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Cooperative Control of Metro Trains to Minimize Net Energy Consumption

Yun Bai, Yunwen Cao, Zhao Yu, Tin Kin Ho *Member*, *IEEE*, Clive Roberts *Member*, *IEEE*, Baohua Mao

Abstract—**With the increasing concerns on energy consumption and operating cost in metro systems, energy saving on train operation attracts significant attentions. Previous studies have mainly focused on optimal control of a single train and energy-efficient train timetabling. The former does not consider the synchronization of motoring and braking trains, which cannot ensure the proper utilization of regenerative energy on the metro lines without energy storage systems. The latter includes scheduling train operations to synchronize motoring and braking trains for better utilization of regenerative energy. However, the overlapping time of motoring and braking trains is usually as short as a few seconds and the energy reduction might be made impossible by train delays which are common in practice. This paper presents a model framework, on the extents of motoring/braking of train acceleration and station stopping, as well as the locations of switching train operation modes, for real-time cooperative control of multiple metro trains. The objective is to minimize the net energy consumption with the consideration of utilizing regenerative energy. A cooperative co-evolutionary algorithm is developed to attain the solution of the proposed model. Case studies on a real-life metro line demonstrate the energy saving performance of the proposed approach compared with separate train control and timetable optimization, from no disturbance to a good range of delays. The results also indicate that partial motoring in train acceleration and partial braking in station stopping achieve better net energy reduction, in comparison with the full motoring/braking preferred in previous studies.**

Index Terms—**Metro train, cooperative control, energy saving, regenerative braking, co-evolutionary algorithm**

I. INTRODUCTION

AS transportation accounts for one third of the total energy consumption in the world, growing concerns on the environment make efficient utilization of energy in transportation systems essential. Metro operation takes up a significant proportion of energy consumption in transport within major cities because of its high traffic volume even though metro is already one of most energy-efficient transport modes. On the other hand, energy consumption generally accounts for a quarter of operating cost across most metro companies. As a result, metro operators around the world are proactively seeking to reduce energy consumption.

Efficient utilization of energy consumed by metro trains is a very popular research topic since train traction takes up half of the energy consumed in metro operation [1]. The objectives of the previous studies can be categorized into infrastructure improvement and train operation optimization. Infrastructure improvement includes energy-oriented design of track profile, weight reduction of vehicles, installation of energy storage devices and reversible substations [2]. Optimization on train operation is a more attractive option for operators because of the lower capital investment and it focuses on optimal train control and energy-efficient timetables [3], [4].

Optimal train control is to identify the driving trajectory to minimize traction energy consumption for the required train movement. In 1960s, the optimal control theory, particularly *Pontryagin's Maximum Principle*, was applied to explore energy-efficient driving strategies for trains without regenerative braking [5]. It was identified that energy-efficient control of a train consists of the optimal switching among four operation modes: full motoring, cruising, coasting, and full braking [6]. Numerical methods were then applied to look for the optimal sequence and switching locations among different operation modes for energy saving under a given run-time [7], [8]. The cruising may not be adopted if the inter-station distances are relatively short. Evolutionary algorithms were widely applied to locate the starting and ending points for coasting in train inter-station runs [9] - [11]. Nonlinear programming models have also been employed to depict the optimal train control problems and solved by commercial software [12] - [15]. Furthermore, fuzzy predictive control and expert systems have found applications in the energy-efficient train control [16] - [18]. Recently, the optimal control of a single train has been extended to include regenerative braking [19], [20]. However, the utilization of regenerative energy was assumed as a constant, which is only suitable for the metro lines equipped with energy storage systems.

While the above studies only focused on the optimal control of a single train, separate control of each train does not

necessarily lead to the minimal net energy consumption, especially in metro systems where regenerative braking is applied without energy storage systems [3]. The net consumption is the difference between traction energy consumed by all trains along the line and the utilized regenerative energy. The regenerative energy was first to supply the auxiliary equipment, such as air-conditioning and lighting onboard. The remaining energy is then fed back into power distribution system and it can be immediately used to supplement motoring of the trains located in the same Power Supply Region (PSR) while a PSR is defined as the area in which power is provided for by the same source. The regenerative energy is otherwise dissipated in heating resistors if there is no energy storage device and the substation is irreversible, which is still quite common in most metro systems [21]. The utilization of regenerative energy is largely determined by the overlapping time between motoring and braking trains in the same PSR [22]. The synchronization of motoring and braking trains can be prolonged by partial motoring for trains accelerating from stations and also partial braking for trains approaching stations. The net energy consumption may be reduced for better utilization of regenerative energy, at the expense of slightly higher traction energy for each train.

Formulation of energy-efficient timetables is another topical research area for metro operation. Early studies included matching transport demand with appropriate means of supply, such as smaller or shorter trains, multiple routing plans and flexible headway [23]. Energy saving is possible through the lower level of service provision in off-peak hours. Another direction was to find the optimal amount of additional run-time to achieve a trade-off between travel time and energy consumption [24], [25]. The allocation of the additional run-time among train inter-station runs, as well as the other timetable parameters, were then optimized for traction energy reduction without considering regenerative energy [26] - [30].

Further energy-efficient timetable studies focus on the utilization of regenerative energy, which may provide up to one third of the energy required for the trains [2]. A power flow model was developed to compute the recovered energy during regenerative braking, and a mathematical programming model was then designed to maximize the utilization of regenerative energy by optimizing timetable configurations [31]. For simple computation of recovered energy, the overlapping time between motoring and braking trains was maximized, by adjusting train headway and dwell time at stations, to enable better utilization of regenerative energy [32]. In subsequent works, the net energy consumption was minimized by identifying the optimal train inter-station run-time and dwell time at stations, as well as headways [33] - [35].

As the overlapping time can be as short as a few seconds, the scheduled synchronization of motoring and braking trains could be shortened or even eliminated completely due to train delays, which are common in practice [36]. On the other hand, service capacity and quality are usually given higher priority than energy performance in practical train scheduling [37]. As a result, there are very few applications of energy-efficient

timetables in practical metro operation to improve the utilization of regenerative energy.

The research on cooperative control of multiple trains begins to emerge on energy reduction for train movements. Liu *et al.* explored the cooperative control of two adjacent trains and presented a control approach for the following train to better utilize the regenerative energy produced by the preceding train [38]. Sun *et al.* optimized the distribution of regenerative energy of the braking train to the neighboring trains and then modified their trajectories to absorb the regenerative energy [39]. However, these studies did not attempt to formulate a general model on the cooperative control of multiple metro trains to minimize the net energy consumption.

In this study, a model framework on real-time cooperative control of multiple trains is proposed, considering the current train status. It aims to find out the optimal locations within the inter-station runs to switch to among train operation modes and the extents of motoring and braking for different trains in their inter-station runs, to minimize the net energy consumption. Practical constraints on train operation, such as punctuality and speed limits, are considered in the proposed model. A cooperative co-evolutionary algorithm is developed to attain cooperative control schemes for multiple trains, satisfying the requirements of real-time solutions. Comparison of energy performance among the proposed approach, separate train control and energy-efficient timetabling is then analyzed.

The remainder of this paper is organized as follows. Section II describes the framework on cooperative control of multiple trains. Section III formulates the optimization model on cooperative control of multiple trains for energy saving. Section IV develops the algorithm to solve the proposed model. Section V discusses the effectiveness of the proposed approach on the basis of case studies on a real-world metro line. Finally, Section VI gives the conclusions of this study.

II. COOPERATIVE CONTROL OF MULTIPLE TRAINS

Communication-Based Train Control (CBTC) has been commonly adopted in modern metro systems. It allows trains to communicate bi-directionally with the Centralized Train Control (CTC) Center. It is thus possible for trains to exchange information through the moderation of the CTC and even liaise on train control decisions. CBTC provides the platform for cooperative control of multiple trains, which is the main theme of this study.

A. Framework on cooperative control of multiple trains

Generally, train status are dynamic parameters. For example, train delay might arise and train weight varies in different inter-station runs due to passengers boarding and alighting at stations. Therefore, it does not make any practical sense to pursue the theoretical global optimal solution for all trains in all inter-station runs, while leaving out the possible changes of train status.

