Nonlinear free vibrations of spider web structures

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Most research into natural spider web structures have considered only the small deformation theory based on small amplitude analysis. In reality, spider web structures, which are slender by nature, are more prone to large-amplitude load effects. This paper firstly presents a numerical study into large amplitude free vibrations of spider web structures. Highly coupled geometry and material nonlinearities have been formulated to establish the finite element models of spider web structures. Validation of the model has been carried out using previous research studies into spider web vibrations. Excellent agreement has been obtained for linear and nonlinear perturbation results. The validated FEM has been further extended to investigate the large amplitude effect on natural frequencies and corresponding mode shapes of highly-slender spider webs. Interestingly, this is the world first to report the dynamic softening and hardening phenomenon in the slender spider web structures. The insight into nonlinear dynamics of slender thin membrane skeleton structures can be used not only for better understanding the nature, but also for improving functionalities of defense technology using ultra-thin plates and shells.

Keywords: Large amplitude, free vibrations, nonlinear dynamics, spider web, membrane structures, biomechanics

1. Introduction

The function of a spider web is to capture and hold a rapidly flying insect, which shows that the spider web has excellent flexibility and resilience. Two aspects of the design of the web make this possible: the optimized spider silk and the design of the web. Spider silk is one of bio-inspired materials that have shown excellent performance exceeding artificial materials in their properties [1]. On a weight to strength ratio basis, its tensile strength is sometimes even stronger than steel and some silks are almost as elastic as rubber. On this ground, silks provide a two to three times toughness of synthetic fibres such as Nylon or Kevlar [1]. Unlike man-made polymers, spider silk can improve strength without compromising fracture toughness [2]. Besides the superior material properties of spider silk, spider web structures themselves can be recognised as a pre-stressed system, it is the so called tensegrity (tensional integrity) structures [3].

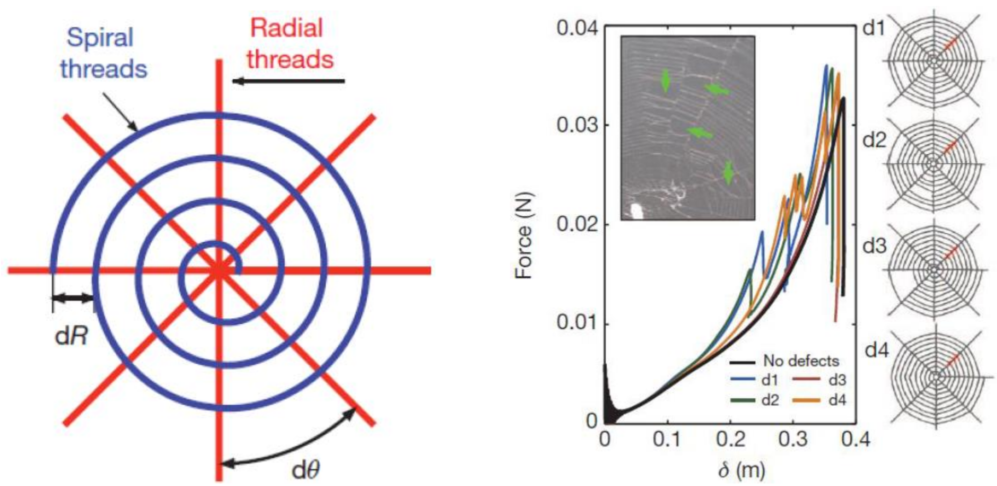

Figure 1: Schematic geometry and behaviour of spider web [4].

