The more the merrier

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The more the merrier: grid based modelling of Kepler dwarfs with 5-dimensional stellar grids

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Abstract. We present preliminary results of our grid based modelling (GBM) of the dwarf/subgiant sample of stars observed with Kepler including global asteroseismic parameters. GBM analysis in this work is based on a large grid of stellar models that is characterized by five independent parameters: model mass and age, initial metallicity ($Z_{\rm ini}$), initial helium ($Y_{\rm ini}$), and mixing length parameter ($\alpha_{\rm MLT}$). Using this grid relaxes assumptions used in all previous GBM work where the initial composition is determined by a single parameter and that $\alpha_{\rm MLT}$ is fixed to a solar-calibrated value. The new grid allows us to study, for example, the impact of different galactic chemical enrichment models on the determination of stellar parameters such as mass radius and age. Also, it allows to include new results from stellar atmosphere models on $\alpha_{\rm MLT}$ in the GBM analysis in a simple manner. Alternatively, it can be tested if global asteroseismology is a useful tool to constraint our ignorance on quantities such as $Y_{\rm ini}$ and $\alpha_{\rm MLT}$. Initial findings show that mass determination is robust with respect to freedom in the latter quantities, with a 4.4% maximum deviation for extreme assumptions regarding prior information on $Y_{\rm ini} - Z_{\rm ini}$ relations and $\alpha_{\rm MLT}$. On the other hand, tests carried out so far seem to indicate that global seismology does not have much power to constrain $Y_{\rm ini} - Z_{\rm ini}$ relations of $\alpha_{\rm MLT}$ values without resourcing to additional information.

1 Introduction

Grid based modelling (GBM) is now routinely employed in the determination of stellar parameters using global asteroseismic quantities, i.e. the large frequency separation $\Delta\nu$ and the frequency of maximum power $\nu_{\rm max}$ (see e.g. [1]). Many quantities need to be determined to construct the large grids of stellar models needed for GBM. Leaving aside variations in the input physics employed in stellar evolution calculations, the most important simplifications made in order to keep the problem tractable are the assumption of a fixed mixing length parameter $\alpha_{\rm MLT}$ and a uniquely relation between the initial helium $Y_{\rm ini}$ and metallicity $Z_{\rm ini}$ of the models. The latter, together with the normalization $X_{\rm ini} + Y_{\rm ini} + Z_{\rm ini} = 1$ and an adopted element mixture leave only one free parameter associated with the initial composition of models. This sets the minimal framework for GBM in which each stellar model in the grid is determined by three independent parameters: its age, the initial mass of the evolutionary sequence to which the stellar model belongs (might differ from its actual mass due to mass loss) and the initial $Z_{\rm ini}$ (or any other parameter defining the initial composition). Needless to say, such construction of models inherently include strong prejudices:

that a solar $\alpha_{\rm MLT}$ is universal and that $Y_{\rm ini}$ and $Z_{\rm ini}$ are uniquely and universally related to each other. These are strong and limiting assumptions.

In the framework of GBM the way to break free is to use stellar grids where $\alpha_{\rm MLT}$ is also a free parameter, together with a second parameter linked to initial composition. Based on this necessity, in this work we present GBM results based on a newly computed grid of stellar models based on a five-dimensional parameter space: age, mass (mass loss is negligible for dwarfs and subgiant stars), $Z_{\rm ini}$, $Y_{\rm ini}$, and $\alpha_{\rm MLT}$ are the parameters that characterize a given stellar model in the grid. The grid of models is combined with `BeSPP`, a Bayesian algorithm [2] developed to determine stellar physical quantities from any appropriate combination of spectroscopic, photometric or asteroseismic quantities. Different assumptions regarding galactic chemical enrichment, e.g. by introducing a relation between $Y_{\rm ini}$ and $Z_{\rm ini}$, or regarding dependences of $\alpha_{\rm MLT}$ on surface stellar parameters ($T_{\rm eff}$, $\log g$ and [Fe/H]) can be tested by considering appropriate prior probabilities in `BeSPP`. We apply our methods to the Kepler sample from [1] in order to answer the following questions. How robust is the determination of stellar parameters given our lack of knowledge regarding $Y_{\rm ini}$ and $\alpha_{\rm MLT}$ in particular? Inverting the problem, can global seismology be used to

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establish a history, e.g. in the form of a $Y_{\mathrm{ini}} - Z_{\mathrm{ini}}$ relation, of galactic chemical enrichment? Can it be used to empirically establish a relation between α_{MLT} and stellar properties to, e.g. test recent results from three dimensional stellar atmosphere models [3]?