In this study, the cooperative control on all trains running on the metro line is decomposed into a series of local optimization processes and each local optimization is triggered whenever a train is going to depart from a station based on the real-time
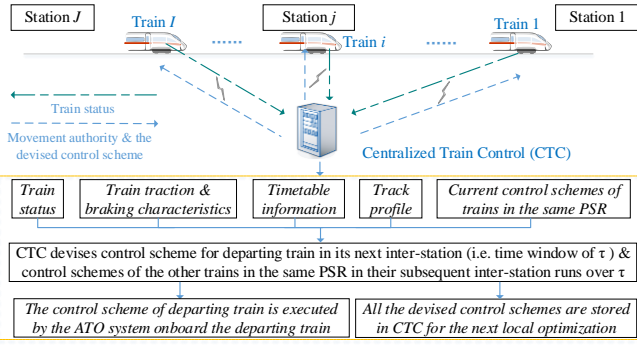
Fig. 1. Framework on cooperative control of multiple metro trains

status. The local optimization is to devise the control scheme for the departing train in the next inter-station run and the control schemes of the other trains in the same PSR over the time window of τ, in which the departing train completes its next inter-station run. The objective of local optimization is to minimize total net energy consumption of all trains involved while the scheduled arrival times and speed limits are observed.

Fig. 1 illustrates the proposed cooperative control framework with I trains running through J stations in a given PSR. With a CBTC system, every train continuously reports its status, such as position, speed, and weight to the CTC. When train i is going to depart from station j, a local optimization is attained in CTC based on the real-time status and pre-stored information. The pre-stored information includes train traction and braking characteristics, timetable, track profile, and current control schemes of other trains in the same PSR which have been obtained in the previous local optimizations.

The local optimization not only devises the control scheme for the departing train i in the next inter-station run, but also develops the future control schemes for the other trains in the same PSR. The control schemes of the other trains in their current inter-station runs have been obtained prior to this local optimization and they are regarded as one of the constraints for the current local optimization. For the other trains, only the control schemes in their subsequent inter-station runs over the time window of τ, are developed in this local optimization. The devised control scheme of departing train will be executed by the Automatic Train Operation (ATO) system, which ensures the train following the devised control scheme strictly in its inter-station run. The developed control schemes for the other trains are stored as projected solutions but they could be altered in the next local optimization due to train status changes.

The formulation of train control scheme is to find the switching locations of train operation modes within an inter-station run and the extents of motoring and braking of the train, which will be illustrated by Section II-B in details. In the case of delay occurrence, the trains will recover delays as soon as possible because service punctuality is more important than energy performance.

The proposed framework enables attainment of the overall optimal control on multiple trains, rather than separate optimal control on individual trains. The control scheme of each inter-station run is still based on switching locations of operation modes, which has been proven to be a useful means

to reduce traction energy [34]. The cooperative control also includes synchronizing motoring and braking trains for better utilization of regenerative energy. In addition, the cooperative control is adaptive to changes of train status.

B. Control scheme for individual train inter-station run

Full motoring, cruising, coasting, and full braking are commonly recognized as the usual sequence of individual train inter-station run for energy saving under a given run-time [40]. Full motoring and full braking enable short acceleration time from standstill to the maximum permissible speed and deceleration time for stops, which allow more time for coasting as the inter-station run-time is fixed in timetable. Coasting implies train movement is carried by its momentum, which presents opportunities to save traction energy [41]. With cruising, a train travels at a constant speed, which helps to reduce energy consumption caused by resistance [42].

However, partial motoring and partial braking may be preferred in cooperative control of multiple trains, as they prolong the time of train motoring and braking and allow more room to synchronize motoring and braking of adjacent trains to obtain better utilization of regenerative energy. The operation sequence of motoring, cruising, coasting, and stop braking is adopted for inter-station runs in this study, as shown in Fig. 2.
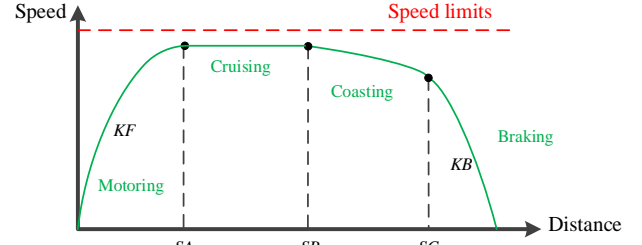
Fig.2 Control scheme for an inter-station run

The extents of motoring and braking and the switching locations among operation modes are the key parameters of a complete control scheme of an inter-station run. KF and KB are the extents of motoring and braking, in term of percentages of the full traction and braking force. SA, SB, and SC are the locations to start cruising, coasting, and braking respectively. SC is obtained by locating the intersection point of coasting and braking profiles. The coasting profile is attained once the extent of motoring and starting points of cruising and coasting are determined. The braking profile can be backtracked from the station stopping point with the extent of braking. Consequently, the control scheme for a train in its inter-station run includes the starting points of cruising and coasting as well as the extents of motoring and braking, i.e. SA, SB, KF, KB.

Fig. 3 illustrates the process of control schemes formulation for train inter-station runs, taking three trains in one PSR as an example. When train 2 stops at station C at time T_k, CTC devises the control scheme for train 2 in the next inter-station run (i.e. from C to D), as well as the control schemes of train 1 from E to F and train 3 from B to C. The development of control schemes takes into account the current control schemes of train 1 from D to E and train 3 from A to B, which were already determined before T_k. As such, the motoring of train 2 when it
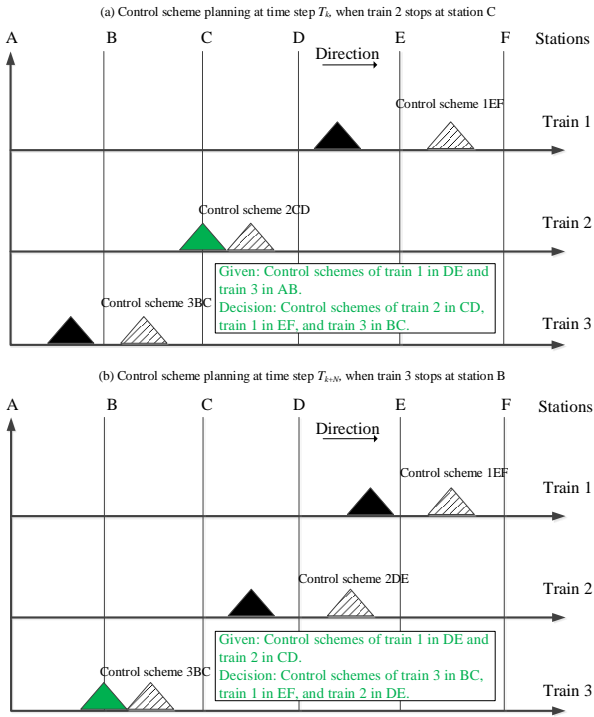
Fig. 3 An illustrative process of control scheme formulation under cooperative control of multiple trains

departs from station C could coincide with the braking control of train 1 and train 3 when they approach stations E and B respectively. In addition, the braking control of train 2 when it approaches station D could be aligned with the motoring control of train 1 from E to F and that of train 3 from B to C. Such possibilities present opportunities for better utilization of regenerative energy.

Upon the control schemes formulated at T_k, all trains proceed accordingly. When train 3 arrives at station B, the next local optimization commences and CTC devises the control schemes of train 3 from B to C, train 1 from E to F and train 2 from D to E, taking into account the current control schemes of train 1 from D to E and train 2 from C to D, as shown in Fig. 3(b). The local optimizations then continue with each train arriving at its respective next station. This approach allows consideration of train delays by changing the scheduled run-time in the next inter-station when delay arises. The train control schemes are therefore developed in response to the latest train status.

It is necessary to point out that this study only illustrates the merits of the cooperative control framework with trains on one direction. It is entirely possible that the proposed framework is applied to cooperative control of trains on bi-directional traffic. In addition, the proposed framework does not impose any limit on the numbers of trains and stations and it is also suitable for the scenario where there are more than one trains between two stations at the same time.

III. MODEL FORMULATION

This section describes the mathematical model on cooperative train control.

A. Model assumptions

The following assumptions are made in the proposed model.

