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We obtain the distribution of a number of atoms in an interval (full counting statistics) for interacting bosons in one dimension. Our results are valid in the weakly interacting regime in a parametrically large window of temperatures and interval lengths. The obtained distribution significantly deviates from a Gaussian away from the quasicondensate regime, and, for sufficiently short intervals, the probability of large number fluctuations is strongly enhanced.

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Introduction.—*In situ* measurements of particle number fluctuations in a one-dimensional (1D) Bose gas have been recently performed in experiments with ultracold ^{87}Rb atoms on a chip [1–3]. In these experiments, absorption images of a 1D gas of interacting bosons were analyzed for many pixels of predetermined size R of the order of several microns, and the number of atoms in each pixel was inferred based on the absorption intensity. Data accumulated over several repetitions of such imaging were then used to extract the second [1] and third [2] moments of the underlying particle number distribution on an interval fixed by the pixel length. This distribution is known as full counting statistics (FCS) and contains full information about many-particle correlations.

Despite the fact that FCS arises naturally in these experiments, there are no theoretical predictions for the experimentally relevant range of interval lengths R. The reason is that such a study involves the calculation of density correlations between an arbitrary number of different spatial points which remains an unsolved problem even for the exactly solvable Lieb-Liniger model [4] describing one-dimensional bosons with contact interactions [5,6]. Even in the impenetrable limit, equivalent to free fermions [7], this is a formidable task involving sophisticated mathematical methods like asymptotics of Toeplitz determinants [8].

If the interval is sufficiently large and can be viewed as a subsystem in contact with the effective bath characterized by temperature T and chemical potential μ, it was suggested first in Ref. [2] that the moments of FCS can be obtained from the appropriate thermodynamic relation involving the mean density of particles $\bar{n}$ as a function of μ and T. The corresponding equation of state $\bar{n}(\mu, T)$ is calculated by using the Yang-Yang thermodynamics of the Lieb-Liniger model [9] and was recently measured experimentally [10]. This approach was later extended in Ref. [11] to a calculation of the fourth moment of FCS.

The results of these studies show that higher moments decay quickly with the increasing of interval sizes and FCS becomes strongly peaked around the mean number of particles $\bar{n}R$, effectively yielding a Gaussian—a result expected from the general thermodynamic argument on the scaling of fluctuations of extensive quantities [12]. This makes large deviations of the particle number from its mean value extremely improbable. In particular, the emptiness formation probability, i.e., the probability to find a void of size R considered in Refs. [13–15], is exponentially small.

In the opposite limit of microscopic intervals $\bar{n}R \ll 1$, the problem of calculating FCS is amenable to an analysis of local correlations. This approach was taken by Bastianello, Piroli, and Calabrese [16], who obtained FCS using exact analytic Bethe ansatz calculations. An expected consequence of these studies is that the most probable particle number is zero and the probability to find N particles decays as $(\bar{n}R)^N$.

In fact, large nonthermodynamic deviations of the number of particles become appreciable already for intervals still containing typically a large number of particles, $\bar{n}R \gg 1$, but shorter than a certain correlation length scale. For such *mesoscopic* intervals, the central limit theorem does not hold, and FCS deviates strongly from the thermodynamic Gaussian distribution expected for a collection of many independent intervals. The importance of such mesoscopic fluctuations for experimental investigations of 3D and 2D systems was addressed in a recent publication [17].

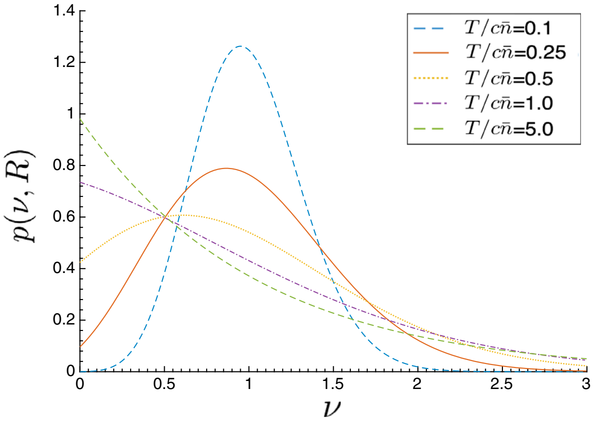

FIG. 1. Normalized FCS $p(\nu, R)$ defined in Eq. (2) in the limit of short intervals. The dimensionless temperature is $T/c\bar{n} = \xi/\ell_\varphi = 0.1$, 0.25, 0.5, 1.0, and 5.0.

