A design of experiments investigation into the processing of fine low specific gravity minerals using a laboratory Knelson Concentrator

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Highlights

- DOE used to investigate the optimum operating conditions when processing a fine (-53 μm) magnetite-quartz synthetic ore using a laboratory 3-in Knelson Concentrator.

- Feeds with three different grades (5 %, 10 % and 15 %) were investigated.

- Optimal operating conditions dependent on feed grade.

- Grade and recovery can be optimized simultaneously for high-grade feeds (15 %), but not for lower-grade (≤ 10 %) material.

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Abstract

Fine grinding, required to liberate valuable minerals in finely disseminated deposits, creates significant challenges for beneficiation. For these deposits, traditional gravity separation techniques are often ineffective, and centrifugal separators, such as the Knelson Concentrator, are required. The Knelson Concentrator is well established for treating gold ores, and due to its relatively low cost and small environmental impact when compared to other separation techniques, it has become an active area of research for the processing of lower specific gravity (SG) minerals. This work investigates the optimum operating conditions when processing fine (-53 μm) low SG material using a laboratory Knelson Concentrator. A synthetic feed comprised of magnetite (SG 5.2) and quartz (SG 2.65), with grades of 5 %, 10 % and 15 % magnetite, was used to mimic a low-density ore. Central composite design was used to design the experiments and response surface method was used for optimization, with the experimental variables being bowl speed (G), fluidizing water rate (L/min) and solids feed rate (g/min). The results indicate, for 5 % and 10 % magnetite feeds, that bowl speed impacts concentrate grade negatively and heavy mineral recovery positively, while the fluidizing water rate has an opposite effect on separation. A trade off between grade and recovery must therefore be made when processing this material. When processing the 15 % feed, maximum concentrate grade and magnetite recovery were achieved at high bowl speeds and low fluidizing water rates.

Keywords

Gravity separation; Knelson Concentrator; Response surface method; Central composite design

1. Introduction

As mineral deposits are becoming increasingly more finely disseminated, the recovery of valuable minerals is exceedingly more difficult. Fine grinding, required for liberation, creates many challenges when trying to treat these ores, and many separation techniques become ineffective. Gravity separation techniques, used to separate minerals based on differences in specific gravity (SG), traditionally require relatively coarse material to efficiently concentrate valuable minerals. However, the development of centrifugal gravity separators, such as the Knelson Concentrator, has allowed for the processing of much finer material. The Knelson Concentrator is a compact centrifugal separator with an active fluidized bed to concentrate high SG minerals (Knelson, 1992; Knelson and Jones, 1994). It was initially developed for gold processing, which commonly employed semi-batch units, as the yield to the concentrate was typically below 1 %. However, due to its relatively low cost, small environmental impact when compared to other separation techniques (such as froth flotation), and the development of the Continuous Variable Discharge (CVD) concentrator, the Knelson has become an active area of research for the processing of many low-SG deposits. A summary of the various low-SG minerals for which the Knelson Concentrator has been applied is shown in Table 1. Although these studies demonstrate that the Knelson Concentrator can be an effective step in the processing of these ores, they are predominately focused on relatively coarse material with little work on optimizing operating variables for separation. Those which do investigate the impact of operating variables, generally do so studying one factor at a time (OFAT). While OFAT analysis can give some basic understanding of how operating variables impact separation, it would require a significant amount of test work to determine optimum conditions and gives no information regarding the interaction of the factors investigated. The type of analysis (single condition, OFAT, experimental design) and the best conditions found by each study are shown in Table 1.

In this study central composite design (CCD) is used to design the experiments and response surface method (RSM) is used for optimization. The experimental variables are bowl speed (G), fluidizing water rate (L/min) and solids feed rate (g/min). A synthetic feed comprised of magnetite and quartz was used to mimic a low-density ore. Three different feed samples with varying magnetite grades (5 %, 10 % and 15 %) were investigated to determine how feed grade impacts the optimum operating conditions. The rational behind this study is to determine ideal operating conditions for processing fine (-53 μm) low-density material using a laboratory Knelson Concentrator; and to serve as a reference for the optimization of plant operations.

Table 1 – Summary of literature focused on the use of a Knelson Concentrator to process low-SG minerals

Value Mineral	Specific Gravity	Feed Particle Size (μm)	Feed Grade (%) [3]	Knelson Unit	Enrichment Ratio	Method of Analysis	Best Operating Conditions			Reference
							BS [4]	FWR [4]	FR [4]	
Cassiterite	7	d_{80} = 160	7 [M]	KC-MD3	2	Single Condition	40 G	1.6 L/min	50 g/min	(Angadi *et al.*, 2017)
Chromite	4.6	d_{80} = 150	25 [O]	KC-MD3	1.7	Experimental Design	60 G	11 L/min	200 g/min	(Akar Sen, 2016)
Colemanite [1]	2.4	-500	2.4 [E]	KC-MD3	0.41	OFAT	11 G	1 L/min	-	(Savas, 2016)
Coal [1]	1.1 – 1.4	-1000	11 [A]	KC-MD3	Not Reported	OFAT	60 G	30 kPa	-	(Butcher and Rowson, 1995)
		d_{70} = 250	36 [A]	KC-MD3	0.47	OFAT	30 G	20 kPa	-	(Majumder *et al.*, 2007)
		-200	19 [A]	KC-MD3	0.79	OFAT	60 G	14 kPa	-	(Rubiera *et al.*, 1997)
		-106	20 [A]	KC-MD3	Not Reported	OFAT	60 G	20 kPa	-	(Uslu *et al.*, 2012)
		-210 +44	39 [A]	CVD6	0.38	Experimental Design	900 rpm	8.2 L/min	-	(Honaker *et al.*, 2005)
		-150 +44	21 [A]	CVD6	0.40	Experimental Design	1100 rpm	7.5 L/min	-	(Honaker and Das, 2004)
Magnetite	5.2	Not Reported	≤ 5 [M]	CVD6	≥ 14	OFAT	40 G	-	-	(Fullam and Grewal, 2001)
		-180 +150	4 [M]	KC-MD3	8	Experimental Design	30 G	34 kPa	300 g/min	(Ghaffari and Farzanegan, 2017)
		d_{80} = 125	4 [M]	CVD6	20	OFAT	700 rpm	23 L/min	-	(McLeavy *et al.*, 2001)
		Not Reported	5 [M]	CVD6	5	OFAT	60 G	30 L/min	-	(Sakuhuni *et al.*, 2015)
		d_{80} = 135	5 [M]	KC-MD3	5	Single Condition	60 G	13 L/min	500 g/min	(Sakuhuni *et al.*, 2016)
Pentlandite	4.6 - 5	d_{80} = 94	0.5 [E]	KC-MD3	1.5	Single Condition	-	-	-	(Klein *et al.*, 2016)
				CVD6	1.5	OFAT	60 G	30 L/min	-	
Rare Earth Minerals	3.8 – 6.3	d_{80} = 40	20 [M]	KC-MD3	1.6 – 3.3	Single Condition	40 G	6 L/min	200 g/min	(Jordens *et al.*, 2016)
Gold Bearing Sulphides [2]	5.0 – 7.6	Not Reported	0.7 ppm [E]	KC-MD3	2 - 7	Single Condition	60 G	13 L/min	500 g/min	(Sakuhuni *et al.*, 2016)
	5.0 – 7.6	Not Reported	0.7 ppm [E]	CVD6	1.5 - 4	Single Condition	-	-	-	(Sakuhuni *et al.*, 2016)
	3.9 – 7.6	d_{50} = 150	16 ppm [E]	CVD6	4	OFAT	30 G	30 L/min	1 t/hr	(Altun *et al.*, 2015)
	3.9 – 4.2	d_{80} = 103	5 ppm [E]	CVD6	7	OFAT	700 rpm	23 L/min	-	(McLeavy *et al.*, 2001)
	5.0 – 6.2	d_{80} = 130	1 ppm [E]	KC-MD3	5	Single Condition	-	-	-	(Klein *et al.*, 2010)
	5.0 – 6.2	d_{80} = 130	1 ppm [E]	CVD6	10	OFAT	60 G	43 L/min	-	(Klein *et al.*, 2010)
Tantalum Bearing Minerals	6.5 – 7.2	d_{45} = 12	0.1 [O]	KC-MD3	6	Single Condition	-	-	-	(Burt *et al.*, 1995)
		d_{85} = 212	0.04 [O]	CVD6	34	OFAT	30 G	33	1.1 t/hr	(Fullam and Grewal, 2001)
Heavy Mineral Sands	4.2 – 4.7	d_{80} = 190	2.4 [M]	KC-MD3	3.3	Single Condition	-	-	-	(Gonçalves and Braga, 2016)
	4.2 – 4.8	- 125 + 63	33 [E]	KC-MD3	1.2	OFAT	60 G	62.1 kPa	-	(Premaratne and Rowson, 2004)

[1] For coal and colemanite the Knelson Concentrator is used to remove relatively high SG contaminates. Enrichment ratios are those of the contaminate in the valuable (Knelson tailings) fraction.

[2] The SG presented for gold bearing sulphides is that of the associated minerals. The actual value may be slightly elevated due to the inclusions of gold. Feed grades reported are that of gold and not the associated sulphide minerals

[3] Feed grades reported as mineral (^{M}), metal oxide (^{O}), elemental (^{E}) or ash (^{A}) content.

[4] BS, FWR and FR refer to bowl speed, fluidizing water rate and feed rate respectively.

2. Materials and Methods

2.1 Materials

Magnetite used for this work was obtained from Gem and Mineral Miners Inc. (USA) and the quartz used in this study was purchased from U.S. Silica (USA). Magnetite was pulverized using a LM2 laboratory pulverizing mill (Labtechnics, Australia) and screened to -53 μm and subsequently purified using a lab-scale WD(20) wet drum permanent magnetic separator (Carpco Inc., USA), equipped with an iron-based permanent magnet (low intensity; 0.03 T at drum surface). The quartz was screened wet at 53 μm to remove any oversize quartz particles. The particle size distributions [determined using a LA-920 particle size analyser (Horiba, Japan)] of magnetite and quartz are shown in Figure 1. Magnetite and quartz were then sampled to produce 1 kg samples with a feed grades of 5 %, 10 % and 15 %. Hydrochloric acid used in this work was purchased from Fisher Scientific (USA).

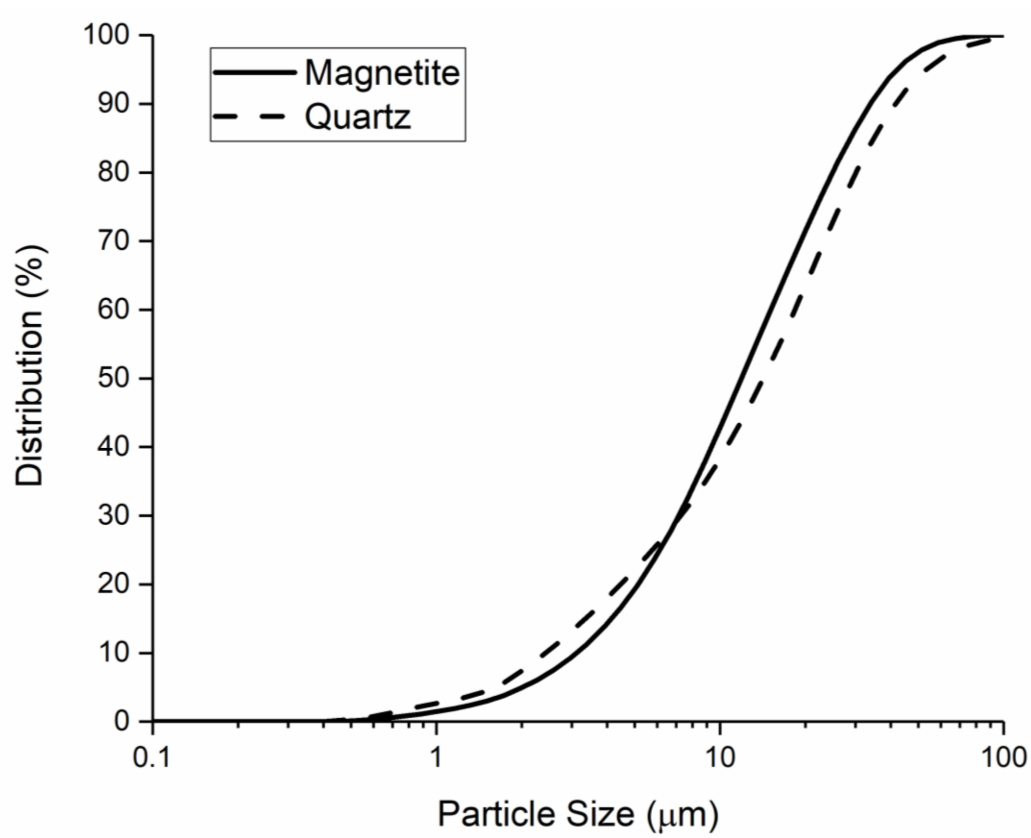

Figure 1 – Particle size distribution of magnetite and quartz samples

2.2 Methods

A lab scale KC-MD3 Knelson Concentrator (FLSmidth, Canada) was used for this study. For each test, 1 kg of synthetic sample was used. Independent variables [bowl speed (G), fluidizing water rate (L/min) and solids feed rate (g/min)] were set to their desired conditions. The range used for bowl speed and fluidizing water rater were based on the limitations of the equipment (maximum and minimum possible speeds of the unit; maximum fluidizing water rate to prevent washing of all the material from the bowl and minimum to maintain for sufficient fluidization for separation). The range of feed rate was based on the work of Prof. Laplante who suggested feed rates of 300 g/min for 75 μm material when performing gravity recovery gold test work (Clarke, 2005; Xiao *et al.*, 2009). This feed rate was set as the maximum value studied here. The feed was fed dry to the feed cone where it was slurried with water at a rate of approximately 1.5 L/min. Slurrying water rate was not considered as an independent variable, as the effect of solids concentration over the range of feed blends investigated is expected to be minimal. Following each test, the bowl was emptied, filtered and dried. Three representative samples from each concentrate were analysed by digesting magnetite with hydrochloric acid. The residual mass was then dried, weighed and compared to the original mass of the digested sample to calculate magnetite grade and recovery.

RSM was used to investigate the relationship between independent variables and the response; and possible interactions between the independent variables and their effects on the separation performance of a Knelson Concentrator. CCD, a well-suited RSM for fitting a second-order response surface, was used to design the experiments (Box and Hunter, 1957; Box and Wilson, 1992; Chen and Parlar, 2013; Montgomery, 2009; Yi *et al.*, 2010). Each variable has five levels ($\pm\beta$, ±1, 0 where $\beta = 2^{3/4} = 1.682$), with grade and recovery of magnetite chosen as the responses.

The number of tests required for the CCD can be calculated using equation 1, which contains the standard 2^k factorial with its origin at the centre, $2k$ points at a distance, β, from the centre to generate the quadratic terms, and replicate tests at the centre (Box and Hunter, 1957; Kökkılıç *et al.*, 2015; Montgomery, 2009; Zhou *et al.*, 2016).

$$N = 2^k + 2k + n_0 \quad (1)$$

With three variables (k) and six replicates at the centre point (n_0), the number of tests required for each feed grade investigated is 20 (Kökkılıç *et al.*, 2015; Montgomery, 2009; Obeng *et al.*, 2005; Zhou *et al.*, 2016). The independent variables are designated as x_1, x_2 and x_3 and the predicted responses, grade and recovery, are designated as y_1 and y_2 respectively. The coded values were calculated as shown in Table 2. These were used to determine the levels of the independent variables for each of the 20 experiments.

Table 2 – Independent variables and their levels

Independent Variables	Symbol	Coded Variable Level				
		Lowest	Low	Centre	High	Highest
		$-\beta^a$	-1	0	+1	$+\beta^a$
Bowl Speed (G)	x_1	10	30	60	90	110
Fluidizing water rate (L/min)	x_2	1	1.8	3	4.2	5
Solids feed rate (g/min)	x_3	100	140	200	260	300

[a] : 1.682

For each Knelson test, the bowl speed, fluidizing water rate and solids feed rate were changed successively during the tests with respect to the central composite design. The mathematical relationship between the three independent variables and responses can be approximated by a second order model, such as equation 2:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \varepsilon$$

(2)

where y is the predicted response; β_0 is the model constant; x_1, x_2 and x_3 are the variables; β_1, β_2 and β_3 are linear coefficients; β_{12}, β_{13} and β_{23} are cross-product coefficients; and β_{11}, β_{22} and β_{33} are the quadratic coefficients (Kökkılıç *et al.*, 2015; Kwak, 2005; Montgomery, 2009; Zhou *et al.*, 2016). Minitab Statistical Software 18® (Minitab, USA) was used to estimate these coefficients [the main effect (β_i), the quadratic effect (β_{ii}) and two-factor interactions (β_{ij})] from the experimental results.