- Trains are able to communicate bi-directionally with the CTC center. All trains are operated on ATO and they use the same type of traction equipment. With ATO, each train strictly follows the devised control scheme in its inter-station run.
- The ratios of traction equipment efficiency in motoring and regenerative braking are regarded as constants.
- Energy loss on transmission is considered negligible as it accounts for a very small proportion of total consumption.
- There is no energy storage system. The regenerative energy can be immediately used to support auxiliary equipment onboard the braking train and to assist motoring of other trains in the same PSR. Unused regenerative energy is dissipated as heat on resistors.
- Trains are considered as points to alleviate the computation burden of real-time cooperative train control. The curvatures and gradients can be re-modelled off-line as a set of effective gradients with consideration of train length, to obtain the equivalent effects.

B. Decision variables

As mentioned in Section II, the cooperative control on multiple trains moving within a PSR is decomposed into a series of local optimization problems. In other words, whenever a train is going to depart from a station, CTC will devise the optimal control schemes of all trains in the same PSR over the time window of τ, in which the departing train completes its next inter-station run.

In a local optimization, the decision variables are a set of train control schemes $\psi=\{\phi_i^j|i=1,2,\cdots,I;j=n(i)+1,\cdots,n(i)+l_i\}$.

$\phi_i^j = \{SA_i^j, SB_i^j, KF_i^j, KB_i^j\}$ represents the control scheme of train i in the j-th inter-station. SA_i^j and SB_i^j are the relative locations of train i to apply cruising and coasting in the j-th inter-station, which are the traveled distances before train starts cruising and coasting over the length of this inter-station run. KF_i^j and KB_i^j are the extents of motoring and braking of train i in the j-th inter-station. All these four variables are defined between 0 and 1, i.e. $0<SA_i^j, SB_i^j, KF_i^j, KB_i^j\leq1$.

I denotes the number of trains in the PSR. $n(i)$ represents the current inter-station where train i is located. l_i denotes the number of whole inter-station runs for train i over the time window of τ, which allows that the other train in the same PSR completes more than one inter-station runs during the next inter-station run of the departing train.

C. Objective function

The goal is to minimize the total net energy consumption of all trains in the PSR which is expressed as

$$E_{net} = \sum_{i=1}^{I} \sum_{j=n(i)+1}^{n(i)+l_i} \sum_{k=1}^{100} \max\{0, E_{tra}(\phi_i^j,k)- \sum_{m=1,m\neq i}^{I} \sum_{u=n(m)}^{n(m)+l_m} \sum_{x=1}^{100} E_{reg}(\phi_m^u,x)\cdot\delta_i^m(x,k)\} \quad (1)$$

where k and x are the indexes of distance step within an inter-station run, which is divided into 100 equal-distance steps. E_{tra} and E_{reg} are the required traction energy for train movement and the regenerative energy produced by the other trains in the PSR. m is the index of other trains and u is the index of inter-station. $\delta_i^m(x,k)$ is a parameter between 0 and 1 and it is attained by

$$\delta_i^m(x,k)=\mathrm{O}(\Delta t_m^u(x),\Delta t_i^j(k))/\Delta t_m^u(x)\,, \qquad (2)$$

where $\mathrm{O}(\Delta t_m^u(x),\Delta t_i^j(k))$ is the overlapping time of train m at the x-th distance step in the u-th inter-station and train i at the k-th distance step in the j-th inter-station, and $\Delta t_m^u(x)$ is the travel time of train m at the x-th distance step in the u-th inter-station.

The net energy consumption is the difference between the required traction energy for train movement and the regenerative energy concurrently produced by the other trains in the PSR. Generally, the former is higher than the latter. However, the regenerative energy might be higher than the required traction energy when more trains are braking than motoring in the PSR. The net energy consumption at each distance step must be nonnegative since the rest of regenerative energy, which cannot be used timely, will be dissipated.

The regenerative energy is attained by

$$E_{\mathrm{reg}}(\phi_m^u,x)=\begin{cases}B_m^u(x)\cdot\Delta s^u\cdot\eta_r-P_a\cdot\Delta t_m^u(x) & \text{if } v_m^u(x)\ge v_r \text{ and } B_m^u(x)>0\\ 0 & \text{otherwise}\end{cases}, \quad (3)$$

where $B_m^u(x)$ is the braking force of train m at the x-th distance step in the u-th inter-station. ΔS^u is the length of distance step in the u-th inter-station. η_r represents the traction equipment efficiency in converting kinetic to electrical energy during regenerative braking. P_a is the power demand of auxiliary equipment onboard the train, which firstly utilizes the regenerative energy produced by the train itself as much as possible. No regenerative energy is produced when the speed of braking train $v_m^u(x)$ is lower than a certain value v_r, in which case train adopts full frictional braking.

The required traction energy is calculated by

$$E_{\mathrm{tra}}(\phi_i^j,k)=\begin{cases}\max(0,P_a\cdot\Delta t_i^j(k)-B_i^j(k)\cdot\Delta s^j\cdot\eta_r) & \text{if } v_i^j(k)\ge v_r \text{ and } B_i^j(k)>0\\ F_i^j(k)\cdot\Delta s^j/\eta_t+P_a\cdot\Delta t_i^j(k) & \text{otherwise}\end{cases}, \quad (4)$$

where $F_i^j(k)$ is the traction force of train i at the k-th distance step in the j-th inter-station and η_t represents the traction equipment efficiency in converting electrical to kinetic energy during motoring. When regenerative braking is applied, the required traction energy is the nonnegative difference between the energy consumed by auxiliary equipment and the regenerative energy produced by the train itself. When the train is motoring, both traction motor and auxiliary equipment consume energy. When the train is coasting or full frictional braking is adopted, the traction force is equal to 0 and only auxiliary equipment consumes energy.

The traction force is calculated by

$$F_i^j(k)=\begin{cases}KF_i^j\cdot F_{\max}(v_i^j(k)) & \text{if } s_i^j(k)\le SA_i^j\\ \max(0,W_i^j(k)) & \text{if } SA_i^j<s_i^j(k)\le SB_i^j\\ 0 & \text{otherwise}\end{cases}, \quad (5)$$

and the braking force is attained by

$$B_i^j(k)=\begin{cases}KB_i^j\cdot B_{\max}(v_i^j(k)) & \text{if } SC_i^j<s_i^j(k)\le 1\\ \max(0,-W_i^j(k)) & \text{if } SA_i^j<s_i^j(k)\le SB_i^j\\ 0 & \text{otherwise}\end{cases}, \quad (6)$$

where $F_{max}(v_i^j(k))$ and $B_{max}(v_i^j(k))$ are the maximal traction and braking force when train speed is $v_i^j(k)$, which are obtained from the train traction and braking characteristics provided by the manufacturers. $s_i^j(k)$ is the relative location of train i at the k-th

distance step in the j-th inter-station, which is the traveled distance over the length of this inter-station run, $0\le s_i^j(k)\le 1$. $W_i^j(k)$ is the resistance acting upon train i at the k-th distance step in the j-th inter-station.

Train resistance contains the basic resistance due to frictions and air drag, and the additional resistance caused by track gradients and curvatures. The resistance depends on train speed and the terrain condition where the train is located [41]. The total resistance acting upon the train at each distance step is

$$W_i^j(k)=M\cdot\{g(s_i^j(k))+r(s_i^j(k))+\alpha+\beta\cdot v_i^j(k)+\gamma\cdot[v_i^j(k)]^2\}\,, \quad (7)$$

where α,β,γ are the coefficients to calculate frictions and air resistance to train per unit mass. $g(s_i^j(k))$ and $r(s_i^j(k))$ are the additional resistance per unit mass on train caused by gradients and curvatures. M is the train mass including both rolling stocks and passengers.

According to Newton's Second Law, train acceleration rate, speed and travel time at each distance step are attained by

$$a_i^j(k)=[F_i^j(k)-W_i^j(k)-B_i^j(k)]/M\,, \qquad (8)$$

$$v_i^j(k+1)=\sqrt{[v_i^j(k)]^2+2a_i^j(k)\cdot\Delta s^j}\,, \qquad (9)$$

$$\Delta t_i^j(k)=[v_i^j(k+1)-v_i^j(k)]/a_i^j(k)\,, \qquad (10)$$

where $a_i^j(k)$ is the acceleration rate of train i at the k-th distance step in the j-th inter-station.

