“Risky” Monetary Aggregates for the UK and US

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Abstract

We extend the scope of monetary aggregation beyond capital certain assets that make up central bank data sets and identify groups of assets that form monetary aggregates composed of both capital certain and risky, capital uncertain, assets. We construct monetary aggregates for the US and UK using a superlative index and relax a key assumption of the Consumption Capital Asset Pricing Model (CCAPM), a one year planning horizon, by using forecasted returns on risky assets. Our new risky monetary aggregates perform well in VAR tests. We recommended exploring risky assets as providers of liquidity services in future research on this topic.

Keywords: Risk, Capital Asset Pricing Model, Liquidity, Divisia Money

JEL Codes: E43, G12, C43, C51

1. Introduction

Many attempts have been made to improve the measurement of money. A well-known suggestion by Friedman and Schwartz (1970) was to apply some form of weighting of the components in the aggregate depending on their relative 'moneyness'. In his pioneering work Barnett (1980) brought together the economics of aggregation over goods and index number theory to propose the construction of monetary aggregates consistent with microeconomic foundations.

Barnett argues that we should treat money as a durable good rendering its owner a flow of services in each period. If money is introduced into the consumer's utility function[1], economic theory of aggregation over goods provides sophisticated methods for choosing which assets to include in a monetary aggregate and how to construct aggregator functions. Index number theory provides parameter and estimation free methods to perform the aggregation. When applying these theories to the construction of monetary aggregates, it becomes apparent that the components included should be weighted depending on the monetary services they provide.

It can be shown that traditional simple sum aggregation is only justified when all asset components are perfect substitutes. As Barnett (1980) suggests, no researcher seeking to create an aggregate transportation index would consider giving equal weights to buses and roller-skates. Belongia (1996), using US data, re-estimated empirical models by replacing simple sum Federal Reserve ad hoc aggregates with Divisia indexes of the same ad hoc group of assets and thereby significantly altered the conclusions about the lack of relationship of money and prices that had been reached by several influential studies.

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[1] The theory has also been extended to including money in the firm's production function (Barnett and Singleton [1987]).

Many studies have focused on the construction or index number aspect of Barnett's (1980) contribution. Barnett (1980) showed that some apparent shifts in money demand were removed when Divisia measures of money replaced simple sum money. Barnett and Spindt (1979 and 1980) showed that Divisia indices have superior information content using the definition given in Bailey *et al* (1982), when compared to simple sum measures. A growing body of literature from around the world is accumulating in support of weighted monetary aggregates. Evidence in support of Divisia aggregation has been forthcoming from Horne and Martin (1989) for Australia; Cockerline and Murray (1981) and Hostland et al (1987) for Canada; Ishida (1984) for Japan; Yue and Fluri (1991) for Switzerland and Giaotti (1996) for Italy. A good overview of the empirical evidence from eleven countries is given in Belongia and Binner (2000). Following this promising line of research, we use a superlative index number to construct the aggregates we identify for the UK and US.

Fewer studies have examined what assets should compose an economic monetary aggregate. Studies by Swofford and Whitney (1987 and 1988) and Hjertstrand et al (2016) on US data and Patterson (1991) and Drake and Chrystal (1994) on UK personal sector data have used the revealed preference approach to identify the components of an economic monetary aggregate. Spencer (1997) used this approach in tests for nine European countries whilst Binner et al (2009) used this weak separability test to determine admissible levels of monetary aggregation for the Euro area. Belongia (2000) used the procedure to identify asset groupings using data from the US, Germany and Japan. Finally, Ewis and Fisher (1984), Fisher and Fleissig (1994) and Fleissig and Swofford (1996 and 1997) and Jadidzadeh and Serletis (2016) for the US and Drake et al (1999) and Drake et al (2003) for the UK are examples of studies that have used econometric approaches to study substitutability among assets.

Far fewer studies have simultaneously examined both the composition and construction of monetary aggregates. Barnett (1980) tested for weak separability of some sub-groups of assets in his data and used those results to build up monetary aggregates. Barnett (1980) then constructed monetary aggregates and compared them to a simple sum aggregate. Swofford and Whitney (1991) identified assets that make up a US economic monetary aggregate, constructed this aggregate using a superlative index number and tested the time series properties on this aggregate. More recently, Duca (1994, 2000) assessed the possibility that adding bond mutual funds, equity mutual funds, or both to M2 in the US would improve this monetary aggregate's ability to forecast nominal GDP growth. Duca found that M2B (M2 plus bond funds) and M2+ (M2 plus bond and stock funds) are statistically significant in ex-plaining past nominal GDP growth. Current work by Anderson et al (2017) which is focused only on US data permits a rich econometric analysis that includes measures of investors' perceptions of risk and of the transaction costs of portfolio switching between M2 and equity mutual funds.

