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Kozai–Lidov cycles towards the limit of circumbinary planets

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ABSTRACT

In this Letter, we answer a simple question: Can a misaligned circumbinary planet induce Kozai–Lidov cycles on an inner stellar binary? We use known analytic equations to analyse the behaviour of the Kozai–Lidov effect as the outer mass is made small. We demonstrate a significant departure from the traditional symmetry, critical angles and amplitude of the effect. Aside from massive planets on near-polar orbits, circumbinary planetary systems are devoid of Kozai–Lidov cycles. This has positive implications for the existence of highly misaligned circumbinary planets: an observationally unexplored and theoretically important parameter space.

Key words: methods: analytical – celestial mechanics – planets and satellites: dynamical evolution and stability – binaries: close.

1 INTRODUCTION

In 1962, there were two independent papers published on an effect seen in the gravitational three-body problem. Lidov (1962) investigated the effect the Moon has on artificial satellites orbiting the Earth. Kozai (1962) looked at asteroids orbiting the Sun under the influence of Jupiter. Both systems were qualitatively the same: an inner restricted three-body problem with initially circular orbits and all of the orbital angular momentum confined to the outer orbit (Fig. 1a). It was discovered that when the two orbital planes were significantly misaligned, between 39° and 141°, there was a high-amplitude modulation in the eccentricity of the inner orbit (in their examples the satellite and the asteroid) and the mutual inclination. Contrastingly, the outer eccentricity was seen to remain constant. Overall, this is known as a Kozai–Lidov (K-L) cycle or effect.

The K-L effect was later generalized to the case of three bodies of comparable mass (Harrington 1968; Lidov & Ziglin 1976) for the application to triple star systems (Fig. 1b). It has subsequently been implicated, for example, in the production of tight stellar binaries (period $\lesssim 7$ d). In this scenario, the K-L cycle induced on the inner binary leads to close encounters between the two stars, at which point tidal friction dissipates orbital energy and shrinks the inner orbit (e.g. Mazeh & Shaham 1979; Eggleton & Kisseleva-Eggleton 2006; Fabrycky & Tremaine 2007).

The theory has been extended to eccentric outer orbits, in which case higher order effects cause the system to be chaotic, possibly inducing flips in the inner orbit orientation. This has been applied to multiplanet systems (Fig. 1c) by Naoz et al. (2011) in order to explain why a surprisingly large fraction of hot-Jupiters are misaligned or even retrograde (Hébrard et al. 2008; Schlaufman 2010; Triaud et al. 2010).

The similarity between the three scenarios in Fig. 1(a)–(c) is that the system's orbital angular momentum is largely confined to the outer orbit. The time-scale, amplitude and limits of K-L cycles do vary between the different configurations, but for initially circular orbits there is qualitatively not a radical departure from the original work of Lidov (1962) and Kozai (1962).

In this Letter, we explore a different type of three-body system: circumbinary planets (Fig. 1d). The discovery of eight transiting systems by the *Kepler* mission (Doyle et al. 2011; Welsh et al. 2014) has given this class of planet widespread scientific exposure and validity. The discoveries so far have been limited to coplanar systems (within ∼4°), so one might question the relevance of K-L cycles in this scenario. However, Martin & Triaud (2014) argued that this narrow distribution is the result of a strong detection bias imposed by the requirement of a consecutive transit sequence.[1] There are also theoretical arguments for the existence of misaligned circumbinary planets, as a result of planet–planet scattering (e.g. Chatterjee et al. 2008 in the context of single stars), a misaligned disc (e.g. 99 Herculis, Kennedy et al. 2012; KH 15D, Winn et al. 2004) or an interaction with an outer tertiary star (Hamers, Perets & Portegies Zwart 2015; Martin, Mazeh & Fabrycky 2015; Mũnoz & Lai 2015).

In a circumbinary planetary system, the outer orbital angular momentum becomes vanishingly insignificant with a decreasing outer mass. This is qualitatively different to the other systems in Fig. 1 and it means that the classic studies of Lidov (1962) and

[1] Circumbinary planets that are misaligned by $\gtrsim 10°$ frequently miss transits, creating a sparse transit sequence which is harder to identify.