D. Constraints

The constraints on train operation are given as follows.

1) Safety constraints

Train speed throughout the inter-station run must not exceed the static civil engineering speed limits $Z(s_i^j(k))$ imposed by the track geometry, as well as the dynamic speed limits $Q_i^j(k)$ determined by the signaling system to guarantee the safe headway between two successive trains. This constraint is given as below.

$$0\le v_i^j(k)\le\min\{Z(s_i^j(k)),Q_i^j(k)\}\ \forall k\in[1,100],\forall i\in[1,I],\forall j\in[1,J], \quad (11)$$

where J denotes the number of inter-stations in the PSR. $Z(s_i^j(k))$ is obtained by looking up the civil engineering speed limits table, which is provided by track maintenance.

Taking moving block signaling system for example, the dynamic speed limit at each distance step is backward calculated from the target point with service braking rate a', as shown in Fig. 4.

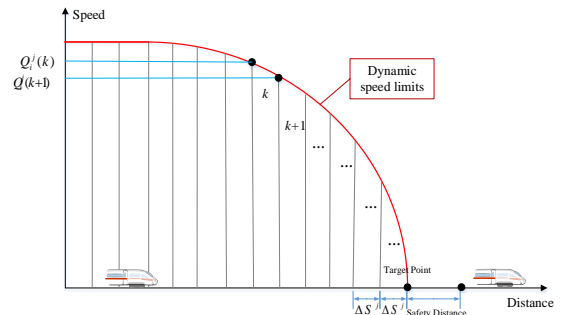$$Q_i^j(k)=\sqrt{2|a'|\cdot\Delta s^j+[Q_i^j(k+1)]^2} \qquad (12)$$

Fig. 4 Dynamical speed limits calculations

2) Punctuality constraint

The difference between train arrival time at stations and the scheduled one in timetable should be limited, as illustrated by

$$\left| TD_i^j - TS_i^j \right| \le \sigma TP_i^j \quad \forall i \in I, \forall j \in J, \qquad (13)$$

where TD_i^j and TS_i^j are the actual and scheduled time of train i arriving at the target stopping point of the j-th inter-station. TP_i^j is the run-time of train i in the j-th inter-station required by the timetable. σ is the allowed deviation of actual inter-station run-time from the scheduled one in term of relative error in percentage. It should be noted that the scheduled run-time in the next inter-station run will be compressed to recover the delay as soon as possible, whenever delay arises.

3) Boundary condition

The starting point for cruising is generally prior to that of coasting, according to the control consequence described in Section II-B. However, the cruising phase might not exist in short inter-station runs. The above boundary constraint is described by

$$0 \le SA_i^j \le SB_i^j \le 1 \quad \forall i \in [1, I], \forall j \in [1, J]. \qquad (14)$$

IV. SOLUTION METHODOLOGIES

The real-time cooperative control on multiple trains for energy saving is a nonlinear optimization problem, which contains a number of decision variables with multiple constraints. It is impractical, if not impossible, to solve this problem by exact methods within the limited computing time. Evolutionary algorithms have been widely applied to solve problems of similar nature as they are able to find the near-optimal solutions quickly. As the complete solution consists of the control schemes of all trains located in the same PSR, the formulation of individual train control scheme should consider the control schemes of other trains in order to incorporate the utilization of regenerative energy. As a result, the co-evolutionary algorithm is applied in this study to solve the proposed model, for its advantage of modeling parallel evolutions of different species when two or more species interact with each other's evolution [43].

A. Cooperative co-evolutionary algorithm

Coevolution is primarily a biological concept, but it has been applied in computer science to improve the efficiency and effectiveness of evolutionary algorithms [44]. Co-evolutionary algorithms can be categorized into competitive and cooperative approaches. In this study, the trains in the same PSR are intended to liaise with each other for better utilization of regenerative energy. Therefore, a cooperative co-evolutionary algorithm is adopted here.

Fig. 5 shows the underlying principle of the proposed algorithm. The complete solution includes the inter-station control schemes of all trains in the PSR. Each inter-station control scheme of a given train is regarded as a species and the possible control schemes in this inter-station run are considered as individuals for this species. The optimal solution of each species is attained by the evolutionary algorithm, as described

in Section IV-B, with parallel computing. Each species should evolve separately according to the fitness of the complete solution, which is the system net energy consumption. Separate evolution means the individuals of a specific species mate amongst themselves, while mating with other species is not allowed. The only interactions among different species are the energy performance evaluation on each individual of any species, which should be combined with individuals in other species to attain the utilized regenerative energy and the net energy consumption.
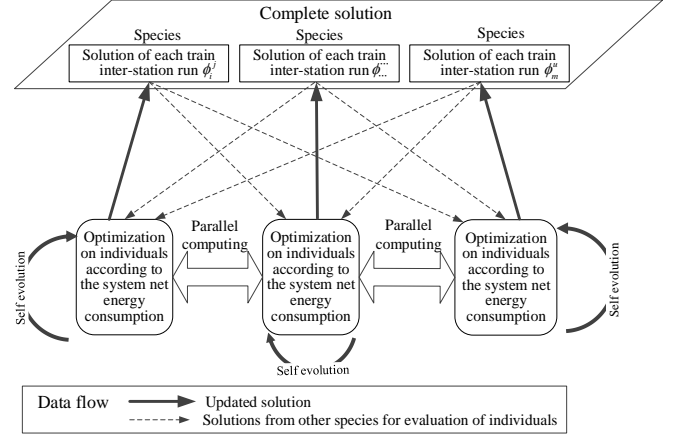
Fig.5 Underlying principle of the cooperative co-evolutionary algorithm

B. Hybrid GA-SA searching for self-evolution

The trade-off between computation efficiency and solution optimality should be taken into account in attaining optimal solution for each species. Inspired by natural evolution such as selection, recombination and mutation, Genetic algorithm (GA) is widely applied in real-time optimization problems as it is able to find the near-optimal solutions quickly [45]. As a population-based evolutionary algorithm, GA has a good global exploration in the search space. However, with traditional GA, the competition only occurs among the individuals in offspring and some excellent parents may be lost due to recombination and mutation. Elitism selection is able to keep excellent individuals in the next generation, but it may lead to premature convergence.

The above drawback can be alleviated by proper selection techniques to maintain a diverse population of solutions. For example, incorporating the Metropolis criterion of simulated annealing (SA) in accepting offspring individuals of GA is commonly adopted [46]. Inspired by annealing in metallurgy, SA allows a decreasing probability of accepting worse solutions, which provides a diverse population for GA without compromising solution convergence.

In this study, a hybrid GA-SA searching procedure is developed to attain the optimal control scheme of each train inter-station run. The process is illustrated in Fig. 6. The key steps of the proposed algorithm are as follows.

Step 1. The parameters, such as population size, recombination probability, mutation probability, the maximum number of generations, annealing initial temperature and cooling coefficient, are given.

Fig. 6 The searching procedure of the hybrid GA-SA algorithm

Step 2. The initial population for each species is generated randomly. Floating-point coding is adopted for chromosome representations of individuals because of its higher efficiency in coding and decoding. All the four decision variables for each inter-station run are normalized to float numb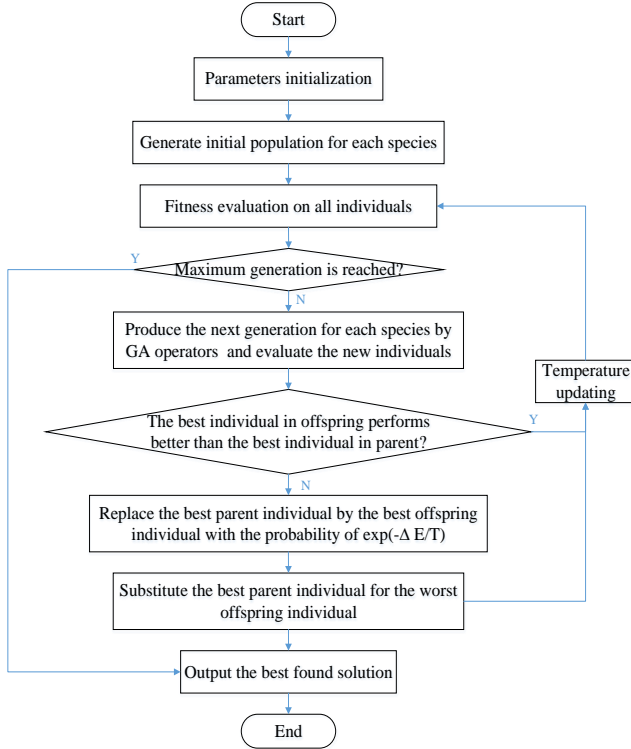ers from 0 to 1. For example, the chromosome $\phi = \{0.15, 0.23, 0.85, 0.87\}$ denotes the extents of motoring and braking are 0.85 and 0.87, respectively, and the locations to start cruising and coasting are 15% and 23% of the overall length of the inter-station distance.