The current paper is one of the few that follows both parts of Barnett (1980) and construct meaningful economic monetary aggregates. Work by Anderson and Jones (2011) and Holmström and Tirole (2011) revisit the construction of monetary aggregates and ask if transaction costs and "sudden stops" in financial markets explain why households and firms choose to hold larger quantities of highly liquid assets than is suggested by models with de minimus asset-market transaction costs. The latter note: "While some forms of equity, such as private equity, may not be readily sold at a 'fair price,' many long-term securities are traded on active organized exchanges…liquidating one's position…can be performed quickly and at low transaction costs" (p. 1). Their analysis implies that not all financial assets are perfect substitutes due to the risks that market trading might suddenly halt, differential user costs can arise in the solution to the optimization problem facing households and firms, and such differential user costs reflect the differing amounts of monetary services furnished by the assets. In this paper we identify the components in economic monetary aggregates for the UK and US, construct them using a superlative index and report some tests of their time-series properties.

Recent developments in the literature have addressed problems associated with the construction of economic monetary aggregates. Notably, Barnett et al (1997) have developed a method to account for risk based on the consumption capital asset pricing model (CCAPM) framework.[2] Barnett and Zhou (1994) suggested that it might be appropriate to include risky assets like ordinary shares, in monetary aggregates. Drake et al (1999) estimated a demand system over both capital certain and risky assets held by the UK personal sector. They showed that risky assets are substitutes for the monetary assets normally included in the Bank of England M4 aggregate. They also found that the substitutability between risky and capital certain assets decreased, when the level of risk aversion increased. Drake et al (1998) studied the leading indicator properties of various UK simple sum, Divisia and risky Divisia monetary aggregates over the time period 1979Q1 to 1994Q2. They find that using risky aggregates offers an improvement over both the simple sum and the standard Divisia monetary aggregates.[3] More recently, Barnett and Wu (2004, 2005) have extended the monetary-asset

[2] One way risk arises in the Divisia index construction due to the fact that the interest rates used as weights are not known with certainty until the end of each period. Since empirical evidence suggests that the risk adjustment is small for the asset components that are usually included in monetary aggregates in this paper we have focused on looking at risky or capital uncertain assets.

[3] Barnett and Xu (1998 and 2000) have investigated the effect of stochastic volatility in interest rates on money velocity. Using simulation data they find that the traditional velocity function becomes unstable if the covariance between consumption and interest rates or between money growth and interest rates change over time.

user-cost risk adjustment and their consequent risk adjusted monetary aggregates of Barnett, Liu and Jensen (1997) to the case of multiple non-monetary assets and intertemporal non-separability. Their model generated potentially larger and more accurate CCAPM user-cost risk adjustments than those found in Barnett et al (1997). This later work showed that risk adjustment to a monetary asset's user cost can be measured easily by its beta and that any risky non-monetary asset can be used as the benchmark asset, if its rate of return is adjusted in accordance with its own formulation. These extensions could be especially useful, when own rates of return are subject to exchange rate risk. See, for example, the recent paper by Ersal-Kiziler and Ha Nguyen (2016) that examines the Eurocurrency risk and the geography of debt flows between core and peripheral members of the European Monetary Union.

We advance the earlier studies by adopting a flexible approach to the construction of the CCAPM proposed by Barnett et al (2004, 2005) for the US and applied by Elger and Binner (2004) for the UK. We relax the restriction that all household decisions are made within one time period by allowing forecasts of expected asset price values to determine the order of the parameters in the construction of the user costs of capital uncertain assets.

Based on weak separability tests we find that capital uncertain assets are part of economic monetary aggregates for the US and UK. We construct these aggregates using a superlative index number. We test these aggregates using time-series analysis and find that these aggregates behave reasonably.

We proceed by reviewing monetary aggregation in section 2. We first discuss aggregation over goods and how we test for an admissible aggregate. We then discuss appropriate index numbers for aggregating the admissible aggregates.

2. Composition and Construction of Economic Monetary Aggregates

The theory of monetary aggregation (Barnett 1978, 1980, 1982, 1987) is based on an optimization framework in which monetary assets are treated as durable goods in the representative consumer's utility function.[4] Let **m** denote a vector of real monetary assets and let **z** denote all other variables in the utility function, so that utility is given by $u(\mathbf{m},\mathbf{z})$. The consumer is assumed to maximize u subject to a budget constraint. The utility function is *weakly separable* in **m** if there exists a macro-function, *U*, and a sub-utility function, *V*, such that

$$u(\mathbf{m},\mathbf{z}) = U\left(V(\mathbf{m}),\mathbf{z}\right).$$

Under weak separability, the marginal rates of substitution between any pair of assets in the separable group of assets, **m**, are functions only of the quantities of those assets. Consequently, the optimal quantities of those assets depend only upon their user costs and group expenditure. Weak separability also implies the existence of an economic aggregate for the separable asset grouping. If the weakly separable group is a group of monetary or financial assets, then the grouping forms a monetary aggregate.[5]

Weak separability can be tested in either a parametric or a non-parametric framework. Both approaches have strengths and weaknesses. The parametric approach requires postulating a functional form and estimating the unknown parameters of that functional form. With the parametric approach the test becomes a joint test of the hypothesized utility structure and the hypothesized functional form. In contrast, the non-parametric revealed preference approach of Varian (1983) does not require a particular functional form and, therefore, avoids problems associated with model misspecification. It can also be used with few data observations and is tractable with a great number of goods and observations.[6] However, since it is

[4] We assume a representative agent or consumer and only explicitly test aggregation over goods.