2 Methods and results

Stellar models have been computed using `GARSTEC`. For the present work, the grid spans a mass range between 0.7 and 1.8 $\mathrm{M}_\odot$ with $\Delta M/\mathrm{M}_\odot = 0.02$ and extends down to $\log g = 3.2$. Y_{ini} and Z_{ini} and α_{MLT} are taken as three independent parameters and the qubic parameter space covered is illustrated in Fig. 1. Note we take Z_{ini}, not [Fe/H] as independent parameter. However, the grid is uniformly spaced in $\log Z_{\mathrm{ini}}$.
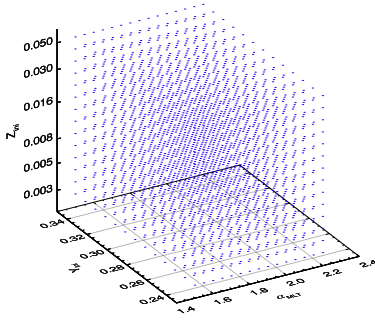
Figure 1: Initial composition and α_{MLT} parameter space of the grid of stellar models used in the present work.

Depending on the problem at hand, priors are included in `BeSPP` that account for different assumptions regarding model parameters. In this work we consider different possibilities regarding priors for initial composition or α_{MLT} as follows:

- **Flat**: flat priors are used for composition and α_{MLT}.

- $\mathbf{\Delta_{YZ} = 1.2}$: a Gaussian prior relates Y_{ini} and Z_{ini} such that

$$p(Y_{\mathrm{ini}}) \propto \exp\left[-\frac{(Y_{\mathrm{ini}} - Y_\Delta)^2}{2\sigma_Y^2}\right], \qquad (1)$$

 with $Y_\Delta = Y_{\mathrm{SBBN}} + \Delta Z_{\mathrm{ini}}$, $\Delta = 1.2$ and $\sigma_Y = 0.01$.

- **3DMLT**: a Gaussian prior relating α_{MLT} with 3D model atmosphere results such that

$$p(\alpha_{\mathrm{MLT}}) \propto \exp\left[-\frac{(\alpha_{\mathrm{MLT}} - \alpha_{3D})^2}{2\sigma_\alpha^2}\right], \qquad (2)$$

 with $\alpha_{3D} \equiv \alpha_{3D}(T_{\mathrm{eff}}, \log g, [\mathrm{Fe/H}])$ from [3] (but shifted by a constant value to match our solar calibrated value $\alpha_{\mathrm{MLT}} = 1.802$) and $\sigma_\alpha = 0.05$.

- **3DMLT + $\mathbf{\Delta_{YZ} = 1.2}$**: combination of Eqs. 1 and 2

$$p(Y_{\mathrm{ini}}, \alpha_{\mathrm{MLT}}) \propto p(Y_{\mathrm{ini}}) \cdot p(\alpha_{\mathrm{MLT}}). \qquad (3)$$

- **SUN**: effectively constraints parameter space to the standard three dimensional space by using

$$p(Y_{\mathrm{ini}}) \propto \exp\left[-\frac{(Y_{\mathrm{ini}} - Y_\Delta)^2}{2\sigma_Y^2}\right] \qquad (4)$$

as before and the solar calibrated $\alpha_{\mathrm{MLT}} = 1.802$.

