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DOI:
10.1061/JTEPBS.0000146

Uncertainty Propagation Assessment in Railway-Track Degradation Model Using Bayes Linear Theory

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Abstract

This paper introduces a semi-probabilistic method driven by the Bayes linear theory to assess uncertainty propagation in parameters of linear model of railway-track-geometry degradation. The parameters were configured in a belief structure before the method updates the prior belief linearly in terms of the first- and second-order moments. Through the updating process, two measures, namely, partial size and bearing adjustment of expectation of prior belief, iteratively displayed how parametric uncertainty propagated at each sample point in the inspection planning horizon. Testing results exhibited a transition point in the horizon, splitting the sample points into two categories: constant and unstable. The latter category consisted of observable quantities that require more observed value (i.e., inspection data to strengthen our belief about the model parameters). Next inspection cycles should keep these quantities in current inspection strategy but lesser attention could be applied to the constant category. A practical use of an assessment of uncertainty propagation is presented and discussed in this paper.

1. Introduction

Recursive implementation of periodic inspections in railway-track maintenance generates data samples for different time (sample) points in a preventive maintenance (PM) cycle. The PM cycle can be defined as an operational interval (expressed in time or accumulated tonnage) starting from the time a track or its components receives restoration until it reaches the next maintenance. The availability of these samples allows the use of a statistical approach to construct regression models that could generate valuable input to a decision-making process, particularly at the design stage of maintenance planning, which aims for a reduction in costs and minutes of train delays (Patra, 2009). In the context of track-geometry maintenance, a degradation model has been developed empirically under a different degree of polynomial; however, a linear-type model has been of interest to researchers for years (Chang, Liu, and Wang 2010). A linear degradation model is apparently simple. It reduces computational complexity dramatically considering the immense size of a railway network.

In the presence of a non-uniform level of parametric uncertainty in the track degradation linear model, model outputs (i.e., predictions) are not fully employed for the entire planning horizon, which leads to a steady dependency on periodic non-destructive in-service inspections. To date, inspection costs are still a substantial percentage of a railway infrastructure company's budget. Thus, addressing the issue of confidence loss in a degradation model that has been proposed initially is necessary to improve (or at least to maintain) the quality of inspection (including maintenance) decisions. The term quality here may refer to precision results and/or fund management. Perhaps a solution of this issue is delivered in the sense of introducing a proper method to estimate sub-intervals on the prediction horizon, in which that the degradation model is considered useful and reliable.

Gligorijevic et al. (2016) argued that the intervals are detectable by properly estimating uncertainty propagation in the model under study. By performing uncertainty propagation, researchers would witness a decreasing trend in the reliability of model prediction caused by the effects of noisiness in input data when predicting further in the future. In order to carry out uncertainty propagation, use of probabilistic representation is common to represent both aleatory and epistemic uncertainty. According to Bedford (2008) and Revie et al. (2010), the fundamental problem of probabilistic representation lies in the selection of prior probability distribution, where in most situations, a parameter of interest is quantified with a poor distribution, causing inaccuracy in the prediction results, forecasting, or inference. This shortcoming can be addressed using the Bayesian approach, which uses new data to update prior distribution. The Bayesian approach provides a theoretical inference framework for updating prior beliefs about uncertain quantities once additional information becomes available (if the decision maker can make observations) from the tests and analyses conducted during the development program. An early work on uncertainty assessment using the Bayesian approach has been reported since early 1970 (Randell et al., 2010)**.** Until now, a wide range of extensions has been developed (see review in Lu and Madanat 1994, Zhang and Mahadevan 2003), and most of the works were developed under a probabilistic Bayesian framework.

When a full detailed probabilistic analysis is too costly to perform, and the belief in parameters of interest is partially elicited, the benefits of conventional Bayesian method is shadowed by the high volume of computational and elicitation effort. In this situation, approximations to the traditional Bayesian analyses, known as Bayes linear analyses, have been proposed as a logical and justifiable framework to express and review on the beliefs about the recognised uncertain quantities. Unlike the conventional Bayesian method--which heavily depends on fully-specified probability distributions--the Bayes linear method linearly adjusted the prior beliefs about these uncertain quantities based upon the theory of Bayes linear statistics (Goldstein and Wooff, 2007). Instead of using probability as a basis (proxy), Bayes linear method uses the first- and second-order moments to model beliefs for the quantities of interest. This means that decision maker's degree of uncertainty regarding a correct value of the quantity under study is represented by variance. Apart from expectation and variance, the Bayes linear method uses covariance to model relationships between quantities which significantly reduces complexity in the need for joint probability distributions in 'traditional' Bayesian approaches.