In this Letter, we study FCS on intervals of arbitrary length and provide an elegant and simple method for its calculation based on the exact mapping of the one-dimensional classical field theory onto a quantum mechanical problem, introduced in Ref. [18]. In the limit of short intervals, the form of FCS is shown to change continuously from a Gaussian to an exponential one as the temperature is increased; see Fig. 1. The limit of long intervals is represented in Fig. 2, and our FCS agrees with the results of previous studies. We also trace FCS as a function of the interval length in Fig. 3. These results show enhanced large deviations of the number of particles for mesoscopic intervals where fluctuations play a major role.

Full counting statistics.—The main quantity studied in this Letter is the FCS defined as the probability $P_N(R)$ to find exactly N particles on an interval of length R. We define it via its generating function:

$$\chi(\lambda, R) = \sum_{N=0}^{\infty} e^{-\lambda N} P_N(R) = \langle e^{-\lambda \hat{N}_R} \rangle. \tag{1}$$

Here, $\hat{N}_R$ is the operator of the number of particles in the interval. Statistical averaging in Eq. (1) is performed in the equilibrium state of a uniform 1D Bose gas with contact interactions [4,9].

For intervals containing a large number of particles, $\bar{n}R \gg 1$, it is convenient to define the rescaled distribution

$$p(\nu, R) = \int_{-i\infty}^{i\infty} \frac{dk}{2\pi i} e^{k\nu} \chi(k/\bar{n}R, R) = \bar{n}R P_{\nu\bar{n}R}(R) \tag{2}$$

of the fraction of particles, $\nu = N/\bar{n}R$, treating it as a continuous variable. As long as the number of particles in the interval is large, $\nu\bar{n}R \gg 1$, the typical values of $\lambda = k/\bar{n}R$ contributing to the integral (2) are small [19]. In this case, the generating function Eq. (1) can be approximated by a normal-ordered average evaluated as a path integral, $\chi(\lambda, R) \simeq \langle : e^{-\lambda \hat{N}_R} : \rangle = Z[\lambda]/Z[0]$, where

$$Z[\lambda] = \int \mathcal{D}\bar{\psi}\psi e^{-\lambda N_R[\bar{\psi},\psi]} e^{-S[\bar{\psi},\psi]}. \tag{3}$$

Here, $N_R[\bar{\psi}, \psi] = \int_0^R \bar{\psi}(x, 0)\psi(x, 0) dx$, and configurations of the complex-valued fields $\bar{\psi}(x, \tau)$, $\psi(x, \tau)$ are weighted by the action

$$S = \int_0^{1/T} d\tau \int dx \left(\bar{\psi}\partial_\tau \psi + \frac{1}{2m} |\partial_x \psi|^2 - \mu |\psi|^2 + \frac{g}{2} |\psi|^4 \right), \tag{4}$$

with m being the atomic mass, g is the strength of 1D contact interaction, and the units are chosen such that $\hbar = 1$ and $k_B = 1$. The integral over space coordinate x runs over

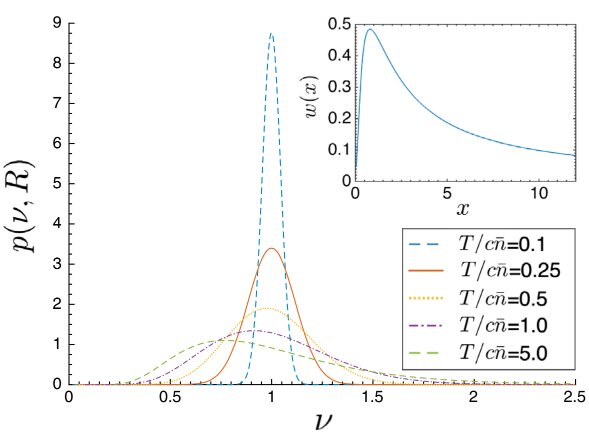

FIG. 2. Normalized FCS $p(\nu, R)$ defined in Eq. (2) in the limit of long intervals. The dimensionless temperatures are the same as in Fig. 1, and $R/\ell_\varphi = 5$. Inset: Reduced width $w(x)$ defined in Eq. (18) characterizing fluctuations of the number of particles.

