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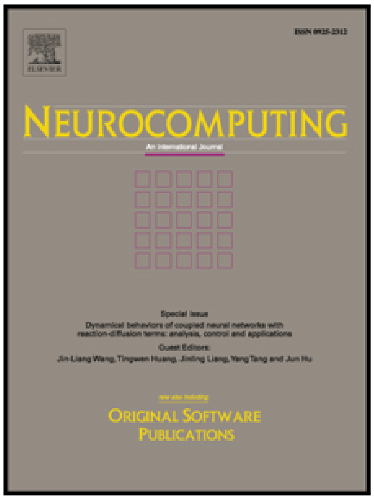

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Recursive Filtering for Communication-Based Train Control Systems with Packet Dropouts

Tao Wen, Lei Zou, Jinling Liang and Clive Roberts

***Abstract*—Accurate information about the train position and velocity is critically important for Communication-based Train Control (CBTC) systems. However, it is practically difficult to obtain the precise information of such information due mainly to the "inaccurate measurements" induced by the measurement noises and the "unreliable communication" caused by the wireless train-ground communication. In this paper, a recursive filtering algorithm is proposed to generate the estimates of the train position and velocity for CBTC systems subject to the measurement noise and packet dropouts. Firstly, the dynamics of a train is modeled based on the Newton's motion equation. Then, a Bernoulli distributed sequence is introduced to describe the packet dropout phenomenon of the wireless communication. The purpose of the problem addressed is to design a recursive filter such that there exists an upper bound for the filtering error covariance. Subsequently, such an upper bound is minimized by properly designing the filter parameter recursively. The desired filter parameter is obtained by solving two Riccati-like difference equations that are of a recursive form suitable for online applications. Finally, an illustrative example is given to show the effective of the proposed filter design scheme.**

***Index Terms*—Recursive filtering; Communication-based Train Control Systems; Packet dropouts; Riccati-like difference equations.**

I. INTRODUCTION

With experiencing fast economic growth, population expansion and urbanisation in worldwide, especially in some major developing counties, the demand for a safer, more efficient and comfortable mass transit system is very urgent [22]. Railway system is a good choice for either urban commuting or intercity transport, as they meet the increasing needs for low emissions and high capacity. In railway systems, it is particularly important to ensure that train control systems could obtain the data from trains including the locations, velocities, identities and other operation information. In early years, railway systems tended to apply the track circuits to realize the communication between trains and the train control center. Such kind of train control systems are known as Track-circuit Based Control (TBTC) systems. TBTC systems would give rise to low detection resolution, which finally leads to long operation headway for trains in order to guarantee there is no possibility for potential collisions. In other words, track-circuit based technology would probably result in low operation efficiency. In recent years, by utilizing modern wireless communication technology, Communication-based Train Control (CBTC) systems have been developed to meet the rapidly increasing demand on efficiency and safety. Compared with the TBTC systems, the CBTC systems could dramatically increase capacity and lower the unreliability in operation.

CBTC systems are automatic train control systems. A typical CBTC system structure is shown in Fig. 1, which contains five subsystems, namely Automatic Train Protection (ATP), Automatic Train Operation (ATO), Automatic Train Supervision (ATS), Computer-based Interlocking (CI) and Data Communication System (DCS). ATP is the most significant subsystem of CBTC systems. The main task of ATP is to trigger the braking when emergencies happen so as to protect the train from dangers. ATO is the driving part of the operation which is utilized to automatically operate trains. ATS is a supervision system managing the railway traffic through commanding the CI. DCS is in charge of exchanging the data flow within each subsystems [15]. Most of the DCSs are distributed systems, which are formed by wayside communication system, onboard communication system and radio communication system respectively. The wayside communication system and onboard communication system are realized through secure wired networks. The radio communication system employs the wireless local area network (WLAN) to exchange data between trains and the wayside access points (APs). Each AP has certain radio coverage. When a train is moving into the coverage of an AP, a bidirectional train-to-wayside wireless connection will be built. When the train is running away the radio coverage of an AP, the connection will be replaced by another one whose radio coverage better covers the current position of the train. Such a phenomenon is referred as the hand-off procedure. In most of the DCSs, the WLAN utilizes the signals with 2.4 GHz industrial, scientific and medical (ISM) band and the IEEE 802.11 family of standards for the media access control (MAC) layer protocol. Obviously, in DCS, APs are very important for the wireless train-ground communication. Continuous data transmissions are carried out between wayside APs and on-board equipment, which enables trains timely receive the moving authorities (MA), speed limit and route data. However, as a radio-based communication