Step 3. The performance of individuals of each species is to be evaluated. In this study, the evaluation of each train control scheme should be combined with the control schemes of other trains. Therefore, any individual in different species should be combined with each other to form a complete solution. Train control schemes are then obtained by decoding the chromosomes of individuals. Finally, the net energy consumption of any combination of individuals is taken as the solution fitness. The complete solution with the lowest fitness value represents the best individual combination. When the operation constraints are not respected, the solution fitness is assigned with a very large value, so that the solution will be discarded in the next generation.

Step 4. GA operators are employed to produce the offspring individuals. The parent individuals are selected by spinning the roulette wheel to breed new individuals through recombination and mutation. The recombination means that two genes, randomly selected from different chromosomes in the same species, are exchanged. For example, if the parent chromosomes are $\phi_g = \{q_1, \cdots q_i, \cdots q_4\}$ and $\phi'_g = \{q'_1, \cdots q'_i, \cdots q'_4\}$, and randomly selected genetic index to be exchanged is i, the offspring individuals are produced as $\phi_{g+1} = \{q_1, \cdots q'_i, \cdots q_4\}$ and $\phi'_{g+1} = \{q'_1, \cdots q_i, \cdots q'_4\}$. Mutation is then adopted to replace the

value of a gene in a chromosome with a random value within valid range. Assuming the chromosome is $\phi_{g+1} = \{q_1, \cdots q'_i, \cdots q_4\}$ and randomly generated genetic index for mutation is i, the new individual is produced as $\phi''_{g+1} = \{q_1, \cdots q^*_i, \cdots q_4\}$, where q^*_i is the new gene generated randomly.

Step 5. The offspring individuals are accepted with Metropolis criterion of SA after the performance evaluation. The best individual in offspring is kept if it is better than the best one in the parent generation. Otherwise, the best offspring individual replaces the best parent with the probability of $\exp(-\Delta E/T)$, where ΔE is the difference on fitness value between the best solutions of offspring and the parent and T is the current temperature in SA. The updated best parent then replaces the worst offspring individual. The excellent offspring is compulsorily accepted while the inferior offspring individual still has a chance to be kept, which helps to improve the diversity of the new population.

Step 6. The evolutionary process from Step 3 to Step 5 is repeated until the maximum number of generation is reached. The temperature T of SA is cooling with a coefficient after each iteration. The cooling temperature determines that the probability to accept inferior solution decreases with the process of evolution, which ensures the convergence of solution in the subsequent evolution.

Step 7. The best solution is the combination of individuals in different species with the lowest fitness in the last generation. The control scheme for the departing train is conveyed to ATO for train control implementation. The control schemes of the other trains are stored as the initial solution for the next local optimization.

V. CASE STUDIES

A segment of Beijing Metro line 5 is adopted to test the performance of the proposed cooperative train control, in comparison with that of separate train control and offline timetable optimization. Extensive analysis is also carried out to assess the impacts of parameters in the proposed model and algorithm on energy performance and computing efficiency. All these studies are conducted by MATLAB R2013a on a PC with 2.6-GHz processor speed and 4-GB memory.

A. Main parameters and set up

The selected track segment consists of 11 stations, passing through key commercial districts in the city center from south to north and including a number of line-transfer stations, as shown in Fig. 7.

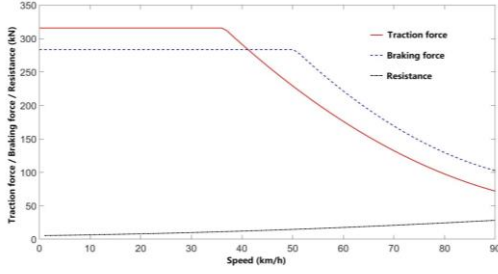Fig. 7 Selected segment from Beijing Metro line 5

Fig.8 Train traction and braking characteristics

Fig. 8 shows the traction and braking characteristics of the trains operation on Beijing metro line 5, where the trains are powered by DC 750 V supply via third-rail. Each train consists of 3 motor cars and 3 trailer cars and it weighs 203 tons. The civil engineering speed limit for train inter-station runs is 80 km/h. The converting efficiency ratios of traction equipment between electrical and kinetic energy on both directions are set at 0.9. The power demand of auxiliary equipment is 6 kWh. A train allows regenerative braking when its speed is higher than 8 km/h while full frictional braking is applied otherwise.

Table I gives timetable information and passenger loading factors in the inter-station runs, which are the actual loaded passengers against the nominal carrying capacity, i.e. 1,424 passengers on one train. It is assumed that all trains carry the same number of passengers in the same inter-station run here, while different loading factors for different trains can be easily adopted in the proposed approach. The scheduled time of each train for the whole journey is 1,135 seconds and the nominal headway is 150 seconds. The deviation of actual inter-station run-time from the scheduled one must be restricted within ±5%. There are 3 PSRs over the selected track and the boundaries are located at the end of inter-stations 3 and 7. There are usually no more than three trains in one PSR at any one time. Therefore, the cooperative control is confined to three trains only here.

TABLE I
NOMINAL RUN-TIME AND DWELL TIME

PSR	Inter-station	Length (m)	Run-time(s)	Dwell time at destination (s)	Passenger loading
I	1	881	70	50	40%
	2	820	70	50	60%
	3	945	75	30	90%
II	4	865	70	45	90%
	5	1000	75	30	60%
	6	792	70	30	100%
	7	875	70	50	30%
III	8	1167	85	30	20%
	9	1050	80	30	30%
	10	1025	75	50	10%

For the GA, the maximum number of generations is 50 and the population size for one generation in each species is 50. The recombination and mutation rates are 0.9 and 0.2, respectively. For the SA, the initial temperature is 100 ℃ and cooling coefficient is 0.98.

B. Energy performance

1) Comparison with separate train control

Table II compares the energy performance between the cooperative train control in this study and the separate train control formulated in [8]. Separate control means each train does not consider the operations of other trains and the synchronization of motoring and braking trains is not considered. The inter-station runs adopt the traditional energy-efficient control consisting of full motoring, cruising, coasting, and full braking. The cruising speed and the switching locations among different operation modes are attained by numerical method, with the objective of minimizing the traction energy consumption while satisfying the scheduled run-time requirement [8]. The cooperative control here contains two different strategies for inter-station runs. The first one is the traditional energy-efficient control with full motoring/braking. The second one adopts motoring, cruising, coasting, and braking, as mentioned in Section II-B. Partial motoring and partial braking are allowed during train acceleration and deceleration of inter-station runs with the second strategy.

TABLE II
ENERGY PERFORMANCE OF COOPERATIVE CONTROL AND SEPARATE CONTROL

Items	Separate control	Cooperative control	
	$KF=KB=1$	$KF=KB=1$	$KF, KB\in[0,1]$
Net energy consumption (kWh)	310.1	279.7	272.8
Traction energy consumption (kWh)	343.1	358.9	365.5
Recovered regenerative energy (kWh)	213.1	226.3	230.2
Utilized regenerative energy (kWh)	33	79.2	92.7
Utilization rate of regenerative energy	15%	35%	40%
Saving on net energy consumption	-	10%	12%
Maximum computing time (s)	9	19	23
Average computing time (s)	7	15	18

This study focuses on the metro lines equipped with ATO system, where each train strictly follows the devised control scheme in its inter-station run. As such, there is no need to regenerate a new control scheme during train inter-station run. In other words, the control scheme computation only needs to be completed well within the minimum dwell time, which is 30 seconds in this study. As shown in Table II, the maximum computing time with any control approach above is less than 25 seconds, which ensures the feasibility of these three control approaches in practice.