[5] See Barnett (1980, 1982) for further discussion.

[6] Varian's non-parametric approach has been criticised in the literature. Firstly, the test's non-stochastic nature means a single rejection suggests rejection of the tested hypothesis as a whole. It may well be the case that the rejection was caused by, for example, a shift in demand or some form of measurement error. Secondly, it has

non-parametric, the revealed preference approach is non-stochastic.[7] To avoid the problem of the joint test of structure and a particular functional form we adopt the revealed preference approach in this paper.

Varian's (1983) test for weakly separable utility maximization builds on the generalized axiom of revealed preference (GARP). GARP can be stated:

$$If\ x^i R x^j\ then\ p^j x^j \leq p^j x^i\ for\ all\ i, j = 1,\ldots, n.$$

where $p^i = (p^i_1, \ldots, p^i_k)$ be the ith observations for the prices of some k goods and assets, $x^i = (x^i_1, \ldots, x^i_k)$ denotes the corresponding quantities of the k goods and assets and R stands for revealed preferred. If the data satisfy GARP there exists a nonsatiated, continuous, monotonic, concave utility function that rationalizes the data.[8]

We now turn to Varian's (1983) revealed preference test of weakly separable utility maximization. Let $\mathbf{m}^i = (m^i_1, ..., m^i_n)$ denote observed real quantities for a set of *n* monetary assets and let $\boldsymbol{\pi}^i = (\pi^i_1, ..., \pi^i_n)$ denote the corresponding observed nominal user costs for these assets, where $i = 1, ..., T$. Further, let $\mathbf{z}^i = (z^i_1, ..., z^i_k)$ denote the observed quantities of all other variables in the utility function (including financial assets not in **m**) with corresponding prices $\mathbf{p}^i = (p^i_1, ..., p^i_k)$. Varian (1983) showed that the following conditions are equivalent:

(*i*) There exists a weakly separable (in **m**) concave, monotonic, continuous nonsatiated utility function, which rationalizes the data $(\mathbf{p}^i, \mathbf{z}^i)$ and $(\boldsymbol{\pi}^i, \mathbf{m}^i)$

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been shown by Barnett and Choi (1989) using Monte Carlo simulations, that the test results are biased towards rejection. Of course, this tests bias toward rejection increases confidence in any identified aggregate.

[7] Non-stochastic extensions have been suggested by Varian (1985a), de Peretti (2005) and Hjerstrand and Swofford (2014) and are the subject of ongoing research.

[8] Thus, a violation of GARP happens when for some $x^i R x^j$, the condition $x^j S x^i$ is true or a violation of GARP happens if x^i is shown to be revealed preferred to x^j but x^j is directly revealed preferred to x^i.

(ii) There exist numbers $U^i, V^i, \lambda^i, \mu^i > 0$ ($i = 1,...,T$) such that;
$U^i \le U^j + \lambda^j \mathbf{p}^j(\mathbf{z}^i - \mathbf{z}^j) + \lambda^j (V^i - V^j)/\mu^j \quad \forall i, j \ \forall i, j$.

We check two necessary conditions for weak separability. First, the combined price and quantity data for the entire sets of goods **x** including **m** and **z** must satisfy GARP, otherwise the data cannot be rationalized by a well-behaved nondegenerate utility function, weakly separable or otherwise. Second, the price and quantity data for the separable group of goods $(\boldsymbol{\pi}^i, \mathbf{m}^i)$ must also satisfy GARP, since otherwise no feasible solution exists for the constraints in condition (ii).

If these necessary conditions are satisfied, then an admissible can aggregate exist. We then check Varian's sufficient but not necessary condition. If the sufficient condition is met then an admissible aggregate exists.

Thus Varian's (1985b) implementation of the above revealed preference test, condition (ii), becomes a three step test.

Step 1: Test if the goods in the hypothesized utility function are consistent with with GARP.

Step 2: Test whether the data in the hypothesized sub utility function are consistent with GARP.

Step 3: Test a sufficient, but not necessary, condition for weak separability that is whether the data with the goods in the hypothesized sub utility function replaced by an aggregate good calculated using the Afriat inequalities are consistent with GARP.