This type of structures shows a unique combination of geometry and mechanics, and as a result of the optimal distribution of structural mass, they are highly efficient structures. It is the nature of spider web structures to absorb quick energy as well as constrain drastic oscillations, which due to prey impact. Moreover, a localized damage is a universal feature of spider webs. When spider web is subjected to local loading (see Fig. 1), failure is limited to the loading threads and the loaded thread becomes a sacrificial element to keep the majority of the web remains intact, and actually spider webs strength after slightly damaged [4]. In 2012, MIT researchers found that the reason why spider webs are so resilient is not only because the silk's exceptional strength, it is the distinctive combination of strength and stretchiness and the geometrical arrangement in a web [4]. However, many researches focus on the outstanding properties of silk rather than the spider structure itself. Hence, this research will investigate large amplitude free vibration behaviour of the spider web structure and the corresponding mode shapes. This better insight in its engineering performance can unleash its applications in battle and defence technology. The large-deformable finite element 3D model of spider web structures with and without geometric nonlinearities is developed by using ABAQUS. According to the virtual work-energy principle, the model is formulated from the strain energy owe to axial deformation, kinetic energy owes to the spider web movement and the virtual work due to the self-weight per unit unstretched length, and the concept of large-strain, large-deformation principle has been used to evaluate natural frequencies and corresponding mode shapes with geometric nonlinearity. In this paper, critical review of the relevant published literature shows that large-amplitude vibrations of spider web are critical to impulsive responses and have not been evaluated. Therefore, we are the first to investigate this nonlinear dynamic behavior of the spider web structural system.

2. Nature of spider webs

Spider silks, as protein polymers, show an excellent physical property. Several studies explored that the spider silk, as a natural material, exhibits exceptional strength (stronger than steel on a pound for pound basis), fineness, toughness, and extensibility. On a weight to strength ratio basis, its tensile strength is sometimes even stronger than and some silks are almost as elastic as rubber [1-4]. Under appropriate conditions, which are often associated with a reasonable balance of low stiffness, extreme extensibility and high strength, a spider web is rubbery and capable of large resistance energies to break, and this balance makes silk ideal for web construction and its applications (e.g. cable roof structures, tensegrity structures, etc.).

In nature, the spider web is the device that helps spiders to trap insects for food. Several different sorts of silk such as viscid, dragline, glue-like, minor, wrapping, attachment and cocoon silk can be used in web structure construction [5]. Several different types of spider webs can be found in the wild, including spiral orb webs, tangle webs or cobwebs, funnel webs, tubular webs, sheet webs, dome or tent webs, and orb web is the traditional type spider web which can be seen frequently around. The function of an orb web is to capture and hold a rapidly flying insect, which shows that the orb web has excellent flexibility and resilience [6]. Fig 2 shows the schematic geometry of spider web. The overall size of different orb webs can change from centimetres to meters. In the real word, spider webs actually are highly organized geometry structures. Different types of spider silks have different mechanical performances and functions, and there are several factors, such as temperature, humidity, and extrusion rate, that have an effect on the strength and the load-deformation response of silks. This study will adopt the properties of dragline silk.