The results in Table II demonstrate that separate train control is able to minimize the traction energy consumption, which is consistent with the previous study [8]. However, the net energy consumption with separate control is significantly higher than that of cooperative control because the utilization of regenerative energy is much lower. The cooperative control allowing partial motoring/braking performs even better on net energy consumption, and the saving reaches 12% when compared to separate control. The reason is that the utilized regenerative energy increases by 59.7 kWh, even though traction energy consumption is 22.4 kWh higher.

To further explain how cooperative control helps save net energy consumption, a direct comparison of train trajectories under different control approaches is given in Fig. 9, taking

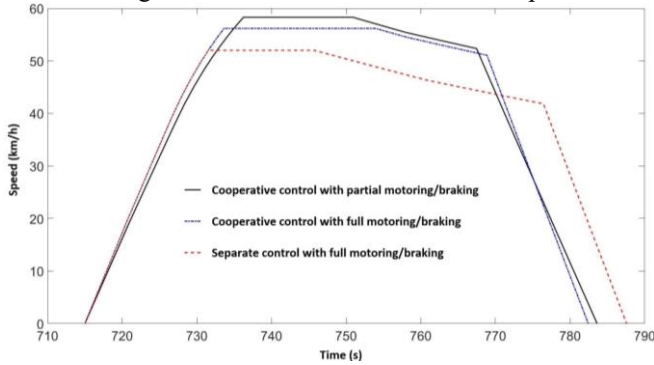train 2 running in the 6-th inter-station as an example.

Fig.9 Train trajectories for an inter-station run with cooperative control and separate control
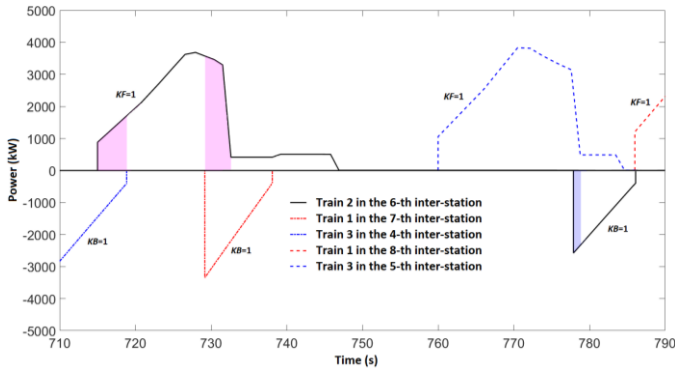

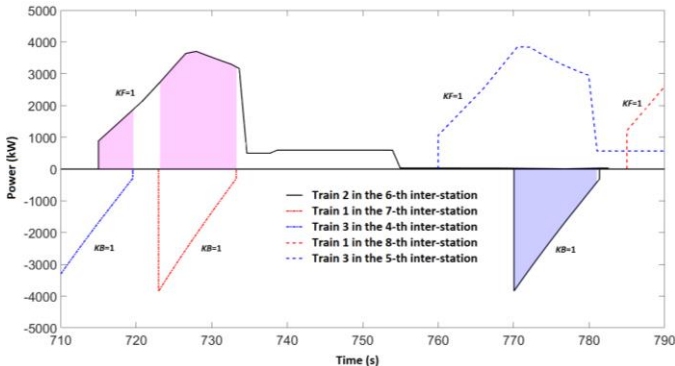Fig. 10 Train power under separate control

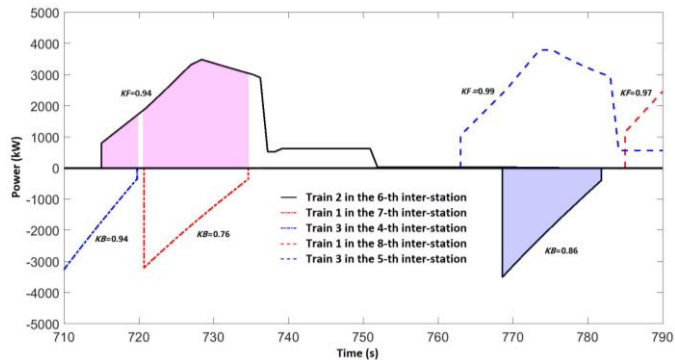Fig. 11 Train power under cooperative control with full motoring and braking

Fig. 12 Train power under cooperative control allowing partial motoring and braking

Separate control prefers to adopt coasting as much as possible to minimize the traction energy consumption, thus the inter-station run-time becomes longer. However, separate control does not consider the synchronization of motoring and braking trains, which leads to less than 10 seconds overlapping time of motoring and braking trains in the same PSR, as shown in Fig. 10, where the power-time diagram of trains under separate control is given. The positive value denotes the train is consuming energy, while negative one represents regenerated energy available to supplement motoring of other trains within the same PSR. The shaded area indicates the overlapping time of motoring and braking trains in the same PSR.

With cooperative control, the timings of motoring and braking of the train consider the operations of other trains in the same PSR. As shown in Fig. 9, train 2 in the 6-th inter-station with cooperative control adopts braking earlier at higher speed than that with separate control, to provide more regenerative energy for acceleration of other trains. In this way, cooperative control with full motoring/braking prolongs the overlapping time to 27 seconds as shown in Fig. 11, where the power-time diagram of trains under cooperative control with full motoring/braking is given.

As this case allows 5% of deviation on train inter-station run-time from the scheduled one, the cooperative control with full motoring/braking might prefer a shorter inter-station run-time to enable synchronization of motoring and braking trains, in comparison with separate control. It consumes 15.8 kWh more traction energy for all trains in their whole trips, as shown in Table II. As a whole, the cooperative control with full motoring/braking reduces the net energy consumption because the utilization of regenerative energy increases by 46.2 kWh.

Partial motoring and braking prolongs the time span of train motoring and braking and provides opportunity for better utilization of regenerative energy. However, train traction energy consumption may increase at the same time, as the coasting distance will be shortened to satisfy the scheduled run-time. A trade-off between utilizing regenerative energy and reducing traction energy is required to minimize the net energy consumption. Fig. 12 gives the optimal KF and KB, i.e. the extents of motoring and braking applied in accelerating and braking of train inter-station runs, under the cooperative control which allows partial motoring/braking.

The cooperative control allowing partial motoring/braking extends the overlapping time of motoring and braking trains to 32 seconds. As a result, the cooperative control allowing partial motoring/braking utilizes 13.5 kWh more regenerative energy in comparison with cooperative control with full motoring/braking, taking all trains in their whole trips into account, as shown in Table II. On the other hand, only 6.6 kWh more traction energy is consumed. As a whole, the net energy consumption is reduced by 6.9 kWh, which accounts for 2.5% of the net energy consumption. In other words, partial motoring/braking allows more space to synchronize motoring and braking trains and thus helps better utilizing regenerative energy and thus reducing the net energy consumption.

2) Comparison with timetable optimization

Offline timetable optimization is another mean to reduce the energy consumed by trains. This section aims to compare the energy performance of timetable optimization and cooperative

train control. The cooperative control here and in subsequent sections allows partial motoring/braking. To ensure fair comparison between cooperative control and timetable optimization, the passenger loading factors in all inter-stations along the journey are set the same at 0.5. Taking the timetable described in Section V-A as the base for comparison, the energy-efficient timetable is attained by optimizing train headway and distribution of run-time among inter-station runs, while the overall travel time for the whole journey is maintained [35]. The objective is to minimize the net energy consumption of all trains. The results show that the energy-optimized headway remains 150 seconds and the optimized inter-station run-time is given in Table III.

TABLE III
OPTIMIZED TRAIN INTER-STATION RUN-TIME

Inter-station	1	2	3	4	5	6	7	8	9	10
Run-time (s)	70	69	74	71	80	64	69	83	78	82

The energy performance of timetable optimization is given in Table IV. In the case of no delays, the optimized timetable saves net energy consumption by 3.9% under separate train control. Nevertheless, the energy performance of timetable optimization is subject to train delays. To verify this argument, 5 to 60 seconds of initial delay is introduced to the middle train at the first station. The delayed train attempts to compress the run-times in its inter-station runs as far as possible until the delay is completely recovered. With 5 seconds delay, the traction energy becomes slightly higher than that without delays, since the run-time of the delayed train is compressed in the first inter-station. With 60 seconds delay, the middle train recovers its delays in the 8-th inter-station, without propagating delays on other trains. The traction energy grows with the increasing delays, because delayed train compresses run-times in more inter-station runs when delay is getting longer.