In this study, we propose the Bayes linear method to estimate uncertainty propagation in parameters of a linear model for railway-track-geometry degradation. The measure produced from the Bayes linear analysis was interpreted in a way to project the trajectory of the defined uncertainty propagates over a planning horizon. The measures that represent the proportionate contribution of each time point in a planning horizon that are involved in the regression analysis (we refer it as a quantity hereafter) were adjusted in prior beliefs about linear model parameters. Graphical representation of these measures exhibits a transition point in the level of parametric uncertainty. Simulation results display the effectiveness of the proposed uncertainty propagation method and offer an attractive way to address the relative importance of each inspection decision made in terms of updating knowledge about an unexplained variance.

2. Background of study

2.1 Bayes linear method

Bayes linear methodology provides a simple structure of belief specifications which allows users to easily add new elements to the model. In fact, users get flexibility to combine lines of evidence of varying quality from many disparate sources of information when assessing uncertainty about elements of quantity of interest, for example, a rate of change of track linear degradation model. Interestingly, adjustments on model specifications are tractable under BL framework where in some cases it can be performed instantaneously; in particular, when multidimensional space needs to be adjusted. Longer computational time is probably taken when using traditional Bayesian approach.

The term 'linear' in Bayes linear method defines a linear relationship between vector $\boldsymbol{B}$ and $\boldsymbol{D}$ in $\boldsymbol{D} = \alpha\boldsymbol{B} + \boldsymbol{R}$ where $\boldsymbol{R}$ represents the unexplained uncertainty between $\boldsymbol{B}$ and $\boldsymbol{D}$. Vectors $\boldsymbol{B}$ and $\boldsymbol{D}$ denote a belief structure representing uncertain quantities of interest, B_i, and is some vector of quantities that might improve decision maker's prior assessment of $\boldsymbol{B}$. The first- and second-order moment of $\boldsymbol{B}$, denoted by $E(\boldsymbol{B})$ and $var(\boldsymbol{B})$ will be adjusted using elicitation and observed values of $\boldsymbol{D}$. Prior to the adjustments, decision maker must construct $E(\boldsymbol{D})$ and $var(\boldsymbol{D})$, and specifies covariance matrix $\operatorname{cov}(\boldsymbol{B}, \boldsymbol{D})$ which address the degrees of relationship between $\boldsymbol{B}$ and $\boldsymbol{D}$. Note that the matrix must satisfy characteristics of non-negative definite matrix. Following the formula in Goldstein and Wooff (2007), the collection $\boldsymbol{B}$, respectively, has adjusted expectation and adjusted variance matrix

$$E_{\boldsymbol{D}}(\boldsymbol{B}) = E(\boldsymbol{B}) + \operatorname{cov}(\boldsymbol{B}, \boldsymbol{D}) \operatorname{var}^{\Psi}(\boldsymbol{D})(\boldsymbol{D} - \boldsymbol{E}(\boldsymbol{D})) \tag{1}$$

$$var_{\boldsymbol{D}}(\boldsymbol{B}) = var(\boldsymbol{B}) + \operatorname{cov}(\boldsymbol{B}, \boldsymbol{D}) \operatorname{var}^{\Psi}(\boldsymbol{D}) \operatorname{cov}(\boldsymbol{D}, \boldsymbol{B}) \tag{2}$$

where $\operatorname{var}^{\Psi}(\boldsymbol{D})$ is the Moore-Penrose generalized inverse. In case of $\operatorname{var}(\boldsymbol{D})$ is non-singular then $\operatorname{var}^{\Psi}(\boldsymbol{D})$ is simply the usual matrix inverse i.e. $\operatorname{var}^{\Psi}(\boldsymbol{D}) = \operatorname{var}^{-1}(\boldsymbol{D})$.

