that mathematics students are often major users of mathematics support [46]. For example, Loughborough University's Mathematics Education Centre Annual Reports, cited in [47], show that typically 25 % of the students who visit mathematics support centres are specialist mathematics students. This is explored further in Section 3.3.

The diversification of the subject specialisms of students accessing mathematics support has resulted in more demand for support with statistics. A report from the British Academy [48, p.2] highlighted the 'varying, and often weak, fluency in statistics' and the need for improvements. Statistics support often takes a different form from mathematics support. For students who are undertaking large scale data gathering projects, drop-in support is not an ideal way of addressing their issues. Although some statistical queries (for example, around basic concepts like measures of central tendency and spread or use of normal distribution tables) can be dealt with well through a short consultation with a tutor in a drop-in centre, many others cannot. For example, where a student is carrying out a major data gathering exercise (for example, as part of a final year project) a longer, more concentrated consultation is needed. For this reason, many institutions offer bookable one-to-one appointments of up to one-hour duration for statistics support [49].

In practice, it is not just students with difficulties at the transition into higher education who engage with mathematics support. Whilst the majority of users have been, and continue to be, first year undergraduates, there are other users too. Lawson et al. [37,p.20] record that

At some universities only foundation and first year students can use the centre (although some indicate that whilst not encouraging others to come they are not actually turned away). More commonly, the support is available to anyone studying a mathematics or statistics module. Finally the most common position is any member of the university can take advantage of the support that is on offer.

Although it was common to make mathematics support available to any member of the university, the expectation was that the overwhelming majority of users would be from foundation and first years. However, this has not always proved to be the case, with some institutions reporting significant engagement with mathematics support by students from later years (for example, [46]). Many students who engage with mathematics support in their first year continue to do so as they progress through their courses. Others who did not encounter serious difficulties with mathematics in their first year, find that the increased level of difficulty in later years means they need to access support. In one institution, 24% of visits to the drop-in centre were from students taking level 3 and level 4 modules (the final two years

of undergraduate study) or from postgraduates. This high level of demand, impacted on the service that could be provided to students from levels 0, 1 and 2. As a consequence, it was felt necessary to restrict access to the drop-in centre to students from levels 0, 1 and 2 [49]. Similar circumstances are reported in another institution [50].

Tolley and Mackenzie [51], in their report of interviews of 23 senior university managers, draw attention to the fact that postgraduate students are increasingly facing challenges in relation to mathematics and statistics. One interviewee stated that 'postgraduates are expected to use and understand statistics to a standard they have not been required to do in the past' [51,p.13]. Particularly in the social sciences, the research literature is much more quantitative than undergraduate curricula [10] and postgraduate students need to engage with this literature without the appropriate statistical knowledge and experience.

The third anticipated characteristic of mathematics support users is that they would be predominantly 'at risk' or weaker students. However, this has not been confirmed by a number of studies. Pell and Croft [52] found that mathematics support was used more by good engineering students seeking better marks than by weak students trying to avoid failure. The results of a study of design students showed that those who made use of mathematics support had a slightly weaker mathematics background than those who did not but engagement with mathematics support was not overwhelmingly dominated by weak students [53]. Another study investigated the use of mathematics support by both arts and science students [54]. This found that at-risk first year students were more likely than stronger students to engage, but that it was the stronger second and third year arts students, seeking to improve their chances of achieving first-class marks, who were more likely to access mathematics support than their weaker counterparts. Pell and Croft commented that mathematics support had moved on from 'remedial support to enhancement' [52,p.172], an idea supported by Rogers, who described the change in mission of the mathematics support centre from being somewhat hidden away and focused on students transitioning into higher education to having a centre stage location in the main library and a new mission 'to enhance the mathematical learning of *all students* [our emphasis] throughout the university' [55, p.2].

There has been growing awareness that the diversity encompassed in the phrase 'all students' requires innovative approaches to offering mathematics support. For example, [56,57] explore effective ways of supporting students with dyslexia in their learning of mathematics. Cliffe sets out ways of creating accessible learning environments to enhance the learning of students with a range of physical disabilities [58]. Thus the mathematics support community has recognised that responding effectively to the needs of all students can require variety in the way support is provided and accordingly has developed appropriate mechanisms to deliver this support.

3.2 Non-engagement with mathematics support

Notwithstanding the widening of the mission of mathematics support provision to one of enhancement for all students and not simply remediation or rescue of weaker students, engagement with at risk students still remains an important part of the aims of mathematics

support providers. One study draws attention to the fact that many students who would be expected to benefit from mathematics support, and indeed for whom such engagement could be the difference between passing and failing their mathematics module, do not engage [59]. This study identifies reasons for non-engagement as revealed through interviews with and focus groups of non-users. The reasons most frequently given for not accessing mathematics support were not being aware that mathematics support was available and not being aware of the location of the mathematics support provision. Such reasons have been described as 'shallow' and it has been suggested that they may mask the real reason [47]. Similar reasons for non-engagement were given by a sample of students from across the island of Ireland [60]. The top two reasons given were that the times when mathematics support was available did not suit and that they did not know where the mathematics support centre was located. In both these studies, a small number of students appear to be either more honest or more self-aware and report reasons for non-engagement with mathematics support such as feeling that they had too many problems, fear of embarrassment, intimidation and demoralisation.