A violation of GARP is illustrated in Figure 1. In this illustration the consumer faces choosing how much of two goods, m_1 and m_2 to consume, given two different sets of prices. If when facing budget set EF the consumer chooses consumption bundle A and when facing budget set GH the consumer chooses consumption bundle B, then this consumer's preferences are inconsistent with GARP. This is a violation of GARP because the consumer purchased bundle A when bundle B was available in the budget set and turned around and purchased bundle B when bundle A was available in the budget set. Logically, bundle A can not be preferred to bundle B, while in a later period bundle B is preferred to bundle A, unless the consumer's preferences have changed. In such a case the consumer does not have stable preferences As stated above if a data set is inconsistent with GARP or contains a violation of GARP, then the consumer's choices can not be rationalized by any well behaved utility function.

Figure 1 also illustrates choices by the consumer that are consistent with GARP. If when facing budget set EF the consumer chooses bundle A and when facing budget GH the consumer chooses bundle C, the consumer's preferences are consistent with GARP. This is because when the consumer chose bundle A, bundle C was not available in the consumer's choice set. When the consumer chose bundle C, bundle A was available in the consumer choice set meaning that bundle C is preferred to bundle A. This lack of inconsistent choices or consistency with GARP implies that there is some nonsatiated, continuous, monotonic, concave utility function that rationalizes data. That is the data are consistent with choices consistent with economic theory.

We check for GARP violations based on the two equations in Varian's (1983) condition (*ii*) above. Since Varian (1983) showed that conditions (*i*) and (*ii*) are equivalent, if condition (*ii*) holds, then there exists a weakly separable concave, monotonic, continuous nonsatiated utility function, which rationalizes the data. When condition (*ii*) is met, then aggregation of the goods in the weakly separably branch of the utility function is consistent with the economics of aggregation over goods.

In terms of the examples in Figure 1, Varian's (1985b) three step test for weakly separable utility maximization checks the conditions in (*ii*) and can be summarize as follows. Step one of Varian's (1985b) three step test outlined above requires that there be no inconsistent choices of bundles chosen by the consumer when all the goods considered. That is step one requires no violations of GARP when all the goods in the data set are considered.

Step two of Varian's (1985b) three step test requires that there be no inconsistent choices of bundles, when looking only at the bundles of goods in the sub-utility function. That is step two requires no violations of GARP when only the goods in the sub-utility function are considered. Step three of Varian's (1985b) three step test outlined above requires that the consumer make no inconsistent choices in the overall data when the goods in the sub-utility function are replaced with an aggregate of those goods. That is step three requires no violations of GARP for the overall data set when the goods in the sub-utility function are replaced by an aggregate of the goods that are in the sub-utility function.

We next turn to constructing aggregates identified as consistent with the economics of aggregation. The first problem associated with constructing a weighted monetary aggregate is knowing which price to impute. Barnett (1978) derived the user cost of monetary assets by solving an intertemporal utility maximization problem where the objective function is a derived utility function that includes money. It is assumed that money produces a flow of services to the bearer, who would otherwise hold assets with a higher expected return. The user cost is similar to the equivalent rental cost used in traditional demand analysis to price durable goods. Following Barnett et al (1997) under the assumption of risk neutrality, or if the expected return on assets is known in advance, the real user cost of the i^{th} asset at time t is defined as:

$$\pi_{it} = \frac{E(R_t) - E(r_{it})}{1 + E(R_t)},$$

where r_{it} is the own rate of return of asset i at time t and R_t is the return on a non-monetary assets, i.e. an assets that yields no monetary (liquidity) services, at time t. One can think of R_t as the rate of return on an assets that yields only investment services, such as human capital.

Following Barnett (1980) exact aggregation is achieved if the aggregator function is linearly homogenous. This is commonly known as the consistency condition[9]. In the general

[9] It can be shown that an appropriate aggregate can be constructed using weaker assumptions (Edgerton [1997]).

case, we have n monetary assets. Under linear homogeneity, the continuous time Divisia line integral is defined as:

$$M = v(\mathrm{m}) = \exp\left(\int \sum_{i=1}^{n} s_i \frac{dm_i(t)/dt}{m_i} \right),$$

where s_i is the expenditure share of the monetary asset i. Since real world data is not available in continuous time, this definition of the index is unproductive for empirical purposes. One discrete time approximation of the Divisia index that is often used in empirical work is the Törnquist-Theil approximation due to Törnquist (1936) and Theil (1967). In log change form, this index is defined as:

$$\ln M_t - \ln M_{t-1} = \sum_{i=1}^{n} s_{it}^{*} \left[\ln m_{it} - \ln m_{it-1}\right],$$

where s_{it}^{*} is the average expenditure share held of asset i in t and t-1. We use the Törnquist-Theil discrete time approximation to construct the monetary aggregates we identify for this paper.[10]

Aggregation with the Törnquist-Theil discrete time approximation of the Divisia index has been standard in the monetary aggregation literature since Barnett (1980). Barnett (1980) in part argued for the use of the Törnquist-Theil index because the growth rate of the resulting monetary aggregate is the weighted average of the growth rates of individual component financial assets in the aggregate.