2.2 Track geometry degradation model

Ride quality has been identified as one of the three important attributes in train passenger services (Wardman and Whelan, 2001). From railway infrastructure manager's desk, a great effort has been put through track geometry maintenance tasks to maintain the quality standards in standard. Besides ride quality, an increase in vehicle safety (i.e. derailment risk reduction), improvement in rail line productivity, better customer satisfaction, and a rise in profit margin are among other benefits of railway maintenance (Hossein et al., 2015). In order to program a cost effective and time efficient maintenance plan, the railway network benefits from the series of inspections assigned systematically across the network at different frequencies, subjected to the accumulated traffic tonnage and speed category (Coenraad Esveld, 2001). An interesting aspect of track geometry inspection is that the track possession is allocated last when the identified tracks are unattended by both passengers and freight trains (Santos et al., 2015). Interrupting scheduled train and freight timetables due to inefficient use of inspection resources should be the last resort of action (Santos et al., 2015). Causing train delays upsets train operators who are major customers to railway infrastructure owners. Thus, it is essential to construct inspection schedules effectively and present the risk estimation of unplanned maintenance due to unexpected failures. One of the key elements for the risk estimation is track degradation models (Dindar et al., 2016).

Receiving axle loading progressively makes an initial state condition of railway tracks deteriorate to lower states, which further end at a state of failure (assuming no rectification during an operational period). In order to estimate properly in which state the track is in degradation, authorities create a model of the state of condition with respect to a track geometric index (TGI) associated with a specific type of geometric defect. Depending on the local railway authority, they may apply different strategies (e.g. roughness, fractal and defectiveness) for TGI formulation based on the mean and standard deviation calculations (Sadeghi, 2010). The selected TGI, when compared with a set of three or four maintenance tolerances (limits), defines a suitable maintenance strategy to restore the quality of the inspected track. In hierarchical order, the alert limit (AL) is the lowest level that is viewable as a separation point between the normal and defective region of track geometry conditions. Upon TQI exceeding the value, the usual completion of a further investigation by means of visual inspection verifies the status before planning a preventive maintenance operation. Avoiding or delaying a tamping preventive maintenance allows the TQI to deteriorate further, which incurs excessive maintenance cost when the TQI passes the boundary value between AL and the intervention limit (Vale et al., 2012).

The trade-off between complexity and readable features is a fundamental issue when presenting a degradation model for decision-making use. Degradation models that capture non-linear characteristics when determining changes in track irregularity often provide a better estimation as compared to a linear model (He et al., 2013). However, a simple description about the relationship between explanatory or predictor and response variable always appear in the latter model type. In fact, updating the state of track quality for a high number of railway tracks consumes a reasonable amount of computational cost. This advantage is transferable when uncertainty associated with model parameters receives an update. Assuming the probabilistic Bayes method drives the updating process as shown in Zhang and Mahadevan (2000), and the complexity of the procedure will rise depending on what assigned probability distributions existed at the prior elicitation. Heavy use of non-normal distribution appeared in Andrade and Teixeira (2012), which probably motivated the authors to introduce track section groups (e.g. switches, bridges, stations, and plain track) before performing uncertainty assessments and propagation in linear model parameters. Realising that localized factors (e.g. overall track structure, groundwater movement and weather patterns) are not included in a linear model, performing uncertainty propagation should occur on each rail track individually. Previous train accident reports have highlighted the importance of having an individual condition assessment. Thus, this paper proposes Bayes' linear method as an approximation of the full-scaled probabilistic Bayes method in the context of parametric uncertainty propagation used in the track geometry degradation linear model.

3. Bayes linear method for uncertainty propagation

3.1 Proposed method

The method proposed in this paper was based on the concept that a time position in a planning horizon, when the inspection data was sampled (refer to a quantity hereafter), has a different degree of importance in terms of propagating uncertainty in the linear model parameters. For example, a quantity near to the beginning of the planning horizon where a restoration is taking place usually has little fluctuation in its observed value compared with quantities far ahead where accumulated tonnage is high. If it is possible to rank quantities in order of their importance to a particular linear degradation model, then exploitation of this information could determine a transition point in uncertainty propagation. In addition, this information was applicable to exclude unnecessary quantities from the sequences upon the arrival of disruptions. Bin Osman et al. (2016) and Osman et al. (2016) explain on potential sources of disruption in the context of track inspection schedules.