Another focused study of engagement with mathematics support used in-depth individual interviews with two groups of students: users and non-users [61]. This work identified that students' reactions to critical events are key in determining their engagement with mathematics support. For example, a key critical event was difficulties with assignments. Students from the group that had engaged with mathematics support reacted in a uniform way to such a critical event – they sought assistance, primarily from the mathematics support centre. However students from the group that did not engage with support reacted differently to this critical event – they decreased their assignment submission rates and attendance at tutorials. When asked if she had simply resigned herself to failure, one student said 'I think I was kind of hoping for some miracle ... I just kind of pushed it to the side' [61,p.14]. This work identifies three factors: fear, social interactions and motivation, as key in determining the way that students respond to critical events.

To address the aforementioned 'shallow' reasons for non-engagement, mathematics support providers have used normal marketing methods to make students aware of the services they provide. Posters, advertisements and links on the institutional VLE, social media and lecture shoutouts are commonly employed. In addition, links with initial diagnostic testing, induction week visits, free calculators collected from the drop in centre and free USB sticks containing mathematics support resources included in the university welcome pack have also been trialled.

In recent years some providers have adopted a different approach to secure engagement from those with 'deep' reasons for non-engagement. They have adapted the method of providing mathematics support by introducing 'embedded support', i.e. moving from an opt-in model to an opt-out one (for example, [44, 62]). Such an approach typically 'attaches' to a module a small number of tutorial or workshop sessions which all students are expected to attend. It may be argued that such an approach is not 'additional' in the sense of the commonly used definition of mathematics support quoted in the Introduction to this paper [1]. However, this approach has been introduced to accommodate to mathematical difficulties students

encounter because of their under-preparedness for their course of study and, as such, it seems reasonable to regard it as mathematics support.

3.3 Mathematics support and the specialist student

Although mathematics support was originally established for students studying disciplines other than the mathematical sciences, as was noted in Section 3.1, in some institutions specialist mathematics students engage extensively with the mathematics support on offer (see also [46]). It has been reported that, at one institution in Australia, specialist students made such extensive use of the support facility (they 'colonised' the space) that the institution provided a separate space for them to use so that the drop-in centre could be more easily accessed by students of other disciplines [63].

A similar phenomenon has been observed in the UK, and the same language of colonisation was used to describe what took place:

An unforeseen consequence of the Support Centre was the mathematics students' colonisation of the physical space and the development of group learning strategies which involve a strong community identity [64,p.421].

In this institution too, a separate facility was made available for the mathematics students.

In this and later work [65], reasons for this colonisation were explored through a series of interviews with some of the students involved. These interviews revealed that the drop-in centre by its very existence and also because of simple physical factors, such as the furniture consisting of round tables, promoted group working and peer to peer support. It also provided a secure environment for students to work in; secure in the sense that students might not go there seeking help but to work on their own or with each other knowing that if they needed it, help was at hand. The one-to-one or one-to-few nature of the interaction with tutors meant that students who, for reasons of embarrassment or peer pressure, would not ask questions in a lecture room in front of the whole cohort would happily ask questions in the support centre. There was also a change in the power dynamic in the relationship with lecturers. Mathematics students who were reluctant to go to a lecturers' office to seek assistance (even during published office hours), because it was 'their space', were much more willing to ask questions in the drop-in centre (to potentially the same member of staff) because the mathematics support centre was viewed as neutral ground. This work showed that female mathematics students particularly appreciated the approach to learning mathematics that the support centre facilitated and promoted, and one which was different from the stereotypical lone and competitive endeavour.

4. Tutoring in mathematics support

In this section we focus on those who deliver mathematics support: the tutors. The two dominant forms of mathematics support are the drop-in centre and the bookable one-to-one appointment. In both of these interactions, the tutor has a key role. A multi-institution investigation of Lawson et al. (2003, p12) highlighted the importance that users of mathematics support placed on the contact with tutors: 'It was clear from the interviews with student users that the one-to-one help was the most valued part of every support centre'[1,p.12]. Similarly, in a study of mathematics support in a single institution, it was found that 'students single out MSC [mathematics support centre] tutors for praise' [66,p.30]. We will explore the role of the tutor and how it has evolved, who undertakes the tutoring, what training is available and conclude by highlighting a specific issue related to staffing of mathematics support.

4.1 The role of the tutor

As has been explained, the original impetus for the establishment of mathematics support provision in universities was to assist students making the transition from school/college into higher education. In particular, the focus was upon engineering undergraduates, particularly those whose prior education meant that they arrived at universities under-prepared for the mathematical demands of their courses. In light of this, it has been suggested that mathematics and statistics support should be delivered by staff who have expertise at the further education/higher education transition and that 'ordinary HE lecturers' will typically not have such expertise and that it might be more appropriate to recruit staff who have taught in the further education (i.e. pre-university) sector [67,p.13]. This echoed the remarks of Sutherland and Dewhurst that 'the teaching needs are more akin to school teaching and university lecturers are not always the best people to be undertaking additional mathematics teaching and support' [68,p.21].