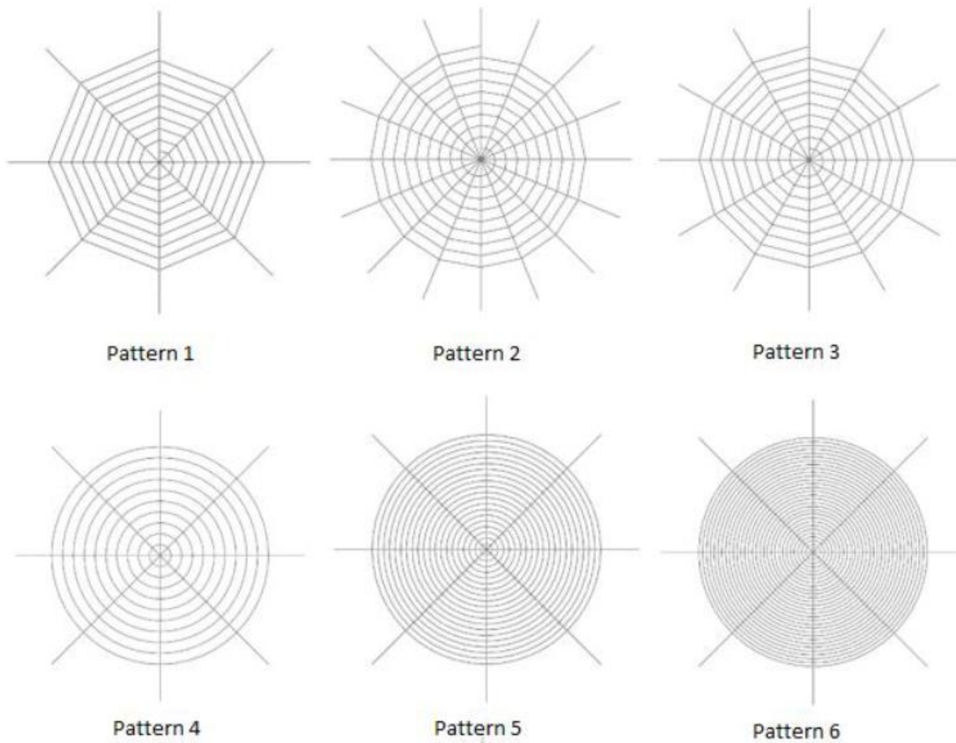

Figure 2: Patterns of spider web in this study.

3. Modelling

Large-deformable finite element models of spider web structures with geometric nonlinearities have been developed using ABAQUS CAE. The spider web model has been developed on a 2-D plane (X and Y axis) in a 3D space where its free vibrations are often induced by out of plane excitations (Z axis). For linearly elastic analysis, five different patterns of spider web models have been built to investigate the change of natural frequencies. For geometric nonlinear model, the ratio of natural frequencies extracted from large amplitude free vibration over linear vibration, and corresponding mode shapes have been investigated. The orb web is the traditional type of spider web structures, which can be seen frequently around the nature; therefore, the pattern of the spider web built in this model is an orb web pattern, as shown in Figure 2. A spider web is a cable structure whose segments only sustain tension.

Therefore, the boundary condition of spider web structure is the same as that of cable structure, which are pin supports (or hinges) that permit rotation [7]. A gravity load in axial Z has been applied in the first step of nonlinear analysis. The initial magnitude is $1\times10^{-9}N$ and each increment is $2\times10^{-10}N$. The models have been validated and very good agreement can be found [8-9]. Considering the behaviours of spider silks (see Fig. 1), the assumption in this study is placed on the large strain formulation within the elastic behaviour region [10].

Many researchers have investigated the mechanical properties of spider silks. In this model, radial and spiral are made of dragline silk of A.anrentia spider with a circular fiber diameter of 4 micron meters. Generally, spiral threads are made of viscid silk, which has lower strength but higher extensibility than dragline silk, and this combination optimises the function of a spider web. In this research, the emphasis is placed on the structural vibration behaviors; therefore, identical material properties have been applied on both radial and spiral thread. The properties are listed in Table 1, which are obtained from the previous research work by Ko and Jovicic [3].

Table 1 Engineering properties of the dragline silk

Material	Density (kg/m^3)	Tensile modulus (GPa)	Poisson's ration	Strength (GPa)	Strain %
Dragline	1098	34	0.49	1.75	26

According to previous studies, a single spider silk thread can be considered as a cable segment. The cable is a type of one-dimensional continuum with perfectly flexible, homogeneous, linearly elastic with negligible torsional, bending and shear rigidities [10]. Therefore, for both a single spider silk thread and a cable, the strain energy is due only to stretching along the axis [11]. Under its own weight, the thread forms a catenary suspension at the initial unstretched state (x, y), then it arrives at the equilibrium position at (x_0, y_0), which is recognized as the initial configuration for dynamic movement due to axial elongation. Then, because of disturbances from external excitation, it moves to the dynamic states by $(x_0 + u, y_0 + v, w)$, in which u, v, w are the displacements calculated from the equilibrium position in the direction of X,Y and Z axis, respectively.