Using experimental data (grade and recovery) from each experiment, a second order regression model which describes the concentration process was produced. Analysis of Variance (ANOVA) was used to determine the regression coefficients and detect the agreement of the model. Statistical importance of each variable on the response was found at a 95 % confidence level by using Fischer (*F*) test and *p*-values. RSM was used to analyse the proposed model and optimization was realized using response surface and contour plots for different interactions of any two independent variables (holding the value of the third variable constant at the centre level). All statistical analyses were conducted using Minitab Statistical Software 17® (Minitab, USA).

3. Results and Discussion

The responses (grade and recovery) for each test run for the 5 %, 10 % and 15 % feed grades are shown in Tables 3 – 5. Due to the difficulty of ensuring the solids feed rate and fluidizing water rate were the same as the coded variables (Table 2), the actual measured values (shown in Tables 3 – 5) were used for statistical analysis. The second order response functions representing grade (y_1) and recovery (y_2) of magnetite in the Knelson concentrate, were expressed as a function of bowl speed (x_1), fluidizing water rate (x_2) and solids feed rate (x_3). The coded model equations are presented in equations 3-8:

$$y_{1\ 5\,\%} = 35.13 - 3.015x_1 + 6.724x_2 - 0.386x_3 + 0.555x_1^2 + 0.469x_2^2 - 0.601x_3^2 - 5.363x_1x_2 + 0.130x_1x_3 + 0.657x_2x_3 \quad (3)$$

$$y_{2\ 5\,\%} = 49.12 + 6.48x_1 + 0.37x_2 - 1.56x_3 - 6.44x_1^2 - 0.36x_2^2 - 0.605x_3^2 - 2.44x_1x_2 - 0.03x_1x_3 + 1.02x_2x_3 \quad (4)$$

$$y_{1\ 10\,\%} = 56.9 - 2.054x_1 + 6.052x_2 + 0.615x_3 - 1.929x_1^2 - 0.686x_2^2 - 0.098x_3^2 + 0.75x_1x_2 + 0.368x_1x_3 - 0.83x_2x_3 \quad (5)$$

$$y_{2\ 10\,\%} = 40.31 + 8.601x_1 - 4.763x_2 + 0.429x_3 - 4.398x_1^2 - 0.835x_2^2 - 0.187x_3^2 + 2.40x_1x_2 - 0.89x_1x_3 + 0.83x_2x_3 \quad (6)$$

$$y_{1\ 15\,\%} = 65.44 + 3.40x_1 + 3.25x_2 + 4.50x_3 + 1.14x_1^2 + 0.22x_2^2 + 1.56x_3^2 - 1.85x_1x_2 + 3.89x_1x_3 - 2.77x_2x_3 \quad (7)$$

$$y_{2\ 15\,\%} = 36.34 + 12.27x_1 - 6.53x_2 + 3.511x_3 - 2.41x_1^2 + 0.329x_2^2 + 1.696x_3^2 - 1.96x_1x_2 + 3.92x_1x_3 - 2.47x_2x_3 \quad (8)$$

Table 3 – Results of magnetite grade and recovery for the 5% feed

Run	Coded Level of Variables			Actual Measured Level of Variables			Response	
	x₁	***x₂***	***x₃***	**Bowl Speed (G)**	**Fluidizing Water Rate (L/min)**	**Solids Feed Rate (g/min)**	**Grade (%)**	**Recovery (%)**
1	1	1	-1	90	4.2	152.1	31.1	41.4
2	0	0	-1.68	60	3.1	94.2	35.5	53.6
3	0	1.68	0	60	5.0	204.1	46.7	49.0
4	-1.68	0	0	10	3.0	197.4	41.8	18.9
5	-1	-1	-1	30	1.9	130.2	26.5	31.2
6	0	0	0	60	3.0	205.5	37.8	50.8
7	-1	1	1	30	4.1	270.3	48.5	36.0
8	1	-1	-1	90	1.9	133.3	32.5	51.1
9	1	1	1	90	4.2	257.5	35.0	46.9
10	0	0	0	60	3.0	210.5	35.5	46.7
11	0	-1.68	0	60	1.0	187.5	28.1	51.7
12	0	0	0	60	3.0	202.7	37.6	53.6
13	0	0	0	60	3.0	199.3	35.8	51.4
14	-1	-1	1	30	1.9	254.2	27.2	32.6
15	1.68	0	0	110	2.9	209.8	33.3	46.8
16	0	0	0	60	3.0	202.0	30.8	45.4
17	0	0	0	60	3.0	192.0	32.7	45.9
18	1	-1	1	90	1.8	263.2	28.0	42.3
19	-1	1	-1	30	4.2	135.1	51.9	40.4
20	0	0	1.68	60	3.0	317.5	32.5	43.8

Table 4 – Results of magnetite grade and recovery for the 10% feed

Run	Coded Level of Variables			Actual Measured Level of Variables			Response	
	x_1	x_2	x_3	Bowl Speed (G)	Fluidizing Water Rate (L/min)	Solids Feed Rate (g/min)	Grade (%)	Recovery (%)
1	1	1	-1	90	4.2	129	58.0	39.7
2	0	0	-1.68	60	3.0	83	54.1	39.1
3	0	1.68	0	60	5.0	190	66.6	29.8
4	-1.68	0	0	10	3.1	210	50.6	7.7
5	-1	-1	-1	30	1.8	103	51.6	34.1
6	0	0	0	60	3.0	237	55.0	38.9
7	-1	1	1	30	4.3	268	63.4	24.8
8	1	-1	-1	90	1.8	135	43.2	45.1
9	1	1	1	90	4.2	243	55.8	38.1
10	0	0	0	60	3.0	196	57.6	40.8
11	0	-1.68	0	60	1.1	180	41.1	45.1
12	0	0	0	60	3.0	178	56.0	41.3
13	0	0	0	60	3.0	206	58.1	40.3
14	-1	-1	1	30	1.8	248	55.1	37.5
15	1.68	0	0	110	3.1	213	51.4	46.7
16	0	0	0	60	3.0	199	57.9	39.7
17	0	0	0	60	3.0	191	57.2	40.5
18	1	-1	1	90	1.8	253	47.5	43.8
19	-1	1	-1	30	4.2	157	61.9	19.1
20	0	0	1.68	60	3.1	390	57.6	39.1

Table 5 – Results of magnetite grade and recovery for the 15% feed

Run	Coded Level of Variables			Actual Measured Level of Variables			Response	
	x_1	x_2	x_3	Bowl Speed (G)	Fluidizing Water Rate (L/min)	Solids Feed Rate (g/min)	Grade (%)	Recovery (%)
1	1	1	-1	90	4.1	131	69.3	38.7
2	0	0	-1.68	60	2.9	103	62.5	35.7
3	0	1.68	0	60	5.1	122	74.8	28.3
4	-1.68	0	0	10	3.0	160	63.9	8.7
5	-1	-1	-1	30	1.7	103	58.2	31.8
6	0	0	0	60	3.0	148	57.0	32.1
7	-1	1	1	30	4.2	226	71.6	21.7
8	1	-1	-1	90	1.7	114	51.6	41.3
9	1	1	1	90	4.1	247	73.6	40.6
10	0	0	0	60	3.0	148	67.3	36.5
11	0	-1.68	0	60	1.0	162	57.0	42.9
12	0	0	0	60	2.9	156	65.0	36.4
13	0	0	0	60	3.1	172	59.7	30.8
14	-1	-1	1	30	1.8	251	60.0	28.7
15	1.68	0	0	110	3.0	167	70.3	45.1
16	0	0	0	60	3.1	195	66.2	35.1
17	0	0	0	60	3.1	175	64.4	36.8
18	1	-1	1	90	1.9	236	78.0	64.9
19	-1	1	-1	30	4.3	139	70.5	20.6
20	0	0	1.68	60	3.1	305	78.9	46.3

To estimate the significance and accuracy of the developed models, ANOVA was applied (Table 6). *F*-values for all cases are greater than the *F*-value found in the *F*-statistics Table with P=0.05 ($F_{0.05(9,10)}$=3.14); *p*-values of the regression models are smaller than 0.05 and standard deviations are relatively low. Normal probability plots of the residuals and a plot of the residuals versus the fitted response are presented in the Appendix (Figures A1 – A6). Residuals generally lie on a straight line, indicating errors are distributed normally; and the residuals scatter randomly, suggesting model predictions are adequate. The quality of fit of the polynomials, expressed by R^2 values (Table 6), is acceptable ($R^2 \geq 0.80$) (Azizi *et al.*, 2012;

Joglekar *et al.*, 1987; Saguy and Graf, 1990). Thus, it can be concluded that regression models are significant and accurate.

Once the model was verified, the Student's *t*-test was performed to estimate the quantitative effects of the variables and their interactions. Tables 7 – 9 show the summarized Student's *t*-test, for each feed grade, which includes the *p*-value and T-value of each variable. The *p*-values indicate the significance of variables and their interactions, with 95 % confidence; and T-values are the result of the Student's *t*-test and indicate whether each significant variable has a positive or negative effect on the response, as well as how significant they are. All variables and interactions with a *p*-value ≤ 0.05 are considered as significant, with the magnitude of the T-values indicating the level of significance (greater the magnitude greater the significance). Response surface plots further demonstrating the impact of an input variable on grade and recovery when processing 5 %, 10 % and 15 % magnetite feeds can be found in the Appendix (Figures A7 – A12).

Table 6 – Summary of ANOVA for regression models

Feed Grade	Response	*F*-value	*p*-value	R^2	R^2_{adj}	Standard deviation
5 %	Grade	14.66	0.000	0.93	0.87	2.58
	Recovery	7.03	0.003	0.86	0.74	4.46
10 %	Grade	8.30	0.001	0.88	0.78	2.94
	Recovery	17.13	0.000	0.94	0.88	3.26
15 %	Grade	6.75	0.003	0.86	0.73	3.86
	Recovery	18.06	0.000	0.94	0.89	3.79

Table 7 indicates that when processing the 5 % magnetite feed, the two responses are affected by the independent variables differently. Bowl speed (x_1) has significant and opposite effects on grade (negative) and recovery (positive); fluidizing water rate (x_2) has a strong positive influence on grade, however, did not affect recovery significantly; and solids feed rate does not have any significant effect on the response. There is a negative interaction effect on grade, between bowl speed and fluidizing water rate (x_1x_2). Bowl speed also has a negative quadratic effect on recovery. The significant and opposite influence of bowl speed on grade and recovery suggests that different operating conditions will be required to obtain a maximum grade or a maximum recovery.

Similar results were observed when processing the 10 % magnetite feed (Table 8), where high grade and high recovery will occur at different conditions. In this case both bowl speed and fluidizing water rates have significant and opposite effects on grade and recovery. Bowl speed also has a negative quadradic effect on both grade and recovery.

Table 9 shows the influence of independent variables on responses for the 15 % magnetite feed. Bowl speed, fluidizing water rate and solids feed rate are all significant variables for both responses. Grade is affected by all three independent variables positively with their significance decreasing according to solids feed rate > bowl speed > fluidizing water rate. The order of significance of the independent variables on recovery is bowl speed positively > fluidizing water rate negatively > solids feed rate positively. An interaction effect, between bowl speed and solids feed rate, is observed for both responses; and bowl speed has a positive quadratic effect on recovery.

Table 7 – Summarized Student's t-test for 5 % feed grade

Term	Grade		Recovery	
	***p*-value**	**T-value**	***p*-value**	**T-value**
x_1	**0.001**	-4.33	**0.000**	5.38
x_2	**0.000**	8.82	0.769	0.30
x_3	0.567	-0.59	0.195	-1.39
x_1^2	0.435	0.81	**0.000**	-5.46
x_2^2	0.521	0.66	0.775	-0.29
x_3^2	0.305	-1.08	0.543	-0.63
x_1x_2	**0.000**	-5.79	0.158	-1.52
x_1x_3	0.886	0.15	0.984	-0.02
x_2x_3	0.481	0.73	0.524	1.02

Table 8 – Summarized Student's t-test for 10 % feed grade

Term	Grade		Recovery	
	***p*-value**	**T-value**	***p*-value**	**T-value**
x_1	**0.029**	-2.55	**0.000**	9.66
x_2	**0.000**	7.33	**0.000**	-5.21
x_3	0.388	0.90	0.582	0.57
x_1^2	**0.035**	-2.44	**0.001**	-5.03
x_2^2	0.425	-0.83	0.382	-0.91
x_3^2	0.776	-0.29	0.627	-0.50

x_1x_2	0.496	0.71	0.067	2.05
x_1x_3	0.720	0.37	0.439	-0.81
x_2x_3	0.434	-0.81	0.477	0.74

Table 9 – Summarized Student's t-test for 15 % feed grade

Term	Grade		Recovery	
	***p*-value**	**T-value**	***p*-value**	**T-value**
x_1	**0.018**	2.82	**0.000**	10.36
x_2	**0.034**	2.45	**0.001**	-5.02
x_3	**0.001**	4.50	**0.005**	3.58
x_1^2	0.294	1.11	**0.039**	-2.38
x_2^2	0.835	0.21	0.749	0.33
x_3^2	0.152	1.55	0.116	1.72
x_1x_2	0.213	-1.33	0.181	-1.44
x_1x_3	**0.017**	2.86	**0.015**	2.95
x_2x_3	0.057	-2.15	0.079	-1.96

Although the findings from Tables 7 – 9 give a general idea of how independent variables affect the responses, and contour plots (Figures A7 – A12) show regions where high grade and recovery can be obtained, they do not indicate the optimum separation conditions. More accurate information about the optimum operating conditions and how they affect both grade and recovery simultaneously can be obtained by drawing overlaid contour plots (Figures 2 – 4). As both grade and recovery cannot be maximized simultaneously for the 5 % and 10 % magnetite feeds, three points are presented on the overlaid plots (a blue dot for maximum grade, a red dot for maximum recovery and a black dot for the conditions where both grade and recovery are maximized simultaneously). In this study, the optimum operating conditions when considering both grade and recovery were chosen to be where an increase in one did not result in a decrease in the other. For the 15 % magnetite feed a single optimum point could be obtained for both grade and recovery (shown in Figure 4). The actual predicted values for the optimum conditions of all three feed grades are shown in Table 10. In some cases, the confidence intervals (95 %) associated with the predicted responses are large, as they are well outside the range of predictor levels (measured responses) used to fit the model. For example, when processing the 15 % magnetite feed the maximum measured grade and recovery were 78.9 % and 64.9 % respectively. However, the model predicts a maximum grade of 91.8 % and recovery 82.7 %, leading to large confidence intervals. To validate the responses at the optimum conditions determined by the model, further experiments were carried out for each feed grade. The validation tests were repeated three times and the results were compared to those predicted by the model (Table 11). It can be concluded that the proposed equations adequately predict magnetite grade and recovery for all three feed grade conditions (Error < 10 %).