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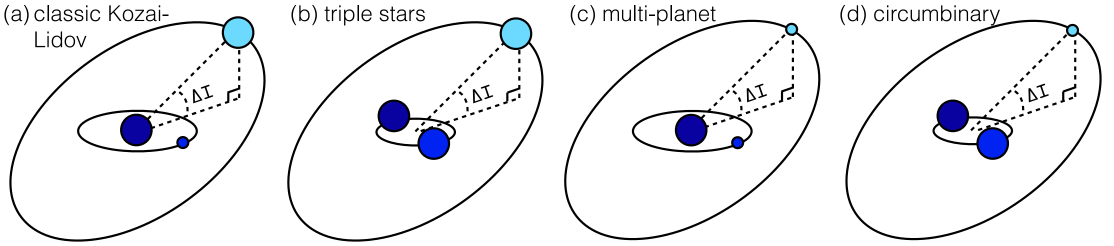

Figure 1. Zoo of different three-body configurations, all with orbits misaligned by ΔI. The orbits are drawn with respect to the centre of mass of the inner orbit. Case (d) is qualitatively different to cases (a)–(c) because in (d) the outer orbit no longer dominates the total angular momentum of the system.

Kozai (1962) are no longer entirely applicable. Furthermore, since the K-L effect only modulates the eccentricity of the inner orbit, not the outer, it is not possible for an inner binary to induce a K-L cycle on a circumbinary planet. Naoz et al. (2013a) and Liu, Mũnoz & Lai (2015) derived equations for the amplitude and limits of the K-L effect for three arbitrary mass bodies, but did not apply them to circumbinary planets. We are motivated to demonstrate the clear and significant difference between circumbinary planets and other three-body systems, and to clear up any possible misconceptions.

The plan of the Letter is as follows: in Section 2, we present equations for the K-L theory, analyse the different limiting cases and consider the competing secular effect of general relativistic precession, which may act to suppress K-L cycles. We then apply the theory in Section 3 to determine what parameter space of circumbinary planets may be able to induce K-L cycles before concluding in Section 4.

2 THEORY

2.1 Equations for the induced eccentricity and critical mutual inclination

Consider a hierarchical triple system like those shown in Fig. 1, which we model as two Keplerian binary orbits. The three bodies have masses M_1 (navy blue in Fig. 1), M_2 (blue) and M_3 (light blue). Both orbits are defined by osculating orbital elements for the period, P, semimajor axis, a, eccentricity, e, argument of periapse, ω, inclination I and longitude of the ascending node, Ω. We use the subscripts 'in' and 'out' to denote each orbit. The mutual inclination between the two orbits, ΔI, is calculated by

$$\cos \Delta I = \sin I_{\rm in} \sin I_{\rm out} \cos \Delta\Omega + \cos I_{\rm in} \cos I_{\rm out}, \quad (1)$$

where $\Delta\Omega = \Omega_{\rm in} - \Omega_{\rm out}$. The orbital angular momenta of the inner binary and outer binaries are

$$G_{\rm in} = L_{\rm in}\sqrt{1-e_{\rm in}^2} \quad \text{and} \quad G_{\rm out} = L_{\rm out}\sqrt{1-e_{\rm out}^2}, \quad (2)$$

where

$$L_{\rm in} = \frac{M_1 M_2}{M_1+M_2}\sqrt{G(M_1+M_2)a_{\rm in}} \quad (3)$$

and

$$L_{\rm out} = \frac{(M_1+M_2)M_3}{M_1+M_2+M_3}\sqrt{G(M_1+M_2+M_3)a_{\rm out}}, \quad (4)$$

and G is the gravitational constant.

We analyse the three-body orbital evolution according to the quadrupole approximation of the Hamiltonian. In this case is no change to $a_{\rm in}$, $a_{\rm out}$ or $e_{\rm out}$.[2] Under certain conditions, there may be a significant variation in the $e_{\rm in}$ and ΔI: a K-L cycle. Outside of K-L cycles $e_{\rm in}$ is constant. Additionally, both orbits will experience an apsidal and nodal precision (variations in ω and Ω, respectively).