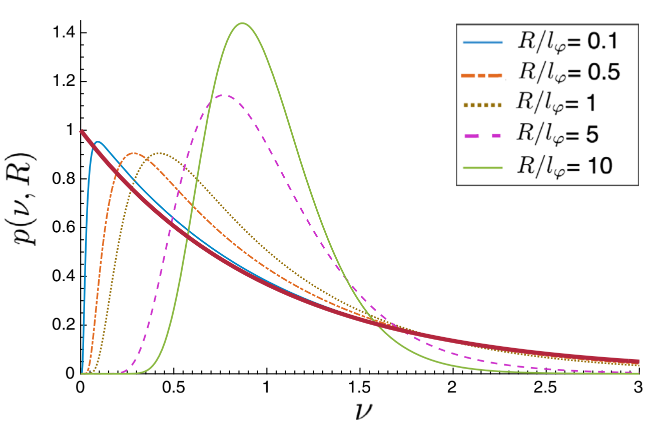

FIG. 3. Normalized FCS $p(\nu, R)$ defined in Eq. (2) for intermediate intervals and high-temperature regime $T/c\bar{n} = \xi/\ell_\varphi \gg 1$. The bold solid line represents exponential distribution, Eq. (9).

an infinite line as thermodynamic limit $N = \bar{n}L \to \infty$ is taken with average density $\bar{n} = \langle\bar{\psi}(x,\tau)\psi(x,\tau)\rangle$ fixed by the chemical potential μ. Having calculated the equation of state $\bar{n} = \bar{n}(\mu, T)$, it is permissible to use $\bar{n}$ as a control parameter and to adjust μ accordingly.

Here, we consider a regime where the mean interparticle separation $1/\bar{n}$ is the smallest of the characteristic length scales. The other two length scales, in addition to the interval length R, are the healing length $\xi = 1/\sqrt{mg\bar{n}}$, which characterizes the interactions, and the phase coherence length $\ell_\varphi = \bar{n}/mT$, which characterizes the temperature. The regime of weak interactions considered here is defined by the dimensionless interaction constant $\gamma = mg/\bar{n} \ll 1$, equivalent to the condition on the healing length $\xi \gg 1/\bar{n}$. The phase coherence length also satisfies $\ell_\varphi \gg 1/\bar{n}$ for a degenerate gas $T < \bar{n}^2/m$. This leaves only two independent dimensionless parameters, which can be chosen to be R/ℓ_φ and ξ/ℓ_φ. The latter equals $T/c\bar{n}$, where $c = \sqrt{g\bar{n}/m}$ is the speed of sound at zero temperature. Below, we obtain FCS as a function of these two parameters.

Classical field theory and effective quantum mechanics.—The main obstacle in calculating FCS is the nonlinearity of the action (4), which is responsible for correlations between the particles and which makes the exact calculation of FCS extremely difficult if at all possible. In the hydrodynamic approach of Refs. [14,15], this difficulty was overcome by expanding the action (4) near configurations of the fields contributing the most to FCS. This method is limited to sufficiently low temperatures and sufficiently large intervals where contributions of quantum and thermal fluctuations are small. Here, we use an alternative classical field method which properly accounts for thermal fluctuations of arbitrary magnitude but not the quantum ones; see Ref. [18] and references therein. The latter can be safely neglected under the condition of sufficiently high temperature equivalent to $\xi/\ell_\varphi \gg 1/\bar{n}\xi$. This condition and the condition of quantum degeneracy $\xi/\ell_\varphi \ll \bar{n}\xi$ define a parametrically wide range of temperatures, $g\bar{n} \ll T \ll n^2/m$, where the classical field method provides reliable results for macroscopic intervals of *any length*.