At the equilibrium, the length ds_0 of the infinitesimal spider silk thread element can be expressed as:

$$d_{s0} = \sqrt{1 + y_0'^2}\, dx_0, \tag{1}$$

where $y_0' = \frac{d_{y0}}{d_{x0}}$,

Based on Lagrangian-strain definition, the silk at the unstretched state ds and dynamic states $d\bar{s}$ are written, respectively, as

$$ds = \frac{\sqrt{1+y_0'^2}}{1+\varepsilon_0} dx_0, \tag{2}$$

$$d\bar{s} = \sqrt{(1+u')^2 + (y_0' + v')^2 + w'^2}\, dx_0, \tag{3}$$

where the initial static strain $\varepsilon_0 = \frac{ds_0 - ds}{ds}$. Hence, the total strain at the dynamic state is

$$\bar{\varepsilon} = \frac{d\bar{s} - ds}{ds} = \frac{(1+\varepsilon_0)}{\sqrt{1+y_0'^2}} \sqrt{(1+u')^2 + (y_0' + v')^2 + w'^2} - 1, \tag{4}$$

In order to validate the spider web model whether it is built in appropriately and whether all the data is reliable, a comparison study of the results derived from previous studies is carried out. In addition, the effect of meshing (the number of elements in the solution) is evaluated to verify that the numerical solutions are non-trivial and convergent. Figure 3 illustrates that the natural frequencies extracted from ABAQUS (in this study) are in very good agreement with the results from a previous study [12]. The maximum discrepancy between numerical results is less than 5%, which is reasonably acceptable.

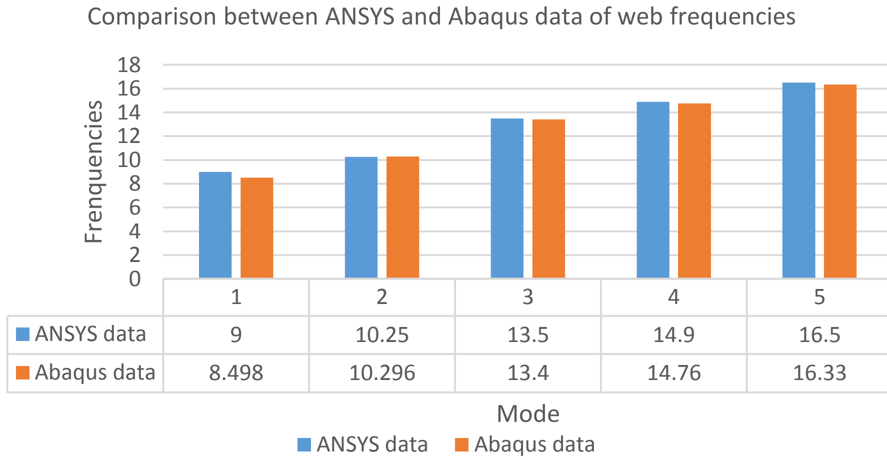

Figure 3: Comparison of linear frequencies between this study (using ABAQUS) and Zhang's ANSYS data [12]

4. Results and discussion

Figures 4 and 5 show the shifting trend of natural frequencies and corresponding mode shapes of spider web structures under different amplitudes of pretension loads. It can be seen that, as the pretension load increasing, the natural frequencies increase gently as well. This is because the stiffness of the whole structure is improved along with the growth of pretension load, as a result of geometric nonlinearity. Surprisingly, mode shapes 8 and 9 have a crossover at the preload amplitude between 1.5625×10-7 N and 7.8125×10-7 N (which cannot be seen clearly due to the natural frequencies of mode 8 and 9 are quite close). This occurrence is attributed to that the hardening phenomenon can influence the performance of spider web structure.