Additionally the Törnquist-Theil index is one index from Diewert's (1976) class of superlative index numbers. As Anderson et al (1997) explain, this means is that Diewert (1976) demonstrated that the Törnqvist-Theil quantity index is exact for the translog flexible

[10] It is common practice in this literature to refer to Törnquist-Theil discrete time approximation as the Divisia index.

functional form. If homotheticity is violated as it usually is, the Törnqvist-Theil index still tracks the monetary aggregator function. Importantly, Caves, Christensen, and Diewert (1982) showed the Törnqvist-Theil index can provide a second order approximation to the Malmquist quantity index, even when homotheticity is violated.

We next describe the data that we use to identify the components of monetary aggregates for the US and UK and to construct the economic monetary aggregates that are used in the time-series analysis. We also describe the other variables used in the time-series analysis of the economic monetary aggregates.

3. Data

We use seasonally adjusted quarterly data covering the period 1998Q1 to 2013Q3 for our weak separability and time-series tests. All data in our studies are real per capita data with the associated nominal prices.[11]

For the UK the goods and assets examined are:

1. Gross Domestic Product (GDP)
2. Leisure (LEIS)
3. Notes and Coins (NC)
4. Non-Interest Bearing Deposits (NIBD)
5. Interest Bearing Bank Sight Deposits (IBSD)
6. Interest Bearing Bank Time Deposits (IBTD)
7. Deposits with Mutual Institutions (DMI)
8. Tax exempt accounts (TESSAS/ISAS)
9. Household unit trust holdings (MFUNDS)
10. Household bonds holdings (BONDS)
11. Household equities holdings (STOCKS)

[11] Thus price time quantity yields expenditure on the good or service.

From the UK Office of National Statistics, we obtain GDP, good 1 above, which is our real sector variable and labor hours worked which is used to calculate leisure, good 2. From the same source we obtain the consumer price index (CPI) and population that are used to convert the data into real per capita or representative agent data. We get the wage rate for the UK from NOMIS – Official UK Labor Market Statistics. We obtain 'capital certain' nominal household sector holdings of the UK components, assets 3 through 8, and the associated deposit rates from the Bank of England interactive database on its website[12]. We obtain 'risky' nominal household sector holdings of UK equities, government bonds and unit trust, assets 9 through 11, at market values from DataStream.

For the US the goods and assets considered are:

1. Gross Domestic Product (GDP)
2. Leisure (LEIS)
3. Currency (CUR)
4. Travellers' Checks (TC)
5. Demand Deposits (DD)
6. Other Checkable Deposits at Commercial Banks (OCDCB)
7. Other Checkable Deposits at Thrift Institutions (OCDTH)
8. Saving Deposits at Commercial Banks (SDCB)
9. Saving Deposits at Thrift Institutions (SDTH)
10. Retail Money Market Funds (RMMF)
11. Small Time Deposits at Commercial Banks (STDCB)
12. Small Time Deposits at Thrift Institutions (STDTH).
13. Household mutual funds holdings (MFUNDS)
14. Household bonds holdings (BONDS)

[12] For details of our UK data source, please see the Bank of England statistical interactive database at http://www.bankofengland.co.uk/boeapps/iadb/index.asp?first=yes&SectionRequired=A&HideNums=-1&ExtraInfo=false&Travel=NIxSTx

15. Household equities holdings (STOCKS)

We obtained all the US the non-monetary data, goods 1 and 2, and associated prices listed above from the Federal Reserve Economic Database (FRED) except Total Hourly Earnings, which are downloaded from DataStream.[13] We obtained US 'capital certain' nominal holdings of assets, assets 3-12 above, from the Centre for Financial Stability.[14] We obtained the 'risky' nominal household sector holdings of US equities, government bonds and unit trusts/mutual funds, assets 13 through 15, at market values from DataStream.[15]

We use FTSE All Shares total return index and S&P 500 Composite total return index for the UK and US respectively for return on equities. We use government-bond price indexes for the UK and US for the return on bonds. We obtain these data also from DataStream. We obtained the returns on unit trusts for the UK and mutual funds for the US from the Morningstar Direct database. For these 'risky' assets, we use the forecasted returns. We consider that individual investors do not change their asset portfolio very quickly. They start with the least expensive way of obtaining a medium of exchange. We assert that equities are the most expensive way and this is the last on the list to obtain medium of exchange, i.e. the last on the list to obtain liquidity. Investors are hesitant to withdraw money from a 'risky' asset even if they incur a loss as argued by Shefrin and Statman (1985) and Barber and Odean

[13] As with the UK data, GDP is our real sector consumption measure and leisure is calculated from average hours worked.

[14] Some of the US data is publically available on the CFS website at this link http://www.centerforfinancialstability.org/hfs.php? and some of the data we obtained in personal contact with the CFS researchers.

[15] It is pertinent to note that the components for the US are not disaggregated into Households and Corporates as done by the Bank of England in the UK. However, to make our US data comparable to those of the UK, we include only those assets that are held by consumers.

