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“My math lessons are all about learning from your mistakes”: how mixed-attainment mathematics grouping affects the way students experience mathematics

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“My math lessons are all about learning from your mistakes”: how mixed-attainment mathematics grouping affects the way students experience mathematics

Mixed-attainment mathematics teaching is not a common practice in England despite evidence that ability grouping is not an effective strategy for improving educational outcomes. This study compares mathematics in School M (mixed-ability groupings) and School S (sets) in relation to student beliefs, and teacher beliefs and practices. Questionnaire data from 286 students and twelve teachers were triangulated with lesson observations and interviews. This article suggests grouping practices could indeed influence students’ mindsets, teachers’ mindsets and teachers’ beliefs and practices. An above average proportion of students in both schools reported growth orientations although these beliefs were held more strongly by students in the mixed-attainment grouping. School M teachers also held stronger growth-mindsets than School S teachers. Mathematics teachers in both schools reported connectionist beliefs but the students’ experiences differed. Most students in School M perceive typical mathematics lessons as involving a substantial problem or challenge worked on collaboratively in pairs or small groups, and having several entry points. Students in both schools valued learning from mistakes but School M students were more likely to both believe this would help them, and have access to this type of learning opportunity. Students taught in sets experienced mathematics as procedures delivered by teachers and reproduced by students. This has implications for further research as mixed-attainment groupings may be a factor in determining the way in which students experience learning mathematics.

Keywords: ability grouping; mathematics; beliefs; mindset; teaching practices; student experience

Introduction/background

There is a “vicious circle” (Taylor et al. 2016) that prevents secondary schools from teaching mathematics in mixed-attainment groups despite evidence that setting or streaming according to some notion of “ability” does not appear to be an effective strategy for raising attainment, particularly for the most disadvantaged students

(Higgins et al. 2016; Steenbergen-Hu, Makel and Olszewski-Kubilius, 2016). Mixed-attainment teaching can be seen as difficult so schools are less likely to group in this way. Consequently there are few examples of good practice and thus little evidence of success so stakeholders are sceptical of the benefits (Taylor et al. 2016). In 2013, Ofsted estimated that around 45% of students were taught in sets (Stewart 2013); figures provided in response to a parliamentary question in 2011 (drawing on data from 2009/2010) suggested 71% of secondary mathematics classes were “ability” sets (Dracup 2014). Even if teachers are initially supportive of the idea of mixed-attainment teaching they are likely to see teaching mixed groups as unworkable unless they change their classroom practices and beliefs (Jackson and Povey 2017).

Mixed-attainment teaching is not a common practice in the UK, where it is widely believed people have an innate, measurable “ability” that is the main determining factor in educational performance (Rattan et al. 2012; Sukhnandan and Lee 1998). This belief continues in spite of evidence achievement is influenced more by social and cultural status (Nash 2006) or financial resources (Jerrim and Macmillan 2015), and evidence from international comparisons which indicates focussing on effort rather than “ability” improves attainment (Rattan et al. 2012; Askew et al. 2010; Stevenson and Stigler 1992; Dweck 1999).

This paper explores the two settings of mixed-attainment mathematics classes and the more common setted mathematics classes. It finds that despite teachers in both setting espousing many similar views about mathematics and ways of teaching, the experiences of the students reveal a different story. Students within the classes which were set by “ability” had a different perspective on their lessons to those of their teachers. They expressed experiencing more of a transmission style of teaching with little opportunity for some ways of working which they valued, such as collaborative

work and learning from their mistakes. We raise the question of whether grouping students in “ability” sets might affect the way in which teachers feel able to carry out the ways of working which fitted their stated beliefs.

Background

Grouping and achievement

“Ability”-grouping does not appear to be an effective strategy for raising attainment. Despite potential issues with combining results several meta-analyses of secondary school setting find small positive effects for higher-attainers and minimal effects for average and below-average students (Kulik and Kulik 1982, 1984; Kulik and Kulik 1987; Kulik 1992). This contrasts with Gutierrez and Slavin (1992) and Linchevski and Kutscher (1998) who find the effect of ability-grouping to be essentially zero and Boaler (2008) and Hallam and Parsons (2013) who find high-attaining students working in mixed-attainment groups progress more rapidly. Reported gains for higher-attainers are at the expense of lower-attainers (Terwel 2005), exaggerate socio-economic differences, and cause underachievement (Boaler 2008; Ireson, Clark and Hallam 2002; Gillborn and Youdell 2000). The overall effect of ability-grouping on attainment is limited; Higgins et al. (2016) found limited impact of setting overall and consistently lower outcomes for middle and low-attainers.

The set a student is placed in impacts their performance and these placements are not always justified. Ireson, Hallam, et al. (2002, 308) found the “extent of setting” experienced by a student and also the set to which a student is allocated can have a significant impact on later achievement. Movement between sets is limited which is significant as “contemporary practices of setting in fact engender low achievement” (Wilkinson and Penney 2014, 425). The same student will do better if placed in a higher

set than if placed in a lower set (Ireson, Hallam, et al. 2002; Lunn and Ferri 1970; Linchevski and Kutscher 1998). Burris, Heubert and Levin (2006) concluded differences in performance of the highest-attaining students were the result of the higher expectations and better teaching experienced. This finding presents an issue of equity as some students may have an arbitrary ceiling on their future attainment (Gillborn and Youdell 2000). Whilst the majority of mathematics departments set using a single assessment, allocations to sets were influenced by factors like behaviour, motivation, ethnicity (Ireson, Clark and Hallam 2002) and socioeconomic status (Wiliam and Bartholomew 2004; Boaler 1997). Even with an excellent test, Black and Wiliam (2006) estimate 50% of students would be placed in the “wrong set” and be unlikely to move (Ireson, Hallam, et al. 2002, Peak and Morrison 1988; Dentzer and Wheelock 1990). Furthermore, different criteria would likely result in different students being categorised as high-ability (Theodoulides 2012).

Students experience mathematics differently if they are taught in sets. Teachers expected students in sets to follow procedures, without detailed help or thinking time in an environment where mistakes are not encouraged, so cannot be learned from, despite the same teachers having used a variety of practices previously when working with mixed-attainment groups (Boaler, Wiliam and Brown 2000). High-attaining groups “focused primarily on mathematical concepts; low-track classes stressed basic computational skills and math facts” (Oakes 1986, 7). Boaler, Wiliam and Brown (2000) found students in both lower and higher sets significantly disadvantaged by setting in terms of their learning and attitudes towards mathematics.

In the following sections, we look at some of the literature concerning the beliefs students and teachers hold about learning and teaching before discussing frameworks which have informed the design of this study.