TABLE IV
ENERGY PERFORMANCE OF TIMETABLE OPTIMIZATION AND TRAIN CONTROL IMPROVEMENT

Initial delay (s)	Energy consumption with separate control (kWh)				Energy consumption with cooperative control (kWh)			
	Original timetable		Optimized timetable		Original timetable		Optimized timetable	
	Traction	Net	Traction	Net	Traction	Net	Traction	Net
0	339.5	294.5	337.0	283.1	359.9	273.9	354.6	272.4
5	342.6	298.6	339.1	288.7	358.0	274.9	355.0	273.7
10	345.7	302.1	343.4	289.3	361.4	275.4	368.3	276.4
15	349.3	302.0	345.7	287.2	365.7	279.7	368.9	276.1
20	350.3	296.7	348.7	286.1	368.1	276.5	372.0	276.0
25	355.0	294.1	351.9	282.2	373.7	279.0	372.6	274.6
30	355.7	296.4	354.9	285.0	374.1	273.1	374.8	280.0
35	358.7	301.7	357.0	289.1	374.3	279.2	369.0	273.1
40	361.0	301.1	359.7	297.1	372.3	280.6	371.6	278.7
45	363.8	304.1	365.1	306.9	378.2	273.7	379.0	276.1
50	366.0	313.2	366.6	312.3	376.3	282.0	372.4	288.1
55	370.9	326.6	367.7	318.0	378.0	299.6	371.5	299.2
60	372.1	336.4	371.9	323.4	374.4	317.1	373.6	306.2
Ave.	356.2	305.2	354.5	296.0	370.3	281.9	369.5	280.8

Table IV shows that timetable optimization achieves energy saving in most situations when delay arises, under separate train control. However, when the initial delay is 45 seconds, the net energy consumption with optimized timetable is even higher than that with the original timetable. With cooperative train control, the net energy consumption with the optimized timetable is also higher than that with the original timetable when delay arises in some cases. The reason is that the delay causes deviation of train trajectory from the scheduled one, which breaks the scheduled synchronization between motoring and braking trains.

The energy performance of cooperative control is also given in Table IV. With both original and optimized timetable, cooperative control always saves net energy consumption from no disturbance to a good range of delays, as compared with separate control. The average saving rates on net energy consumption are 8% and 5% with original and optimized timetables respectively.

Fig. 13 gives a more intuitive comparison on energy performance between timetable optimization and cooperative control. Taking separate control with original timetable as the base point, the cooperative control with original timetable always achieves more savings on net energy consumption than separate control with optimized timetable. As shown in Table IV, the average net energy consumption of the former (i.e. 281.9 kWh) is about 5% lower than that of the latter (i.e. 296 kWh).
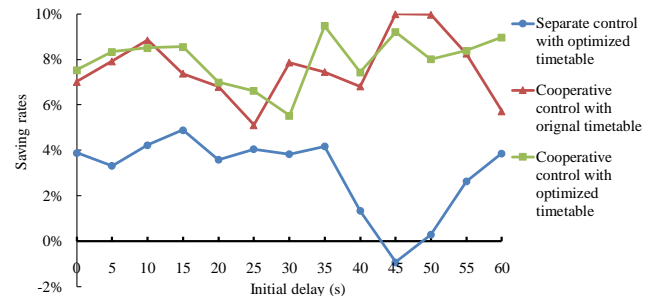

Fig. 13 Energy savings of timetable optimization, cooperative control and integrated optimization compared to separate control with original timetable

It is also found that the integrated optimization (i.e. cooperative control with optimized timetable) performs even better on net energy saving than cooperative control with original timetable, when no delay occurs. The extent of reduction depends on the quality of original timetable. Since the scheduled synchronization of motoring and braking trains might be broken by delays, the net energy consumption of cooperative control with optimized timetable may be higher than that of cooperative control with original timetable when delay arises, as shown in Table IV and Fig. 13. On average, the net energy consumption of integrated optimization (i.e. 280.8 kWh) is slightly lower than that of cooperative control only (i.e. 281.9 kWh). In other words, timetable optimization integrated with cooperative control is able to further reduce net energy consumption and it is particularly necessary when the original timetable is not specifically geared toward energy reduction.

C. Impacts of model parameters on energy performance

This section is to investigate the effect of regenerative efficiency and service headway on the energy performance of the proposed model through a sensitive analysis on Beijing metro line 5.

Fig. 14 illustrates the variation on the energy performance of cooperative control, when the regenerative efficiency decreases from 0.95 to 0.5. The results show that the regenerated energy in train braking drops sharply with the regenerative efficiency. However, the utilized regenerative energy decreases relatively slowly with the regenerative efficiency. On the other hand, the regenerative efficiency carries no significant impact on the traction energy consumption. As a result, the net energy consumption increases slowly with the decreasing regenerative efficiency under cooperative control.
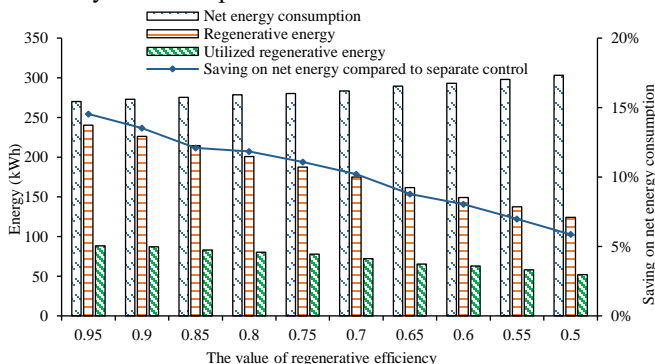

Fig. 14 Energy performance of the proposed cooperative control with different values of regenerative efficiency

As cooperative control includes the synchronization of motoring and braking trains, the utilization of regenerative energy in cooperative control is much higher than that in separate control. When the regenerative efficiency declines, the regenerated energy during braking decreases sharply, which in turn reduce the optimization space of cooperative control for better utilization of regenerative energy. As a result, the energy saving of cooperative control compared to separate control declines with the reduction of regenerative efficiency. However, it should also be noted that the energy saving still exceeds 5% across the range of regenerative efficiency as shown in Figure 14.

Fig. 15 compares the energy performance of cooperative control and separate control under service headways from 80 to 150 seconds. The results show that the system net energy consumption is closely related to the headways, with both cooperative control and separate control. The reason is that the headway, together with the inter-station distances of individual lines, has a direct impact on the amount of synchronization of motoring and braking trains, which determines the utilization of regenerative energy. For example, when the headway is around 110 - 120 seconds, the trains usually concurrently stop at stations on this particular line as the sum of scheduled run-time and dwell time in most inter-stations is close to the headway. In such case, it is almost impossible to synchronize motoring and braking trains, which results in very low utilization of regenerative energy and leads to relatively higher system net energy consumption.
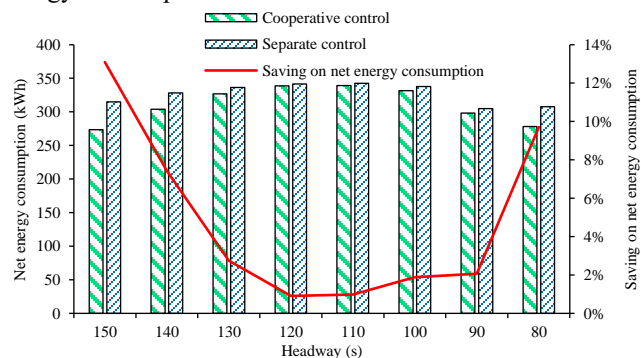

Fig. 15 Energy saving of the proposed cooperative control compared to separate control with different train headways

It is also found that the cooperative control always saves system net energy consumption when compared to the separate control, although the extent of saving fluctuates with train headways. When the headway is around 110 - 120 seconds, it is very difficult to synchronize the motoring and braking of different trains in this study, which in turn reduces the optimization room for the cooperative control. The proposed cooperative control yet outperforms separate control in reducing the net energy consumption of metro system in all headways as shown in Fig. 15.