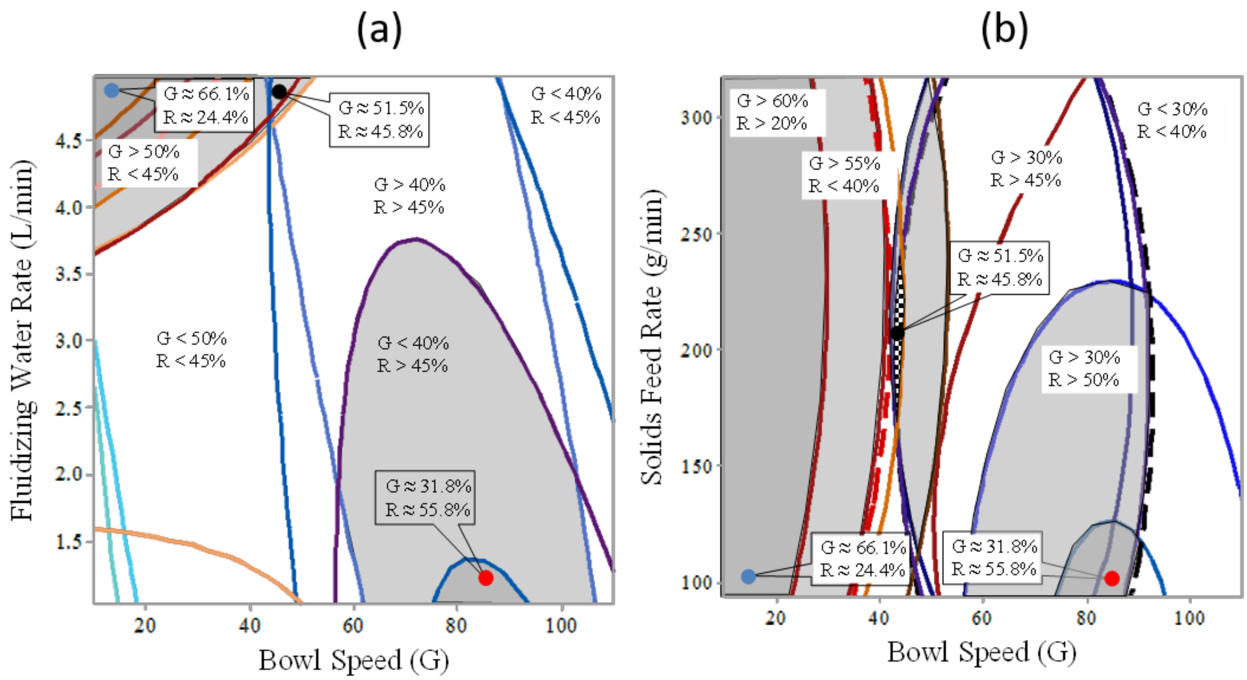

Figure 2 – Grade and recovery behaviour at different (a) fluidizing water rates and bowl speeds and (b) solids feed rate and bowl speeds for the 5 % magnetite feed

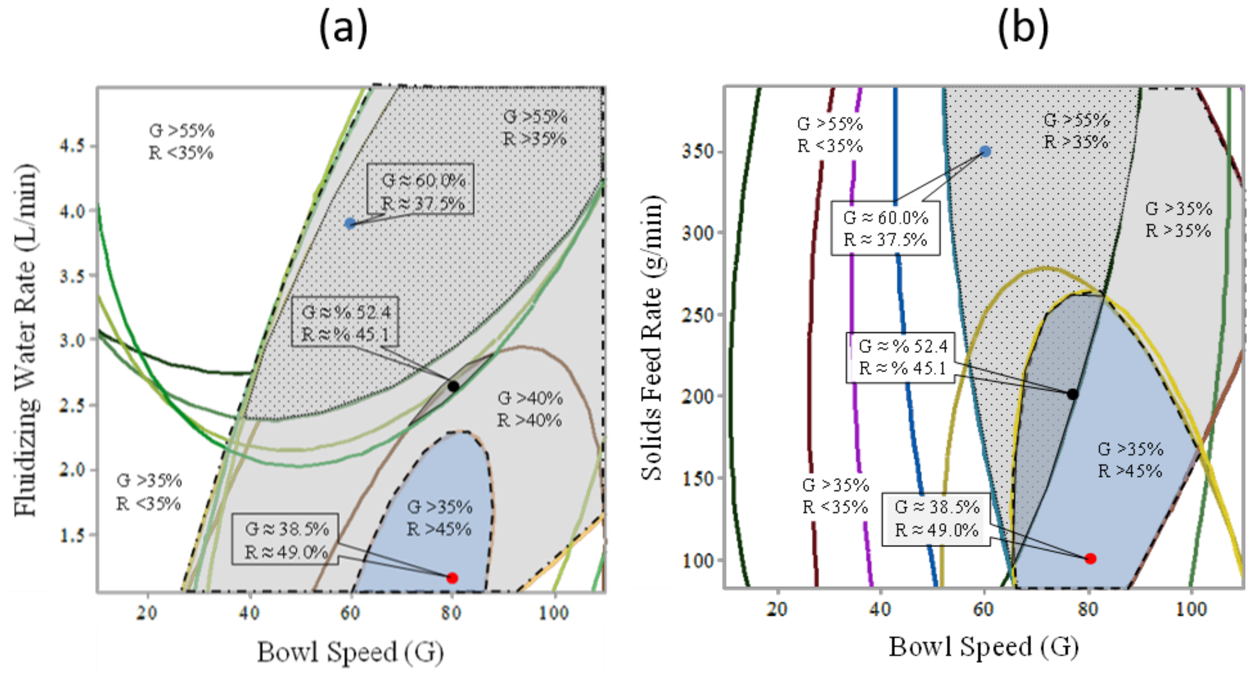

Figure 3 – Grade and recovery behaviour at different (a) fluidizing water rates and bowl speeds and (b) solids feed rate and bowl speeds for the 10 % magnetite feed

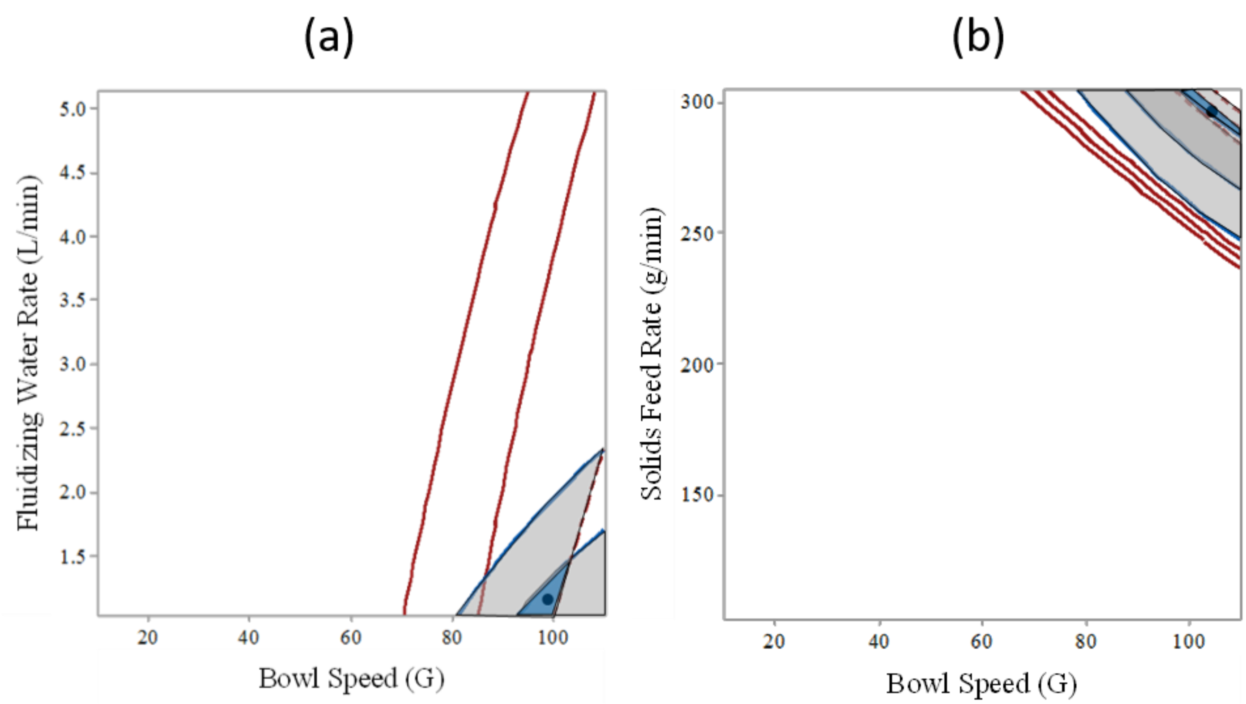

Figure 4 – Grade and recovery behaviour at different (a) fluidizing water rates and bowl speeds and (b) solids feed rate and bowl speeds for the 15 % magnetite feed

Table 10 – Optimum conditions for grade and recovery

Feed Grade	Optimized for	Operating Variables: Bowl Speed (G)	Fluidizing Water Rate (L/min)	Solids Feed Rate (g/min)	Predicted Response: Grade (%)[1]	Recovery (%)[1]
5 %	Grade	10	5.0	100	66.1 ± 11.7	24.4 ± 20.0
	Recovery	85	1.0	100	31.8 ± 8.9	55.7 ± 15.0
	Grade and Recovery	45	4.8	200	51.5 ± 4.2	45.8 ± 7.2
10 %	Grade	60	3.9	350	60.4 ± 5.8	37.7 ± 6.9
	Recovery	80	1.1	100	38.4 ± 8.3	49.0 ± 9.3
	Grade and Recovery	80	2.6	200	52.4 ± 2.6	45.1 ± 2.9
15 %	Grade and Recovery	100	1.1	275	91.8 ± 16.6	82.7 ± 16.2

[1] Error shown for predicted responses represents 95 % confidence intervals

The above findings demonstrate that when processing relatively low grade (≤ 10 %) feed a balance between both bowl speed and fluidizing water rate is required. For high grade feeds (15 %), high grade and high recovery is realized at high bowl speeds, low fluidizing water rates and high solid feed rates. Comparing the significant parameters from each data set it can be concluded that with low feed grades, the force balance acting on particles plays a much greater role in optimizing separation; whereas, with high feed grades optimizing the properties of the fluidizing bed becomes more important.

Table 11 – Comparative data at optimum conditions for validation

Feed Grade	Operating Variables: Bowl Speed (G)	Fluidizing Water Rate (L/min)	Solids Feed Rate (g/min)	Predicted Response: Grade (%)	Recovery (%)	Validation Tests: Grade (%)[1]	% Error	Recovery (%)[1]	% Error
5 %	45	4.8	200	51.5	45.8	49.6 ± 2.7	-3.7	41.7 ± 1.9	-9.0
10 %	80	2.6	200	52.4	45.1	52.4 ± 1.4	0.0	44.3 ± 2.1	-1.8
15 %	100	1.1	275	91.8	82.7	91.3 ± 1.2	-0.5	84.6 ± 1.2	2.3

[1] Error shown for validation tests represents 95 % confidence intervals

For the 5 % magnetite feed, product grade is influenced by only bowl speed (negative), fluidizing water rate (positive) and an interaction between bowl speed and fluidizing water rate (negative); whereas, recovery is influenced by only bowl speed (positive) and a quadratic effect of bowl speed (negative). This suggests that high grade is predominantly achieved with low centrifugal force and high drag force to reject as much low SG material as possible and recover only the heaviest material (resulting in low recovery). High recovery is obtained by using high bowl speeds to induce a high centrifugal acceleration on particles and recover as much high SG material as possible. However, this also results in high recovery of low SG material (and therefore low product grades) and after a certain point, further increases in bowl speed result in the process becoming completely unselective and becomes detrimental to recovery.

For the 10 % magnetite feed, it is similarly suggested that a balance between drag force and centrifugal acceleration is required to achieve ideal separation. However, in this case there is a larger quantity of high SG material to replace the low SG particles which are recovered in the concentrating bed throughout the process. This allows for greater substitution of material and the properties of the fluidizing bed become more important. In this case slightly higher bowl speeds are recommended (bowl speed now has a quadratic influence on both product grade and recovery) and fluidizing water rate becomes an important variable for the recovery of magnetite. The negative influence of fluidizing water rate on recovery suggests a tightly packed fluidized particle bed is beneficial.

For the 15 % magnetite feed there is now sufficient high SG material to achieve high grade and recovery simultaneously through the optimization of the properties of the fluidizing bed. In this case, the recovery process occurs mainly through the substitution of low SG particles with high SG material in the concentrating bed. There is no need to increase the fluidising water to achieve higher drag force acting on particles to reject low SG material. At this feed grade, elevated bowl speeds are recommended along with low fluidizing water rates to keep a tightly packed concentrating bed. Feed rate is also an important variable with high feed rates desired for optimum separation. There is also a positive interaction between bowl speed and feed rate suggesting higher feed rates are recommended at higher bowl speeds. The influence of feed rate suggests that bringing high SG material into the concentrating bed quicker likely prevents the settling of low SG material allowing for easier substitution.

It is important to note that the findings of this study are specific for the processing of 1 kg feed samples. For low-grade feeds (≤ 10 %), processing more material is likely to result in an improved concentrate grade, as feeding more material will result in greater substitution of quartz with magnetite in the concentrating bed. This, however, is not likely to have a beneficial effect on recovery and after a certain point (when the bowl is overloaded) will be detrimental to the recovery of magnetite. It is also important to note that the industrial application of a Knelson Concentrator to feeds like those studied here, would require the use of a continuous system, however, work by Sakuhuni et al 2016, has demonstrated a lab scale Knelson Concentrator can be used for predicting CVD performance.

The findings from this work could also be extended to the processing of a high-grade low-SG deposit where any loss of value may have a significant impact on the profitability of a mineral processing plant. A high grade (15 % or 10 %) feed could first be processed at its optimum conditions and then the tailings could be reprocessed at more optimal conditions for a low-grade feed (5 %). An example of such a flowsheet is shown in Figure 5, where the values shown for the second Knelson concentrator are those found for the 5 % magnetite feed.

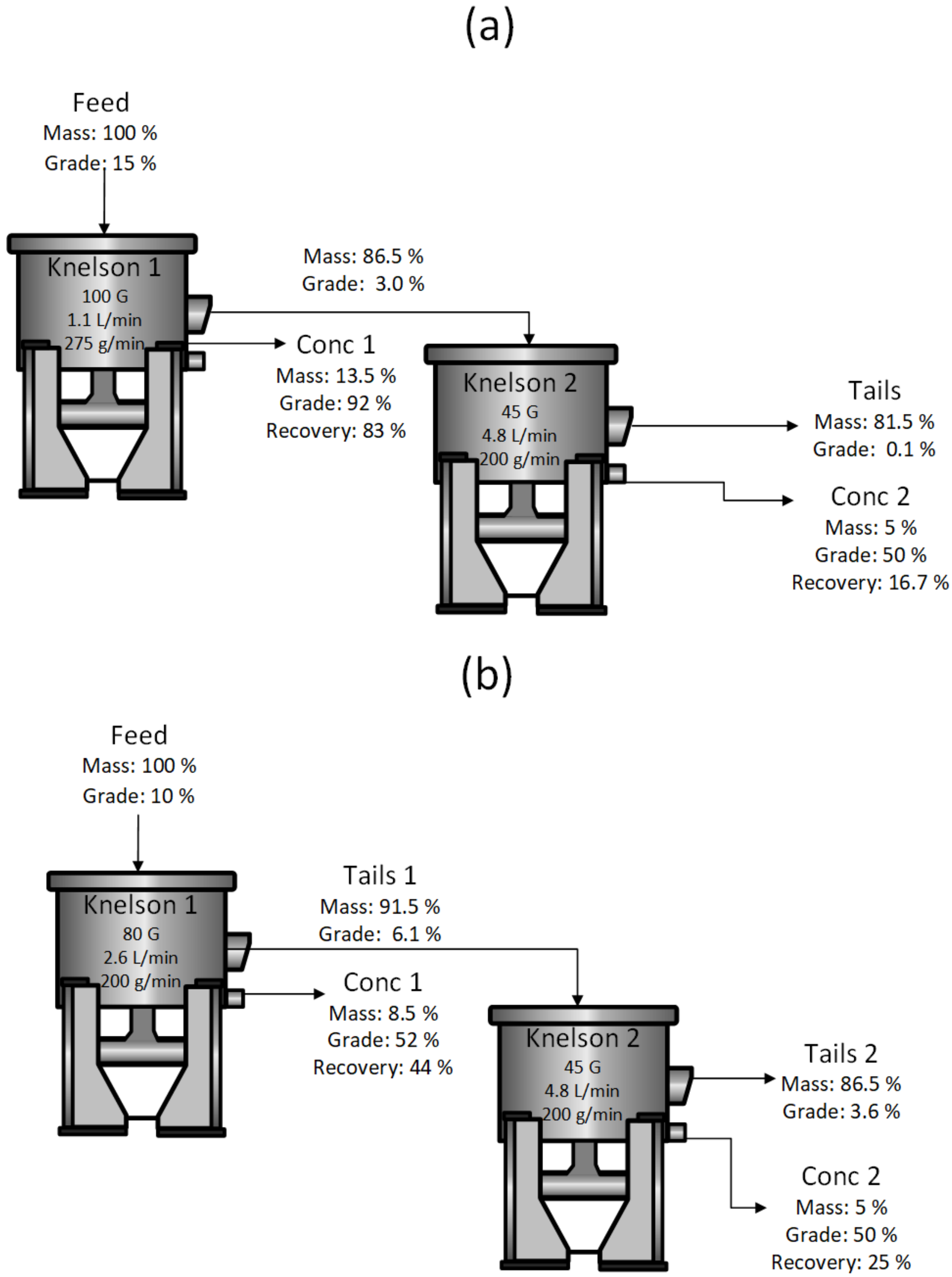

Figure 5 – Example flowsheet for processing (a) a 15 % and (b) 10 % feed with a series of Knelson Concentrators

Conclusions

This study utilized response surface design experiments to determine the effect of Knelson operating variables (bowl speed, fluidizing water rate and solids feed rate) on grade and recovery of magnetite from a synthetic sample consisting of magnetite and quartz with three different feed grades. The conclusions are as follows:

1. The empirical regression equations as a function of the independent variables were derived by the RSM model for the grade and recovery of magnetite from feeds with 5 %, 10 % and 15 % magnetite.

2. The regression models are considered acceptable and fit well for all three feed grades examined. The regression models for each feed grade have p – values less than 0.05 for magnetite grade and recovery, indicating that the selected models are significant to the responses.

3. Feed grade has a significant effect on the optimal operating conditions for grade and recovery. Bowl speed and fluidizing water rate are significant operating variables for all three feed grades examined. Solids feed rate only had a significant impact when processing the 15 % magnetite feed.

4. A trade off between grade and recovery must be made when processing material with low feed grades (≤ 10 %). Grade and recovery can be simultaneously maximized for the 15 % magnetite feed.

5. Comparing the significant parameters from each data set it can be concluded that with low feed grades, the force balance acting on particles plays a much greater role in optimizing separation; whereas with high feed grades optimizing the properties of the fluidizing bed becomes more important.