However, as we have seen in Section 2, mathematics support has evolved from its early focus on engineering undergraduates making the transition into higher education, rendering the decision about who should tutor more complicated than simply recruiting someone with a further education teaching background. Nowadays, a student seeking mathematics support could be studying any of the wide range of courses that the university offers, and at any level from foundation through to postgraduate. They may have a very strong mathematical background or have studied very little or no mathematics since the age of 16. They may be very confident about their mathematical ability and visit the centre for clarification on some advanced topic. Conversely, they may have mathematics anxiety and be disorientated with regard to their studies. Some will have additional needs and neuro-diversities that affect their learning. The mathematics support tutor will know none of these things before the first encounter. Additionally, tutors usually have no prior knowledge of the questions they are likely to be asked. Consequently, they must be able to think on their feet and be willing to explore and research possible solutions in partnership with the student seeking help. They must try to understand the student's mathematical problems and then offer guidance, motivation and support to help the student successfully work through their issues. These reasons make it difficult to prepare for sessions. Furthermore, they illustrate how support centre tutoring is very different in nature from mainstream university teaching. Given the broad spectrum of students that mathematics and statistics support serves, tutors have diverse, challenging and vital student-facing roles.

Tutors should be welcoming and non-judgmental. They need to be able to cope with a wide range of mathematical content and able to discuss it at a level appropriate to the student. They need to know how to deal with situations when the subject matter is not known to them. Individually, each of these characteristics brings challenges, but taken together they demonstrate how difficult it is to be a `good' support centre tutor. A key point is that building students' confidence is of huge importance [37] and this is particularly true in the light of findings cited earlier that indicate very large numbers of students embark on university courses with insufficiently developed skills [28].

The required characteristics of a mathematics support tutor have been summarised as follows:

The tutor must be:

- \circ Able to deal with any aspect of mathematics or statistics
- Aware of their applications over a wide range of main subjects
- An expert analyser and re-synthesiser of students' minds
- An interpreter of students' class notes
- Able to create the feel-good factor out of an apparent disaster area [69,p5].

It should be pointed out that this list was written at a time when the focus of mathematics support was the transition from pre-university to university mathematics. It was therefore reasonable to expect tutors to be able to deal with any aspect of mathematics or statistics at this level. As the level of students accessing mathematics support has broadened (as described in Section 3.1) so this required characteristic has become somewhat unreasonable. The other desirable tutor characteristics listed above still remain valid in the broader context within which tutors now operate.

Ireland points out that tutors need to realise that often the student's problems do not lie with the topic for which they are seeking help [70]. She gives as an example a student seeking help with applying the quadratic formula. A perceptive tutor will take time to explore the nature of the difficulty and may identify that the problem is not with the student's understanding of the quadratic formula but rather springs from their inability to manipulate negative numbers confidently and competently. An investigation of tutoring in mathematics support through analysis of video recorded tutor-student interactions found positive features including the use of open questions, corrective questioning, the avoidance of judgmental language and negative non-verbal clues [71].

Croft and Grove re-iterate these themes:

Providing mathematics support is not about 'telling' the student the answer, but about encouraging them to identify their own mathematical problems, helping them tackle these for themselves with support and guidance, and providing suggestions and strategies for independent study. It requires individuals who are comfortable working on a one-to-one basis, who are patient, able to explain mathematical ideas in multiple

ways, have excellent interpersonal skills, and are able to work with students of a range of abilities and from different disciplinary areas [72,p.12].

4.2 The tutors and their development

Given the complexity of the tutoring task, it is natural to ask who the tutors are. A number of national surveys have sought to address this question as outlined in Table 1 below.

Insert Table 1 near here

It should be noted that the percentages in each row of Table 1 sum to more than one hundred since many institutions use more than one type of tutor.

In Table 1, departmental staff refers to academic staff, usually from the mathematics department, who have other mainstream teaching duties as well as providing mathematics support; dedicated staff refers to full-time, part-time or hourly paid staff employed specifically to provide mathematics support. The all-Ireland survey [73] did not distinguish between these types of staff. It did however record that 36% of institutions providing mathematics support have a full-time mathematics support manager/co-ordinator with a further 8% having someone with such duties as a separate part of their contract (i.e. these are dedicated staff as noted in Table 1).

The data in Table 1 imply that in England and Wales there has been a move away from departmental academic staff providing mathematics support and towards the employment of dedicated staff. It is also clear that significant use is made of postgraduate students to deliver mathematics support. The latest survey work found that there were seven institutions where mathematics support is provided *solely* by postgraduate students [62]. These institutions were predominantly research intensive universities.