This study adopts Bayes' linear theory to measure the relative importance of all observable quantities in terms of their contribution to reducing uncertainty in parameters of linear degradation models. Simply, a quantity that has contributed more to uncertainty reduction should receive a higher assigned value of recognized measures and should remain for the next PM cycle. Having the measures, we could rank the quantities and point out a time position where the parametric uncertainty starts to propagate actively. We splitted quantities into two groups: a group for before the transition and a group for after the transition point.

Given a linear model equation written in $Y_i = \beta_o + \beta_1 X_i + \varepsilon_i$, where ε_i is an unobserved error term, a priori was expected to have a mean of zero. Our interest was the collection $\boldsymbol{B} = (\beta_o, \beta_1)$. Given observations on a collection of observable quantities $\boldsymbol{D} = (D_1, D_2, \ldots, D_m)$, prior belief was a vector $\boldsymbol{B}$ updates via the adjusted expectation, $E_{\boldsymbol{D}}(\boldsymbol{B})$. By calculating the size of adjustment over $\boldsymbol{B}$ given by the observed values of D using an equation (3), we were able to quantify how deviation of the adjusted expectation was from the prior expectation. Application of a similar principle then occurred to calculate an adjustment over $\boldsymbol{B}$ given by a portion of $\boldsymbol{D}$. For an individual assessment, the size of partial adjustment may have referred to and derived from the Equation (4).

$$Size_D(\boldsymbol{B}) = [E_D(\boldsymbol{B}) - E(\boldsymbol{B})]^T \operatorname{var}^{\Psi}(\boldsymbol{B})[E_D(\boldsymbol{B}) - E(\boldsymbol{B})] \tag{3}$$

$$Size_{[F/D]}(\boldsymbol{B}) = [E_{F\cup D}(\boldsymbol{B}) - E_D(\boldsymbol{B})]^T \operatorname{var}^{\Psi}(\boldsymbol{B})[E_{F\cup D}(\boldsymbol{B}) - E_D(\boldsymbol{B})] \tag{4}$$

We used this measure as a proxy to measure relative importance to each quantity in $\boldsymbol{D}$. Ideally, a quantity with large value of $Size_{[F/D]}(\boldsymbol{B})$ has a larger chance to remain in the next inspection cycle. Another aspect that we considered in a weight assignment was a partial bearing for the partial adjustment, denoted by $Z_{[F/D]}(\boldsymbol{B})$. This measure expressed both the direction and the magnitude of the changes over $\boldsymbol{B}$ when we additionally adjusted $\boldsymbol{B}$ by $\boldsymbol{F}$ given a preceding adjustment by $\boldsymbol{D}$, through the relation

$$cov_D(B_i, \boldsymbol{Z}_{\boldsymbol{F}/\boldsymbol{D}}(\boldsymbol{B})) = E_{D\cup F}(B_i) - E_D(B_i); \forall B_i \in \boldsymbol{B} \tag{5}$$

3.2 An example

The researcher applied the proposed methodology to a generic example of a single track geometric parameter, which was responsible for a specific isolated track geometric defect. A list of the defects commonly appeared in railway networks reside in (Coenraad Esveld 2001). Eight data samples, each corresponding to a short time series for an individual plain track, extracted from (Andrade and Teixeira, 2011) were used in the testing. A time series has a length of 14 independent observations (data points) representing a standard deviation of the chosen parameter for a 200-meter track segment. With this description, we have 14 quantities for a set of $\boldsymbol{D}$. An open-access application called WebPlotDigitalizer (Rohatgi, 2010) helped to execute data extraction and the total of 112 observations appeared in a plotted chart in Figure 1. Errors between the real and plotted values are expected to result from the extraction process and settled somewhere around 5% as reported in (Moeyaert et al., 2016). From the figure, it is clear that there is a missing record between $D_{i=1}$ and D_{i+1} for all samples. To update the prior belief about $(intercept, rate)$ Bayes linear method also requires prior moments regarding every quantity, $D_i; i = 1,\ldots,14$. Due to small samples gathered from D_i, a careful examination requires completion to avoid the findings from becoming irrelevant. As suggested in Ghasemi and Zahediasl (2012), a parametric test on each D_i occurred using the Shapiro-Wilks test. In brief, the Shapiro-Wilks test has a high power to reject H_o at nominal alpha. H_o entails the definition that follows:

H_o: *The quantity* $D_i = (d_{1,i}, d_{2,i}, \ldots, d_{m,i})$ *is a random sample from a specified distribution if the p-value associated with the Shapiro-Wilks statistics is not less than the chosen alpha value.*

Mean and variance from the fitted distribution applied as in the prior belief of D_i. In case H_o is rejected at nominal α=0.01, 0.05, 0.10 for all suggested distribution, their *p*-values are compared and used as a basis to choose an appropriate distribution for D_i. At this point, the moments are presented in a range of values instead of a single value. The core process of updating beliefs repeats for many values. Table 1 shows the initial belief about $\boldsymbol{B}$ as recommended in Goldstein and

Wooff (2007). This implies that the users have little idea on where the true $\boldsymbol{B}$ lies over a given planning horizon *T*. Prior to updating the belief, moments of each quantity in $\boldsymbol{D}$ revealed the results of hypothesis testing as described in the previous paragraph. The values gathered in Table 2 were obtained through Monte-Carlo simulations as default settings in Matlab.

Using prior belief about the moments in $\boldsymbol{B}$ and $\boldsymbol{D}$, as viewed in Table 1 and 2, 150 runs tests of BLM employed a random observation $\boldsymbol{d} \in \boldsymbol{D}$ to capture an overall changing in Equation **(3-5).** The size of $\boldsymbol{d}$ follows a number of quantities involved when calculating these measures. The term $\boldsymbol{d}$ needs at least one quantity and its size can rise up to a maximum size of $|\boldsymbol{D}|$, i.e. when full quantities were involved in a test. For example, $\boldsymbol{d}_{1,2,3} = (d_1, d_2, d_3)$ indicates that a test will be performed using the first three quantities in $\boldsymbol{D}$**,** in which their value is randomly assigned from their respective prior information in Table 2. The median of boxplot statistics that summarised test results appear orderly plotted in Figure 2, where values in brackets are 25-th and 75-th percentile values.

The belief about $\boldsymbol{B}$ overall updated to an expectation of $E_{\boldsymbol{D}}(\beta_{\boldsymbol{o}})$ and $E_{\boldsymbol{D}}(\beta_1)$ with variances of $var_{\boldsymbol{D}}(\beta_{\boldsymbol{o}})$ and $var_{\boldsymbol{D}}(\beta_1)$, respectively. In Figure 2(a), comparing to the maximum value of the size of adjustment, i.e. using the first 11 quantities, a decision of using a full $\boldsymbol{D}$ has extremely decreased the highest $Size_D(\boldsymbol{B})$ about 95%. However, the $Size_D(\boldsymbol{B})$ associated with full $\boldsymbol{D}$ has a percentage increment about 360% as compared to a decision using only the first quantity. We see that there is no significant change in the $Size_D(\boldsymbol{B})$despite extending the initial test to include more quantities (up to six quantities). An average individual adjustment on (intercept, rate), as shown in Figure 2(b), shows that all of the first eight D_i fairly have similar information gains. However, there is a clear fluctuation in the size of adjustments when $d_{1,\ldots,8\cup j}; \mathrm{j} = 9,\ldots,14$ was tested. Among all j quantities, the tests showed that D_{11} has adjusted the prior belief the most and followed by D_{12} as the next best informative quantity to use for belief updating. Adding D_{13} into $\boldsymbol{d}_{1,\ldots,12}$ dramatically reduces the $Size_D(\boldsymbol{B})$ but the value is likely unchanged with a participation of D_{14} in tests. Moving to Figure 2(c), testing results show that prior belief updated in a different direction from what it experienced with $\boldsymbol{d}_{\boldsymbol{F}/\boldsymbol{D}}$. In fact, a direction of change can be seen in the negative region of Bearings itself, for example, $d_{D_{11}/D_{10}}, d_{D_{12}/D_{11}}$ and $d_{D_{13}/D_{12}}$.