6. Optimum operating conditions were obtained at a bowl speed, fluidizing water rate and solids feed rate of 45 G, 4.8 L/min and 200 g/min, for the 5 % feed; 80 G, 2.6 L/min and 200 g/min, for the 10 % feed; and 100 G, 1.1 L/min and 275 g/min for the 15 % feed.

Acknowledgements

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Appendix

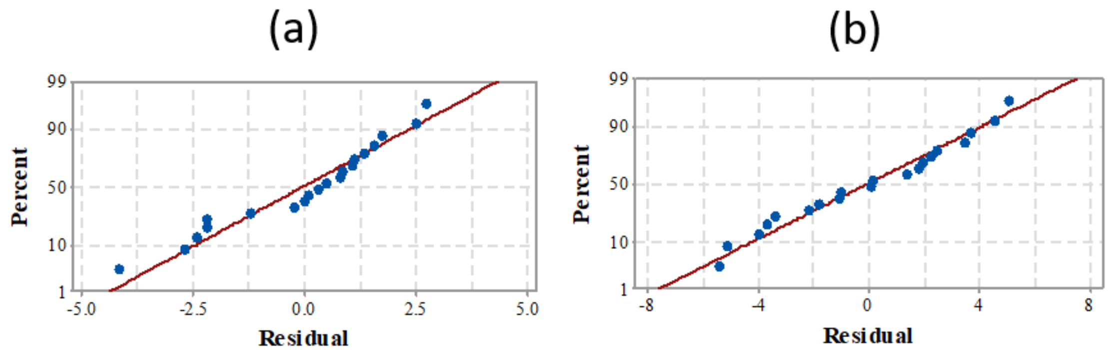

Figure A1 – Normal probability plot of the residuals for (a) grade and (b) recovery for the 5 % magnetite feed

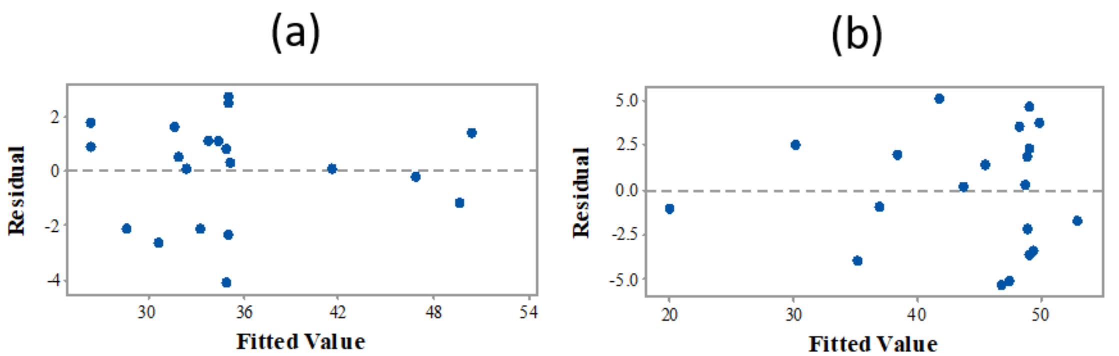

Figure A2 – Plot of the residuals versus fitted response for (a) grade and (b) recovery for the 5 % magnetite feed

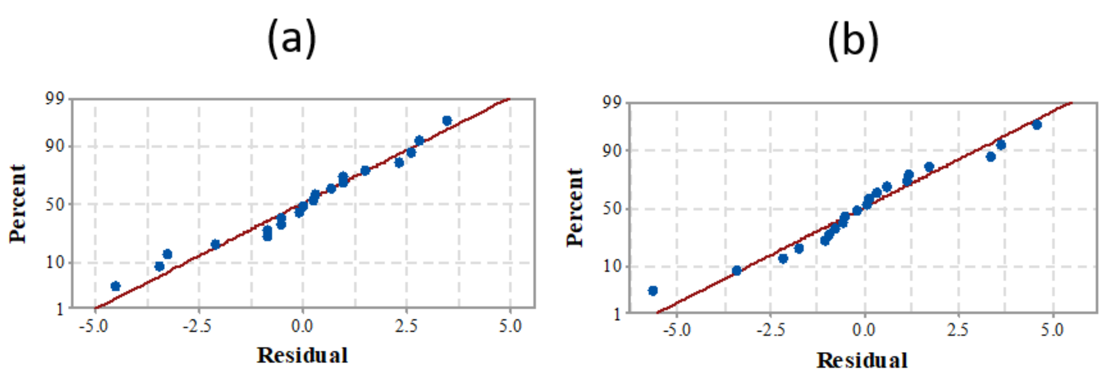

Figure A3 – Normal probability plot of the residuals for (a) grade and (b) recovery for the 10 % magnetite feed

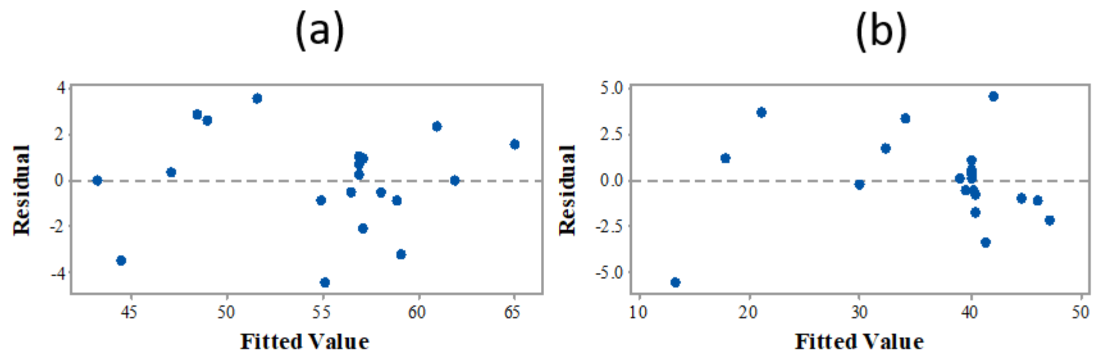

Figure A4 – Plot of the residuals versus fitted response for (a) grade and (b) recovery for the 10 % magnetite feed

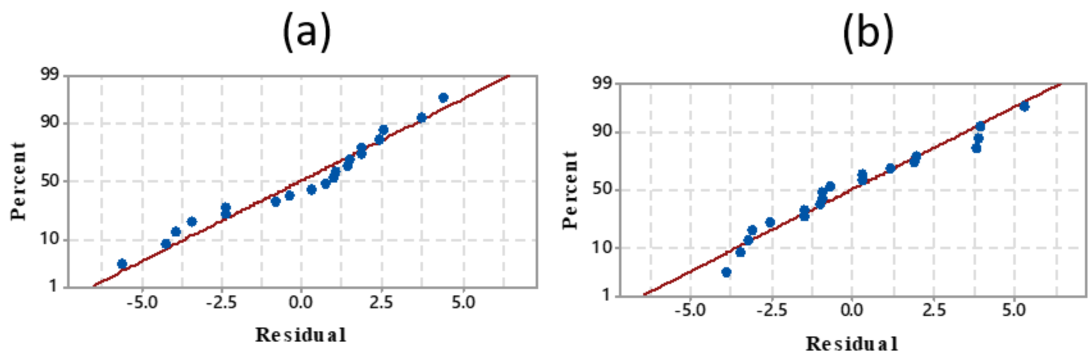

Figure A5 – Normal probability plot of the residuals for (a) grade and (b) recovery for the 15 % magnetite feed

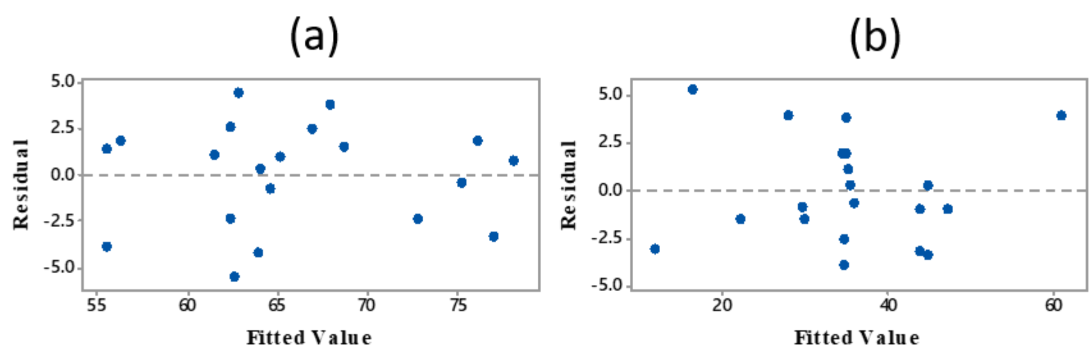

Figure A6 – Plot of the residuals versus fitted response for (a) grade and (b) recovery for the 15 % magnetite feed

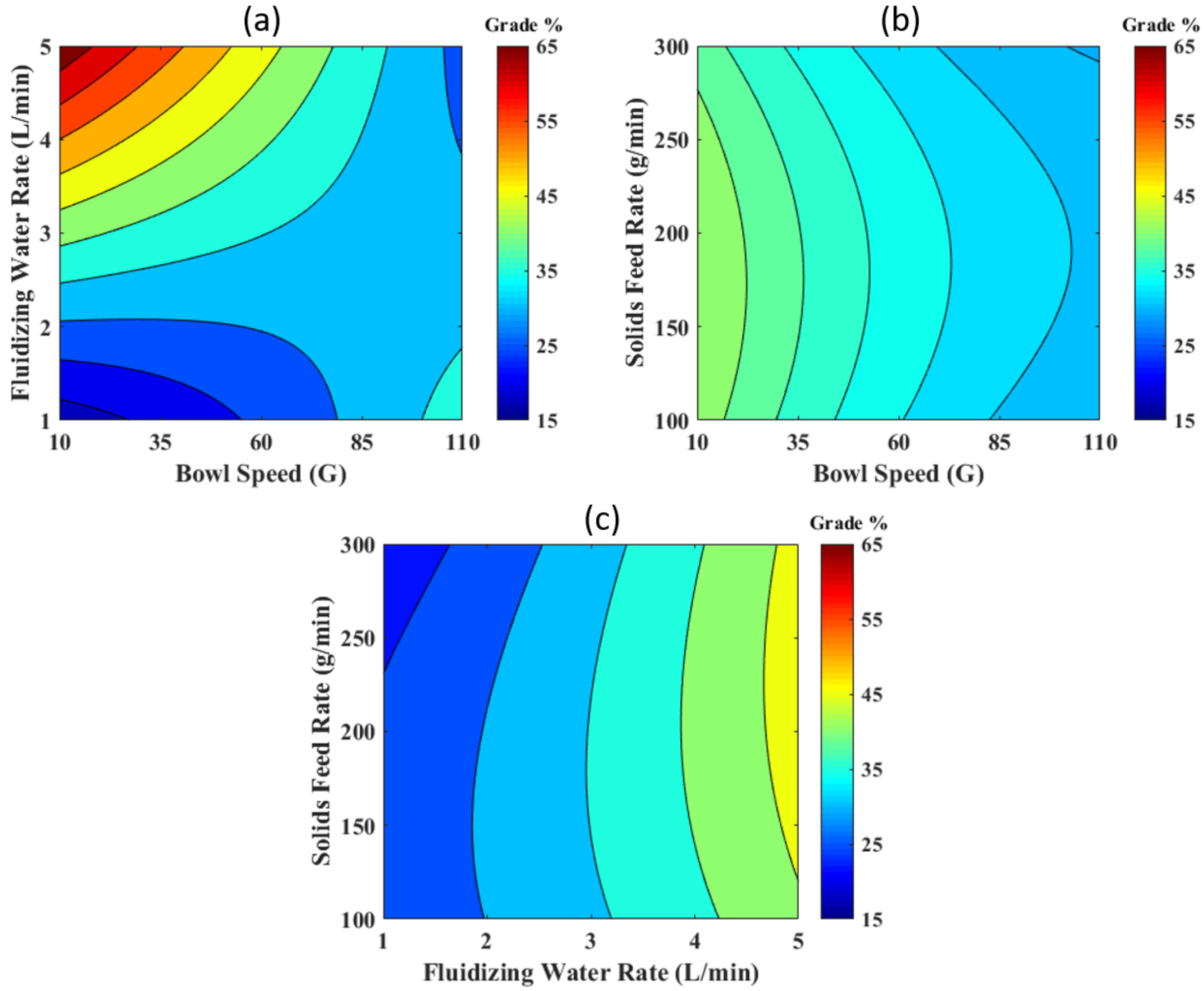

Figure A7 – Response surface plots for grade showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 5 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

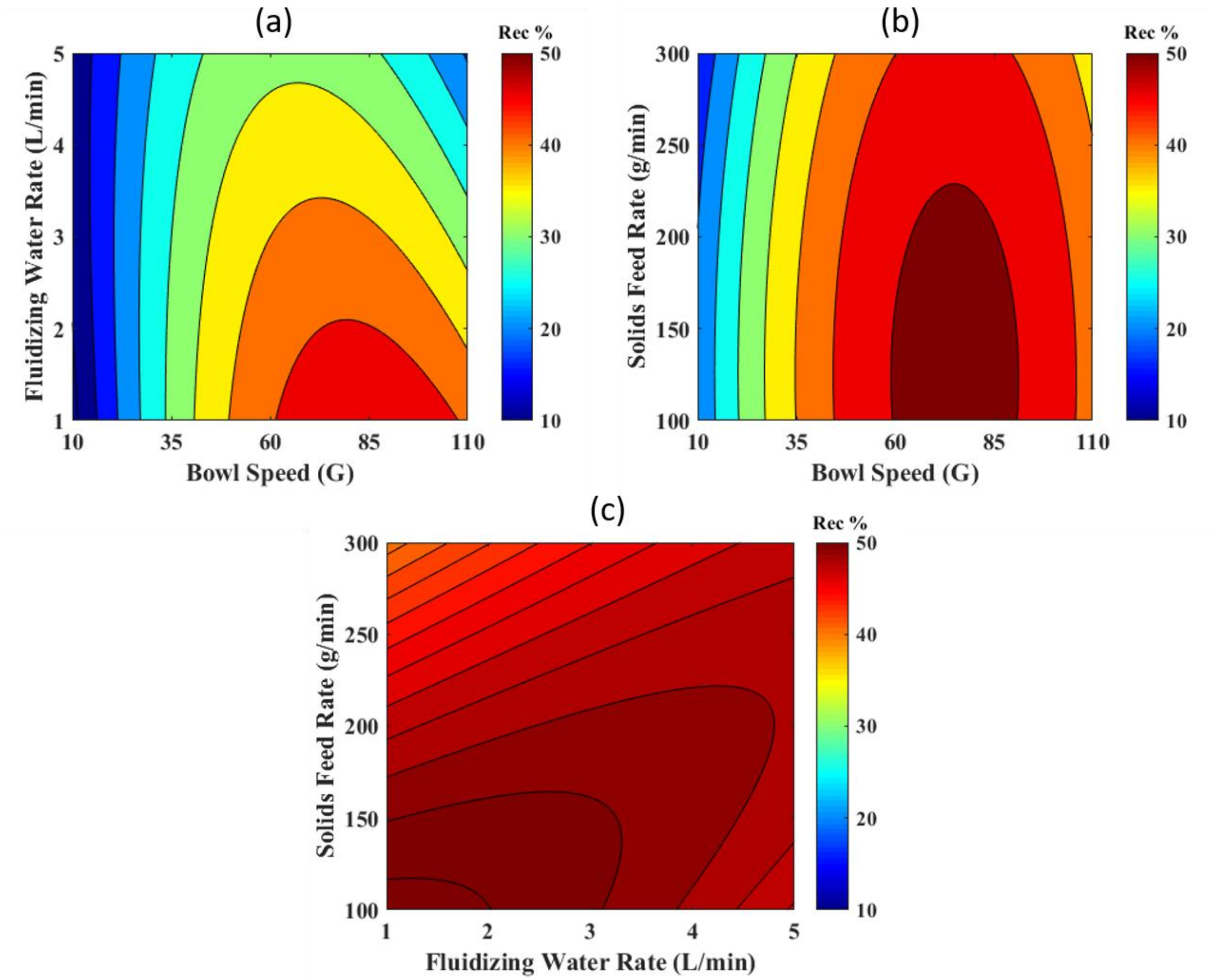

Figure A8 – Response surface plots for recovery showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 5 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