Student beliefs

Students' beliefs can have a significant impact on their learning of mathematics. Dweck and Elliott (1983) found students were motivated differently when completing tasks. Some have a "performance orientation" to do well on tasks, contrasting with "mastery orientation" where the goal is to learn from tasks (Dweck 1999). Students sometimes attribute academic failure to internal factors outside their control, for example lack of ability and attribute success to external factors such as luck or tasks being "easy" (Prawat and Anderson 1994). This "attributional bias" can be a barrier to success in mathematics (Eccles 1986). Dweck (1999) suggested motivation and attributional style are both influenced by self-theories and contrasted two "theories of intelligence". An "entity theory" or "fixed-mindset" is the belief qualities like intelligence or mathematical ability are permanent traits that cannot be developed. These students are driven by performance goals, devaluing effort, "if you're good at something, you shouldn't need effort" (Dweck 1999, 40). An "incremental theory" or "growth-mindset" views intelligence as malleable and increasable through effort (Bandura and Dweck 1985), yet ability-grouping has been shown to put students into a "fixed mindset" (Boaler 2009). The labelling of students implicit in setting can be seen as relating to them having "fixed ability" (Francis et al. 2017, 103). Blackwell, Trzesniewski and Dweck (2007) showed even simple messages could put students into a fixed mindset. They found growth-mindset students improved in mathematics following challenges and they were significantly more orientated towards learning goals. Following setbacks, fixed-mindset students were less persistent and found work less enjoyable, they also reported these fixed-mindset students would "try to avoid the subject in the future" or "try to cheat on the next test" (Blackwell, Trzesniewski and Dweck 2007, 250). This supports earlier work by Butler (1987) who found learning goals fostered greater

intrinsic motivation. Studies found interventions putting students in a growth-mindset reversed trends for declining grades (Blackwell, Trzesniewski and Dweck 2007; Good, Aronson and Inzlicht 2003). How teachers view students impacts upon students' self-image, view of themselves as students (Allen 2003), motivation (Pollard et al. 2001) and self-confidence (Maclellan 2014).

Teacher beliefs

Teachers' beliefs depend on their "conception of the nature of mathematics and mental models of teaching and learning mathematics" (Ernest 1989, 249). Lakatos (1978) suggests two dominant views: firstly mathematics as a body of knowledge built on universal foundations; secondly that mathematics develops out of conjecturing, then proving or refuting conjectures. Beliefs about the nature of mathematics influence how teachers work with children and studies suggest if teachers perceive mathematics as abstract and requiring correct answers then they view it as less suitable for mixed-attainment teaching (Ireson and Hallam 1999; Kerry 1980; Reid et al. 1981). The "traditional" conception of mathematics as a body of knowledge, skills and techniques for students to acquire is prevalent in western society (Swan 2006). US and German mathematics teachers wanted to teach a particular skill whereas Japanese teachers wanted students to "understand a new concept or transform their thinking" (Stigler 1999). Stevenson and Stigler (1992) found parents and teachers in Japan place more emphasis on effort than Americans. More recently it was shown by Boaler, Wiliam and Brown (2000) and Ireson, Hallam, et al. (2002) that teachers change their practices when teaching mixed and setted groups. Setting practices created fixed mindsets (Boaler 2009, 2013; Dweck 2006) and teachers tended to possess fixed-mindsets, believing for example "top-set" students ought to understand methods given without explanations, "You should be able to, you're in the top set" (Boaler, Wiliam and Brown

2000, 640). These statements send fixed-mindset messages to students and also suggest the teacher's view of students' intelligence is fixed. This is even more damaging for low-attaining students (see Linchevski and Kutscher [1998] for example). As Dweck (2006, 108) notes, teachers with a fixed-mindset "don't believe in improvement, so they don't try to create it". Ireson and Hallam (1999) find teachers stereotype and label students based on their group allocation and this is supported by others (Hargreaves 1967; Lacey 1970; Keddie 1971; Ball 1981; Burgess 1983).

Teachers' beliefs are associated with their practices and further with student attainment (Askew et al. 1997). More recently, enacted practices were strongly related to teachers' dispositions and "students whose teachers had reported teacher-centered beliefs and teacher-centered practices had significantly lower gain scores on curriculum-based assessments" (Polly et al. 2013, 23).

The aim of this study is to consider how teachers' beliefs relate to the similarities and differences in mathematical experienced by students in their schools when comparing mixed-attainment and setted mathematics groupings. Also, to explore whether there are differences in the mindsets of students learning mathematics in these two settings.

Theoretical framework

Askew et al. (1997, 29) identified three orientations which they suggest "shape rather than directly control [teacher] behaviour". The "transmission", "discovery" and "connectionist" orientations emerged initially from a large study of primary teachers (Askew et al. 1997), but were later used by Swan (2006) when investigating Further Education practices suggesting the characterisations may be useful at all levels of mathematics teaching (for more detail see Francome [2015]).

Transmission teachers view mathematics as a body of knowledge and standard procedures to be “covered” (Swan 2006, 133). Explanations of methods are given before students attempt problems to minimise misconceptions. Students are believed to learn best by watching, listening and imitating methods then practising until fluency is achieved (Askew et al. 1997). Askew et al. (1997) suggest this stems from beliefs that students vary in ability and failure to learn results from lack of aptitude if methods are logically explained. The *discovery* orientation involves students “working things out for themselves” (Askew et al. 1997, 5) and using their own concepts and methods. The teacher’s role involves waiting for students to be “ready” to learn and facilitating a stimulating environment for them. These practices have been found to be less effective than teachers taking a connectionist view (Askew et al. 1997; Kirschner et al. 2006).

Connectionist teachers view mathematics as a “coherent collection of interrelated concepts and procedures” (Thompson 1984, 109). Connectionists believe the most effective ways to teach children mathematics require a non-linear dialogue, working on problems before explanations and learning from mistakes (Swan 2005) and were found to be the most effective in increasing student gains (Askew et al. 1997; Swan 2006; Polly et al. 2013). This orientation is also associated with a higher degree of collaboration. Effective collaboration requires group goals and individual accountability (Slavin, Hurley and Chamberlain 2003) and can reduce the cognitive-load of appropriately challenging problems (Paas and Sweller 2012). The three orientations outlined are ideal types and although no teacher is likely to perfectly fit one, teachers may hold “clusters of beliefs” in certain situations (Cooney 2002).

Swan (2006) asked teachers to record the perceived frequency of different practices and gave percentage weightings to three different belief statements. It was used as Swan (2006, 194) describes as “a crude measure of teachers” relative

orientations towards transmission, discovery and connectionist belief systems. Previous observational work by Swan (2006) highlighted a number of “teacher centred” and “student centred” behaviours. Twenty eight statements were devised to offer “insights indirectly, through low inference statements concerning practices” (Swan 2006, 198). These statements were utilised within this study.

Blackwell, Trzesniewski and Dweck (2007, 249) reported on US data collected during a four-year longitudinal study and designed a “motivational questionnaire assessing theory of intelligence, goals, beliefs about effort, and helpless versus mastery-orientated responses to failure”. These items were adapted to shift the focus from intelligence to mathematics.

Methods

This small-scale study took place across two comprehensive schools in the West Midlands during March 2014. The schools were selected to provide contrast in terms of their grouping practices whilst having similarities in other aspects. School M taught mathematics to mixed-attainment groups whilst School S students were taught in attainment sets.