Neglecting quantum fluctuations amounts to retaining only τ-independent configurations of fields in Eq. (4), leading to a $1+0$-dimensional field theory described by the action

$$S \simeq S_{\mathrm{cl}} = \frac{1}{T}\int dx\left(\frac{1}{2m}|\partial_x\psi|^2 - \mu|\psi|^2 + \frac{g}{2}|\psi|^4\right). \quad (5)$$

This action can be reformulated as an effective quantum mechanical problem if we treat the rescaled spatial coordinate x/ℓ_φ as an effective imaginary time. The components of the complex field $\psi = \sqrt{\bar{n}}re^{i\theta}$ are parametrized by dimensionless polar coordinates (r,θ) of a fictitious quantum particle with the unit mass moving in a rotationally symmetric potential:

$$V(r) = -\mu m\ell_\varphi^2 r^2 + \frac{1}{2}\frac{\ell_\varphi^2}{\xi^2}r^4. \quad (6)$$

The coefficient of the first (quadratic) term in Eq. (6) depends on the chemical potential μ and is fixed from the condition $\langle 0|r^2|0\rangle = 1$, where $|0\rangle$ is the ground state of the Hamiltonian $H_0 = -\nabla^2/2 + V(r)$. It was shown in Ref. [18] that, for low temperatures where $\xi/\ell_\varphi \ll 1$, the chemical potential is positive and the potential experienced by the effective particle has a characteristic "Mexican hat" shape, with the effective particle localized near $r \simeq 1$. This temperature range corresponds to the quasicondensate regime [21,22]. For high temperatures, $\xi/\ell_\varphi \gg 1$, corresponding to the quantum degenerate regime of Refs. [21,22], the chemical potential is negative, $\mu < 0$, and the effective particle explores the vicinity of the minimum at $r = 0$ where the potential is almost harmonic.

In this language of effective quantum mechanics, the generating function (1) has the following meaning. The ground state $|0\rangle$ is evolved for imaginary time $t_R = R/\ell_\varphi$ by the modified Hamiltonian $H_s = H_0 + sr^2$, where $s = \lambda\bar{n}\ell_\varphi$. This results in the modified state $e^{-t_R H_s}|0\rangle$. The generating function is then given by the normalized overlap

$$\chi(\lambda, R) = \chi(s/\bar{n}\ell_\varphi, t_R\ell_\varphi) = \langle 0|e^{-t_R(H_s - E_0)}|0\rangle, \quad (7)$$

where E_0 is the ground state energy of H_0.

Short intervals.—We first consider the case of a short interval R. In this limit, the imaginary time evolution of the ground state in Eq. (7) is obtained by a multiplication of the rotationally symmetric ground state wave function $\langle r|0\rangle = \Phi_0(r)$ by an exponential factor $e^{-t_R s r^2}$ so that the corresponding probability distribution is proportional to the ground state probability density of the effective 2D quantum mechanical problem:

$$p(\nu, R) = \int_{-\infty}^{\infty} dk \int_0^{\infty} r dr e^{ik(\nu - r^2)}|\Phi_0(r)|^2 = \pi|\Phi_0(\sqrt{\nu})|^2 \quad (8)$$

and is independent of the interval length R.

For high temperatures, $\xi/\ell_\varphi \gg 1$, the Hamiltonian H_0 is that of a two-dimensional harmonic oscillator with frequency $\omega_0 = 1$, and its ground state wave function is simply $\Phi_0(r) = e^{-r^2/2}/\sqrt{\pi}$. Using Eq. (8), we see immediately that FCS is exponential;

$$p(\nu, R) = e^{-\nu} \quad (9)$$

and analogous to the intensity distribution in a speckle pattern resulting from interference of Gaussian random complex fields.