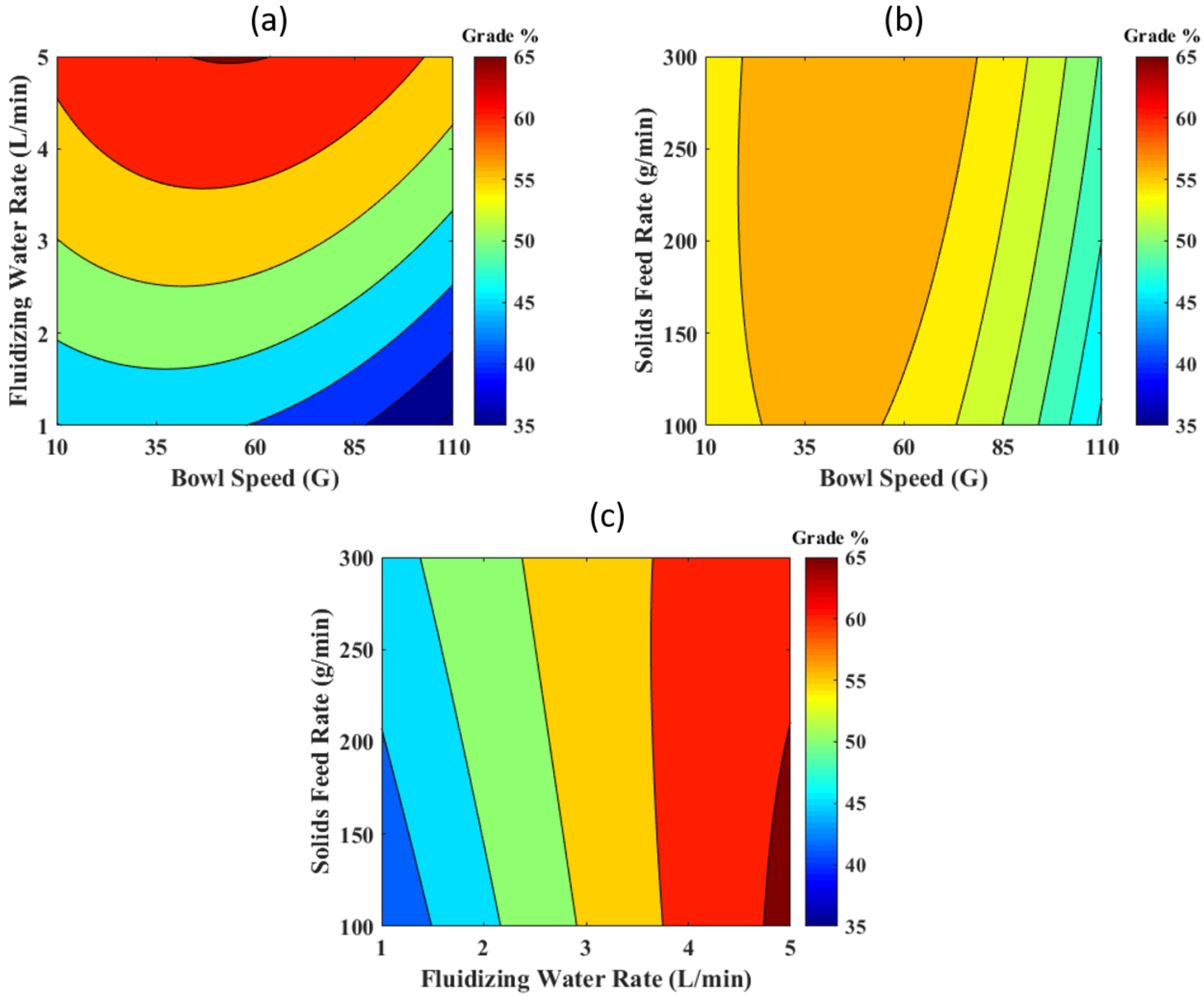

Figure A9 – Response surface plots for grade showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 10 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

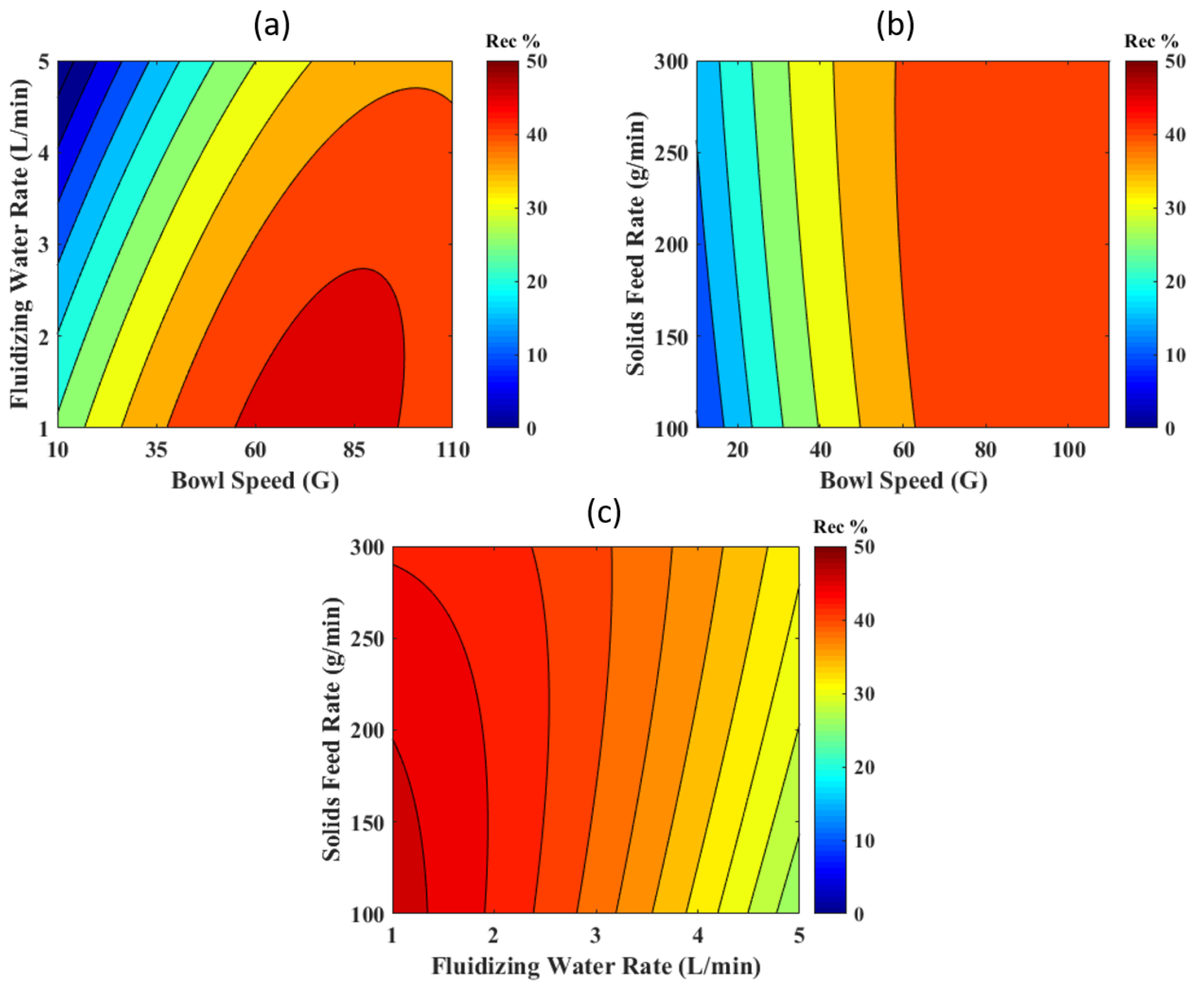

Figure A10 – Response surface plots for recovery showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 10 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

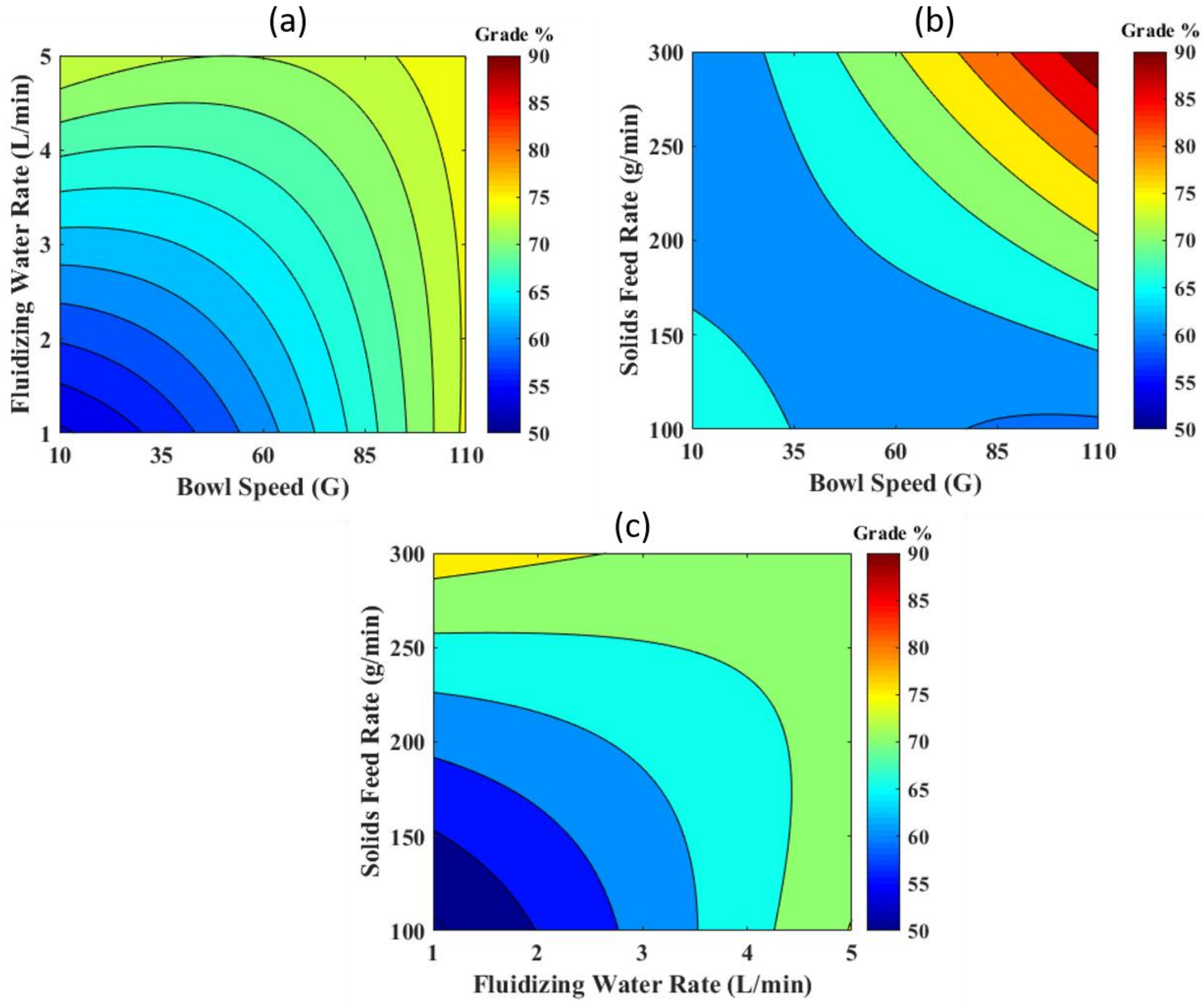

Figure A11 – Response surface plots for grade showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 15 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

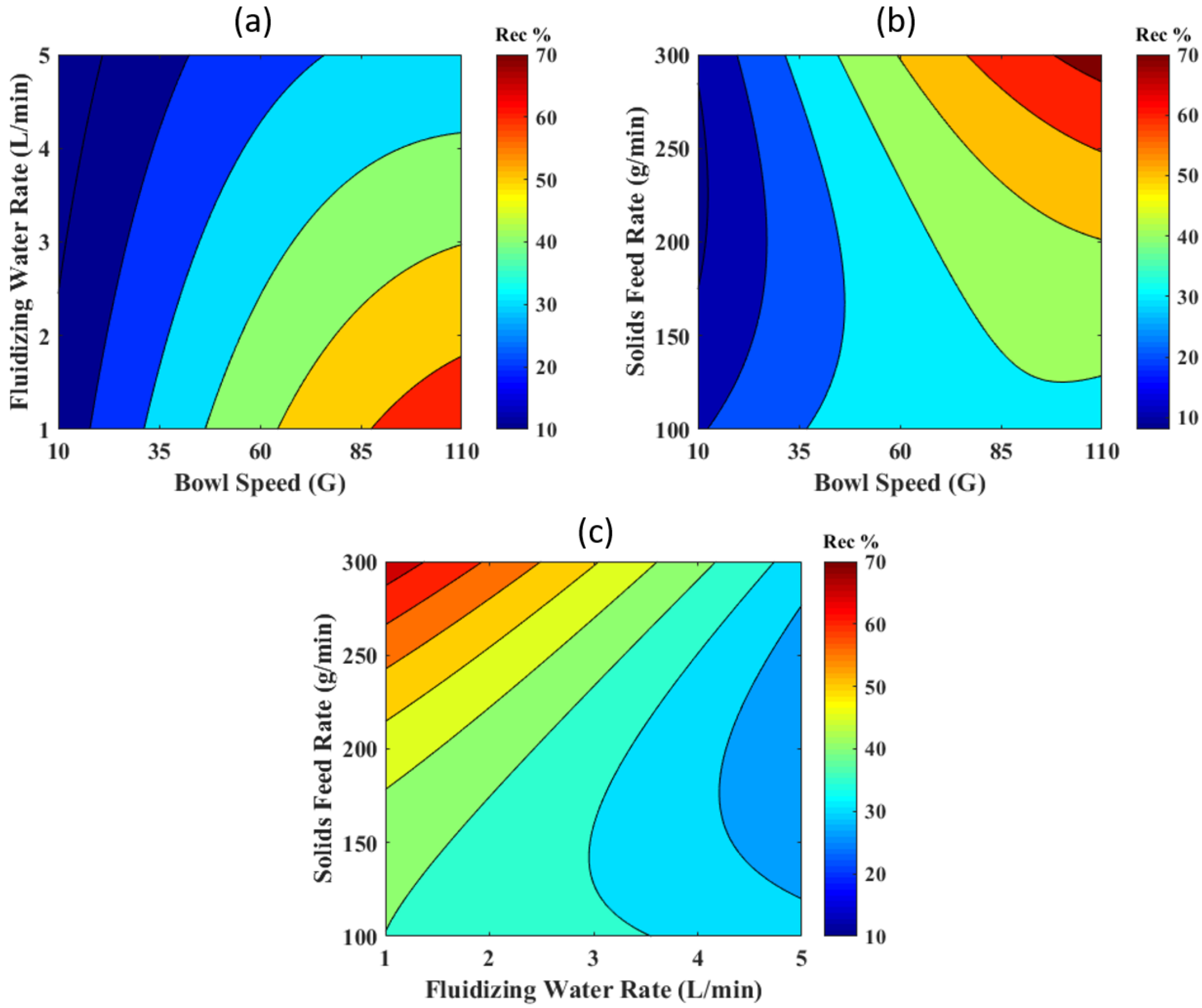

Figure A12 – Response surface plots for recovery showing the relationship between (a) bowl speed and fluidizing water rate, (b) bowl speed and solids feed rate and (c) fluidizing water rate and solids feed rate when processing the 15 % magnetite feed. In all cases the third variable is held constant at the centre (0) level

Figure 1

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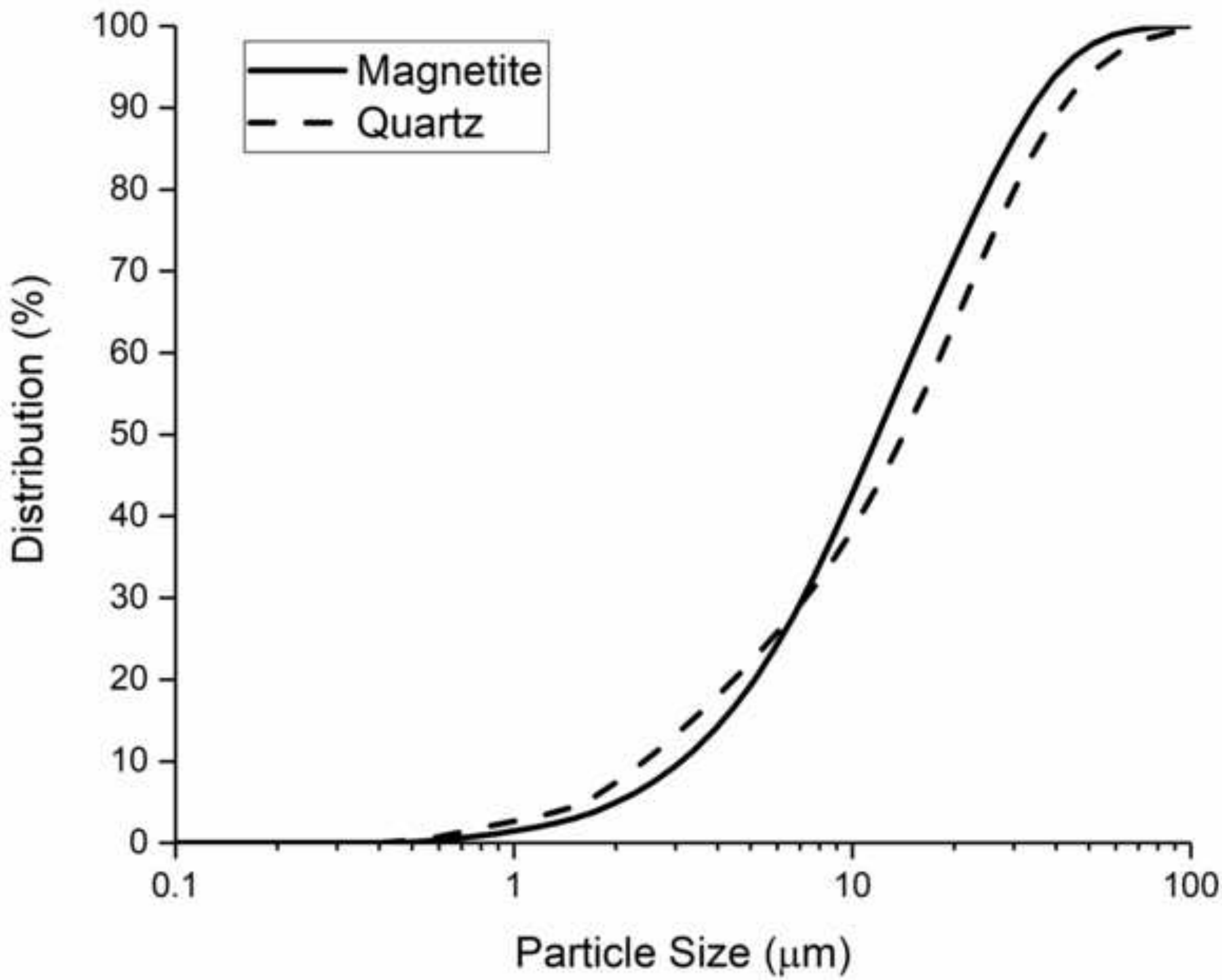