Both schools taught the mathematics curriculum in half-term blocks with the aim of “deep progress” (Watson, De Geest and Prestage 2003) in contrast to a more standard practice of changing topics frequently and repeating similar work each year. The schemes of work were designed to allow all students full access to the curriculum (rather than a lower/higher split for some students) and to spend enough time on topics so key ideas could be explored in depth laying a firm foundation for later work. The schools had similar proportions of students eligible for Pupil-Premium funding (School S, 25.0%; School M, 26.7%) and the number of students from black and minority ethnic

backgrounds for both was around the national average. They also had comparable examination results and OFSTED gradings regarding the judgement of the quality of education the schools provided. A potentially significant difference was that School S was co-educational whilst School M was an all-girls school and this is a limitation of the study. A pragmatic issue was that there were relatively few secondary schools which operate mixed attainment grouping in mathematics across a number of years (Tereshchenko, et al. 2018) whilst having similarities in other aspects. As Gorard (2013) notes, it is never possible to match comparator groups on all factors and we return to this particular difference between the schools later within our conclusions. It should be noted that the first author was Head of Mathematics in School M and this affected choices made within the data gathering, as noted in places below.

Data were collected from four sources. These were student questionnaires, teacher questionnaires, lesson observations and brief teacher interviews following lesson observations. As the purpose of this study was to describe the experiences of students and teachers of mathematics, two questionnaires were devised. The teacher questionnaire was administered electronically to teachers in two schools (six teachers in School M and 18 teachers from School S). Six responses were received from School M and six from School S. It was acknowledged that responses from teachers in School M might be affected through their relationship with the first author, and so the student questionnaire was used partly to triangulate the results from the teacher questionnaires. The results from School S also need to be interpreted in the light that responses were received from just six of the eighteen teachers. These results have similarities but may not be representative of all teachers at School S.

The teacher questions asked were heavily influenced by the literature. Theoretical frameworks were adapted from Blackwell, Trzesniewski and Dweck (2007),

Boaler, Wiliam and Brown (2000) and, in particular, teachers' beliefs were accessed via Swan's (2006, 198) "low-inference" questions on their practices. Teachers were asked to assign percentage weightings to beliefs about mathematics, learning, and teaching and then to record the relative frequency of a number of teacher behaviours in order to gain insights into their practices. Another section of the teacher questionnaire was adapted from Blackwell, Trzesniewski and Dweck (2007) and designed to access beliefs related to mindset, and the final section allowed space for some more detailed responses on teachers' opinions on setting by ability, and aims when planning for and teaching mathematics lessons. This was to generate rich data and give teachers an opportunity to mention anything they felt had been omitted.

Questionnaire data was also collected from 286 year seven (age 11/12) students in School M (129 students) and School S (157 students). Teachers' professed beliefs and what happens in practice can be quite different (Fang 1996) so data was collected from students regarding their experiences in mathematics lessons. Students' perceptions were targeted as students' own learning strategies were inevitably constrained to an extent by the teacher; for example, students cannot choose to discuss in groups if a teacher insists on silence. Tutors administered the questionnaires during form period. This was so that students would feel freer to express their thoughts about mathematics lessons outside of the context of the mathematics lesson. The first section of the student questionnaire, again adapted from Swan (2006), contained items similar to the teachers' as a mechanism for evaluating to what extent the teachers' beliefs matched up with the students' experiences. An example of corresponding items on the two questionnaires is given below:

Students work on their own, consulting a neighbour from time to time (teacher)
The teacher expects us to work mostly on our own, asking a neighbour from time to time (student)

Student mindsets were accessed via questions devised by Blackwell, Trzesniewski and Dweck (2007) and adapted to focus on mathematics rather than intelligence. Effort beliefs items tested whether students thought increased effort led to more favourable outcomes. Positively and negatively phrased items were used throughout both questionnaires to avoid pattern answering (Denscombe 2010).

The final section of the questionnaire used items from Boaler, Wiliam and Brown (2000) and regarded enjoyment, pace and identity as a mathematician. Dichotomous responses were included to allow for chi-squared analysis later (Gillham 2008). There were also two longer response sections to allow for some student voice (Rudduck, Chaplain and Wallace 1996).

The questionnaire was piloted in two stages. Firstly, the questionnaire was administered to a small group of Year 8 students (12-13 year olds) to see if they had any difficulties and some revisions to wording were implemented. The second stage involved getting a small group of students to complete the questionnaire without assistance. The 100% response rate implied the majority of significant issues in the questionnaire design had been resolved. Participants in the study were clear about how their responses would be used and their right to withdraw.

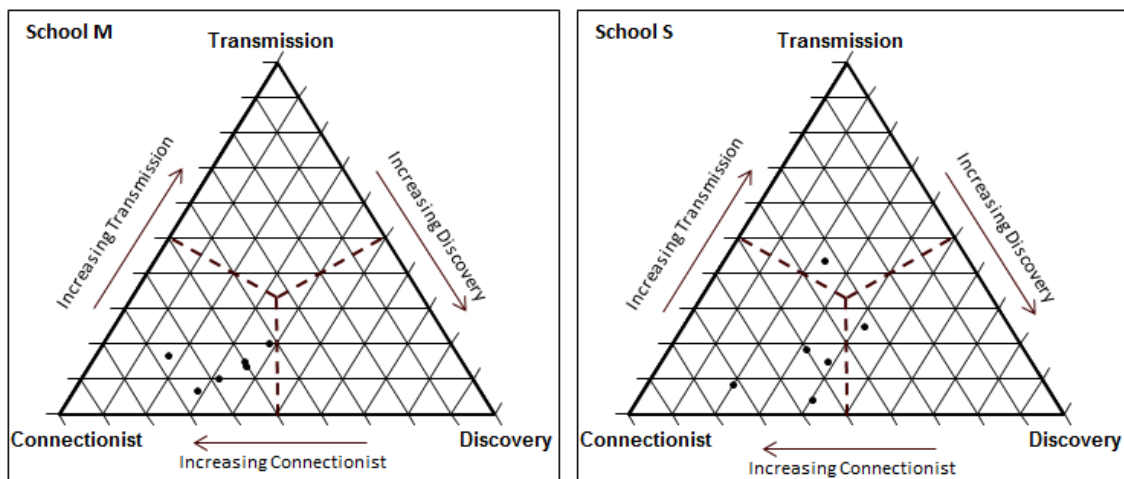
Questionnaire data was triangulated with observations of three lessons in each school and each observation was followed up with a semi-structured interview with the teacher. Structured observations were used where the type of student activity and the type of dialogue were recorded at one-minute intervals throughout the lessons using an observation schedule. The coding was linked to the questionnaire material. The codes related to the degree of collaboration, the type of task and whether students were following a routine they had been taught or discussing alternative approaches. The lessons were audio-recorded so they could be analysed in further detail. Brief recorded

interviews were given with teachers following observed lessons. These asked teachers to reflect upon some of the decisions made during the lesson. These were semi-structured interviews designed to minimise possible bias as the teachers from School M were known well to the first author.

Analysis of findings

Most of the 12 teachers (six from each school) professed overall connectionist beliefs from their responses to the questionnaire. School M teachers had very low transmission scores ($\bar{x} = 13\%$) and very high connectionist scores ($\bar{x} = 55\%$). Teachers from School S tended to profess slightly more of a discovery orientation (33%) and a higher degree of transmission beliefs (20%) with one teacher stating significantly more transmission beliefs (see Figure 1).

Figure 1: Triangular plots showing the overall professed beliefs of each teacher. Points in the top third, bottom right and bottom left represent a predominantly transmission, discovery or connectionist approach respectively.