Input data in our analysis is taken from [1] and includes T_{eff}, [Fe/H], $\Delta\nu$ and ν_{max}. In that work, [Fe/H] values for all stars were taken equal to a mean solar neighborhood value and do not reflect any spectroscopic or photometric determination.
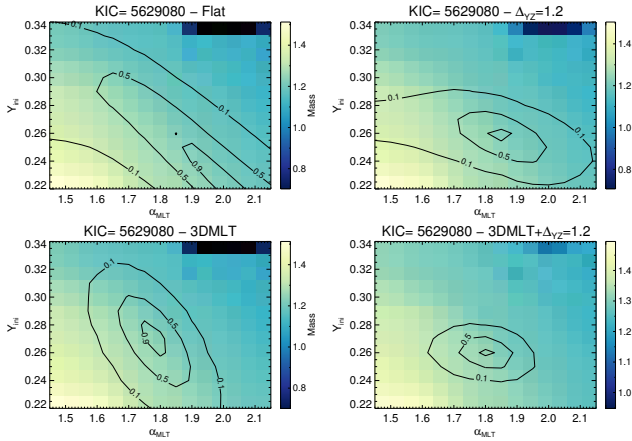
Fig. 2 shows results for the mass determination of KIC 8547279 in the $Y_{\mathrm{ini}} - \alpha_{\mathrm{MLT}}$ plane. Mass is color coded, and at each point in the plot, the mass value marginalized over Z_{ini}, and age is shown. Contours correspond to 0.9, 0.5, and 0.1 probability levels. Priors applied in each case are given in each plot title. Very little constraining power comes from the asteroseismic analysis for Y_{ini} and α_{MLT} when flat priors are applied. This is probably due to a degenerate behavior than α_{MLT} and Y_{ini} have on T_{eff}, as can be seen by the diagonal orientation of the contours. Increasing either Y_{ini} or α_{MLT} leads to higher T_{eff} in models, but given that T_{eff} is bound by observations, then our results show a compensation effect: higher Y_{ini} correspond to lower α_{MLT} values and viceversa.
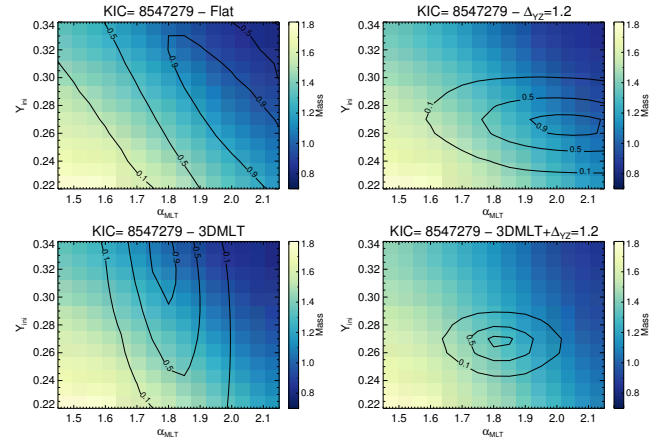
Figure 2: Impact of using different priors in the mass determination of KIC 8547279. For each $Y_{\mathrm{ini}} - \alpha_{\mathrm{MLT}}$ point, mass value is the mean value of the mass PDF after marginalizing the 5-dimensional PDF over age and metallicity. Contours depict isoprobability curves for the mass as a function of Y_{ini} and α_{MLT}.

As priors are added, results become more constrained for Y_{ini} and α_{MLT}, as expected. It is interesting to note, however, that the estimated mass determination is not too sensitive to the choice of priors. In fact, KIC 8547279 is one of the worst cases we have found in this study because the estimated mass varies by about 15% depending on the priors used. Note that the comparison between the flat and the 3DMLT + $\Delta_{YZ} = 1.2$ priors is an extreme one, so this level of variation in mass determinations due to ignorance regarding Y_{ini} and α_{MLT} is probably an upper limit. In Fig. 3 we show one more example, this time for KIC 5629080, which shows almost no variation in its mass

estimate. However, as for the previous case, constraints for Y_{ini} and α_{MLT} come mostly from priors; asteroseismic data is in fact providing little or no information about these two quantities.


**Figure 3:** Same as Fig. 2 but for KIC 5629080.

Fig. 4 summarizes results for the mass determinations in the whole sample by comparing results obtained for the 5 different assumptions on priors given above. In all cases, the x-axis corresponds to priors in Eq. 4. The maximum dispersion is seen in the comparison Flat-Sun (top left panel) and it is at most 15% (except for a few cases). In fact, the fractional mass difference between the two cases has $\sigma_{F,\odot} = 4.4\%$, which can be taken as an estimate of the uncertainty between assuming no prior info on initial composition and α_{MLT}. We believe this is a very encouraging result. The prior on α_{MLT} (bottom left panel) leads to almost the same mass determination as the flat prior, i.e. constraints on α_{MLT} do not affect mass determinations by themselves. On the contrary, relating Y_{ini} to Z_{ini} (Eq. 1) influences mass estimates. Interestingly, the interplay between Y_{ini} and α_{MLT} due to their degeneracy in relation to T_{eff} values, as described before, make α_{MLT} priors relevant if used in combination with constrained initial composition, as shown by comparision of the upper and lower right panels in Fig. 4.