Figure 2
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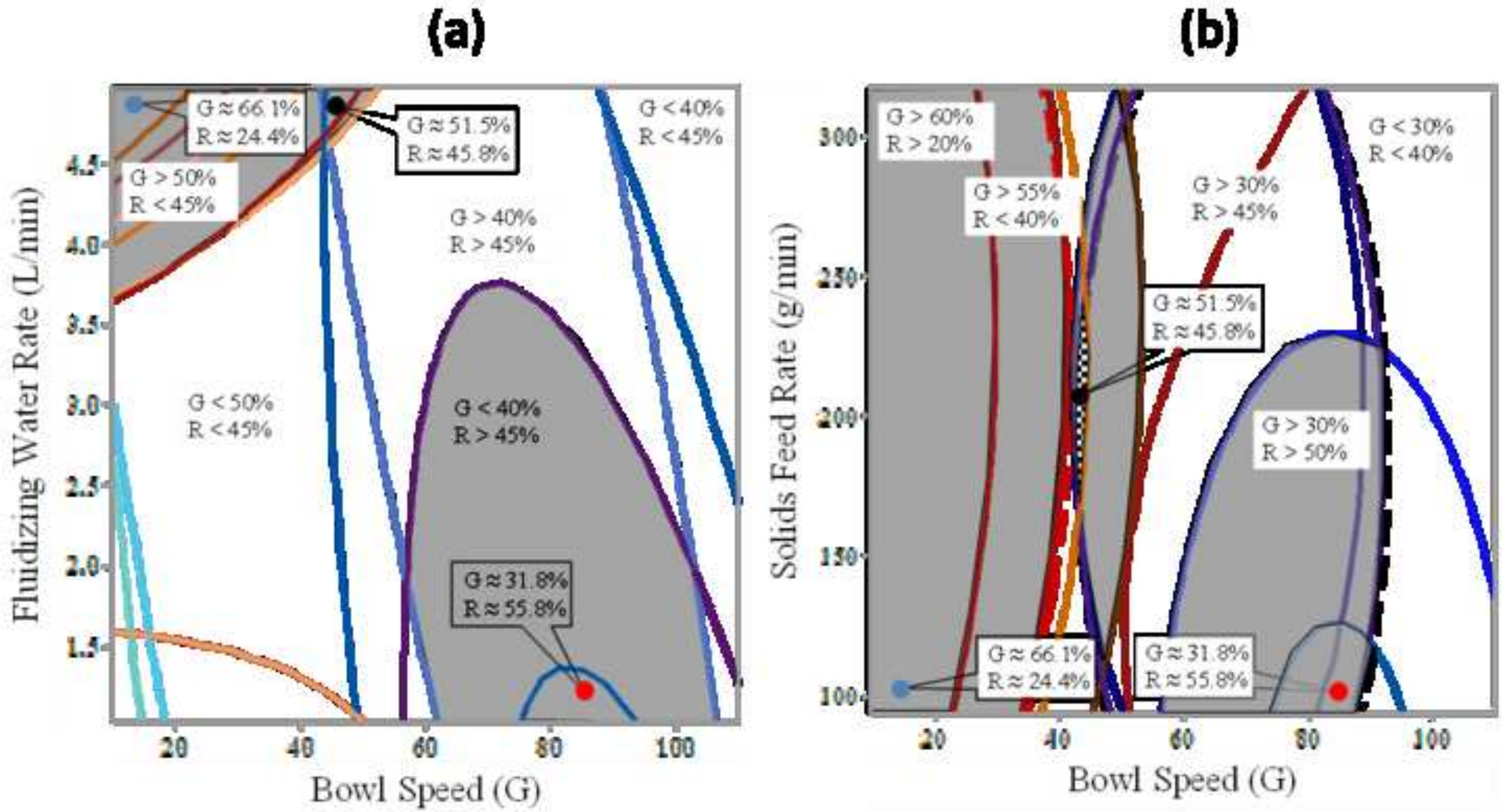

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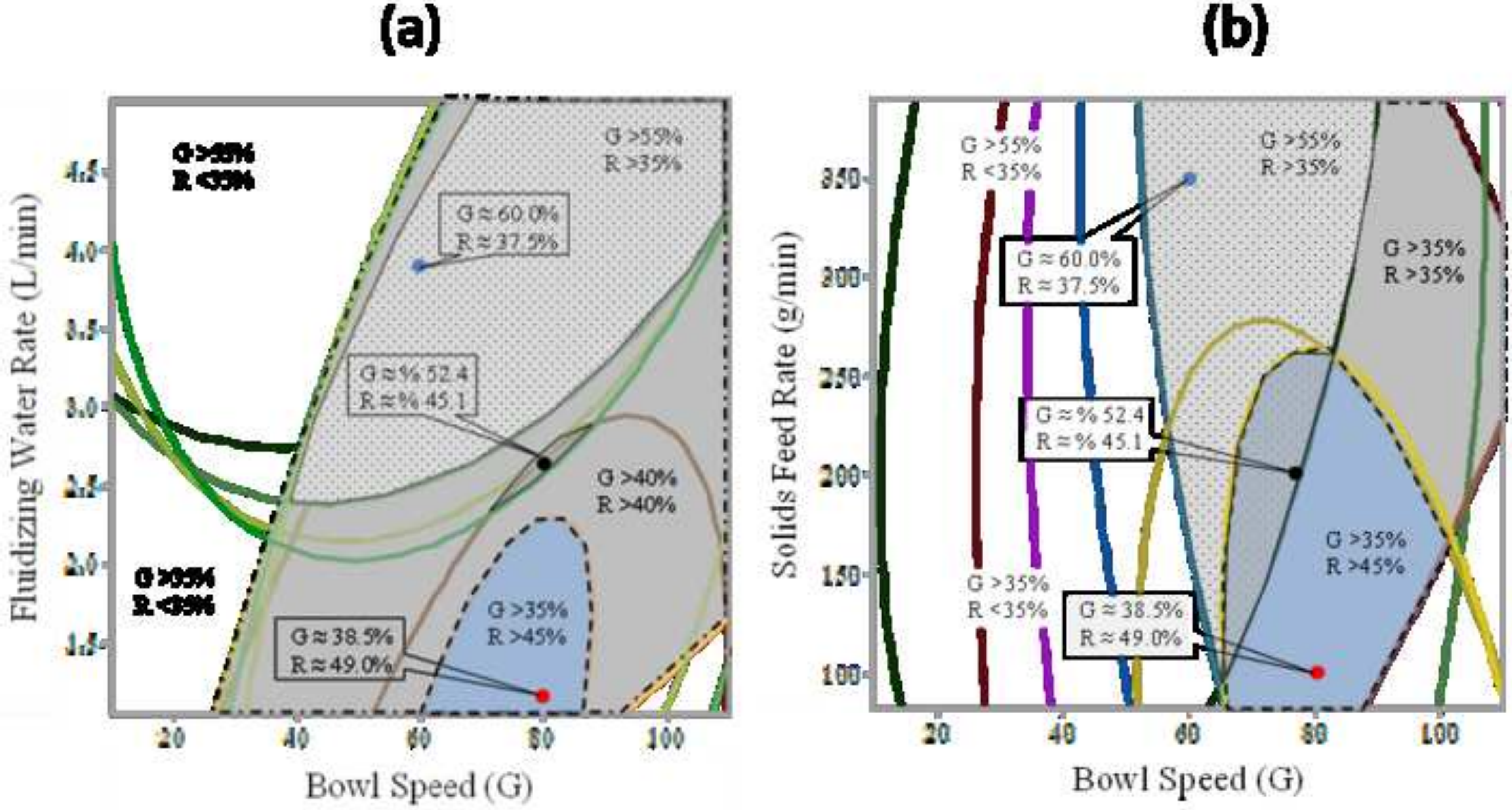

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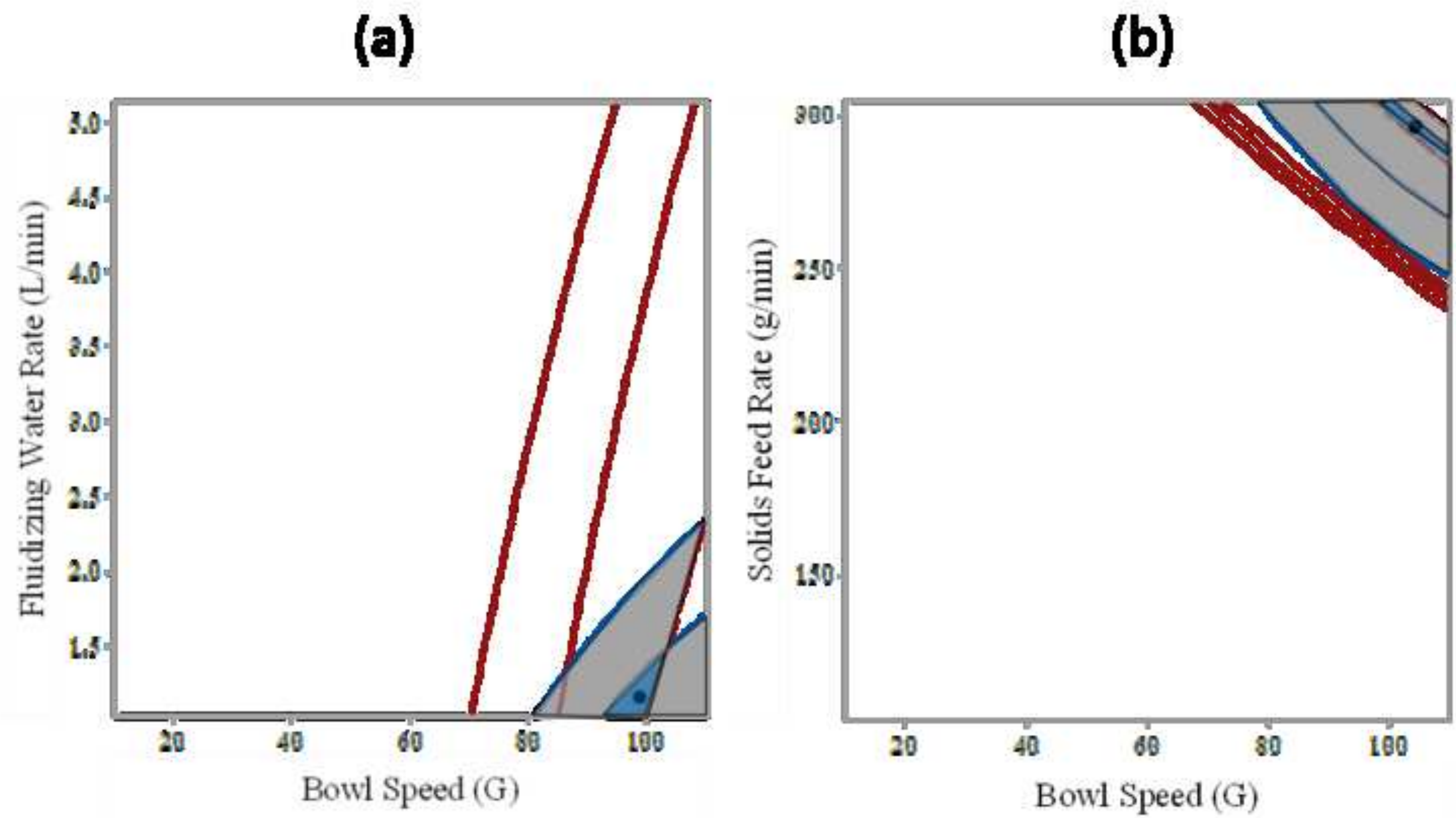

Figure 5
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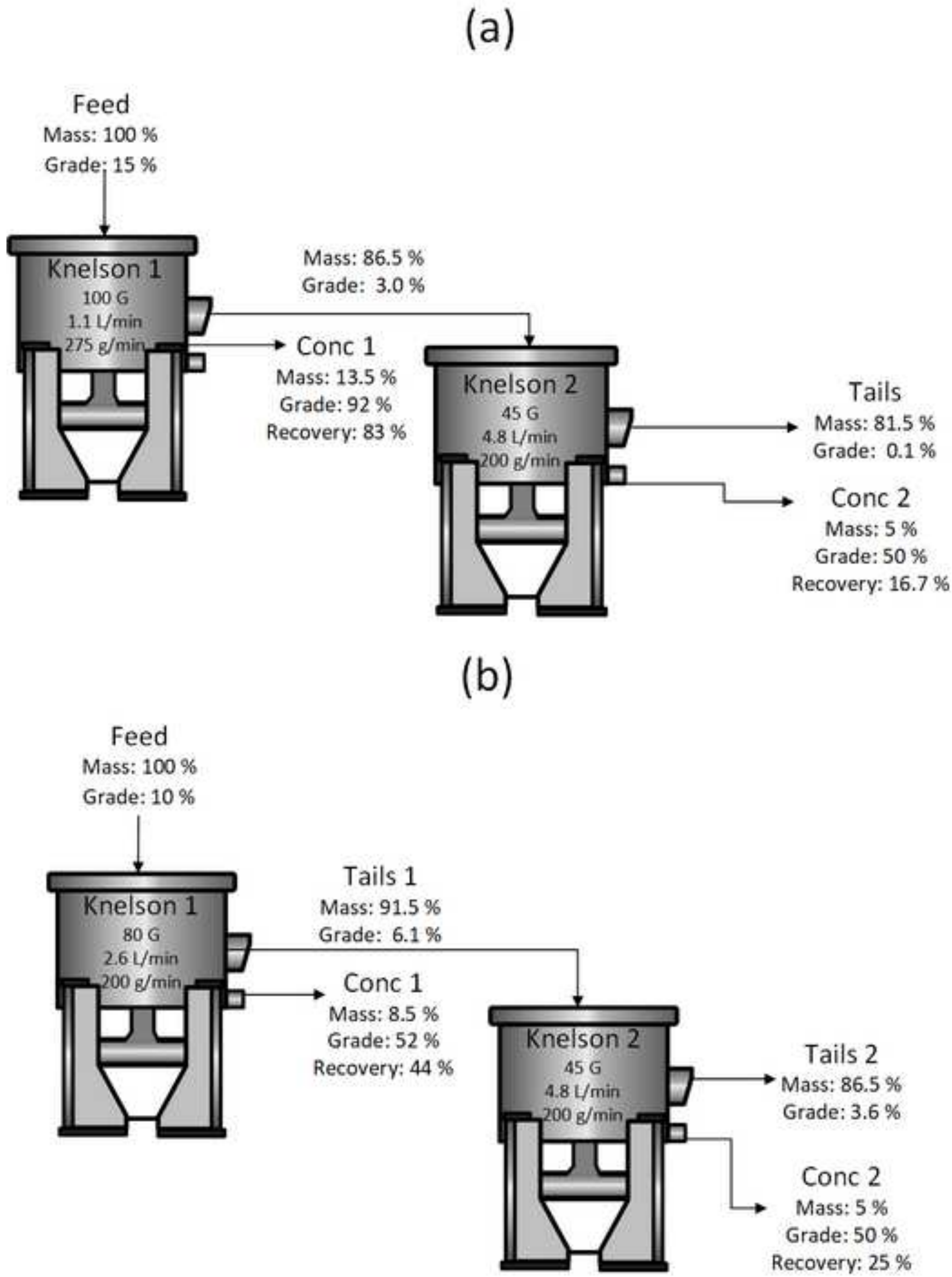