Given the complexity of the nature of the tutor role, as outlined in Section 3.1, it is perhaps surprising that postgraduates are so widely used in a role that, ideally, requires welldeveloped teaching skills. It is therefore important to be careful in the recruitment and development of postgraduate mathematics support tutors. Gillard, Robathan, and Wilson's (2011) An email survey of 40 UK mathematics support providers identified that one of the disadvantages of using postgraduate students as tutors is their lack of teaching experience. One of the respondents to their survey stated that 'they deliberately recruit PG [postgraduate] tutors with a more accommodating personality' [74,p.48]. In other words, mathematical knowledge alone is not sufficient to make a good postgraduate tutor. A detailed study of postgraduate mathematics support tutors found that these tutors often adopted a didactic style giving minimal opportunities for students to attempt problems or ask questions [75]. This work makes several recommendations relating to the training of postgraduate tutors.

The mathematics support community has, for some time, recognised the importance of tutor training and for over ten years has organized regional tutor training workshops, developing freely available training resources [72]. The 2014 evaluation of mathematics support in

Ireland [76] recommended that the provision of bespoke mathematics support tutor training should be a priority. Subsequently, the Irish Mathematics Learning Support Network secured funding to deliver such training in several locations across Ireland. An evaluation of this training pointed to the importance of team building, as many tutors reported feeling isolated in their work, and highlighted the need to further develop the questioning skills of tutors [77].

Despite the recognition of the importance of tutor training and its availability, recent investigations in the UK [79] and in Ireland [73] both found that only about 50% of institutions which provide mathematics support have tutor training in place. The logistics of arranging such training in a timely manner may be reasons why tutor training is not more widespread.

Although postgraduate tutors need training, there are also benefits in using such tutors. The tutors themselves develop personally and professionally. Some tutors work as volunteers in a support centre because they felt this developed their ability to be good tutorial assistants in mainstream teaching and because they found it personally enriching [63]. Postgraduate tutors can make a significant contribution to policy, practice and resource generation [79]. In a detailed study of nine postgraduate mathematics support tutors, the tutors themselves identify multiple personal benefits including developing a 'more professional' attitude, becoming better not only at teaching but also at mathematics and growing in confidence so that they felt able to suggest changes to university teaching practices [80].

4.3 A staffing issue

In the early 1990s, when institutional mathematics support was in its infancy, those who delivered mathematics support were typically academic staff from the mathematics department who provided mathematics support as an adjunct to their main roles of teaching and research. As mathematics support has become more embedded across the higher education sector and larger in scale, the staffing of support provision has diversified as revealed in a number of multi-institution surveys and single institution studies (for example, [66,73,82]).

As shown in Table 1, many institutions now employ dedicated mathematics support managers and/or staff, who may be supported by hourly paid staff and postgraduate tutors who typically contribute a few hours per week. Often many of the staff have fixed-term (typically lasting for a year) contracts because the funding for the mathematics support provision is subject to renewal. The short-term nature of contracts means that many mathematics support staff move on to other positions after a few years as they seek better job security. Furthermore, and as is discussed in more detail in Section 5, in several institutions, responsibility for the provision of mathematics does not lie with the mathematics department but with a central student learning support facility. In such circumstances, it is often the case that mathematics support staff are not on academic contracts. Frequent changes in staffing can impact on the consistency, availability and quality of support [82]. Similar issues are raised in [83].

5. The position of mathematics support within HE structures

It is difficult to be precise about when organised mathematics support (as opposed to individual members of academic staff offering ad hoc additional support to their students) began in the UK. It would appear to be in the late 1980s / early 1990s in response to problems identified 'at the chalkface' which were later formally acknowledged within national reports [19,20].

One of the first large scale mathematics support centres was at Coventry Polytechnic (now Coventry University). This was established in 1991 following the receipt of a grant from the oil company BP and focused initially on supporting engineering students [84]. This was an initiative from the mathematics department which was responsible for teaching mathematics to engineering students.

In 1993, the first National Conference on Supporting Mathematics in Further and Higher Education took place at the University of Luton and one of the outcomes of that conference was the establishment of a biannual Mathematics Support Newsletter [85]. Nine issues of this newsletter were published during the period 1994 – 1999. Articles in these newsletters show how mathematics support in universities appeared to be initiated by and be the responsibility of the mathematics department (for example, [24,86]). A survey at this time found that the funding for mathematics support in universities came mainly from mathematics departments, sometimes with additional central funding being provided [87].

It appears that in its early stages of development, mathematics support tended to be provided at a departmental level as a local response to wider national problems. Lane, one of the delegates at the aforementioned first National Conference on Support Mathematics in Further and Higher Education, indicated that mathematics support was delivered by 'dedicated enthusiasts struggling to cope with a desperate situation which is getting worse each year, usually with inadequate resources' [88,p.23]. This view of mathematics support being driven by individuals was later echoed by Kyle:

Although I might not have put it in these terms at the time, I probably regarded mathematics support as a form of cottage industry practised by a few well meaning, possibly eccentric, individuals, who may themselves have been hard pushed to offer a credible rationale for this work [13,p.103].