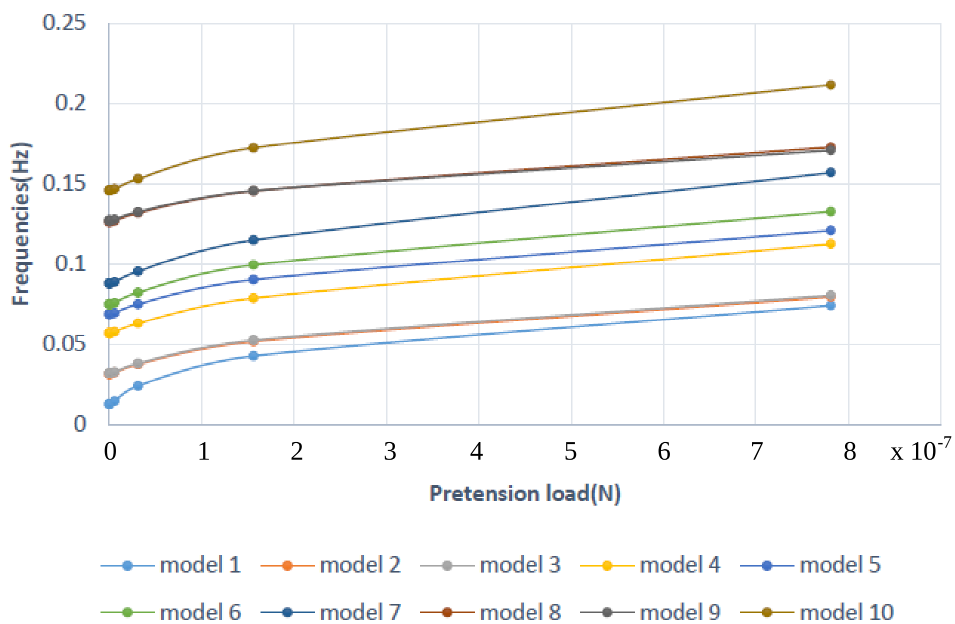

Figure 4: Effects of pretension on nonlinear frequencies.