The presence or absence of K-L cycles has significant implications for the stability of a three-body system. For circular and coplanar triple systems there exists a rule of thumb that $a_{\rm out} \gtrsim 3a_{\rm in}$ for stable orbits (Dvorak 1986; Holman & Wiegert 1999; Mardling & Aarseth 2001). Around circular binaries, the stability limit generally moves inwards as ΔI increases and is only a weak function of the stellar mass ratio (Doolin & Blundell 2011). If the inner binary is eccentric, for example during a K-L cycle, then this stability limit is pushed outwards. An eccentric binary also makes the stability limit a more complex function of ΔI and the mass ratio (Doolin & Blundell 2011; Li et al. 2014). Seven of the known eight circumbinary systems have $e_{\rm in} \le 0.2$, with *Kepler*-34 being the only highly eccentric case ($e_{\rm in} = 0.5$; Welsh et al. 2012).

Naoz et al. (2013a) and Liu et al. (2015) derived a relation for the maximum inner eccentricity obtained, $e_{\rm in,max}$, as a function of the starting mutual inclination, ΔI_0, the ratio of orbital angular momenta and the outer eccentricity,

$$5\cos^2\Delta I_0 - 3 + \frac{L_{\rm in}}{L_{\rm out}}\frac{\cos\Delta I_0}{\sqrt{1-e_{\rm out}^2}} + \left(\frac{L_{\rm in}}{L_{\rm out}}\right)^2 \frac{e_{\rm in,max}^4}{1-e_{\rm out}^2}$$
$$+ e_{\rm in,max}^2\left[3 + 4\frac{L_{\rm in}}{L_{\rm out}}\frac{\cos\Delta I_0}{\sqrt{1-e_{\rm out}^2}} + \left(\frac{L_{\rm in}}{2L_{\rm out}}\right)^2\frac{1}{1-e_{\rm out}^2}\right] = 0, \quad (5)$$

where it is assumed that the inner binary is initially circular. Solutions for $e_{\rm in,max}$ only exist in a 'K-L active' region, which is bounded by lower and upper limits on the initial ΔI, which we call $\Delta I_{\rm lower}$ and $\Delta I_{\rm upper}$.

The limiting mutual inclination for K-L cycles to occur is calculated by setting $e_{\rm in,max} = 0$ in equation (5), leaving a quadratic

$$5\cos^2\Delta I_{\rm lower,upper} - 3 + \frac{L_{\rm in}}{L_{\rm out}}\frac{\cos\Delta I_{\rm lower,upper}}{\sqrt{1-e_{\rm out}^2}} = 0. \quad (6)$$

From solving the quadratic, the lower limit of ΔI_0 is defined as

$$\cos\Delta I_{\rm lower} = \frac{1}{10}\left[-\frac{L_{\rm in}}{L_{\rm out}}\frac{1}{\sqrt{1-e_{\rm out}^2}} + \sqrt{\left(\frac{L_{\rm in}}{L_{\rm out}}\right)^2\frac{1}{1-e_{\rm out}^2} + 60}\right]. \quad (7)$$

[2] In the octupole level Hamiltonian there may be some variation in $e_{\rm out}$ but generally this is small. An interesting exception, however, was found by Li, Zhou & Zhang (2014), who showed that an eccentric inner binary ($e_{\rm in} \gtrsim 0.4$) may induce moderate eccentricity variations (up to $e_{\rm out} \sim 0.3$) on a massless outer body if it has a near-polar orbit.

The upper limit is simply the other solution to the quadratic,

$$\cos\Delta I_{\rm upper} = \frac{1}{10}\left[-\frac{L_{\rm in}}{L_{\rm out}}\frac{1}{\sqrt{1-e_{\rm out}^2}} - \sqrt{\left(\frac{L_{\rm in}}{L_{\rm out}}\right)^2\frac{1}{1-e_{\rm out}^2}+60}\right]$$

$$\text{for}\quad \frac{L_{\rm in}}{L_{\rm out}}\frac{1}{\sqrt{1-e_{\rm out}^2}} < 2, \qquad (8)$$

but with the caveat that it is only defined when the outer orbit contains the majority of the angular momentum. Otherwise one must invoke a different upper limit, taken from Lidov & Ziglin (1976):

$$\cos\Delta I_{\rm upper} = -\frac{1}{2}\frac{L_{\rm out}}{L_{\rm in}}\sqrt{1-e_{\rm out}^2}$$

$$\text{for}\quad \frac{L_{\rm in}}{L_{\rm out}}\frac{1}{\sqrt{1-e_{\rm out}^2}} \geq 2. \qquad (9)$$

This second upper limit is not inherently respected by equation (5) when calculating $e_{\rm in,max}$ so it must be imposed manually.