Recently, 3-dimensional model atmospheres have been used to determine the dependence of α_{MLT} on stellar surface properties [3, 4]. But it is not easy to test these results because observational tests have to rely on T_{eff} determinations and, in stellar models, the T_{eff} scale does not depend solely on α_{MLT} but also, for example, on both the metal and helium abundance of stars. Here, we consider whether or not global seismic quantities, in combination with T_{eff} and [Fe/H] measurements, can constrain α_{MLT} in the $T_{eff} - \log g$ plane.

We use BeSPP to determine posterior values for α_{MLT} for our whole sample, using as before different assumptions for priors. Ideally, we would like to obtain good constraints for α_{MLT} in the case of flat priors that could be used to test theoretical determinations of α_{MLT} based on stellar atmosphere models. This has been attempted before, with different data and methods, by [5]. Our initial results are
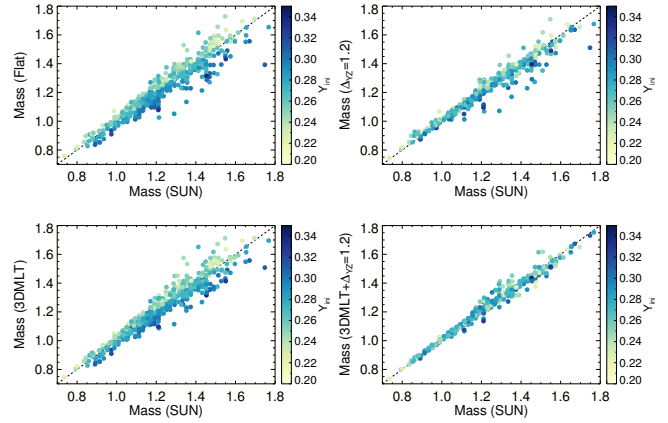
Figure 4: Mass determinations for the whole [1] sample depending on the different priors used in BeSPP as indicated in the axes.
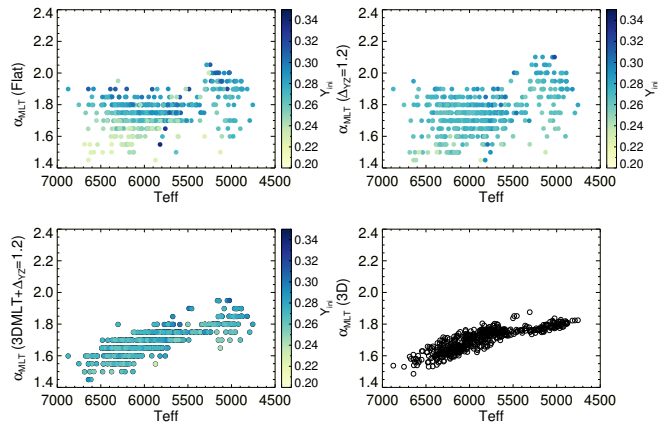

Figure 5: Determination of α_{MLT} using different priors in BeSPP. Bottom right panel shows results directly obtained from α_{MLT} values based on 3D stellar atmospheres [3].

shown in Fig. 5, where α_{MLT} is shown as a function of T_{eff} and color-coded to represent Y_{ini}. For the Flat case, there is a clear correlation between α_{MLT} and Y_{ini}, regardless of T_{eff}. Although it is not impossible that there is a physical dependence of α_{MLT} on the helium abundance of stars, i.e. on their composition, we are more inclined to believe that seismic data and the [Fe/H] from [1] are not enough to give too meaningful constraints on α_{MLT}. The results change partially once the Δ_{YZ} prior is added, but still a correlation between α_{MLT} and Y_{ini} seems to be present. Only after adding the **3DMLT** prior this correlation is almost completely removed (bottom left panel). The lower right panel shows the α_{MLT} values corresponding to our whole sample that correspond to [3] 3D atmosphere models, directly determined from the stars T_{eff}, [Fe/H] and $\log g$ values using formulae given in that reference but with a shift applied so that α_{MLT} values match for the Sun. Clearly, the bottom two panels look quite similar, because our **3DMLT** prior is constructed from [3] results. A more rigurous study is required, at this point, to better assess the possibilities that global seismology has to offer for determin-

ing α_{MLT} values and testing 3D hydrodynamic atmosphere models. Given the correlation with Y_{ini}, i.e. with the stellar composition, it might be crucial to use actual [Fe/H] values in the analysis before firm conclusions can be drawn.