The low-temperature limit $\xi/\ell_\varphi \ll 1$ corresponds to the quasicondensate regime. The value of chemical potential $\mu = 1/m\xi^2 = g\bar{n}$ is fixed from the requirement that the Mexican-hat-shaped potential has a minimum at $r = 1$. Expanding $V(r)$ near the minimum, we get a one-dimensional harmonic potential with the frequency $\omega = 2\ell_\varphi/\xi \gg 1$. Substituting its ground state wave function $\Phi_0(1+\delta r) = (\omega/4\pi^3)^{1/4} e^{-\omega\delta r^2/2}$ in Eq. (8) gives

$$p(\nu, R) = \sqrt{\frac{\ell_\varphi}{2\pi\xi}} e^{-(1/2)(\ell_\varphi/\xi)(\nu-1)^2}. \tag{10}$$

The quadratic approximation for $V(r)$ fails for large deviations $\nu - 1 \sim 1$ already in the low-temperature regime and becomes worse in the regime of intermediate temperatures. In these cases, the effective quantum mechanical problem can be solved numerically. We find the ground state $\Phi_0(r)$ and plot the corresponding distributions in Fig. 1 for several values of dimensionless temperature $T/c\bar{n} = \xi/\ell_\varphi$. The plots show how the exponential distribution in Eq. (9) transforms into the Gaussian distribution of Eq. (10) with a decreasing temperature. The distinctive feature of these plots is the enhanced probability to find the fraction ν well below its average value 1.

The above results are valid provided R is smaller than correlation length R_c, which can be estimated as

$$\frac{R_c}{\ell_\varphi} = \min(1, \xi/\ell_\varphi). \tag{11}$$

For very small values of $\nu \ll \min(1, \xi/\ell_\varphi)$, we have a more stringent condition $R/\ell_\varphi \ll \nu$ equivalent to replacing mean density $\bar{n}$ with $\nu\bar{n} = N/R$ in the definition of ℓ_φ. This fact makes the result in Fig. 1 unreliable for the corresponding small values of the fraction ν.

Long intervals.—For $R \gg R_c$, the evolution operator in Eq. (7) becomes a projector:

$$e^{-t_R(H_s - E_0)} \simeq |s\rangle e^{-t_R\Delta(s)}\langle s| \tag{12}$$

onto the ground state $|s\rangle$ of the modified Hamiltonian, $H_s|s\rangle = [E_0 + \Delta(s)]|s\rangle$. In this limit, the generating function (7) has the following form:

$$\chi(s/\bar{n}\ell_\varphi, t_R\ell_\varphi) = A(s)e^{-t_R\Delta(s)}, \tag{13}$$

where the ground state energy shift $\Delta(s)$ and the overlap $A(s) = |\langle s|0\rangle|^2$ are independent of R. For long intervals, the distribution (2) can be found by the saddle point method:

$$p(\nu, R) = \frac{R}{\ell_\varphi}\int \frac{ds}{2\pi i} A(s) e^{(R/\ell_\varphi)[s\nu - \Delta(s)]} \simeq D(\nu)e^{(R/\ell_\varphi)\Gamma(\nu)}, \tag{14}$$

where Legendre transform $\Gamma(\nu) = s\nu - \Delta(s)$ and the prefactor $D(\nu) = \sqrt{R/2\pi\ell_\varphi}A(s)/\sqrt{|\Delta''(s)|}$ are calculated at the saddle point obtained from the condition $\nu = \Delta'(s)$.

In the high-temperature limit, $\xi/\ell_\varphi \gg 1$, the rescaled ground state energy shift and the overlap become independent of the temperature $\Delta(s) = \sqrt{1+2s} - 1$, $A(s) = 4\sqrt{1+2s}(1+\sqrt{1+2s})^{-2}$ [20], and we obtain

$$p(\nu, R) = \sqrt{\frac{R}{2\pi\ell_\varphi}} \frac{4e^{(R/2\ell_\varphi)(2-\nu-1/\nu)}}{\sqrt{\nu}(1+\nu)^2}. \tag{15}$$

For small deviations $|\nu - 1| \ll 1$, this expression for FCS becomes a Gaussian with variance $\overline{\delta\nu^2} = \ell_\varphi/R$.