The questionnaire asked teachers to consider 27 statements and say how frequently each occurred within their lessons. Table 1 shows that the most common practices in School M were connectionist with almost all lessons containing students discussing ideas, working collaboratively and using mistakes and misconceptions.

Although School S teachers' most common practices include some "teacher-centred" practices they were also broadly connectionist aligning with their professed beliefs above. As Swan (2006, 199) notes "teacher-centred describes practices one would expect to arise from a transmission-oriented belief system".

Our results were markedly different from those of the Further Education teachers in Swan's (2006) study where, in the 63 teachers sampled, *every* teacher-centred practice was rated more highly than *every* student-centred practice. So the teachers within our study appeared to favour more "student-centred" statements. However, School S teachers varied much more than those of School M as evidenced by the lack of any statements where all six teachers felt a particular practice occurred almost always or most of the time in their lessons. Also, there were fewer statements where at least half of the School S's teachers agreed were common practices.

Table 1: Teachers were asked to say how often each practice occurred on the following scale: 1 = Almost Never 2 = Occasionally 3 = Half the time 4 = Most of the time 5 = Almost always. The table shows all those with at least three occurrences of 4 or 5. "S" stands for "Student-centred" and T for "Teacher-centred" practices.

School M			School S		
Statement		Common practices	Statement		Common practices
S	Students work collaboratively in pairs or small groups	6	S	I encourage students to make & discuss mistakes	5
S	Students learn through discussing their ideas	6	T	I teach the whole class at once	4
S	I encourage students to make & discuss mistakes	5	S	Students choose which questions they tackle.	4
S	Students work on substantial tasks that can be worked on at	4	T	I know exactly what maths the lesson will contain	4

	different levels				
S	Students compare different methods for doing questions	4	S	Students work on substantial tasks that can be worked on at different levels	4
T	I teach the whole class at once	4	S	I jump between topics as the need arises	4
S	I find out which parts students already understand and don't teach those parts	4	T	Students start with easy questions and work up to harder questions.	4
T	I know exactly what maths the lesson will contain	4	S	I draw links between topics and move back and forth between topics	3
S	Students choose which questions they tackle.	3	S	Students work collaboratively in pairs or small groups	3
S	I draw links between topics and move back and forth between topics	3	S	I am surprised by the ideas that come up in a lesson	3
T	I teach each topic from the beginning, assuming they know nothing	3			
S	I teach each student differently according to individual needs	3			

Teachers at School M believe students learn almost all of the time by discussing ideas collaboratively in pairs or small groups. Most of the time students are encouraged to make and discuss mistakes, work on substantial tasks accessible at different levels and compare alternative methods. These statements fit with the triangular plot (Figure 1) suggesting the majority of teachers at School M hold connectionist beliefs. This is further illustrated by the following:

“Misconceptions must be drawn out, highlighted and worked on. It is through conjectures and misconceptions that the majority of learning takes place within my lessons.” (Teacher M4)

“[When I plan mathematics lessons I think about] ...how to give rise to misconceptions, what tasks will allow for students to explore the idea and develop understanding and methodology for it, how students might access a rich task and how it may be used to extend their mathematical thinking (e.g. generalising, extending the question with a 'what if...?'), what resources will support their learning, how to create a sense of need of the maths...” (Teacher M6)

These statements contrast to a certain extent with some of the statements made by teachers at School S. Although in general School S teachers professed connectionist beliefs, some responses to more open questions indicated there may be some tensions between what they believe and what they feel they need to do when teaching mathematics. For example, one teacher from School S said:

I do realise very much so that it was a very led lesson like it was do this, do this, follow this example do this. I get that and I get that that's not like the best way to teach things but for this class I feel like it's worked well to get them to be doing maths and focused. But I know that it's not the best way to teach that lesson.
(Teacher S3)

Another teacher in School S expressed an intention to “get through” as much as possible and “learn the rules” as well as doing some practice:

...if they can't do it, obviously go through some examples and give them some practice exam questions. (Teacher S2).

In summary, teachers in both schools profess connectionist beliefs and practices, although some responses to open questions raise the issue of whether practice matches those professed beliefs. We will now consider to what extent teachers' views on teaching correlate with students' perceptions of lessons.

Students' perceptions

Students' perceptions of teachers' practices were included to investigate the degree to which teachers' professed beliefs about teaching were reflected in their day-to-day actions as experienced by the students they teach. However this presented some issues as a number of students were taught by teachers other than those who chose to take part in the study. Consequently we compare between the two schools. Not all the statements from Table 1 were transferred into statements for the students (e.g. "I find out which parts students already understand and don't teach those parts") in their questionnaire but many of them were. The top three statements identified by the teachers in School M (Table 1) were corroborated by the students as they also felt that these practices were happening either most of the time or almost always. All six teachers in School M felt they asked their students to work collaboratively in pairs or small groups and 52.4% of the students felt this happened most of the time or almost always compared to 30.1% of pupils in School S. All but one of the teachers felt they encourage students to make and discuss mistakes most of the time or almost always and 68.8% of the students agreed with this compared to just 45.7% in School S. Some student quotations illustrate these connectionist practices:

"I work hard and I sometimes make mistakes but [the teacher] helps me learn from them." (Student M117)

"I like discussing my answers with other classmates because I like to see if we came up with similar strategies" (Student M29)

In response to the question "how do you think maths lessons could be improved for you?" one School M student said "I think maybe just occasionally we could have work set out of the textbook" (Student M67) suggesting the practising experienced does not contain a lot of traditional exercises as in a more transmission approach.

At School S, the majority of pupils surveyed were taught by teachers outside of the study and the practices they experienced were quite different to those in School M. Students tended to feel that a more transmission style of teaching was experienced most of the time or almost always, with the most common practices being “the teacher expects us to follow the textbook or worksheet closely” (75% vs 54% at School M) and “the teacher shows us which method to use, then asks us to use it” (73.2% vs 61.2% at School M). This contrasts with the more connectionist experiences of pupils at School M.

Some further evidence of the transmission approach can be found in some typical School S student responses when asked how they would describe mathematics lessons:

In my maths lessons we always have a worksheet to do but before we start our teacher gives us some examples on the board (Student S100)

Miss tells us the task and explains what to do and the method that needs to be used (Student S111).

Some typical improvements suggested by School S students:

I would like to be able to discuss ideas (Student S48)

Having fun tasks instead of miss just explaining everything and us writing it down in our books. It would help us in tasks and activities to show what we know, and then learning from our mistakes (Student S145).

These quotations from students illustrate they perceive transmission style teaching as commonplace within mathematics lessons and would prefer to work collaboratively and discuss ideas rather than individually copy examples off the board and repeatedly mimic a prescribed method. The differences between the schools were significant (χ^2 , $p=0.009$) as 73% of students in School S thought this practice occurred “almost always” or “most of the time” compared to only 61% in School M. Overall analysis seems to suggest

transmission is the dominant approach experienced by students in School S whereas School M students experience more discussion and opportunities to learn from mistakes. Analysing individual items in more depth suggests students perceive mathematics lessons as containing mostly individual work far more frequently in the sets than the mixed groups. In School S, 69% of students thought they worked mostly alone “almost always” or “most of the time” compared to 52% in School M which was a significant difference (χ^2 , $p=0.038$). This was borne out through the lesson observations (Table 2) which were strikingly different although all described by the teachers as “typical”. School M lessons involved a high level of discussion to expose and discuss misconceptions. The level of student discussion represented a key difference, with School M undertaking a far more collaborative approach.