Figure A1
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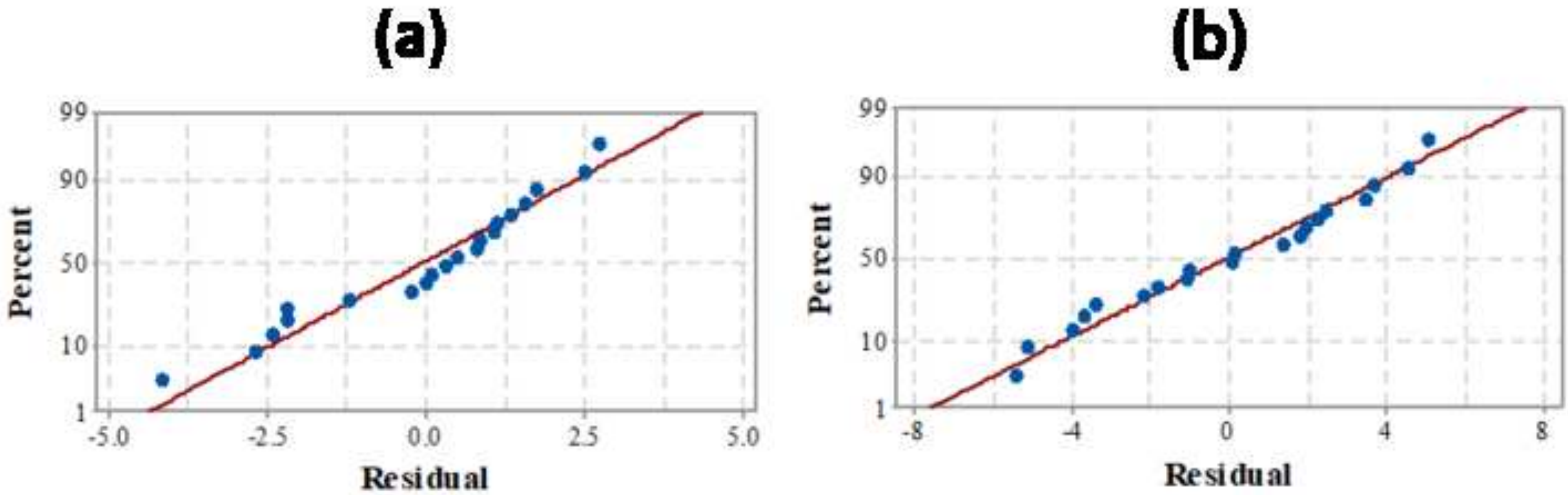

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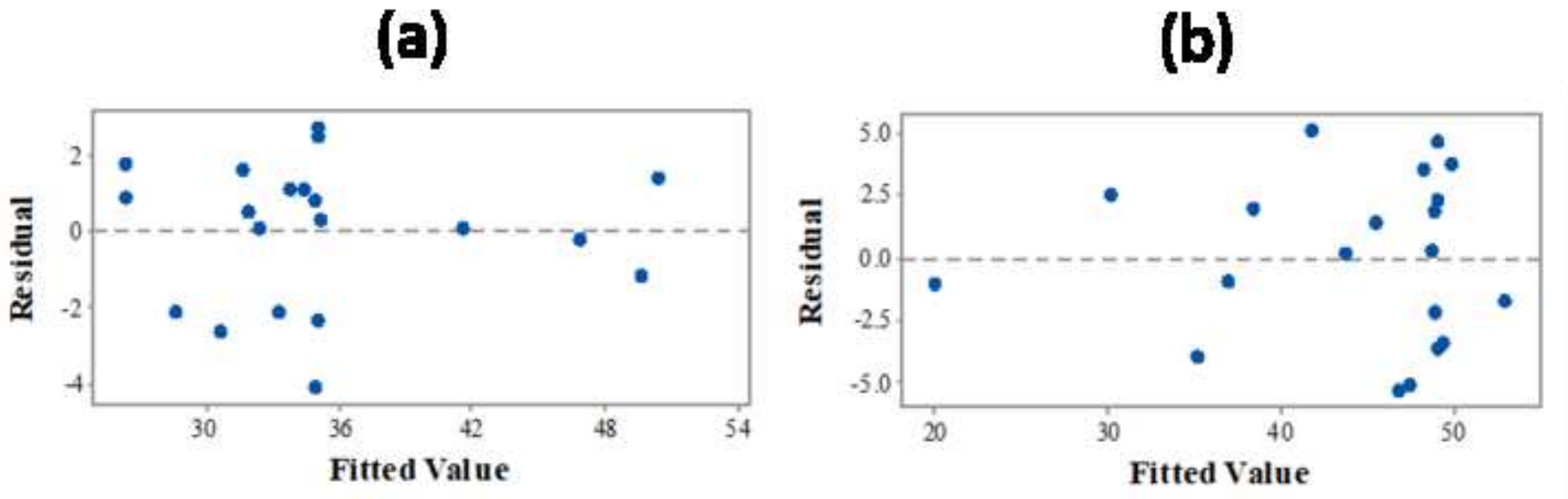

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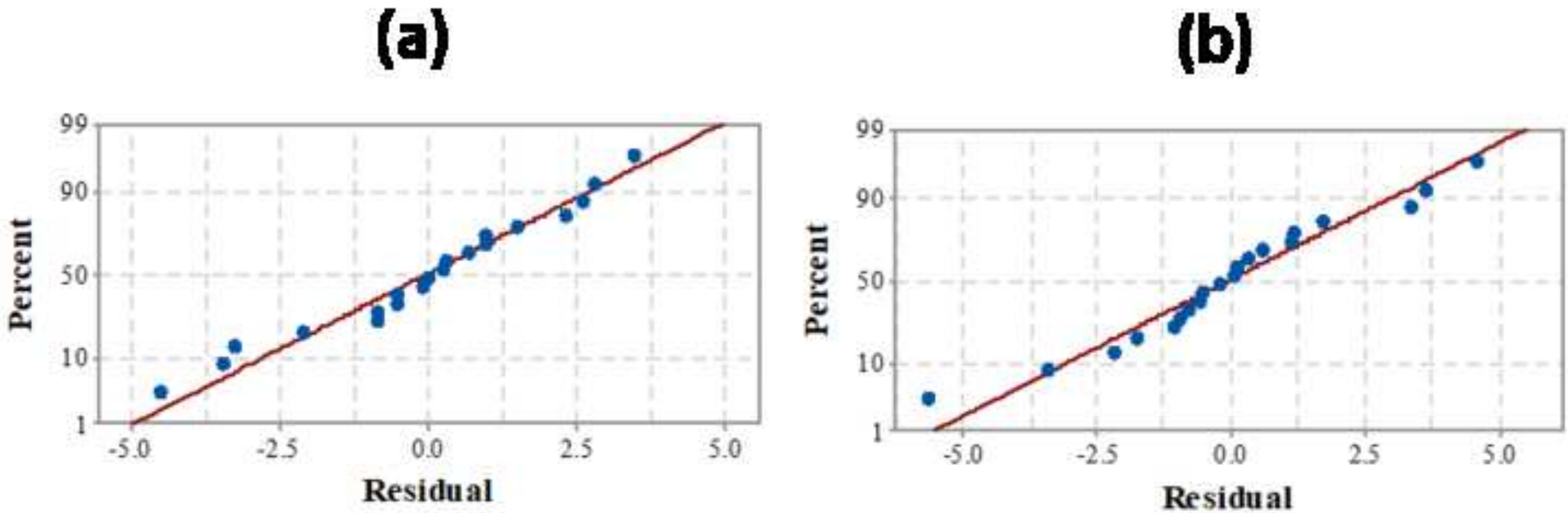

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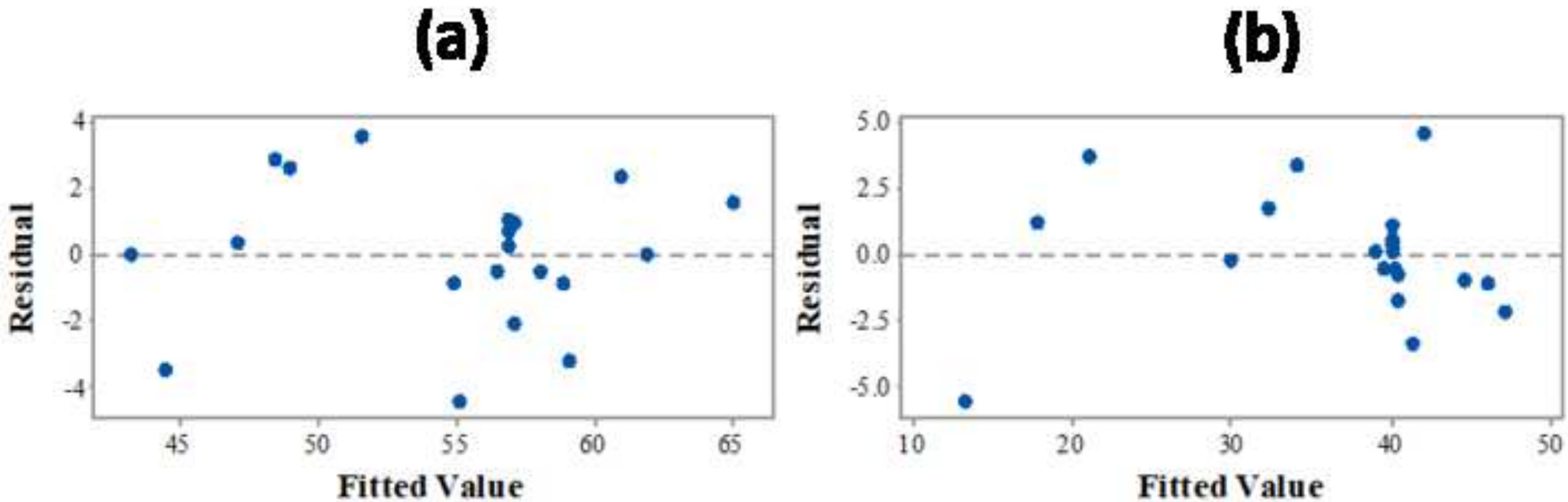

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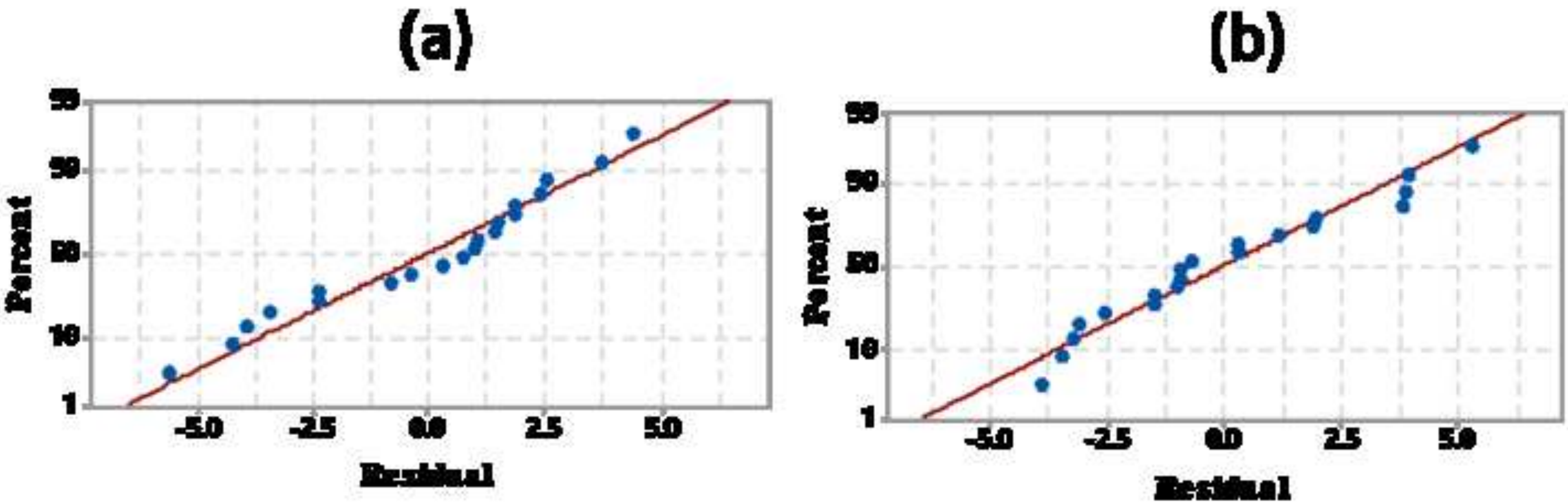

Figure A6

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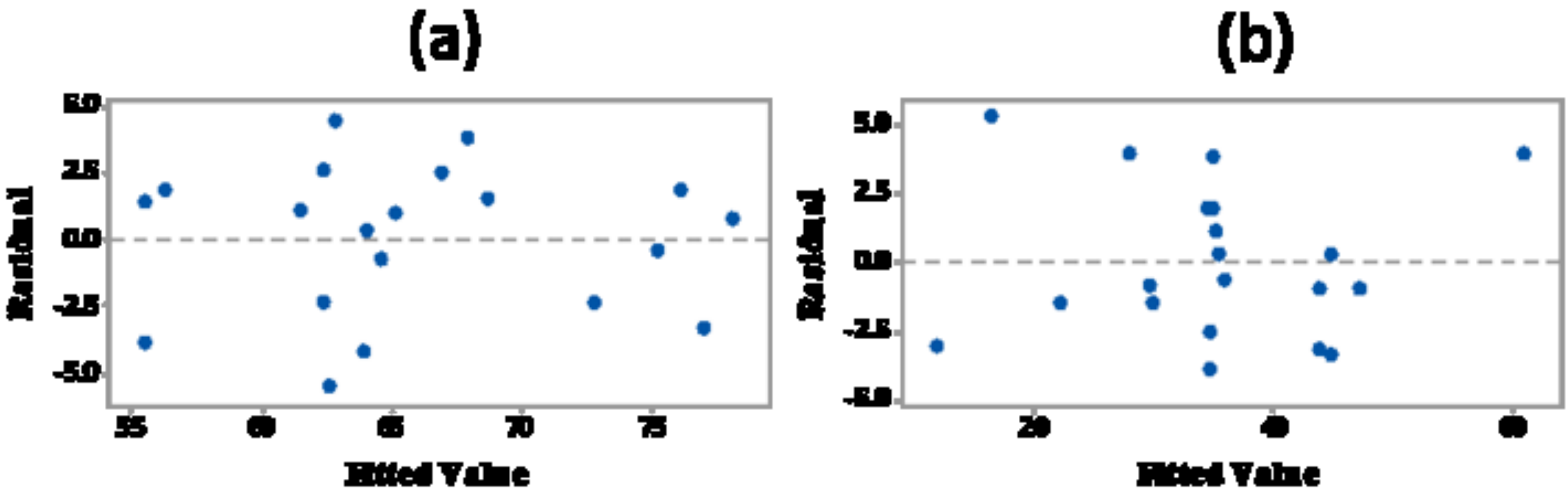

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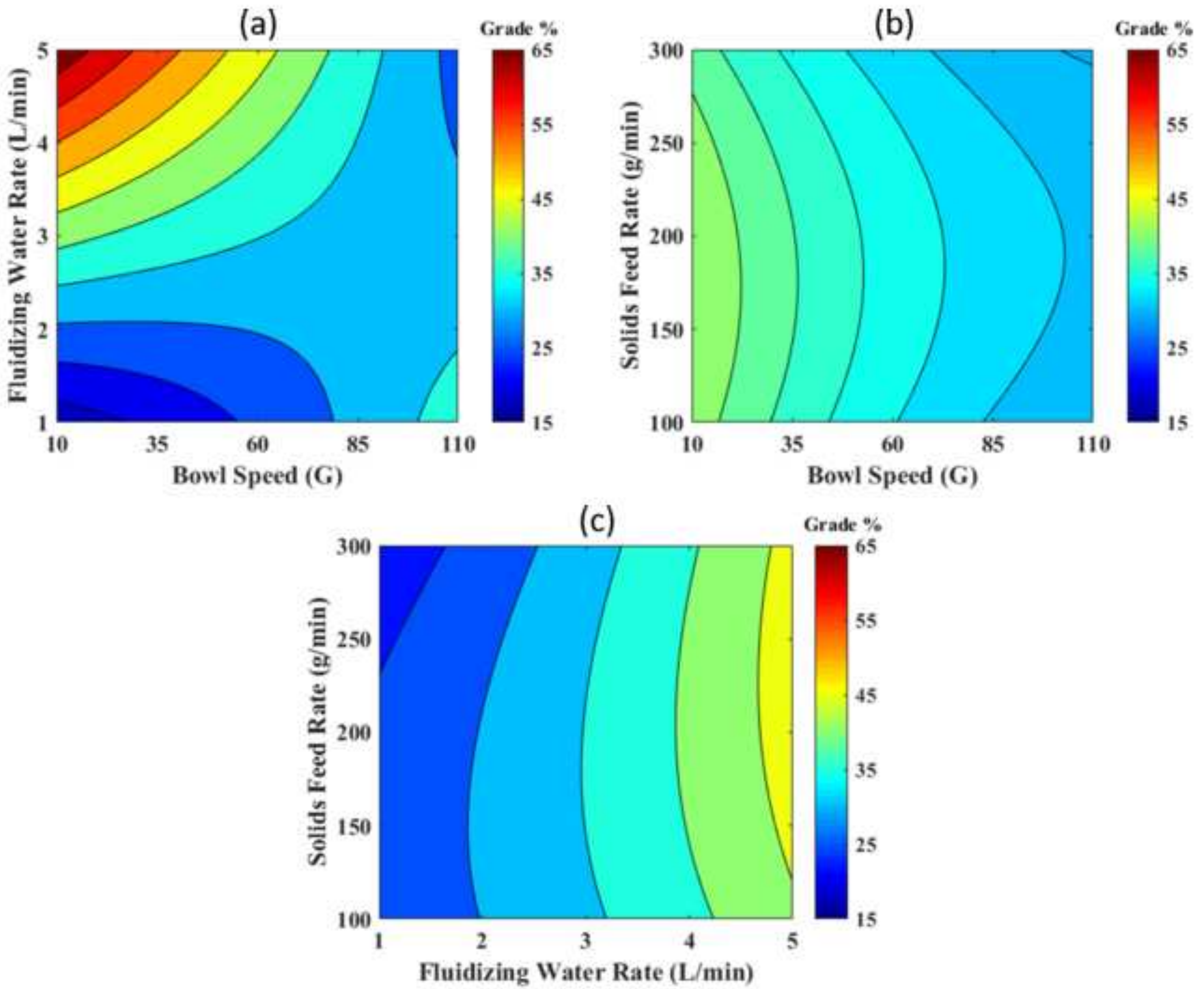