Liu et al. (2015) compared numerical simulations with the analytic calculations using equation (5), showing a very close match, but they only used equation (8) as an upper limit on ΔI_0, not equation (9). However, in their tests the outer orbit always had a significant portion of the angular momentum, and hence equation (9) was never applicable.

2.2 Limiting cases

K-L cycles were originally derived in the limit of the inner restricted three-body problem, in which case $M_2 \to 0$ and hence $L_{\rm in}/L_{\rm out} \to 0$. In this case $\cos\Delta I_{\rm lower} \to \sqrt{3/5}$, and for the upper limit we use equation (8) to obtain $\cos\Delta I_{\rm upper} \to -\sqrt{3/5}$. This recovers the classic limiting mutual inclinations for K-L cycles: $\Delta I_{\rm lower} = 39\overset{\circ}{.}23$ and $\Delta I_{\rm upper} = 140\overset{\circ}{.}79$. The calculation for $e_{\rm in,max}$ reduces from equation (5) to the simple well-known formula

$$e_{\rm in,max}\Big|_{L_{\rm in}/L_{\rm out}=0} = \sqrt{1-\frac{5}{3}\cos^2\Delta I_0}, \qquad (10)$$

which is symmetric around $\Delta I_0 = 90°$.

The opposite limiting case is the outer restricted three-body problem, where $M_3 \to 0$, and hence $L_{\rm in}/L_{\rm out} \to \infty$. In this limit $\cos\Delta I_{\rm lower} \to 0$ and hence $\Delta I_{\rm lower} \to 90°$. This means that the prograde K-L active region disappears. For the upper limit, we use equation (9) to see that $\cos\Delta I_{\rm upper} \to 0$ and hence $\Delta I_{\rm upper} \to 90°$, so the retrograde region disappears too. We recover the trivial limit of the outer restricted three-body problem, where there can be no K-L cycles since the binary feels no influence from the orbiting body, matching the work of Pilat-Lohinger et al. (2003), Farago & Laskar (2010) and Doolin & Blundell (2011). The only dynamical effect on the circumbinary planet's orbit is an apsidal and nodal precession, at a rate calculated by Farago & Laskar (2010).

2.3 Competing secular time-scales

Even if a circumbinary system is said to be in a K-L active regime according to Section 2, it is still possible that K-L cycles will not occur. This is because general relativity (GR) induces a competing precession on the inner binary, and out of the two effects the slower one will be suppressed.[3] The K-L time-scale is

$$\tau_{\rm K\text{-}L} = \frac{4}{3}\left(\frac{a_{\rm out}^2}{a_{\rm in}}\right)^{3/2}\sqrt{\frac{M_1+M_2}{GM_3^2}}, \qquad (11)$$

which is taken from Fabrycky & Tremaine (2007) but has been converted to be a function of semimajor axes. The GR time-scale is

$$\tau_{\rm GR} = \frac{2\pi}{3}\frac{a_{\rm in}^{5/2}c^2}{G^{3/2}(M_1+M_2)^{3/2}}, \qquad (12)$$

where c is the speed of light (Fabrycky 2010).

An approximate limit for the suppression of K-L cycles is

$$\left.\frac{\tau_{\rm K\text{-}L}}{\tau_{\rm GR}}\right|_{\rm lim} \sim 1. \qquad (13)$$

This criterion assumes a sharp cut-off of K-L cycles, whereas in reality there is likely a smooth transition between a K-L-dominated regime and a GR-dominated regime. We are also ignoring the possibility of resonances between the two secular effects, which may in fact boost the modulation of $e_{\rm in}$ (Naoz et al. 2013b). Nevertheless, this criterion is deemed sufficient for the purposes of the simple analysis presented in this Letter. Evaluating equation (13) using equations (11) and (12) yields

$$\left.\frac{a_{\rm out}}{a_{\rm in}}\right|_{\rm lim} \sim \left[a_{\rm in}\frac{M_3}{(M_1+M_2)^2}\frac{c^2\pi}{2G}\right]^{1/3}. \qquad (14)$$

This provides an upper limit on the tightness of circumbinary systems for which K-L cycles may be excited, lest they be quenched by general relativistic precession.