In the low-temperature regime, $\xi/\ell_\varphi \ll 1$, to the lowest order, the rescaled ground state energy shift is a quadratic function $\Delta(s) \simeq s - (\xi/\ell_\varphi)^2 s^2/2$ and $A(s) \simeq 1$, so that by Gaussian integration we get

$$p(\nu, R) = \sqrt{\frac{R\ell_\varphi}{2\pi\xi^2}} \exp\left(-\frac{R\ell_\varphi}{2\xi^2}(\nu-1)^2\right) \tag{16}$$

in full agreement with the hydrodynamic result of Refs. [14,15]. The variance is $\overline{\delta\nu^2} = \xi^2/\ell_\varphi R$. In Fig. 2, the results based on the numerical calculations of Eq. (14) are shown for intermediate values of the temperature.

Intermediate intervals.—In the limiting cases of high and low temperature, the probability distribution can be obtained for an interval of arbitrary length R. The method is based on the exact evolution of harmonic oscillator wave functions under a time-dependent variation of the frequency and external force [23] as explained in Supplemental Material [20]. For high temperatures, $\xi/\ell_\varphi \gg 1$, FCS is shown in Fig. 3. It interpolates between Eqs. (9) and (15) and has a distinctive non-Gaussian shape. For small intervals, the distribution approaches the exponential distribution Eq. (9) everywhere apart from the interval $\nu < R/\ell_\varphi$, where it drops to zero.

For low temperatures, $\xi/\ell_\varphi \ll 1$, the distribution remains very close to a Gaussian with variance depending on the interval length,

$$p(\nu, R) = \sqrt{\frac{1}{2\pi F_c(R/\xi)} \frac{\ell_\varphi}{\xi}} e^{-[1/2F_c(R/\xi)](\ell_\varphi/\xi)(\nu-1)^2}. \tag{17}$$

The crossover function $F_c(x) = (2x - 1 + e^{-2x})/2x^2$ behaves as $F_c(x) \simeq 1$ for $x \ll 1$ and $F_c(x) \simeq 1/x$ for $x \gg 1$ and interpolates between Eqs. (10) and (16). This result could have otherwise been obtained by including gradient terms in the hydrodynamical approach of Ref. [15].

Variance of the particle number.—For long intervals $R > R_c$, the above results suggest the following scaling form for the variance of the number of particles:

$$\frac{\overline{\delta N^2}}{\bar{n}R} = \bar{n}R\overline{\delta\nu^2} = \bar{n}\xi w(\xi/\ell_\varphi), \tag{18}$$

where the universal function has the limiting behavior $w(x) = x$ for $x \ll 1$ and $w(x) = 1/x$ for $x \gg 1$. For intermediate values of x, the numerical results for $w(x)$ are shown in the inset in Fig. 2 and confirm the non-monotonic dependence of the particle number variance on the temperature anticipated from the limiting behaviors of $w(x)$. The right-hand side of Eq. (18) is greater than 1 in the whole range of validity of our approach, $1/\bar{n}\xi < \xi/\ell_\varphi < \bar{n}\xi$, and thus the fluctuations of particle number are super-Poissonian, in agreement with findings of Ref. [3].

Higher moments of FCS can also be obtained from the knowledge of generating function $\chi(\lambda, R)$, and we calculate the third and the fourth moments in Supplemental Material [20]. They are in full agreement with the results of previous studies [2,11].

Conclusions.—The departure of FCS from Poisson distribution characteristic of a classical ideal gas [20] is a direct consequence of quantum statistics and constitutes manifestation of bosonic bunching beyond second-order fluctuations. A distinctive feature of distribution we found at high enough temperatures is an enhanced probability to find large (on the scale of mean interparticle separation) regions of depleted numbers of particles. One can directly verify our results by counting the number of atoms in pixels of the size R of the order of a few microns in many realizations of the one-dimensional cloud of atoms. For typical values of parameters in the experiments in Refs. [1,2], $1/\bar{n} \sim 0.02\ \mu\mathrm{m}$ and $\xi \sim 0.3\ \mu\mathrm{m} \gg 1/\bar{n}$ as required in the weakly interacting regime. The phase coherence length ℓ_φ is similar to ξ but can be changed by adjusting the temperature. Thus, the observation of mesoscopic FCS in the various temperature regimes described above is indeed within the reach of current experiments. Such measurements can provide a novel way to characterize the temperature and interaction strength due to the strong dependence of FCS on these parameters.

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