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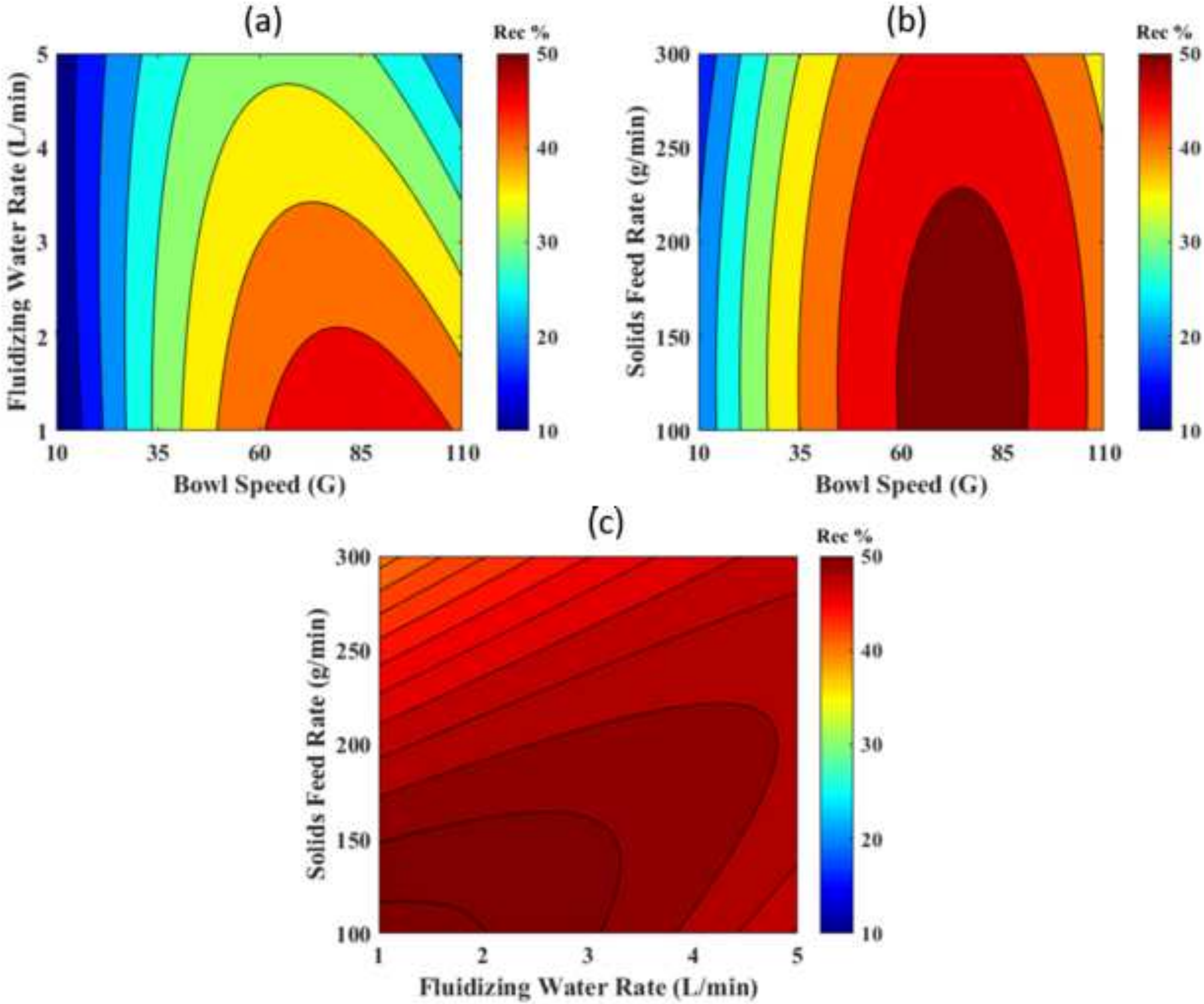

Figure A9

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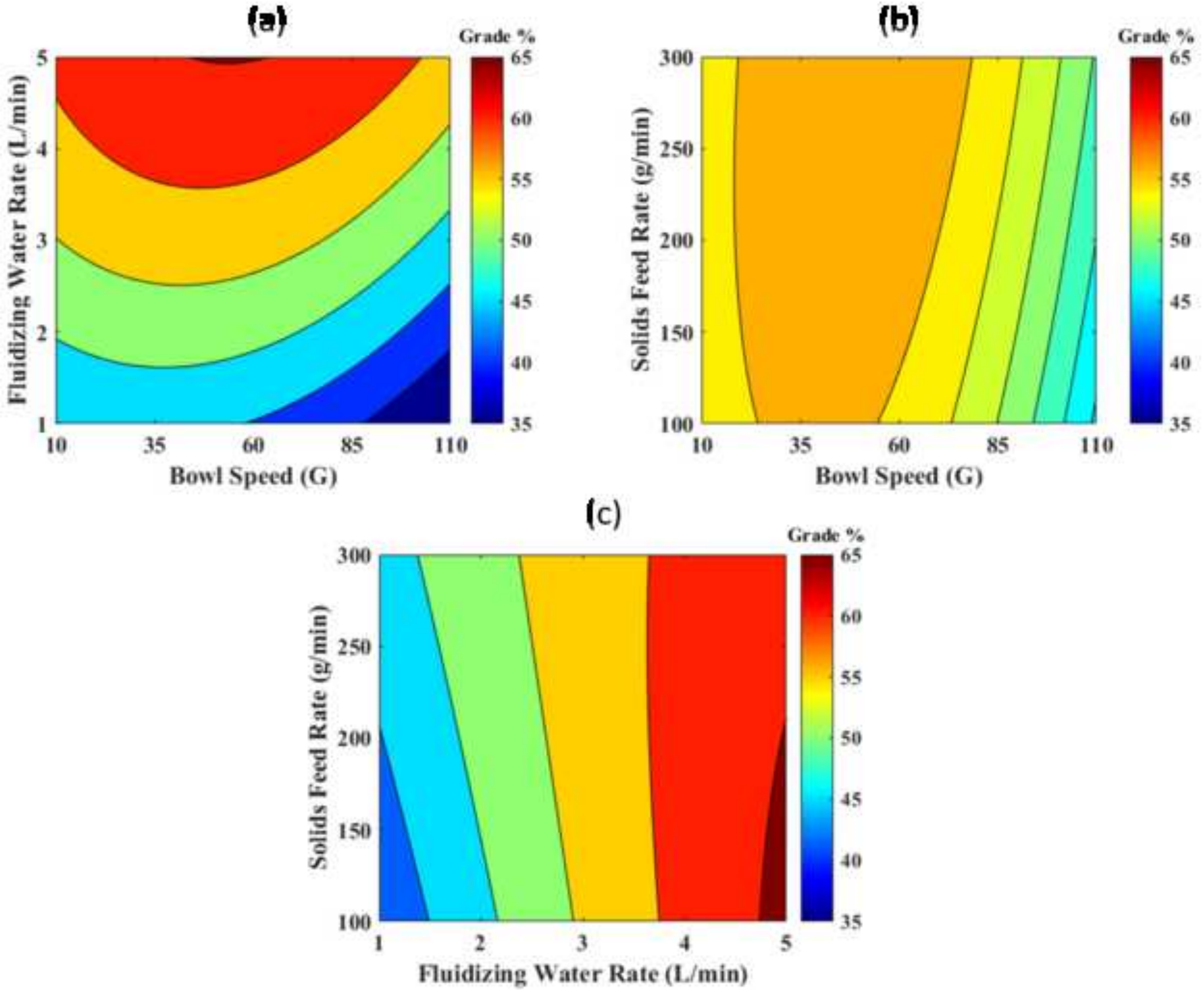

Figure A10
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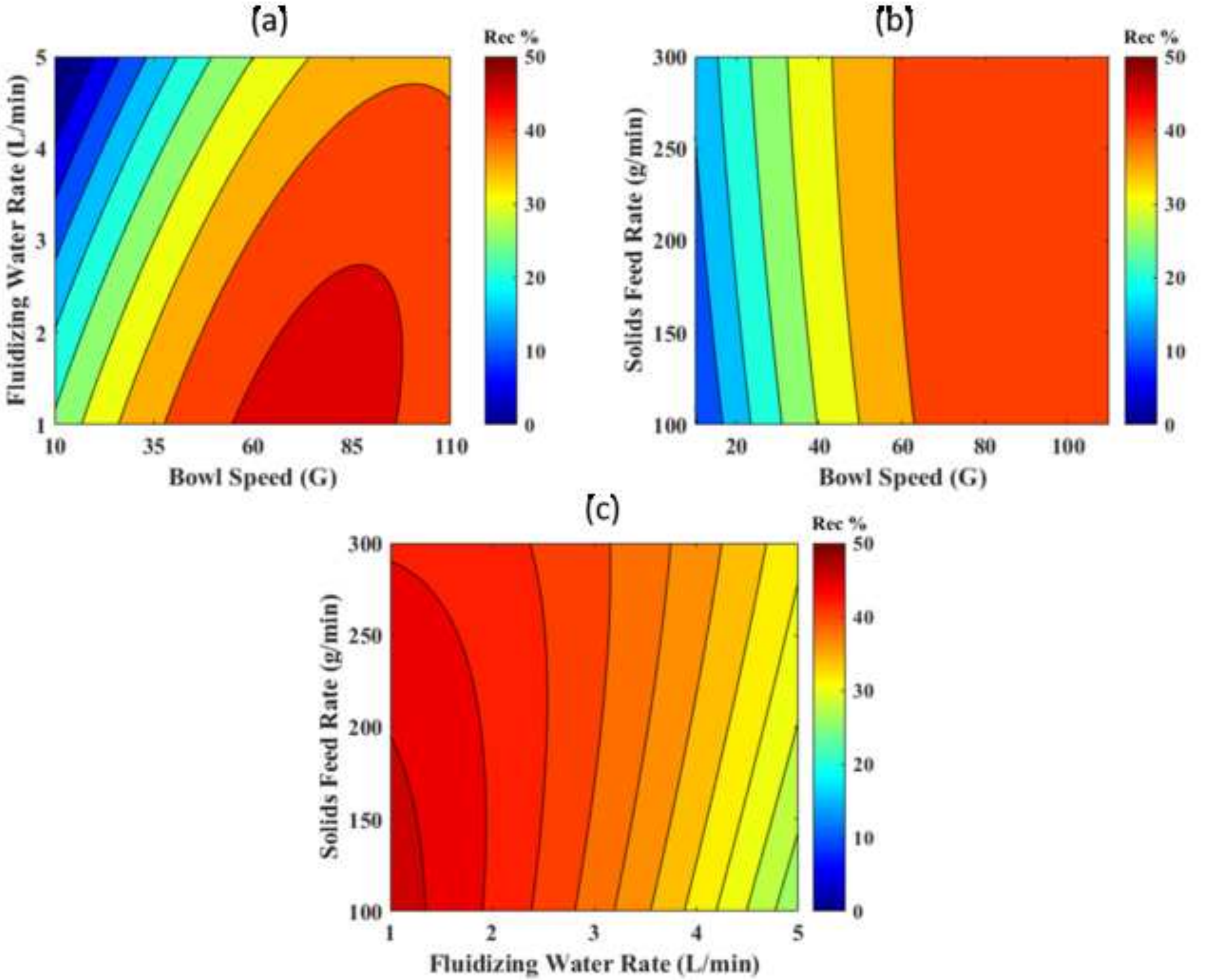

Figure A11
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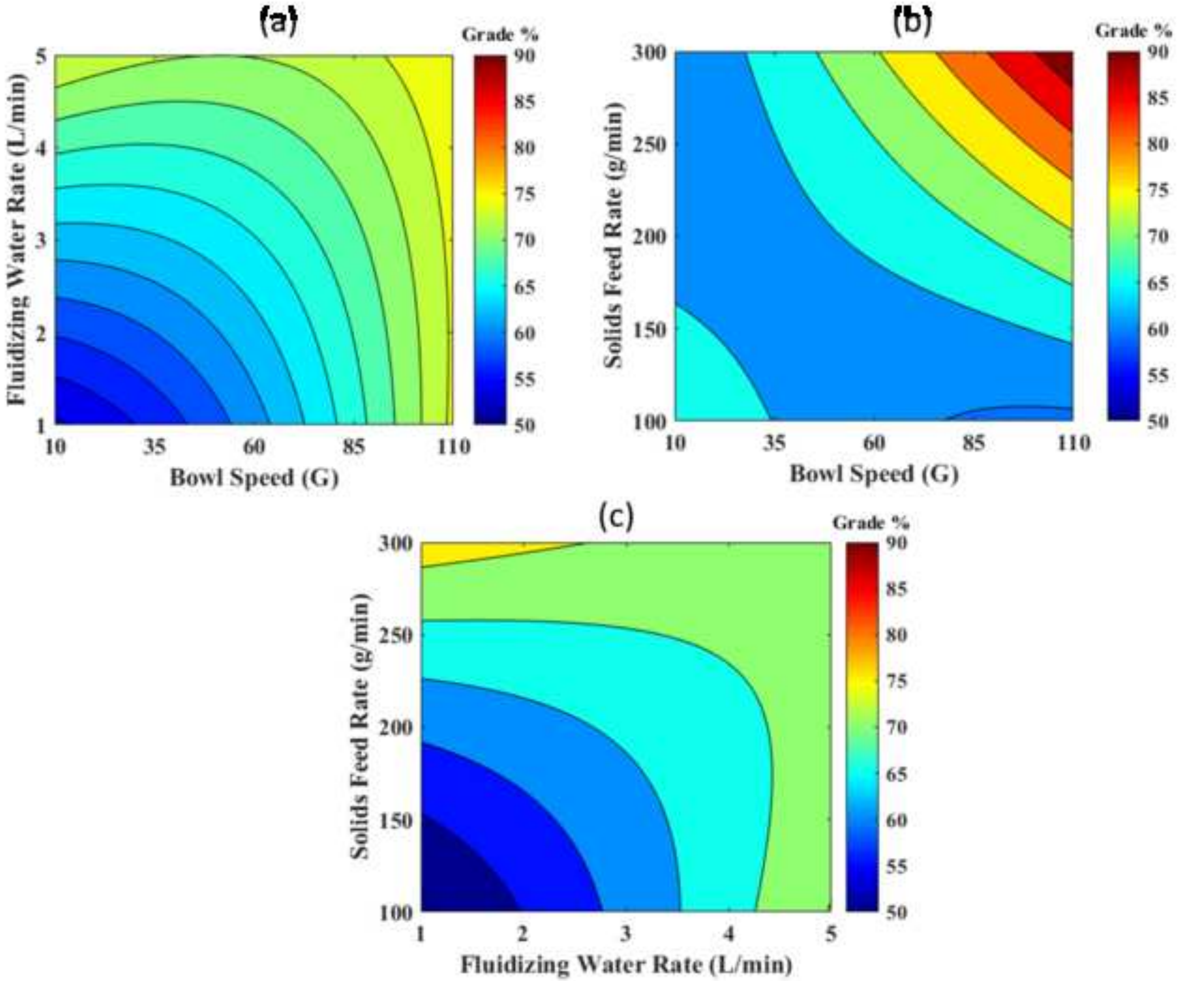

Figure A12
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