The Mathematics Support Newsletter covered both Further and Higher Education. The last newsletter was published in 1999 and from then most published scholarship relating to mathematics support focused on higher education. A survey of only the HE sector in 2001 showed that mathematics support was still predominantly departmentally focused, but there were early signs that mathematics support was beginning to move up institutional agendas [37]. For example, 5% of institutions (i.e. 2 institutions) reporting that they had a

mathematics support provision indicated that this provision was part of a general student support centre.

In a reflection on lessons learnt in implementing mathematics support, one of the key lessons highlighted is the importance of securing proactive senior management support [89]. However, this work is from the perspective of the need for mathematics support being a local initiative originating in a mathematics or engineering department. This was the original mode of development. But external drivers were leading to the provision of mathematics support being an increasingly important strategic priority at institutional level and therefore being initiated and overseen, not just championed, by senior management. For example, at Loughborough University, the Strategic Plan included an aspiration for the Mathematics Education Centre: 'nationally the Centre will be known for leading-edge curricula including cross-campus specialist support in areas such as mathematics and statistics' [90,p.116].

Government policies relating to widening participation referred to earlier in this paper, caused university managers to start to view the provision of mathematics support as more than an issue local to the mathematics department. Widening participation funding was used to set up a mathematics support centre at Nottingham Trent University; this was however a school-level initiative [91]. In contrast, South Bank University took a strategic decision to establish central academic support for all of its students as an indication of the University's commitment to widening participation [67]. This central provision covered English language, study skills, disability and dyslexic support as well as mathematics and statistics support. Although not necessarily focused on widening participation, campus-wide mathematics support at the University of Hull was similarly co-ordinated by a central unit, Study Advice Services, rather than being a departmental or school-level provision [92].

To further promote its widening participation policy, the Government established the Office for Fair Access (OFFA) in 2004. Each English university was required to produce an 'access agreement' which had to be approved by OFFA. The access agreement had to set out how the university was approaching widening participation, not just in terms of recruiting students from under-represented backgrounds but also in terms of ensuring such students successfully completed their studies. At least 14 universities identified in their access agreements that the provision of mathematics support was part of their widening participation strategy [12].

In addition, during the first decade of this century, government policy focused on greater accountability for higher education through agencies such as the Quality Assurance Agency and instruments such as the National Student Survey (NSS) and more lately the Teaching Excellence Framework. Alongside these official measures, several newspapers began to create league tables of universities giving unofficial rankings of institutions by combining a range of publicly available metrics such as NSS scores, drop-out rates, proportion of students achieving 'good degrees' (i.e. a first class or upper second class honours degree) and the proportion of students securing graduate-level employment. This combination of official and unofficial scrutiny forced universities to give more attention to these issues.

A series of interviews with senior managers from 23 universities representing all mission groups within the UK higher education sector showed that

All of the HEIs questioned reported having students who are challenged by mathematics and statistics ... all the universities questioned recognised that unless they provide appropriate forms of learning support for mathematics and statistics it is inevitable that there will be an adverse impact on their students' satisfaction, retention, achievement and employability ... mathematics support is now more visible and high-profile within HEIs and is seen as important for enhancing the student experience and aiding success [51,p.2].

A consequence of the greater strategic importance within institutions of mathematics support has often been that it has been moved from being managed and operated by the mathematics department to being part of a wider central student support provision. A survey of the operation of mathematics and statistics support at 48 institutions showed that in 33 institutions (69%), mathematics and statistics support was integrated with other institutional services and concluded that

The level of integration, collaboration or alignment with other institutional services again reinforces the findings from Tolley and MacKenzie (2015) that institutions are becoming more strategic about how their provision of mathematics and statistics support operates [78,p.178].

The recognition of the strategic importance to a university of mathematics support provision has had both benefits and disadvantages. Previously, those providing mathematics support had reported in national surveys that their on-going existence was somewhat precarious as there was no guarantee of the level of on-going funding [93,94]. However, the integration within wider student support provision appears to make funding more secure and information from the latest survey of mathematics support provision in England and Wales shows evidence of greater longevity of mathematics support in most institutions, with only a very small number closing such provision [62].

On the other hand, the development of mathematics and statistics support may be hindered by being part of wider student support provision. In one survey it was found that in only 13 out of 48 (27%) institutions studied did complete management responsibility for mathematics support lie with an academic department [78]. Frequently this resulted in mathematics support being managed by someone who had no role in actually delivering this support to students. Also, since staff in central support units are not usually on academic contracts, there is a fear that the burgeoning research and scholarship in the field which took place during the early 2000s may be curtailed. It is this research and scholarship which led Kyle to conclude that his previously somewhat dismissive view of mathematics support could no longer be sustained: 'Mathematics support came of age in the first decade of the 21st century. What might once have been described as a cottage industry now plays a respected and widely adopted role in Higher Education' [13,p.104].