(2000). Therefore, we forecasted a period of 12 quarters, or a 3 years ahead expected return on equities, bonds and unit trusts.

GDP for the UK and the US have base years of 2010 and 2009 respectively. The base year for the UK CPI is 2005 and for the US is 2010. We use the price indexes and population to convert the GDP, leisure and assets series into real per capita terms.

Having discussed the data and how and what we did, we turn to empirical results. We first present the monetary aggregates we found for the UK and US using revealed preference tests. We then report some empirical results from time-series tests including these economic monetary aggregates.

4. **Empirical Results**

The data described above were first checked with Varian's revealed preference test described in section 2. The results presented in Table 1 show the representative consumer for each country chose consumption bundles in each period of the data that are consistent with each other. Thus for these structures of the economy there were no violations of GARP for the overall data set, for the sub-utility function goods or overall data set with a monetary aggregate of the goods in the sub-utility functioned substituted for the sub-utility function component goods. That is the data for the UK and US were found to be consistent with Varian's 1983 condition (*ii*) using Varian's (1985b) three step revealed preference test outlined in section 2.

Thus the results presented in Table 1 imply that the data for each country are consistent with a well behaved utility function for the representative consumer. Further as discussed in section 2 above, the results in Table 1 imply that for each country there exists a weakly separable grouping of monetary and financial assets that form a monetary aggregate consistent with economic theory.[16]

This means that broad monetary aggregates consistent with the economics of aggregation over goods were identified for both the UK and US. For the UK the broad monetary aggregate from Table 1 is composed of the goods from sub-utility function: V(NC, NIBD, ITSD, IBTD, DMI, TESSAS/ISAS, MFUNDS, BONDS). For the US the broad monetary aggregate from Table 1 is composed of the goods from the sub-utility function: V(CUR, TC, DD, OCBCB, OCDTH, SDCB, SDTH, RMMF, STDDCB, STDTH, MFUNDS, BONDS).

For the UK data the necessary conditions for weakly separable utility maximization are met. For the US data the necessary and sufficient conditions for weakly separable utility maximization are met. Hence these results on the representative consumer in each country support the inclusion of bonds, equities and unit trusts (mutual funds in the US) in the broad money that includes a basket of monetary and financial assets.

There are three particularly interesting aspects of these results. The first is that these are the first monetary aggregates identified that we know about that go beyond central bank data. There is no logical reason researchers in monetary economics should restrict themselves to data provided by central banks.

A second interesting aspect of these results is that the identified monetary aggregates include risky capital uncertain assets such as bonds. As these assets are relatively liquid, it is not surprising that as hypothesized by Duca (1994) such assets provide liquidity services to the representative consumer. There is no particular reason from economic theory why only capital certain assets should provide the representative consumer monetary services. Again

[16] The Varian revealed preference approach has been used by Swofford and Whitney (1987, 1988 and 1991) and Hjertstrand et al (2016), on US data, and Patterson (1991) and Drake and Chrystal (1994), on UK data, to identify economic monetary aggregates.

as far as we know, no other monetary aggregates have been identified in other prior research that includes such risky capital uncertain asset.

A third interesting aspect of these results is that the forecasted return on stocks or shares empirically is the benchmark return in every period for both the UK and US data. Theoretically, there should be a single bench mark asset that provides no liquidity services and whose return is the benchmark return in each period.[17] In practice many researchers in the area of monetary aggregation have used an envelope approach taking whatever happens to be the highest return in each period as the benchmark return. That means that in many prior studies, there was no single benchmark asset. In this study the forecasted return on stocks or shares was the highest return in each period. This also means that empirically neither stocks nor shares provide liquidity services nor enter into the monetary aggregates constructed for the VAR analyses below. [18]

As discussed above in section 2 we construct the monetary aggregates identified above and to be used for the empirical results below using a reputable index number that Diewert (1976 and 1978) showed to be in his superlative class of index numbers. The index we use is the Törnquist-Theil discrete time approximation of the continuous time Divisia index. By using a superlative index number we do not treat all the component assets as perfect substitutes, but rather allow substitution among assets as relative use costs (prices) change.

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[17] The textbook example of such an asset is human capital. The characteristics of a plausible proxy for the benchmark assets needs (i) to be as good or better store of value than the components of the money aggregate, (ii) to provide no (or at least minimal) transactions services, and as an implication of (i) and (ii) have a certainty equivalent rate of return greater than that of any of the components of the money supply.

[18] Thus, the monetary user cost of stocks is zero indicating a zero weight, i.e. no transactions service, in the economic monetary aggregates.

We next turn to modelling the monetary policy shocks including the identified monetary aggregates for each country. We used a model proposed originally by Christiano, Eichenbaum and Evans (1999) and later adapted by Keating, Kelly and Valcarcel (2014) and Keating, Kelly, Smith and Valcarcel (forthcoming) that can be used both in times when the policy rate of interest is at or nears its zero lower bound and when the policy rate of interest is away from the zero lower bound.