Table 2: Lesson observation points showing the nature of collaboration in School M and School S. (School M, $n = 165$; School S, $n = 135$. Percentages are accurate to the nearest integer.)

	Whole class	Work alone	Consult occasionally	Work collaboratively
School M (mixed-attainment)	38%	2%	25%	35%
School S (sets)	49%	22%	24%	5%

Table 2 shows similar proportions of time were spent in both schools with students consulting occasionally with peers, such as checking they had the same answer, and whole class teaching still made up a significant proportion of the observed lessons. The vast majority of the remaining time was spent on individual work in School S and on collaborative work in School M. School M students still work individually “We work in

groups/pairs to solve problems but other times we might work on our own” (Student M25). A School S student describes a typical lesson “In my maths class I sit alone and get on with my work” (Student S127).

Although teachers reported similar connectionist beliefs, students taught mathematics in mixed-attainment groups experienced their learning in different ways to those taught in sets. The next section discusses the impact of these differing experiences on teacher and student mindsets.

Mindsets

Teachers in School M and School S have similar views on intelligence and mathematical ability; both groups believe ability/intelligence can be increased in line with the growth-mindset. However, opinions are held more strongly by the teachers of mixed-attainment groups. School M teachers agreed more strongly with growth-mindset statements and disagreed more strongly with fixed-mindset statements. Some quotations illustrated School M teachers’ growth-mindsets and how they wanted to encourage students to learn from mistakes:

“I want my students to feel confident and not worry about making mistakes.”
(Teacher M4)

“I want them to get stuck and see how much there is out there. I want them to feel that they can ask. But I want them to feel that they can persist and unstick themselves...” (Teacher M3)

Both sets of teachers viewed “mathematical ability” as being more susceptible to change through effort than intelligence but again more so in School M. Some things School S teachers actually said during interviews hint at a more fixed-mindset. One School S teacher, working with what she described as a “*bottom-set*” said “...*most of them always start like struggling with the easiest because that’s the nature..., that’s*

their... I don't know why...” (Teacher S3), suggesting she may have some fixed views of the students’ “ability” and some awareness of them as she checks herself and says she is not actually sure why students always start with easy questions. Another described a class as being “*Top-set, so the strongest ability*” (Teacher S2) implying mathematical ability may be viewed as a fixed trait. Teacher responses to the questionnaire suggested that, despite a difference in degree, teachers from both schools lean towards a growth-mindset. We now consider the views of students.

A larger proportion of students in School M (73%) reported overall growth mindsets than those in School S (67%) and held these views more strongly. It should be noted that although the difference was significant (χ^2 , $p=0.001084$), these figures are both much more similar to each other and well above than the often quoted 40% typically observed (Dweck 2014, 2). Questionnaire responses revealed that 64% of School S students gave a fixed-mindset response to the statement “You have a certain amount of maths ability, and you can’t really do much to change it” compared to only 43% in School M. Some students were explicit about this:

“My maths lessons are fun and interesting. My maths lessons are helping me get better at maths” (Student M12)

This illustrates the growth-mindset that mathematical ability can be increased through effort and contrasts with this statement from a School S student implying he has a fixed ability identified by his teacher:

“I’m always doing work at my level” (Student S135).

These responses are interesting because the overwhelming majority of students believe that “effort” is more important than “being naturally clever” (School M, 98%; School S, 94%). Students’ views are closer on the statement “You can learn new things in maths,

but you can't really change your basic maths ability" with 63% of School S students giving a fixed-mindset response compared to 54% in School M. However, both sets of students think "You can change your maths ability level quite a bit" (School M, 90%; School S, 87%). This apparent contradiction may be due to the inclusion of the word "level" as students at this time in the UK system had already had eight years of progressing through national curriculum "levels" in school. So students may be more inclined to think levels can increase. In the US system, where these questions were first used, "level" had no such specific meaning.

Students in both schools felt learning was a good motivator even when it conflicted with short-term performance. No significant difference was found between the two groups' agreement with the statement "I like maths work I'll learn from even if I make a lot of mistakes" (School M, 94%; School S, 88%; χ^2 , $p=0.073627$). However, comments suggest that whilst students in both schools appear to value learning from mistakes students from the two schools have different levels of access to this type of learning. For example, two students from School M said:

"My math lessons are all about learning from your mistakes and improving on your mathematics skills" (Student M103).

"I enjoy my maths lessons, I am trying to enjoy challenges and to enjoy thinking hard. I'm sure this will develop my brain" (Student M9)

In contrast with a quote mentioned earlier from a student in School S:

"[...] instead of miss just explaining everything and us writing it down in our books. It would help us in tasks and activities to show what we know, and then learning from our mistakes" (Student S126).

However it is worth noting some students do not mind a lack of challenge:

"[Lessons] are fun because they're not that difficult" (Student S4).

Students at School M believe more strongly that effort is the key to improvement in mathematics. 84% of School M students disagreed with the statement “If you’re not good at maths, working hard won’t make you good at it” compared with 73% of School S which is a significant difference between the schools (χ^2 , $p=0.033$). The strength of disagreement was also notable, around 43% of students displayed a “pure incremental theory” (Blackwell, Trzesniewski and Dweck 2007) by strongly disagreeing with this statement compared to 34% of School S students.

The data suggests students and teachers in both schools tended to have growth-mindsets. These beliefs were held more strongly by students and teachers in School M than School S. One teacher commented:

“I think the most important lesson for anyone to learn in maths is the harder you work at it, the better you’ll do” (Teacher M4).

Teachers in School M possess stronger growth orientations and were more likely to structure lessons so students make and learn from mistakes. School M students appreciate the level of challenge, for example:

“My maths lesson is really good and is hard” (Student M91).

“[teachers] make me work hard in a good way” (Student M14).

School M students want work that is challenging enough for them to make mistakes they can learn from, as illustrated by this remark:

“I learn from my mistakes in lessons” (Student M34).

Students in School S are also motivated by learning but in contrast to School M students, they do not necessarily feel like they are receiving challenging work:

“Maths could be improved like to have harder work and to work in groups more and to have more tasks” (Student S40)

“They could stop giving you as many questions that you are already good at”
(Student S87).

Conclusion

A surprisingly higher than average number of students in both schools reported having a growth-mindset although the beliefs tended to be stronger amongst the mixed-attainment students. Students taught in mixed-attainment groups had a stronger view of intelligence as improvable, were more strongly motivated by “learning goals” and held stronger beliefs that effort was key to success in mathematics. This raises the issue of whether the way in which students are grouped in mathematics might affect the mindset they have with regard to their potential learning.