3 PARAMETER SPACE EXPLORATION

We explore how the K-L effect behaves in an example triple system consisting of an inner stellar binary with $M_1 = 1\,{\rm M}_\odot$ and $M_2 = 0.5\,{\rm M}_\odot$ and an outer body with M_3 ranging from a star down to a planet. Different values of M_1 and M_2 would change the limits and amplitude of the K-L effect, but in our example we use the mean stellar masses of the known circumbinary systems from *Kepler*, in order to make it as representative as possible. From Section 2.1, the amplitude and limits of K-L cycles are functions of the ratio of angular momentum, and consequently in our tests we only need to probe different values of the ratio $a_{\rm out}/a_{\rm in}$, not the individual values. We start $e_{\rm in}$ and $e_{\rm out}$ at 0, but $e_{\rm in}$ may rise significantly during a K-L cycle.

In Fig. 2, we calculate $e_{\rm in,max}$ from equation (5) as a function of $a_{\rm out}/a_{\rm in}$ and ΔI_0, for six different outer body masses: $1\,{\rm M}_\odot$, $0.1\,{\rm M}_\odot$, $0.05\,{\rm M}_\odot$, $0.01\,{\rm M}_\odot$, $0.005\,{\rm M}_\odot$ and $0.001\,{\rm M}_\odot$. The smallest value of $a_{\rm out}/a_{\rm in}$ was 3, corresponding to the rough stability limit. The lower limit (left-hand limit) of the K-L active region is demarcated in a white dashed curve (equation 7). The two different upper limits (right-hand limits), chosen according to the value of M_3 and $a_{\rm out}/a_{\rm in}$, are depicted as green (equation 8) and pink (equation 9) dashed curves.

The relative time-scales of K-L cycles and GR precession set an upper limit in $a_{\rm out}/a_{\rm in}$ on the K-L active region, as a function of $a_{\rm in}$ (equation 14). In Fig. 2, two light blue horizontal lines denote the limiting ratio $a_{\rm out}/a_{\rm in}$ for $a_{\rm in} = 0.0655$ au ($P_{\rm in} = 5$ d, lower line)

[3] Tidal and rotational bulges also induce a precession on the binary's orbit but these effects are generally only significant for very tight binaries.