We follow the general methodology outlined in Keating, et al (2014 and forthcoming) to obtain Z_t, where Z_t is an n-vector of variables. The variables in the model are subdivided into three blocks:

$$Z_t = \begin{bmatrix} EA_t \\ imp_t \\ MI_t \end{bmatrix}$$

where EA_t represents a vector of *Economic Activity* variables, imp_t represents a single variable that serves as an indicator of monetary shock and MI_t represents a vector of *Monetary Information* variables all providing information to policy makers. In our implementation the first block, EA_t, contains real gross domestic product and the implicit GDP price deflator. We use our risky economic monetary aggregates that includes capital uncertain assets as the monetary indicator, imp_t. Finally, in our implementation of the Keating, et al empirical model MI_t contains total reserves and the user cost of money.

When as in our case the monetary instrument is a single variable, Christiano, et al (1999) show that a Cholesky factorization will identify the dynamic responses of all variables in Z to monetary shocks for any ordering of EA_t or MI_t blocks. We use the Bayesian information criterion (BIC) to determine the lag order of the VAR. The BIC is minimized by a lag order of one for both the UK and US.

Figure 2 shows plots of the impulse responses from the risky monetary aggregate VAR estimates for both the US and UK. The overall impression from these results is that the risky monetary aggregate provides a plausible interpretation of the data, free of puzzles common in the VAR literature. For example, other researchers that have not used monetary aggregates consistent with the economic theory of aggregation over goods have had to include commodity price indexes on an ad hoc basis to avoid counter intuitive results with some variable. Our results based on monetary aggregates constructed in a manner consistent with economic theory of aggregation over goods are free of such counter intuitive puzzles without resorting to such ad hoc solutions. [19]

The US results are presented in the five plots in the first column in Figure 2. The first plot for US shows the response of US GDP to a positive monetary shock in the US. There is a strong significant, positive expansion of GDP. That is US GDP grows in response to a positive monetary shock, when the monetary aggregate is constructed consistently with the economics of aggregation over goods.

The second plot in the left hand column of Figure 2 shows the US price level response. There is a strong significant increase in price level in response to the positive monetary shock after a relatively short time lag. Thus we find for the US that the price level is related a monetary aggregate consistent with macroeconomic theory.

The third plot in the left hand column of Figure 2 shows the US own response of the risky money aggregate. This effect is the effect of the positive monetary shock on the US risky money aggregate. In the US data the positive monetary shock to the risky monetary aggregate has a positive effect that dissipates slowly.

[19] Including a commodity price index in EA_t is a common solution in the literature to the so called "price puzzle" in VAR models. Keating, et al (2014) show that such "ad hoc" solutions are unnecessary when liquidity is measured properly using economic monetary aggregates like we use in this paper.

The fourth plot in the left hand column of Figure 2 shows the effect of a positive monetary shock on total bank reserves. US total reserves first jump in response to the positive monetary shock and then decline, but this effect is insignificant.

The last plot in the left hand column of Figure 2 shows response of user cost of money to a monetary shock. The US interest rate or user cost at first decreases due to the liquidity effect but the user costs soon rises in response to the rising price level, i.e. we observe the Fisher effect. Thus the overall effect is plausible, but insignificant.

The results for the UK data are presented in the five plots on the right hand side of Figure 2. The UK results are similar in direction to the US results. As seen in first plot on the right hand side of Figure 2, there is a positive effect on GDP in the UK due to the positive monetary shock, but the effect is not significant.

The results of UK price level is shown in plot two on the right hands side of Figure 2. After an initial decline for a few quarters, the effect on the UK price level is positive, but not significant. Thus, the UK price level did not respond to the positive monetary shock during the period of time shown in this plot.

The UK own monetary response is shown in the third plot on the right hand side of Figure 2. This is the response of the positive monetary shock on the UK risky monetary aggregate. The UK monetary aggregate exhibits a strong positive response that dies out much quicker in the UK economy than in the US case. The fact that monetary shocks appear to be much less persistent in the UK may explain why price and output response are insignificant.

The fourth plot on the right hand side of Figure 2 shows the effect of the positive monetary shock on total reserves in the UK. At about two quarters there is a positive and significant effect of the positive monetary shock on UK total reserves, but as the monetary shock dies away, so too does the response of Total Reserves.

Finally, the last plot on the right hand side of Figure 2 shows the UK response of user costs to the positive monetary shock. As in the US, UK user cost declines due to the liquidity effect. Unlike the US there is not even an insignificant Fisher effect in the UK data, which is expected given the insignificant response of price level. The UK user cost drops at first due to the added liquidity and unlike the US remains lower. Again as in the US case the user cost response is not statistically significant.