Teachers’ professed beliefs in the two settings were similar although their practices differed in some noteworthy ways. Teachers in both schools reported mainly “connectionist” beliefs (Askew et al. 1997) regarding the nature of mathematics, children and teaching. Teachers in the mixed-attainment setting claimed they used mainly student-centred practices whereas there was a greater mix of student- and teacher-centred practices claimed by teachers teaching setted classes. When teaching practices were analysed, typical mixed-attainment lessons involved students discussing ideas collaboratively in small groups or pairs, and being encouraged to make and discuss mistakes whilst working on substantial tasks accessible at different levels. This was corroborated by students who expected lessons to involve working together and sharing ideas to solve problems. Mistakes were encouraged, learnt from and students developed understanding through discussion.

A key aspect of the teaching in School M concerned beliefs about mistakes and misconceptions which were associated with both the “connectionist” teaching

orientation and the growth mindset. Teachers actively planned for mistakes to arise so that they could be discussed and worked on:

“[when planning, I think about] activities which will draw out the misconceptions or the maths that I want to occur” (Teacher M4).

Utilising mistakes as a tool for learning has been shown to increase mathematics attainment (Dweck 2008; Blackwell, Trzesniewski and Dweck 2007; Boaler 2008; Boaler 2009; Boaler 2013). Students in School M were conscious of this strategy being used by teachers demonstrating that learning from mistakes has translated from belief into practice, as one student succinctly said:

“my math lessons are all about learning from your mistakes and improving on your mathematics skills” (Student M103).

The most common experiences of pupils in the “ability” grouped setting, despite at least some teachers professing connectionist orientations, were teacher-centred with the whole class being kept together and the teacher knowing exactly what mathematics the lesson would contain. Students in “ability” groups reported expecting to work mostly on their own, mimicking a method shown by the teacher and following a textbook/worksheet closely. Many of these students professed a desire to work on more challenging mathematics by discussing their ideas in groups.

There was a difference in the students’ experiences of mathematics lessons in the two settings. In the mixed-attainment classes, students’ opinions tended to align with those expressed by their teachers. The overall picture was one of working collaboratively and learning from discussing their ideas. In contrast, students in setted classes were experiencing lessons with fewer connectionist features such as discussion of mistakes. Instead they expressed views which fitted a more transmission style of

teaching. In general, students in “ability” classes tended to work on their own whilst students in the mixed-attainment classes worked more collaboratively and discussed their ideas more.

Students in both schools felt learning motivated them even if it resulted in reduced short-term performance. The overwhelming majority in both schools expressed a belief that they learn from their mistakes. However, there was a difference in access to this form of learning with students in setted classes reporting less opportunity to learn from mistakes as the work was either not hard enough or that everything was explained by the teacher and they had just to copy what they were told. There was a greater sense of students feeling less challenged by the work they received in the setted classes compared with those in the mixed-attainment classes.

Although this research was conducted carefully and attempts were made to triangulate findings by collecting data from multiple sources there are noteworthy limitations. This work compared only two schools and they differed in ways other than their grouping practices. It may be useful to apply Gorard’s (2013, p.47) ‘inverse warrant principle’ to conjecture about what could explain the differences if it is not grouping practices.

School M was single-sex and it may be that the school’s ethos and philosophy could have had more impact on mindsets and teachers’ practices. However there are a number of reasons for choosing grouping practices as the favoured explanation for the differences. Firstly, School-level effects are typically smaller than classroom-level effects (Hattie, 2009). Secondly, growth mindset interventions alone, for example, don’t tend to work beyond the short term (Donohoe, Topping and Hannah, 2012). Thirdly, School M only had mixed groups for mathematics and not for other subjects, so it is unlikely that the school ethos is the cause of observed differences. Finally, the literature suggests that setting can put pupils into a fixed-mindset (Boaler, 2009; 2013; Dweck,

2006) so there is a theoretical basis for the claim. If the reader accepts the differences are a result of grouping practices, then the conclusions follow. However, the work could be strengthened using random allocation of grouping conditions to account for both known and unknown factors although recent attempts have not been straightforward (Taylor et al. 2016).

Such a small study cannot be generalised but it raises some important questions around whether a connectionist orientation leads to seeing teaching mixed groups as more appropriate or whether teaching in mixed groups encourages alignment with connectionist views. We will consider how the fact of a class being mixed-attainment or setted, can lead to or discourage connectionist orientations. The fact that there was little difference in the mixed-attainment classes gives a sense of the environment being such that the teachers were able to carry out the way of working which fitted their beliefs. Mixed-attainment teaching requires the variety of students' current levels of awareness and expertise to be addressed in the planning and teaching of those classes. As such attempting to teach "at a particular level" is unlikely to succeed and so working on substantial tasks which can be worked on at different levels allows each student to feel challenged mathematically. This, in turn, is likely to lead to different methods being employed by students. Working collaboratively allows ideas to be communicated and shared with discussion playing a crucial role. Since the tasks are substantial, mistakes are more likely to occur and this allows greater opportunity for students to learn from their mistakes. Such a variety of student perspectives allows for greater opportunity to make connections between the different mathematical aspects of what students have done.

In contrast the fact that a class is setted can give a teacher a false sense of the students being "at the same level". This can lead to more directed and traditional

teaching where the students are told a procedure which they are then asked to follow. Since the procedure has been explained, the students are more likely to be able to reproduce this and so not feel challenged mathematically, making fewer mistakes and not gaining a sense of learning from mistakes. There can also be less variety in the mathematics taking place within a lesson and so less opportunity for making connections between different aspects of mathematics. There is less need for discussion or collaborative working as often students are completing less substantial tasks and have already got the same answers. A consequence of this is that the actual experiences students have of learning mathematics may be at odds with the beliefs expressed by teachers within the school.

The issue of whether the way in which students are organised into mixed-attainment or setted classes might affect the way in which teachers go about their teaching and consequently, the learning experiences gained by students is one which deserves greater attention. Further research on this would be welcome.

References

- Allen, B. 2003. "Pupils' Perspectives on Learning Mathematics." In *Mathematics Education: Exploring the Culture of Learning*, edited by B. Allen and S. Johnston-Wilder, 233-242. London: RoutledgeFalmer.
- Askew, M., M. Brown, V. Rhodes, D. Johnson and D. Wiliam. 1997. *Effective Teachers of Numeracy*. London: Kings College.
- Askew, M., J. Hodgen, S. Hossain and N. Bretscher. 2010. *Values and Variables: Mathematics Education in High-Performing Countries*. London: Nuffield Foundation.
- Ball, S. J. 1981. *Beachside Comprehensive*. Cambridge: Cambridge University Press.
- Bandura, M. and C. S. Dweck. 1985. *The Relationship of Conceptions of Intelligence and Achievement Goals to Achievement-Related Cognition, Affect and Behavior*, Unpublished Manuscript, Harvard University.
- Black, P. and D. Wiliam. 2006. "The Reliability of Assessments." In *Assessment and Learning*, edited by J. Gardner, 119-131. London: SAGE Publications.
- Blackwell, L. S., K. H. Trzesniewski and C. S. Dweck. 2007. "Implicit Theories of Intelligence Predict Achievement Across an Adolescent Transition: A Longitudinal Study and an Intervention." *Child Development* 78 (1): 246-263.
- Boaler, J. 1997. *Experiencing School Mathematics: Teaching Styles, Sex and Setting*. Buckingham: Open University Press.
- Boaler, J. 2008. "Promoting 'Relational Equity' and High Mathematics Achievement Through an Innovative Mixed-Ability Approach." *British Educational Research Journal* 34 (2): 167-194.
- Boaler, J. 2009. *The Elephant in the Classroom: Helping Children Learn and Love Maths*. London: Souvenir Press.
- Boaler, J. 2013. "Ability and Mathematics: The Mindset Revolution that is Reshaping Education." *Forum* 55 (1): 143-152.
- Boaler, J., D. Wiliam and M. Brown. 2000. "Students' Experiences of Ability Grouping: Disaffection, Polarisation and the Construction of Failure." *British Educational Research Journal* 26 (5): 631-648.
- Burgess, R.G. 1983. *Experiencing Comprehensive Education: A Study of Bishop McGregor School*. London: Methuen.