4. Conclusions

Understanding on how parametric uncertainty in a linear degradation model propagates over time is necessary to effectively plan track geometry inspections. Bayesian approach has been used to address this issue but heavy use of probabilistic computations creates another dimension of complexity in track inspection planning. In this study, we argue that there is a much simpler method to construct prior beliefs and performing an adjustment on them upon arrival of new information. Bayes linear method uses the first- and second-order moments as a proxy when reliably adjusted prior belief about quantities of interest. The research also presented on how the method is able to assign relative importance measures to a set of quantities in terms of uncertainty propagation in parameters of linear degradation model. By plotting adjusted expectation measures in a sequential order, we can view how parametric uncertainty evolves along the planning horizon. We also obtained a quick way of estimating a new level of uncertainty. For further exploration using the same data, we would extend variance learning from a static linear combination of observations to multiple linear combinations. This might create a longer process due to evaluations of variance and covariance between those linear combinations. Apart from that, measures used in this study should be weighted with respect to class type and location of rail tracks. By having weighting function, relative importance of each quantity could be represented more adequately while taking complexity of decisions in reality into practical consideration. Lastly, a performance comparison between two types of Bayesian approach in terms of assessing uncertainty propagation should be presented to demonstrate practicality when dealing with a large size of components.

Acknowledgments

The authors are sincerely grateful to European Commission for the financial sponsorship of the H2020-RISE Project No. 691135 "RISEN: Rail Infrastructure Systems Engineering Network", which enables a global research network that tackles the grand challenge in railway infrastructure resilience and advanced sensing under extreme conditions. The corresponding author would like to acknowledge scholarship from the Ministry of Higher Education of Malaysia and Univesiti Kebangsaan Malaysia.

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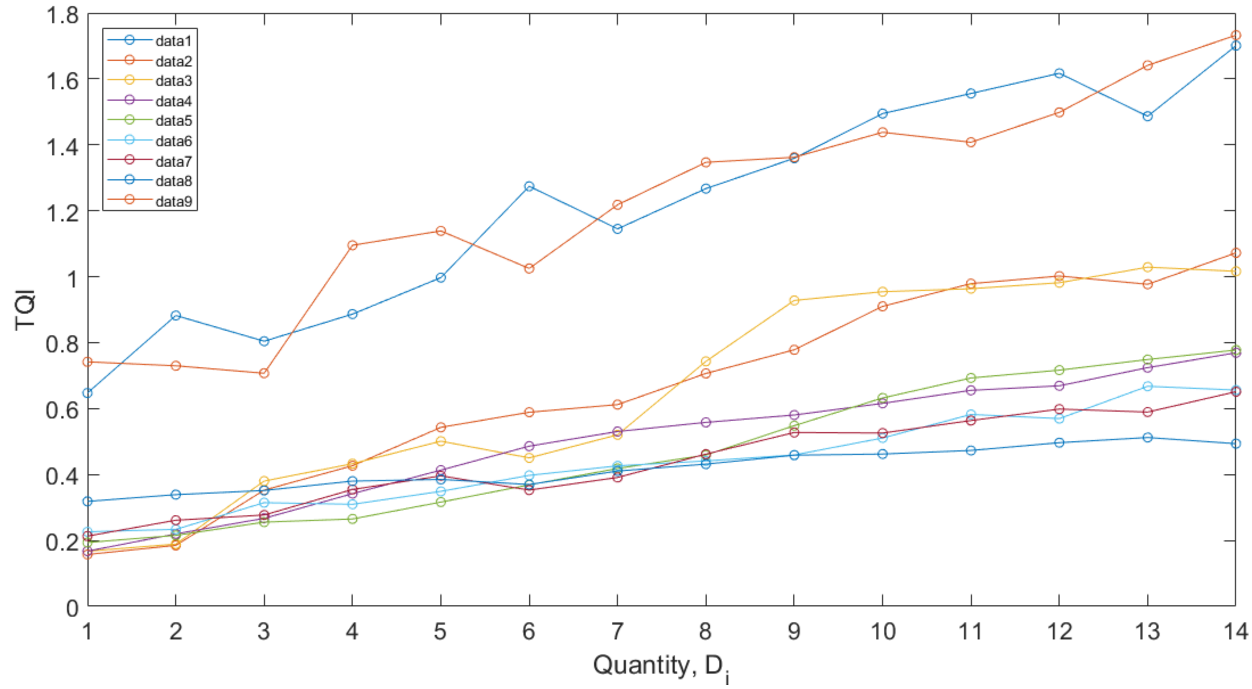