6. Evaluating the effectiveness of mathematics support

Kyle's observations as to how the acceptance of mathematics support has been influenced by the way in which those involved in its delivery have extended their efforts to include gathering 'data on the way students use such resources and [to] look for optimal strategies for the delivery of this support' [13,p.104]. The mathematics support community has sought evidence-based answers to questions such as 'Do students value mathematics support?' and 'Is mathematics support effective in improving student learning outcomes?' This move towards a scholarly or research-based perspective [47] has contributed to substantial recent growth in the body of research literature relating to the effectiveness of mathematics support.

A comprehensive review of the published literature relating to the evaluation of mathematics support that existed up until the end of 2012 cited some 56 sources and studies [95]. We will not repeat their work here, although we do use their review framework. Instead we highlight a further series of works, published from 2013 onwards, that contain additional evidence of the impact of mathematics support upon learners.

6.1 The development of evaluation in mathematics support

As Matthews et al. note, 'there is general agreement in the MS [mathematics support] community that evaluation is both necessary and possible' [95,p.177]. Evaluation enables those delivering mathematics support to have confidence that what they are doing is making a difference to students and also provides evidence to university management to support the continued funding of such provision [96]. As shown in Section 5, in many institutions the strategic importance of mathematics support has been recognised and funding is relatively secure. As such, the focus for the evaluation of mathematics support has moved from justifying its existence towards identifying optimal strategies for delivery, and the exploration of approaches that better engage those students who are most at risk from a lack of mathematical preparedness or fluency. This exploration of alternative approaches is an indication of ways in which the provision of mathematics support is diversifying and how mathematics support is responding to specific institutional needs [62].

The difficulties of evaluating mathematics support are well known [95]. Studies continue to note the significant challenges of establishing causal relationships [97] and in demonstrating statistically significant findings [98,99]. Indeed, whilst in one study a regression analysis demonstrated that for at-risk students, their qualifications prior to entry, diagnostic tests scores, and either their number of visits or time spent within mathematics support were predictors of their final mathematics grade, these variables did not explain the level of the variance observed within their overall grades [100]. As such, whilst engagement with mathematics support can be shown to be important, its effects cannot be readily isolated from other factors that also influence student success, confidence and motivation.

The majority of published studies cited in [95] referred to usage data (that is who uses mathematics support, when and how often) within their evaluations. It is noted that whilst such data 'may show who is using a MSC and what topics are being discussed...it does not provide any indication of the quality or success of this support or insights into the student experience' [95,p.178]. This is reinforced by Croft who comments that whilst such usage data is relatively easy to collect and analyse, and over a suitable time period, may show trends in demand and usage by learners, it does not provide insight into what occurs within a mathematics support centre, the quality of the student experience, or the impact of mathematics support on student retention or achievement [101]. Some publications (for example, [102,103]) present student feedback as evidence. However, such student comments must be interpreted cautiously as they are rarely negative because many students are grateful for the efforts of staff to help them [52]. There is also now an increasing range of studies that seek to use statistical methodologies to establish causal links between the use of mathematics support and student success rates (for example [45,99,104,105]).

In recent times cross-institutional studies have become increasingly commonplace with the benefit of offering increased insight into how mathematics support operates and its impacts. A nine-institution evaluation of mathematics support that captures the views of over 1,600 students; it demonstrates the significant impact mathematics support can have on learners as

22% of respondents who had availed of [mathematics support] had considered dropping out of their course due to mathematical difficulties and almost two thirds of these students stated that availing of [mathematics support] had a positive impact on their retention on their course [76,p.11].

The extent and uptake of mathematics support is explored in [62] which shows the extensive engagement now made with mathematics support services by learners across England and Wales; in total, some 85,000 engagements are collectively reported with mathematics support in an academic year by just under 34,000 students. If the situation in England is similar to that quantified in Ireland regarding propensity to drop out, then approximately 7,500 of these students will have considered dropping out due to mathematical difficulties and approximately 5,000 will have been influenced to continue with their studies by the availability of mathematics support.

6.2 Impact of mathematics support upon learners

Several evaluations that demonstrate the impact of mathematics support upon student performance, retention and confidence are cited in [95]; more recent studies continue to reinforce its impact in these areas. Mulligan and Mac an Bhaird comment upon the role of their mathematics support centre in influencing 26 students who had considered dropping out of university due to their mathematical difficulties to remain [66]. Another study not only links the higher retention rates (when compared with national benchmarks) of engineering students to the widespread implementation of mathematics support in the institution, but also

reports that there are the associated effects of 'a significant increase in student success for those accessing support...and an increase in student engagement' [106,p.107].

Some studies explore the impact of mathematics support upon mature learners [98,104]. One shows that the average mark amongst a cohort of mechanical engineering students is slightly higher for those who regularly attended mathematics support, and more significantly notes that mature learners are motivated to use it because they 'are not just interested in passing the exams...they wish to gain a deeper understanding of the subject' [98,p.19]. The other found that the majority of mature learners within the study who engaged with mathematics support indicated a positive change in their study habits, and felt either a greater level of confidence, or a change in their attitude, towards mathematics [104]. A further study from within an institution where 'many students are from "non-traditional" backgrounds such as mature-aged, first in family, and Indigenous' [107,p.A-92] compared the level of students' mathematical confidence before and after obtaining mathematics support: 'Different aspects of confidence, as suggested in the literature, were examined, and it was found that there was an increase in the levels of each of them' [107,p.A-91].