- Burris, C. C., J. P. Heubert and H. M. Levin. 2006. "Accelerating Mathematics Achievement Using Heterogeneous Grouping." *American Educational Research Journal* 43 (1): 137-154.
- Butler, R. 1987. "Task-Involving and Ego-Involving Properties of Evaluation: Effects of Different Feedback Conditions on Motivational Perceptions, Interest, and Performance." *Journal of Educational Psychology* 79 (4): 474-482.
- Cooney, T. 2002. "Mathematics Teacher Change and Developments". In *Beliefs: A Hidden Variable in Mathematics Education?* Mathematics Education Library Vol 31, edited by G. C. Leder, E. Pehkonen and G. Törner, 127-147. Dordrecht: Springer.
- Denscombe, M. 2010. *The Good Research Guide: For Small-Scale Social Research*. 4th edn. Maidenhead, Berkshire: McGraw Hill.
- Dentzer, E. and A. Wheelock. 1990. *Locked In/locked Out: Tracking and Placement Practices in Boston Public Schools: A Report*: Massachusetts Advocacy Center.
- Donohoe, C., Topping, K. and Hannah, E. 2012. "The impact of an online intervention (Brainology) on the mindset and resiliency of secondary school pupils: a preliminary mixed methods study." *Educational Psychology* 32 (5): 641-655.
- Dracup, T. 2014. "The Politics of Setting." *Gifted Phoenix*, November 12. Accessed 14 July 2017. <https://giftedphoenix.wordpress.com/2014/11/12/the-politics-of-setting/>
- Dweck, C. S. 1999. *Self-Theories: Their Role in Motivation, Personality, and Development*. Hove, East Sussex: Psychology Press.
- Dweck, C. S. 2006. *Mindset: The New Psychology of Success*. New York: Ballantine.
- Dweck, C. S. 2008. *Mindsets and Math/Science Achievement*. New York: Carnegie Corporation of New York, Institute for Advanced Study, Commission on Mathematics and Science Education.
- Dweck, C. S. and E. S. Elliott. 1983. "Achievement Motivation." In *Handbook of Child Psychology* 4, edited by P. H. Mussen and E. M. Hetherington, 643-691. New York: Wiley.
- Eccles, J. S. 1986. "Gender-Roles and Women's Achievement." *Educational Researcher* 15 (6): 15-19.
- Ernest, P. 1989. "The Impact of Beliefs on the Teaching of Mathematics." In *Mathematics Teaching: The State of the Art*, edited by P. Ernest, 249-254. New York: Falmer Press.

- Fang, Z. 1996. "A Review of Research on Teacher Beliefs and Practices." *Educational Research* 38 (1): 47-65.
- Francis, B., P. Connolly, L. Archer, J. Hodgen, A. Mazonod, D. Pepper, S. Sloan, B. Taylor, A. Tereshchenko and M.-C. Travers. 2017. "Attainment Grouping as Self-Fulfilling Prophecy? A Mixed Methods Exploration of Self Confidence and Set Level Among Year 7 Students." *International Journal of Educational Research* 86: 96-108.
- Francome, T. J. 2015. *Experiences of Teaching and Learning Mathematics in Setted and Mixed Settings*. Unpublished MRes Thesis. Birmingham: University of Birmingham.
- Gillborn, D. and D. Youdell. 2000. *Rationing Education. Policy, Practice and Equity*. Buckingham, Philadelphia: Open University Press.
- Gillham, B. 2008. *Developing a Questionnaire*. London: A&C Black.
- Good, C., J. Aronson and M. Inzlicht. 2003. "Improving Adolescents' Standardized Test Performance: An Intervention to Reduce the Effects of Stereotype Threat." *Journal of Applied Developmental Psychology* 24 (6): 645-662.
- Gorard, S. 2013. *Research design: Creating robust approaches for the social sciences*. London: Sage.
- Gutierrez, R. and R. E. Slavin. 1992. "Achievement Effects of the Nongraded Elementary School: A Best Evidence Synthesis." *Review of Educational Research* 62 (4): 333-376.
- Hallam, S. and S. Parsons. 2013. "Prevalence of Streaming in UK Primary Schools: Evidence from the Millennium Cohort Study." *British Educational Research Journal* 39 (3): 514-544.
- Hargreaves, D. 1967. *Social Relations in a Secondary School*. London: Tinling.
- Hattie, J. 2009. *Visible Learning*. London: Routledge.
- Higgins, S., M. Katsipataki, A. Villanueva-Aguilera, R. Coleman, P. Henderson, L. Major, R. Coe and D. Mason. 2016. *The Sutton Trust-Education Endowment Foundation Teaching and Learning Toolkit*. Durham: Education Endowment Foundation.
- Ireson, J., H. Clark and S. Hallam. 2002. "Constructing Ability Groups in the Secondary School: Issues in practice." *School Leadership and Management* 22 (2): 163-176.

- Ireson, J. and S. Hallam. 1999. "Raising Standards: Is Ability Grouping the Answer?" *Oxford Review of Education* 25 (3): 343-358.
- Ireson, J., S. Hallam, S. Hack, H. Clark and I. Plewis. 2002. "Ability Grouping in English Secondary Schools: Effects on Attainment in English, Mathematics and Science." *Educational Research and Evaluation* 8 (3): 299-318.
- Jackson, C. and H. Povey. 2017. "'No, it just didn't work': A Teacher's Reflections on All Attainment Teaching." In *CERME 10: 10th Congress for European Research in Mathematics Education (CERME10, February 1-5, 2017)*, edited by Thérèse Dooley and Ghislaine Gueudet, 1545-1552. Dublin, Ireland: DCU Institute of Education and ERME.
- Jerrim, J. and L. Macmillan. 2015. "Income Inequality, Intergenerational Mobility, and the Great Gatsby Curve: Is Education the Key?" *Social Forces* 94 (2): 505-533.
- Keddie, N. 1971. "Classroom Knowledge." In *Knowledge and Control*, edited by M. Young, 133-60. London: Collier-MacMillan.
- Kerry, T. 1980. "RE: A Suitable Case for Mixed Ability." *British Journal of Religious Education* 3 (2): 46-52.
- Kirschner, P. A., J. Sweller and R. E. Clark. 2006. "Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching." *Educational Psychologist* 41 (2): 75-86.
- Kulik, C.-L. C. and J. A. Kulik. 1982. "Effects of Ability Grouping on Secondary School Students: A Meta-Analysis of Evaluation Findings." *American Educational Research Journal* 19 (3): 415-428.
- Kulik, C.-L. C. and J. A. Kulik. 1984. "Effects of Ability Grouping on Elementary School Pupils: A Meta-Analysis." Paper Presented at the Annual Meeting of the American Psychological Association Ontario, Canada, October 24-28. (Washington DC: ERIC Document Reproduction Service No. ED255329.)
- Kulik, J. A. 1992. *An Analysis of the Research on Ability Grouping: Historical and Contemporary Perspectives. Research-Based Decision Making Series*. Storrs, CT.: National Research Center on the Gifted and Talented. (Washington DC: ERIC Document Reproduction Service No. ED350777.)
- Kulikand, J. A. and C.-L. C. Kulik. 1987. "Effects of Ability Grouping on Student Achievement." *Equity and Excellence in Education* 23 (1-2): 22-30.
- Lacey, C. 1970. *Hightown Grammar*. Manchester: Manchester University Press.