There might be two trains located within one inter-station at the same time when the service headway is small enough. For example, in the case that the headway is 80 seconds in this study, train 1 and train 2 are located within the 8-th inter-station at the same time. The corresponding speed trajectories, as well as the start and end times of these two trains in the 8-th inter-station, are shown in the Fig. 16.
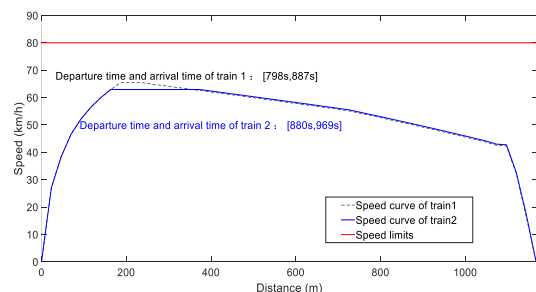

Fig. 16 The speed trajectories of two trains in the same inter-station at the same time when service headway is 80 seconds

Fig. 16 shows that the start timing of the following train 2 in the 8-th inter-station is 880s, which is earlier than the end timing of the preceding train 1 in the same inter-station (i.e. 887 s). In other word, train 1 and train 2 are simultaneously located within the same inter-station between the time window from 880 s to 887 s, which demonstrates that the proposed framework is suitable for the scenarios where there are more than one trains between two stations at the same time.

D. Impact of algorithm parameters on energy performance and computing efficiency

This section aims to analysis the impacts of algorithm parameters, such as population size in each species and the number of generations in GA-SA searching, on energy performance and computing efficiency of cooperative control.

To reduce the randomness of the results, 10 tests are conducted for each group of parameters. The average net energy consumption and computing time with different algorithm parameters are given in Table V. N/A denotes that the solution with the group of parameters requires more than 30 seconds computation time, which is longer than the minimum dwell time at stations and leads to infeasible cooperative control.

It is shown that the computation time strictly increases with growing population size and generations. The net energy consumption generally decreases when the number of generations becomes higher. However, a bigger population size does not necessarily lead to better solutions, when the number of generations remains the same. The reason is that a bigger population size will slow the convergence rate of the proposed algorithm, where the incorporated Metropolis criterion of SA eliminates the need of a bigger population size to maintain the diversity of population to search for the optimal solution.

The solution of the algorithm with 20 chromosomes and 100 generations is the best one in Table V. It is much better than that with 100 chromosomes and 20 generations, while the computation time differs little. It is concluded that having more generations is more effective to find optimal solutions, when compared with allowing more individuals in each generation.

TABLE V
IMPACTS ON ENERGY PERFORMANCE AND COMPUTING TIME BY PARAMETERS OF PROPOSED ALGORITHM

Population size / Generation number	20	30	40	50	60	70	80	90	100
	Net energy consumption (kWh) / Computing time (s)								
20	282.0/4	280.5/5	279.9/6	280.1/8	279.2/9	280.1/10	279.8/13	279.6/14	279.7/16
30	278.1/5	277.9/7	278.3/9	278.7/12	278.9/14	279.9/17	278.2/20	278.1/21	278.0/23
40	277.0/6	276.8/9	276.6/12	278.1/16	277.3/18	277.5/22	277.6/26	277.5/28	N/A
50	274.7/8	276.4/11	276.2/15	276.1/19	276.9/23	278.9/25	N/A	N/A	N/A
60	274.3/9	274.2/14	275.4/18	277.0/24	276.6/27	N/A	N/A	N/A	N/A
70	273.5/11	274.1/16	276.4/22	276.0/26	N/A	N/A	N/A	N/A	N/A
80	273.8/12	274.0/18	275.4/24	N/A	N/A	N/A	N/A	N/A	N/A
90	273.0/13	272.7/20	275.6/27	N/A	N/A	N/A	N/A	N/A	N/A
100	**271.9/15**	272.3/23	274.1/29	N/A	N/A	N/A	N/A	N/A	N/A

TABLE VI
ENERGY PERFORMANCE AND COMPUTING TIME OF GA

Population size / Generation number	20	30	40	50	60	70	80	90	100
	Net energy consumption (kWh) / Computing time (s)								
20	287.0/3	283.3/5	282.2/6	280.3/8	280.1/9	278.5/11	278.5/12	278.9/13	278.1/15
30	283.3/6	282.6/7	281.0/9	281.0/11	278.4/14	279.6/15	278.6/17	278.1/20	278.5/24
40	285.7/7	280.6/9	280.0/12	278.3/14	278.4/18	278.9/21	277.9/24	277.5/26	N/A
50	283.1/8	280.1/11	279.8/15	278.5/19	278.6/22	277.9/26	277.9/29	N/A	N/A
60	281.2/9	278.9/14	278.1/18	278.5/22	278.6/26	N/A	N/A	N/A	N/A
70	279.3/11	279.3/17	278.6/21	278.0/26	N/A	N/A	N/A	N/A	N/A
80	280.4/13	277.8/18	277.7/25	277.3/28	N/A	N/A	N/A	N/A	N/A
90	277.7/14	277.9/20	276.9/26	N/A	N/A	N/A	N/A	N/A	N/A
100	278.5/16	277.8/22	**275.8/30**	N/A	N/A	N/A	N/A	N/A	N/A

To demonstrate the effectiveness of the hybrid GA-SA searching, the energy performance and computing time of GA are given in Table VI for comparison. There is only one species in each generation and the control schemes of all trains in the same PSR are denoted by one chromosome in the GA, where the Metropolis criterion of SA is out of consideration.

It is found that the minimal net energy consumption with GA is 275.8 kWh, while the optimal one with the GA-SA is 271.9 kWh. In other words, the proposed algorithm saves 1.5% of the net energy consumption than GA. On the other hand, the proposed algorithm is able to find a solution consuming 280 kWh less within 5 seconds, while GA needs more than 10 seconds. The computation efficiency is more important if there are more than three trains in one PSR when trains from up and down directions are considered. In summary, the proposed algorithm performs better in both solution optimality and computing efficiency when compared to the GA.

VI. Conclusion

With the application of regenerative braking, separately control of single train for energy saving does not lead to the minimization of system net energy consumption. To this end, recent studies attempt to optimize train timetable for better utilization of regenerative energy, by synchronization of motoring and braking trains. However, the actual train trajectory may deviate from the schedule as train delays are inevitable in metro operation, which breaks the scheduled synchronization of motoring and braking trains since the overlapping time can be very short. The main contribution of this paper is a model framework for real-time cooperative control of multiple trains to minimize system net energy consumption, considering possible changes of train status. A cooperative co-evolutionary algorithm is developed to attain the solution of the proposed model.

Case studies on Beijing Metro line 5 confirm the feasibility and effectiveness of the proposed model and algorithm. The cooperative control is able to reduce system net energy consumption in comparison with separate train control. It also performs better than train timetable optimization in energy saving, considering no traffic disturbance and different extents of train delays. Case studies further reveal that allowing partial motoring/braking in train control helps to reduce net energy consumption, by substantially improving the utilization of regenerative energy at the expense of consuming slightly more traction energy. Sensitive analyses on model and algorithm parameters demonstrate the effectiveness of the proposed cooperative control in different scenarios.

In this study, the energy performance of cooperative train control is only demonstrated by a series of case studies, in comparison with that of separate train control [8] and timetable optimization [35]. A mathematical proof that the results of the cooperative control satisfy the necessary optimality conditions of Pontryagin's Maximum Principle will be explored in the further research. It also should be noted that only the traffic in one direction of a metro line is discussed in the cooperative train control. In most real-world metro lines, trains are able to use the regenerative energy produced by other trains running on opposite direction in the same PSR. The cooperative control of trains on two-way traffic will be explored in the future study. In addition, the traction energy consumption and regenerative energy are computed from a mechanical perspective, without considering the configuration of electrical power distribution system in the metro operation. An electrical simulation model to accurately attain the energy consumption and regenerative energy will be developed in the future work.

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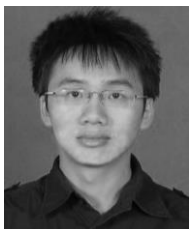
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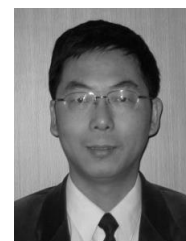
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