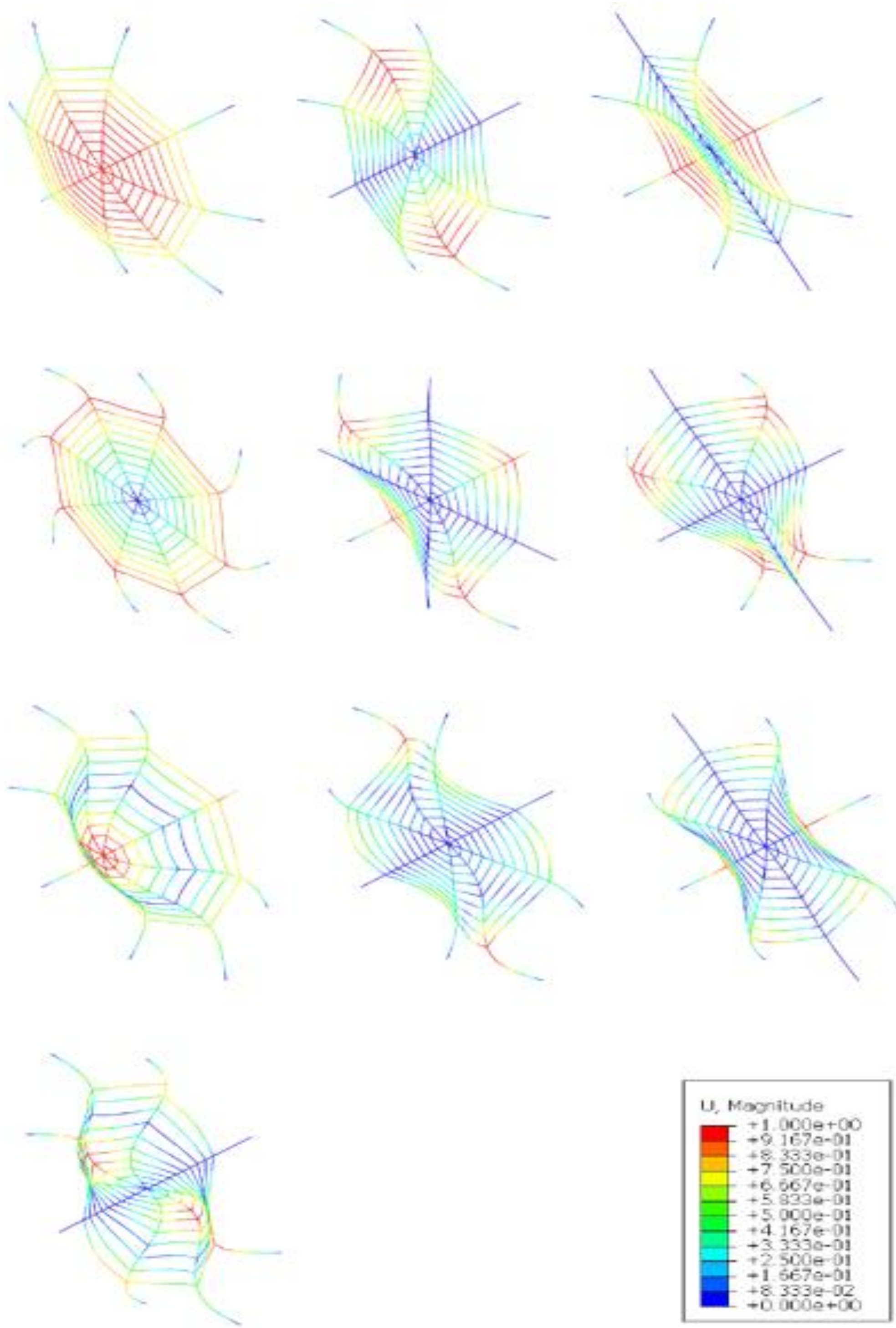

Figure 5: Corresponding mode shapes of spider web structure.

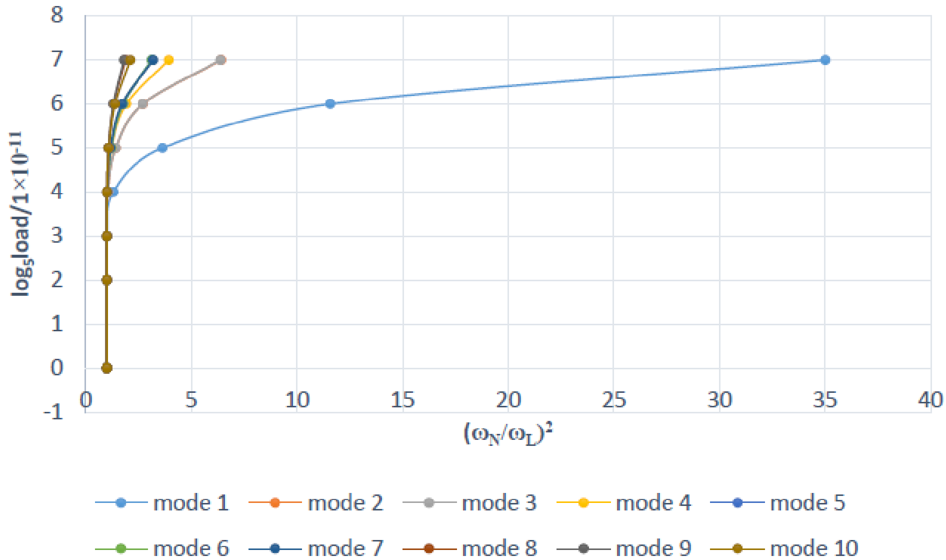

Figure 6: Hardening effect in nonlinear frequencies.

Figure 6 demonstrates the nonlinear frequency ratios $(\omega N/\omega L)^2$ of the spider web and it exhibits that the nonlinear vibrational behavior of the spider web is categorized into hardening type. For mode shape 1 (the lowest mode of vibration), the hardening phenomenon is the most obvious one (which is similar to beam [13-15] but quite different to plate problems [16-17]).

5. Conclusion

Nonlinear geometric finite element models of spider web structures have been developed and validated with previous research. The numerical results in this study show an excellent agreement in frequency predictions with others. The model incorporates the geometric nonlinearity of the thin web structures by applying initial large pretensions (to create an initial excitation from large amplitude displacement of spider web structure). By using the FEMs, nonlinear strain-displacement relationship of spider webs has also been adopted. By increasing the pretension load, the nonlinear vibration behaviour is categorized into hardening type when natural frequencies increase with the amplitude of free vibration, demonstrating as the dynamics of hardening structures. It should be noted also what the amplitude of vibration can signify the crossover behaviour of the spider web structures.

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