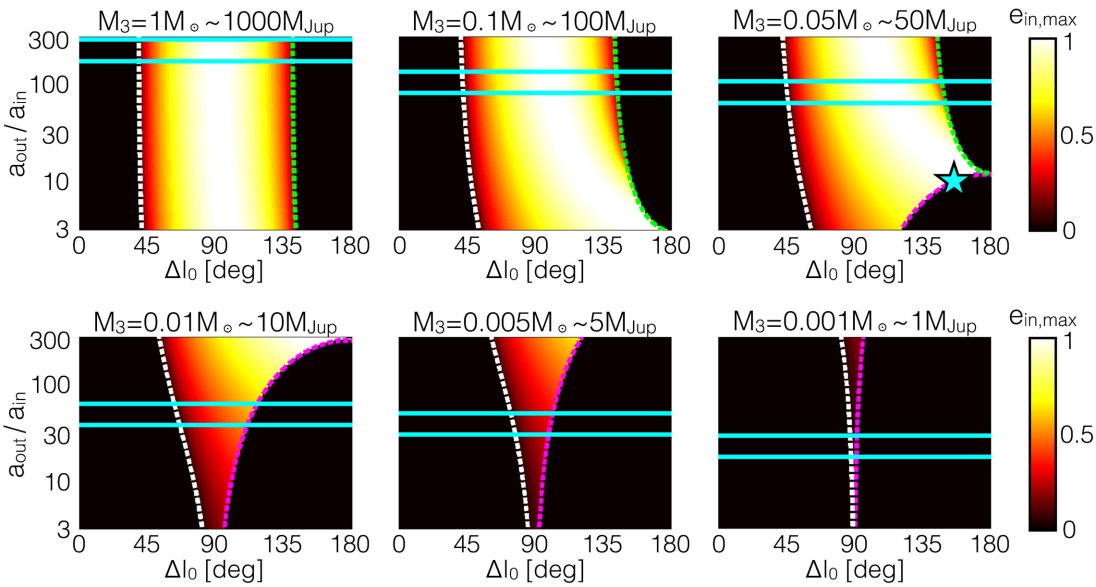

Figure 2. Maximum inner eccentricity obtained during K-L cycles according to equation (5), as a function of $a_{\rm out}/a_{\rm in}$ and ΔI_0, for six different values of the outer mass M_3. The inner masses are set at $M_1 = 1\,{\rm M}_{\odot}$ and $M_2 = 0.5\,{\rm M}_{\odot}$. The initial eccentricities are both zero (and $e_{\rm out}$ is constant). The black regions correspond to where K-L cycles do not occur. These are separated from the K-L active regimes by a lower limit of ΔI_0 (white dashed line on the left-hand limit from equation 7) and an upper limit of ΔI_0 (green and/or pink dashed lines on the right-hand limit from equation (8) and/or equation (9), respectively). The light blue horizontal lines are upper limits on $a_{\rm out}/a_{\rm in}$ for K-L cycles to occur without being quenched by GR precession, for $a_{\rm in} = 0.0655$ au ($P_{\rm in} = 5$ d) as the lower line and $a_{\rm in} = 0.3041$ au ($P_{\rm in} = 50$ d) as the upper line. The light blue star symbol in the top right subplot corresponds to the set of N-body simulations in Fig. 3.

and $a_{\rm in} = 0.3041$ au ($P_{\rm in} = 50$ d, upper line). These two example values of $a_{\rm in}$ are not completely arbitrary as they cover the range of binaries known to host circumbinary planets (Welsh et al. 2014).

For $M_3 = 1\,{\rm M}_{\odot}$ we have a triple star system like in Fig. 1(a), which is well modelled by a classic K-L regime with limits at 39° and 141°. Here $e_{\rm in,max}$ is solely a function of ΔI_0 and is well approximated by equation (10). The GR time-scale only impinges upon the widest systems within this parameter space. As M_3 decreases to $0.1\,{\rm M}_{\odot}$, an M-dwarf, the K-L active regime is relatively unchanged for wide systems but shifts towards the retrograde region for tight systems. This asymmetry was noted by Liu et al. (2015).

Within the brown-dwarf regime ($M_3 = 0.05\,{\rm M}_{\odot}$), there is a significant change in the K-L behaviour for tight triple systems ($a_{\rm out}/a_{\rm in} < 15$). The high-amplitude eccentricity modulations ($e_{\rm in,max} \gtrsim 0.9$) are confined to retrograde orbits. Also the second upper limit (equation 9) becomes applicable, causing the upper limit of the K-L active region to 'turn around' and move back towards 90°.

Within the planetary regime between roughly 1 and 10 $M_{\rm Jup}$, we see that K-L cycles become restricted to a narrow slither of angles either side of a polar $\Delta I_0 = 90°$ orbit. The amplitude also decreases as the outer body becomes smaller. Furthermore, the K-L time-scale is also so slow that GR precession suppresses it in all but the closest systems. For reference, the observed circumbinary planets have been found predominantly near critical stability limit ($a_{\rm out} \sim 3a_{\rm in}$), with low eccentricities ($e_{\rm out} < 0.2$), have mass $M_3 < 1M_{\rm Jup}$ and are nearly coplanar ($\Delta I \lesssim 4°$; Welsh et al. 2014).

One interesting property is that the upper limit of K-L cycles calculated by equation (9) (pink dashed curve in Fig. 2) imposes a sharp transition between K-L active and inactive regions. As an example, for $M_3 = 0.05\,{\rm M}_{\odot}$ and $a_{\rm out}/a_{\rm in} = 10$ there is a sharp transition at $\Delta I_0 = 159°$ (marked with a light blue star symbol in the top right subplot in Fig. 2). We demonstrate this with an N-body numerical solution of the exact equations of motion[4] in Fig. 3, showing the time-evolution of the osculating orbital elements ΔI, $e_{\rm in}$ and $e_{\rm out}$ for three different values of ΔI_0: 157° (light blue), 158° (blue) and 159° (navy blue). The N-body code does not include GR, which is reasonable since the K-L time-scale is significantly faster in this example. There is a sharp transition between K-L active and inactive regions at $\Delta I_0 = 159°$. The maximum numerical value $e_{\rm in}$ matches the analytic prediction to within 0.04 per cent error.