A series of articles (for example, [108]) have been published based on the data from a national survey in Ireland [76]. The large scale nature of their survey means there are a number of robust findings about the impact of mathematics support upon learners: 56% of respondents who had made use of it indicated that it had been 'helpful' or 'extremely helpful' to their mathematical confidence; 56% indicated it had 'some' or a 'large' impact upon their performance in class tests or examinations; and, 65% indicated that it had been 'some' or a 'huge' help with the mathematical demands of their course. It is concluded that 'the results of this survey strongly indicate that students identify mathematics support as having a positive impact on their mathematical experience' [108,p.965]. Whilst this paper focuses on first year students studying service mathematics, another study focused on specialist mathematics students and showed that 70% of users reported that mathematics support had impacted upon their success whilst at university [46].

Whilst the provision of mathematics support through a drop-in model remains dominant within higher education, a recent survey has identified that around 70% of institutions responding to their study across England and Wales indicate they are also delivering mathematics support in different ways (often alongside drop-in centres), either via organised lectures or workshops, or through support opportunities embedded within mainstream teaching [62]. Work from Germany [109] investigates four types of mathematics learning support: bridging courses, mathematics support centres, redesigned lectures and support measures that parallel courses. This study explores 44 projects of mathematics learning support, with mathematics support centres being in a relatively small minority compared to the other approaches. In addition to [109], there is a growing body of literature identifying the impact of alternative approaches to mathematics support that extend beyond use of the drop-in model alone.

Little compares two different approaches to supporting student nurses [45]. The first intervention, aimed at learners who did not achieve full marks on an initial foundation numeracy assessment, offered one-to-one or small group support prior to a re-assessment. The second intervention of a workshop and drop-in session focused upon the drug calculations themselves rather than the numeracy aspects alone. It is noted that 'there is stronger evidence for the first intervention having impact...in the second intervention the highest scoring group of students made use of both the workshop and the drop-in' [45,p.43].

Lingham and Baughan describe a similar, and collaborative, workshop-based approach between the mathematics support centre and the careers service to help students prepare for graduate-level numerical reasoning tests [110]. They find that whilst the centre 'has seen fewer "last minute" numerical reasoning test students', the participants at the workshop over the last two years report an increased knowledge and confidence in numerical reasoning as a result of their attendance. They also identify that 66% of the participants at the workshop were female and this aligns with the findings of another study of student use of drop-in provision where 'female students were almost two and a half times more likely to engage with mathematics support than male students' [111,p.297].

Alternative models of mathematics support have been developed for first year science students within a university lacking an institutional mathematics support centre [112]. This voluntary and extra-curricular programme used a blended learning approach, consisting of drop-in sessions, student worksheets and a commercial online tutoring programme, and allowed students to select the learning activities most suitable for them. In addition to students appreciating this flexibility, 90% of the students who completed both the pre- and post-diagnostic tests improved their scores. Amongst students from the first-year chemistry, biology, physics and statistics cohorts, pass rates 'were higher, often by considerable margins, for the groups of those who engaged measurably in the programme' [112,p.860], with differences between participating and non-participating students from chemistry and biology being statistically significant. A follow-on study, considering the results from a survey of students, reports how embedding mathematics within the context of their disciplines not only helped students understand its relevance but had an 'immediate pay-off for participation' [113,p.77]. A similar blended learning support approach for students studying a first-year mathematics subject which assumes no prior mathematical knowledge is presented in [53]. Additional support was offered by an online platform, workshops, and drop-in support. Whilst they provide as evidence of its positive effects the 15% difference in pass rates between those students who made use of the support and those who did not, they also urge a degree of caution by noting 'it was not possible to disentangle support from tutorial attendance and use of the [learning management system] as there is a statistically significant correlation between the three' [113,p.72].

Several studies highlight the importance of students engaging with mathematics support in a meaningful way in order for it to offer observable benefits to them as learners. In [97], an optional mathematics support programme is described; the programme is for engineering students identified as being at high risk of failing a first-year mathematics course in calculus

and linear algebra on the basis of either their performance in a prior mathematics module or because they were repeating the course. The programme took the form of a structured onehour tutorial and incorporated successful support strategies identified from a review of effective practice elsewhere. Their results show that amongst those students who consistently accessed this support, there existed a pass rate of 79% compared with 43% for those who briefly accessed support, and 46% for those who were unsupported. A similar approach to supporting first-year engineering students with their learning of mathematics is described in [99]. Based upon the results of an initial diagnostic test, students were either directed towards a 'standard mathematics subject' or a 'foundation mathematics subject'. Support for those studying the foundation subject was provided by the mathematics support centre and consisted of problem-solving workshops, support workshops, an online question and answer service and drop-in sessions. It was found that almost 60% of students accessed some form of learning support, and amongst those that were high users of this support there existed a significant difference when compared to the no/low users in that 'their average [learning management system] activity was just over one standard deviation higher and they did better overall in the subject by almost 10 marks'. However, splitting the cohort into different ability groups can cause problems as reported in [114]. Here, different support approaches are used for students with a vocational rather than more traditional academic background. It is noted that some students felt 'like they had been put in a remedial class' and 'perceived that staff had lower expectations, which in turn led to reduced motivation'. Some also reported finding the act of dropping into the mathematics support centre 'to sometimes be intimidating, due to having "simple" questions in an environment often dominated by students studying more complex topics' [114,p.25].