- Lakatos, I. 1978. *Philosophical Papers, Vol.2*. Cambridge: Cambridge University Press.
- Linchevski, L. and B. Kutscher. 1998. "Tell Me with Whom You're Learning, and I'll Tell You How Much You've Learned: Mixed-Ability versus Same-Ability Grouping in Mathematics." *Journal for Research in Mathematics Education* 29 (5): 533-554.
- Lunn, J. C. B. and E. Ferri. 1970. *Streaming in the Primary School: A Longitudinal Study of Children in Streamed and Non-Streamed Junior Schools*. Slough: National Foundation for Educational Research in England and Wales.
- Maclellan, E. 2014. "How Might Teachers Enable Learner Self-Confidence? A Review Study", *Educational Review* 66 (1): 59-74.
- Nash, R. 2006. "Controlling for 'Ability': A Conceptual and Empirical Study of Primary and Secondary Effects." *British Journal of Sociology of Education* 27 (2): 157-172.
- Oakes, J. 1986. "Keeping Track, Part 1: The Policy and Practice of Curriculum Inequality." *Phi Delta Kappan* 68 (1): 12-17.
- Paas, F. and J. Sweller. 2012. "An Evolutionary Upgrade of Cognitive Load Theory: Using the Human Motor System and Collaboration to Support the Learning of Complex Cognitive Tasks." *Educational Psychology Review* 24 (1): 27-45.
- Peak, B. and K. Morrison. 1988. "Investigating Banding Origins and Destinations in a Comprehensive School." *School Organization* 8 (3): 339-349.
- Pollard, A., P. Broadfoot, E. McNess, M. Osborn and P. Triggs. 2001. *What Teachers Do: Changing Policy and Practice in Primary Education*. London: Continuum.
- Polly, D., J. R. McGee, C. Wang, R. G. Lambert, D. K. Pugalee and S. Johnson. 2013. "The Association Between Teachers' Beliefs, Enacted Practices, and Student Learning in Mathematics." *Mathematics Educator* 22 (2): 11-30.
- Prawat, R. S. and A. L. Anderson. 1994. "The Affective Experiences of Children during Mathematics." *The Journal of Mathematical Behavior* 13 (2): 201-222.
- Rattan, A., K. Savani, N. V. Naidu and C. S. Dweck. 2012. "Can Everyone Become Highly Intelligent? Cultural Differences in and Societal Consequences of Beliefs about the Universal Potential for Intelligence." *Journal of Personality and Social Psychology* 103 (5): 787-803.
- Reid, M. I., L. R. Clunies-Ross, B. Goacher and C. Vile. 1981. "Mixed Ability Teaching: Problems and Possibilities." *Educational Research* 24 (1): 3-10.

- Rudduck, J., R. Chaplain and G. Wallace. 1996. *School Improvement: What Can Pupils Tell Us?*: London: David Fulton.
- Slavin, R. E., E. A. Hurley and A. Chamberlain. 2003. "Cooperative Learning and Achievement: Theory and Research." In *Handbook of Psychology*, edited by W. M. Reynolds and G. E. Miller, 177-198. Hoboken, NJ: Wiley.
- Stevenson, H. and J. W. Stigler. 1992. *Learning Gap: Why Our Schools are Failing and What We can Learn from Japanese and Chinese Education*. New York: Simon and Schuster Paperbacks.
- Steenbergen-Hu, S., Makel, M. C. and Olszewski-Kubilius, P. 2016. "What one hundred years of research says about the effects of ability grouping and acceleration on K–12 students' academic achievement: Findings of two second-order meta-analyses." *Review of Educational Research* 86 (4): 849-899.
- Stewart, W. 2013. "Streaming Is Out – Now You Can Go With The Flow." *Times Educational Supplement [online]*, April 19. Accessed 20 February 2014. <http://www.tes.co.uk/article.aspx?storycode=6329982>
- Stigler, J. W. 1999. *The TIMSS Videotape Classroom Study: Methods and Findings from an Exploratory Research Project on Eighth-Grade Mathematics Instruction in Germany, Japan, and The United States*. Washington DC: DIANE Publishing.
- Sukhnandan, L. and B. Lee. 1998. *Streaming, Setting and Grouping by Ability: A Review of the Literature*. Slough: NFER.
- Swan, M. 2005. *Standards Unit. Improving Learning in Mathematics: Challenges and Strategies*. London: Department for Education and Skills. Accessed 24 December 2017. <https://www.ncetm.org.uk/files/224>
- Swan, M. 2006. *Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices*. London: NRDC.
- Taylor, B., B. Francis, L. Archer, J. Hodgen, D. Pepper, A. Tereshchenko and M.-C. Travers. 2016. "Factors Deterring Schools from Mixed Attainment Teaching Practice." *Pedagogy, Culture & Society* 25 (3): 1-16.
- Tereshchenko, A., Francis, B., Archer, L., Hodgen, J., Mazenod, A., Taylor, B., Pepper, D. and Travers, M.-C. 2018. "Learners' attitudes to mixed-attainment grouping: examining the views of students of high, middle and low attainment." *Research Papers in Education*, DOI: 10.1080/02671522.2018.1452962

- Terwel, J. 2005. "Curriculum Differentiation: Multiple Perspectives and Developments in Education." *Journal of Curriculum Studies* 37 (6): 653-670.
- Theodoulides, A. 2012. "Personalized Learning in Physical Education". In *Equity and Inclusion in Physical Education and Sport*, 2nd ed., edited by G. Stidder and S. Hayes, 34-50. Abingdon: Routledge.
- Thompson, A. G. 1984. "The Relationship of Teachers' Conceptions of Mathematics and Mathematics Teaching to Instructional Practice." *Educational Studies in Mathematics* 15 (2): 105-127.
- Watson, A., E. De Geest and S. Prestage. 2003. *Deep Progress in Mathematics-The Improving Attainment in Mathematics Project*. Oxford: University of Oxford, Department of Educational Studies.
- Wiliam, D. and H. Bartholomew. 2004. "It's Not Which School But Which Set You're In That Matters: The Influence of Ability Grouping Practices on Student Progress in Mathematics." *British Educational Research Journal* 30 (2): 279-293.
- Wilkinson, S. D. and D. Penney. 2014. "The Effects of Setting on Classroom Teaching and Student Learning in Mainstream Mathematics, English and Science Lessons: A Critical Review of the Literature in England." *Educational Review* 66 (4): 411-427.