Fig. 3 shows that $e_{\rm out}$ remains close to zero but not exactly. For binaries undergoing a K-L cycle, the outer eccentricity experiences low-amplitude periodic rises, in phase with the K-L cycle of the inner binary. This is not a K-L cycle being induced on the outer body itself, but rather a response to the high-amplitude eccentricity modulation of the orbited binary.

Relaxing the requirement of a circular outer orbit moves the sharp upper limit towards 90°. In Fig. 3 example with $e_{\rm out} = 0.5$ there is a shift in $\Delta I_{\rm upper}$ from 159° to 144°, according to equation (9), which we verified in an N-body simulation. Like before, the simulation showed little variation in $e_{\rm out}$ over time.

4 CONCLUSION

The near-complete absence of potentially destabilizing K-L cycles in the context of circumbinary planets has positive ramifications for the existence of misaligned systems. It frees us from the narrow confines of coplanarity and broadens the parameter space for potential discoveries. Based on studies of the abundance of circumbinary planets, if a presently hidden population of misaligned systems were

[4] REBOUND (Rein & Liu 2012, http://github.com/hannorein/rebound), with a 14th–15th-order adaptive time step integrator (Rein & Spiegel 2015).

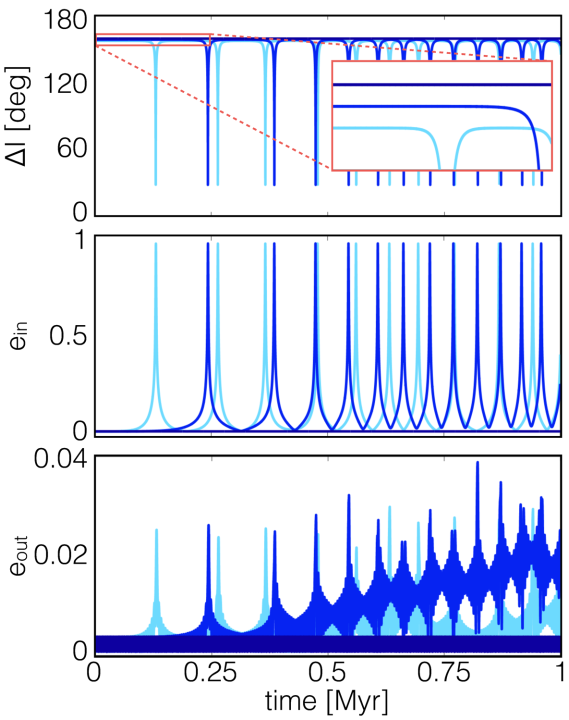

Figure 3. Numerical simulations of K-L cycles, showing the evolution of ΔI (top panel), $e_{\rm in}$ (middle) and $e_{\rm out}$ (bottom). The parameters are $a_{\rm in} = 0.5$ au, $a_{\rm out} = 5$ au, $M_1 = 1\,{\rm M}_{\odot}$, $M_2 = 0.5\,{\rm M}_{\odot}$, $M_3 = 0.05\,{\rm M}_{\odot}$ and $e_{\rm out}$ and $e_{\rm in}$ initially set at zero. Three different values of ΔI_0 were tested: 157° (light blue), 158° (blue) and 159° (navy blue). All other orbital angles were set to zero. The top panel includes a zoomed inset to show the small difference in starting conditions. The numerical values of $e_{\rm in,max}$ are 0.9894 ($\Delta I_0 = 157°$), 0.9899 ($\Delta I_0 = 158°$) and 1.63×10^{-4} ($\Delta I_0 = 159°$), and hence there in the last case there is no K-L cycle. The analytic predictions for $e_{\rm in,max}$ are correct to within 0.04 per cent error.

to exist, this would imply that planets are surprisingly more abundant around two stars than one (Armstrong et al. 2014; Martin & Triaud 2014).

There are also theoretical implications in the context of close binary formation by a combination of K-L cycles and tidal friction. Circumbinary planets cannot activate this mechanism. Circumbinary brown dwarfs may but only in close, retrograde orbits.

Observational evidence of such misaligned systems is presently lacking, but fortunately there exists several methods for their discovery: eclipse timing variations (Borkovits et al. 2011), astrometry (Sahlmann, Triaud & Martin 2015) and sparse transits on both eclipsing and non-eclipsing binaries (Martin & Triaud 2014, 2015).

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