The last exercise we present here relates to determination Y_{ini} from asteroseismic data, much as done above for α_{MLT}, but present results in terms of the enrichment parameter $\Delta = \Delta Y/\Delta Z$, where $\Delta Y = Y_{ini} - Y_{SBBN}$, $Y_{SBBN} = 0.2485$ is the standard Big Bang Nucleosynthesis (SBBN) value and $\Delta Z = Z_{ini}$ because $Z_{SBBN} = 0$. Attempts to constrain Δ from asteroseismology, based on modeling stars based on individual frequencies or combinations, have been plagued with difficulties linked to problematic determination of helium abundances. In fact, in many cases helium abundances are too low, below the SBBN value [6, 7], which makes us wary of the robustness of results for the whole sample. Asteroseismic modelling of stars using individual frequencies is known to introduce biases. This might also be the case when using frequency separation ratios. Is the situation different when global seismic quantities are used instead?

Unfortunately, our first tests show than in fact Y_{ini} values are also at odds with SBBN. Results for Δ are shown in Fig. 6 where, for different assumptions regarding priors, we show resulting Δ distributions. $\Delta < 0$ indicates $Y_{ini} < Y_{SBBN}$. For the flat or the 3DMLT priors, clearly there are many stars that lead to low unrealistic Y_{ini} values. Also, and despite the fact that most Δ values nucleate between 0.5, the distribution looks very broad. When adding the Δ_{YZ} prior, the Δ distribution is peaked around 1.2, but obviously this simply reflects the prior information. The combination of the Δ_{YZ} and 3DMLT priors (lower right panel) gives the best behaved results, actually improving over the Δ_{YZ}-alone case (top right panel). Of course, Δ depends critically on Z_{ini} and, as stated before, [Fe/H] values in our sample are just fiducial values. Only after we improve this aspect, we can make more firm statements about this.
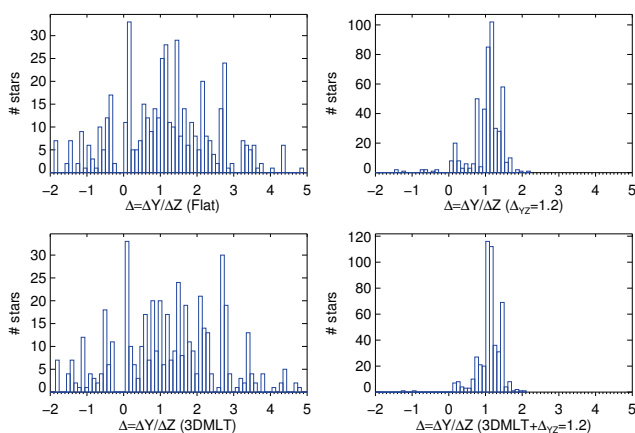
Figure 6: Histograms showing the distribution of $\Delta = \Delta Y/\Delta Z$ for different assumptions regarding priors.

3 Conclusions

We have extended the capabilities of `BeSPP` to determine stellar physical parameters by adding Y_{ini} and α_{MLT} as additional free parameters in its grid of stellar models. The fully new 5-dimensional grid of models relaxes the strong constraints imposed by: 1) the standard assumption that there is a one-to-one correlation between initial metal and helium abundance in stars; 2) that α_{MLT} takes a universal value that is solar calibrated. By adding priors in `BeSPP`, it is possible in principle to study different galactic chemical enrichment models, test α_{MLT} results from 3D model atmospheres and, very importantly, assess the impact of the usual assumptions (one-to-one $Y_{ini} - Z_{ini}$ relation and solar α_{MLT}) in determination of stellar parameters such as mass, radius and age.

In this preliminary exercise we have used [1] sample, so we do not use reliable [Fe/H] values. Therefore,results here are *experimental*. However, one interesting and probably robust conclusion is that mass (and radius - not shown here) determinations using global seismic quantities is robust with respect to our ignorance on α_{MLT} and Y_{ini}.An estimate of the uncertainty in mass determinations yield a typical 4.4% deviation between the two most extreme cases: flat priors on Y_{ini} and α_{MLT} and priors that mimic typical GBM results (e.g. [1]). This is an encouraging result. On the other hand, constraining $Y_{ini} - Z_{ini}$ relations or α_{MLT} values that can be used to test galactic chemical enrichment or stellar atmosphere models seems at the moment a very daunting task because either Y_{ini} or α_{MLT} can introduce changes in the model T_{eff} scale, leading to degenerate results. However, this needs further investigation, particularly by considering actual [Fe/H] in the analysis.

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