The importance of sustained student engagement with mathematics support is evidenced in [105] through analysis of data gathered from the visits of some 10,500 students over 12 years to the mathematics support centre. The results, although requiring a note of caution as they show correlation rather than causation, indicate that 'the odds of a student who attended mathematics support once passing their module were 1.63 times higher than for one who had never engaged with the service' but 'the odds for those who attended 15 or more times were almost 14 times higher' [105,p.1].

The considerable volume of publications since the seminal literature review paper [95] indicates that evaluation of mathematics support continues to be a live area of research activity. Large-scale studies of 'traditional' mathematics support have provided strong evidence of its effectiveness. Alongside these, smaller studies of emerging forms of mathematics support indicate value in these approaches too.

7. Conclusions

The sheer extent of the literature relating to mathematics and statistics support is indicative of an active community which is engaging extensively in research and scholarship to seek to enhance the service it provides to learners. Most of the research reviewed is practice-based

and innovations that are reported almost exclusively emerge from the professional practice of those involved in delivering support to students, rather than beginning from a theoretical standpoint and seeking to translate theoretical perspectives into practical application. Nonetheless, the widespread scholarly approach to mathematics support has been instrumental in moving mathematics support from the status of 'cottage industry' through 'Cinderella service' and on to its current status of being 'respected and widely adopted'.

In their abstract for a conference talk in 2010, Greenhow and Namestnikova wrote

Given the changing face of the student population and the views of the various stakeholders in Higher Education (students, parents, government, employers and university staff) that eventually get reflected in league tables and NSS⁷scores, we propose that financially secure, long-lasting and fully-embedded mathematics support will relatively quickly form an unquestioned part of the provision of any well-found university' (cited in [115,p.7]).

The national extent of provision surveys in Australia [116], England and Wales [62], Ireland [73] and Scotland [117] indicate that Greenhow and Namestnikova's proposition has been proved correct within these countries.

One of the strengths of mathematics support being underpinned by practice-based scholarship is that it can be dynamic and responsive. This paper has shown how mathematics support has rapidly evolved to meet the challenges presented by changes to the environment in which it operates. The scale of mathematics support has increased not only in terms of the number of institutions now providing it, but also in terms of both the subject and level of study of the student users. Furthermore, it has evolved from being a remedial service to being about enabling enhancement of learning for all students. One important on-going area of research is 'reaching the hard to reach' or, to put it another way, securing the engagement of those who are reluctant to avail themselves of mathematics support for reasons (such as embarrassment, fear or belief) from the affective rather than the practical domain. Innovative ways of delivering support, such as embedded support are in their infancy, but they offer the prospect of securing some engagement from those who do not respond to the opt-in model of a drop-in centre or bookable appointments.

Staffing of mathematics support appears to be gradually moving from being dominated by academic staff who undertake mathematics support duties as part of their wider academic role to dedicated mathematics support staff aided by postgraduate students. Tutoring in a mathematics support provision is a complex and demanding pedagogical undertaking. Those staff who have a dedicated role may be recruited for their expertise, but postgraduate students need training and, ideally, opportunities for mentoring and peer development to offset their lack of pedagogical experience.

⁷ National Student Survey – an annual Government-mandated student satisfaction survey

External accountability measures, in some jurisdictions at least, have contributed to mathematics support being viewed increasingly as a strategic priority of HEIs, rather than a response to a local departmental difficulty. This has helped achieve Greenhow and Namestnikova's 2010 vision of *'financially secure'* and *'fully-embedded'* mathematics support.

The most fundamental question concerning mathematics support is '*Does it work*?' This is a complex and multi-faceted question which cannot be easily answered. However, as with the other aspects of mathematics support reviewed in this paper, there has been a definite advance in approaches to answering this question over the last twenty years. The seminal large-scale evaluation in Ireland [76] produced the very powerful datum that 63% of mathematics support users who had considered dropping out of university felt that the availability of mathematics learning support had influenced their decision to continue with their studies. Notwithstanding this important finding, robust evaluation of the effectiveness of mathematics support alongside effective ways of engaging the disengaged remain the most important research areas in mathematics support.

8. Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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Tables

Survey	Nation	Departmental Staff	Dedicated Staff	Postgraduates
Lawson et al. [37]	UK	65%	41%	23%
Cronin et al. [74]	Ireland	72%		48%
Grove et al. [62]	England & Wales	33%	63%	42%

Table 1: Percentages of institutions using different kinds of tutors within mathematics support.