Figure 1. Eight examples of rail track quality index degradation over a fixed planning horizon *T*. A collection of data points at position *i*-th in *T* associates with a quantity D_i

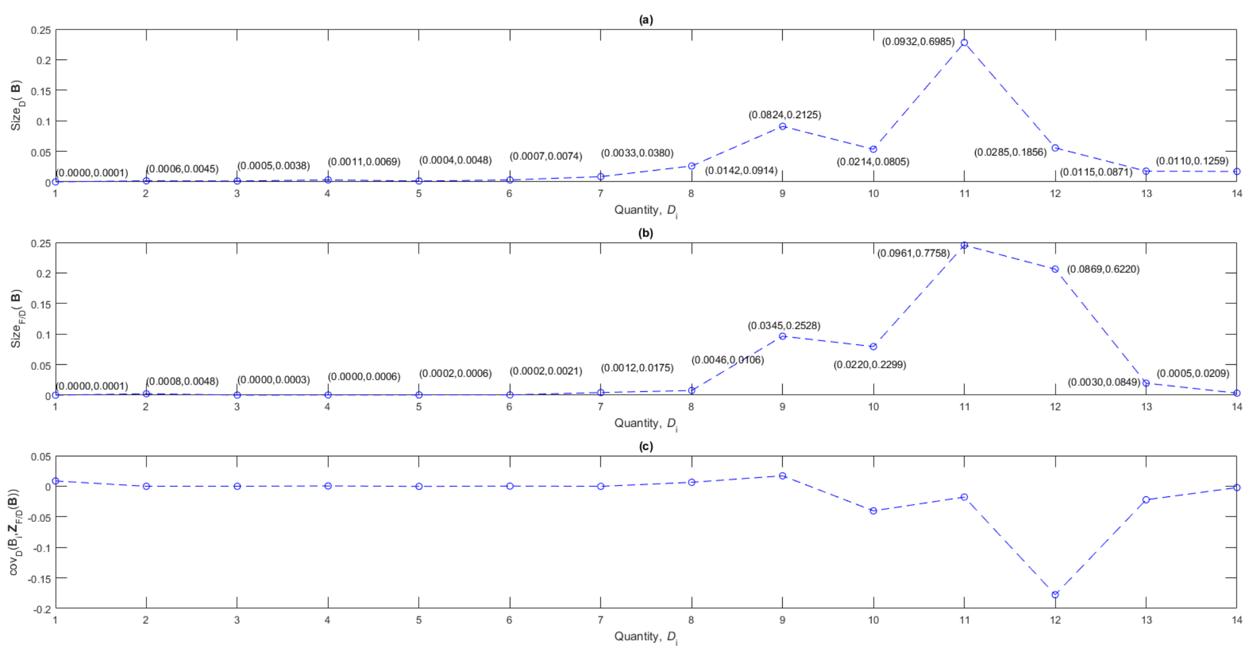

Figure 2. Evolution in uncertainty propagation in the belief structure over a defined planning horizon represented in three modes; a) Size of adjustment, b) partial size of adjustment, and c) partial bearing of adjustment

Table 1. Prior Specifications About ***B*** Structure

Variable	Expectation	Variance
β_0	0	2
β_1	0	1

Table 2. Prior Specifications About ***D*** Structure

Variable	Prior distribution	Expectation	Variance	Variable	Prior distribution	Expectation	Variance
$D_{i=1}$	Exponential	0.4060	0.1648	D_8	Normal	0.6934	0.1956
D_2	Exponential	0.4448	0.1979	D_9	Normal	0.7241	0.2197
D_3	Exponential	0.4615	0.2130	D_{10}	Normal	0.7817	0.2524
D_4	Exponential	0.4817	0.2320	D_{11}	Exponential	0.8293	0.6878
D_5	Exponential	0.5288	0.2796	D_{12}	Normal	0.8248	0.2839
D_6	Normal	0.6090	0.1596	D_{13}	Normal	0.8495	0.2941
D_7	Exponential	0.6344	0.4025	D_{14}	Normal	0.9051	0.3358