Overall our results are qualitatively similar in the UK and US and closely corroborate the findings of Keating et al (2014 and forthcoming), although the UK policy responses are much faster. Therefore we can conclude that the effects of the UK market liquidity shock die very rapidly and as a result, whilst the policy responses are similar to those that we observe in the US, they are less statistically significant. We can see that although insignificant, we do see a pronounced liquidity effect with little or no Fisher effect which makes sense given that there is no impact upon the price level in the UK. Our reasoning is that the monetary assets with higher levels of liquidity, e.g. cash, checkable deposits and savings accounts, have lower own rates of return and, as a result, the response of those own rates to price level changes, i.e. the Fisher effect, is smaller than that of the monetary assets with lower levels of liquidity, e.g. bonds equities and unit trusts. Take the extreme case of cash, whose user cost is;

$$UC_{cash} = \frac{E(R_t)}{1 + E(R_t)}$$

Hence, the Fisher effect exhibited by the user cost of cash is identical to that of the bench-mark asset. At the other extreme, the asset with the highest own rate will have zero Fisher effect.

5. Summary and Conclusions

In this paper, we constructed economic monetary aggregates for the UK and the US. We identified these aggregates using revealed preference tests and constructed them using the Törnquist-Theil discrete time approximation of the continuous time Divisia index which is a superlative index. Among the key characteristics of the economic monetary aggregates we constructed is that they:

- come from analysis that examines data beyond central bank data,
- include capital uncertain assets,
- relax a key assumption of the CCAPM model, i.e. a one year planning horizon, and introduce forecasted returns on risky assets,
- discover a plausible benchmark asset and return in the data rather than having to adopt the ad hoc envelope approach.

Further this is also one of only a handful of papers to address both the composition and construction of monetary aggregates. We explore the time-series properties of our aggregates and find they yield plausible results in time-series analysis without having to adopt ad hoc solutions to price puzzles. We find that our economic monetary aggregates including capital uncertain assets provides a plausible interpretation of the data, free of puzzles common in the VAR literature.

These extensions are especially useful when own rates of return are subject to exchange rate risk, thus our work is particularly timely, given the increased uncertainty and volatility in the international money and financial markets and turbulence in exchange rates following the British exit from Europe referendum. Further extensions of this current research on the construction and composition of the monetary aggregates could be applied to consider e.g. the pricing of the fees charged on payment card services, see e.g. recent work by Valverde et al (2016).

Based on our findings we would recommend that future monetary researchers not restrict themselves to central bank data sets and explore the possibility that capital uncertain assets provide liquidity services. We would also recommend that researchers examine economic monetary aggregates constructed using the Törnquist-Theil discrete approximation of the Divisia index number formulation before drawing strong conclusions about the role or lack of a role of money in the economy.

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Table 1

Structures and Monetary Aggregates For Which Weak Separability Does Obtain

For the UK:

U(GDP, LEIS, V(NC, NIBD, ITSD, IBTD, DMI, TESSAS/ISAS, MFUNDS, BONDS))

For the US:

U(GDP, LEIS, V(CUR, TC, DD, OCBCB, OCDTH, SDCB, SDTH, RMMF, STDDCB, STDTH, MFUNDS, BONDS)

Figure 1

Examples of Choices Consistent and in Violation with GARP

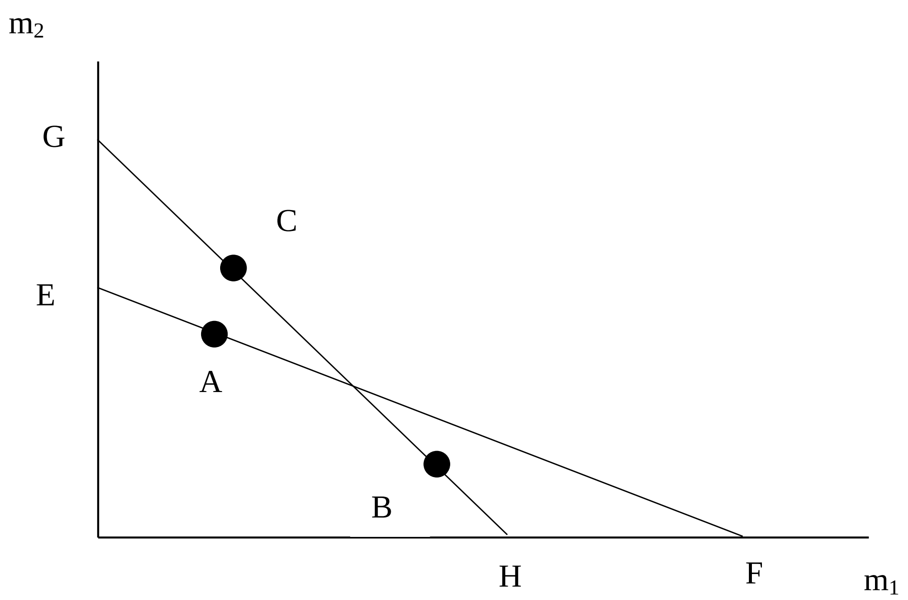

Figure 2

Plots Vector Auto Regression Results