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technology, the communication performance of such a technology is not always perfect. Packet dropouts might happen and result in operational risks in railway systems.

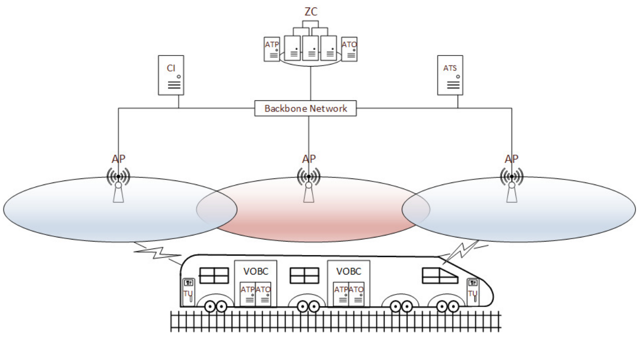

Fig. 1: The typical system structure of CBTC

Due to only a limited spectrum allocated in WLANs and unlicensed ISM band used as the working frequency, the co-channel interference could be the biggest threat for packet dropouts. A main resource of co-channel interference comes from the unwell-planned AP deployment in the DCS. A number of research works have been done on exploring how to decrease the packet dropout rate by planning a more reasonable AP deployment [23]. For example, in [4], a novel AP placement planning method in railway context has been developed. In [24], [16] and [8], various AP placement planning strategies focusing on indoor Code Division Multiple Access (CDMA) networks have been discussed. In [9], [10] and [12], the planning strategies focusing on WLANs are proposed. To further reduce the affection caused by the co-channel interference, channel assignment in WLNAs is discussed in [2], [3], [21] and [13], in which channel assignment has been considered alongside the AP deployment aiming to further improve the DCS system performance in terms of decreasing the packet dropout rate. In addition to co-channel interference, hand-off could be regarded as another important reason of packet dropouts in DCS. In [28] and [7], improved hand-off schemes have been proposed, which have been designed to minimize the packet dropouts when hand-off phenomena happens. However, the aforementioned research results could not eliminate packet dropouts in DCS due to the wireless communication nature, thereby affecting the reliability and Quality of Service (QoS) of CBTC systems. Obviously, packet dropouts would greatly reduce the precision of the train data (e.g. location and velocity) obtained by the Zone Controller (ZC). On the other hand, it is worth mentioning that the train location for most of CBTC systems is measured by the combination of balises and axle counting based speed odometers. In CBTC systems, balises are intermittently placed on the track. When a train runs in the interval between two adjacent balises, train data will be measured by counting the rotations of wheel axel, and when the train passes a balise, the train data will be adjusted. However, due to the unavoidable wheel slip, the estimated train data might be polluted by the measurement noise before the adjustment is made. Accordingly, it is practically difficult to achieve the exact train data under the influence of "inaccurate measurements" and "unreliable communication".

In order to achieve accurate information of the train data, in this paper, a recursive filtering algorithm is developed to generate the estimate of the train data based on the received measurements. The filtering (or state estimation) problems have long been fundamental issues in control and signal processing fields, whose purpose is to derive an estimate of the internal state for a given system based on the obtained measurements. So far, various filtering methods have been reported in the literature (e.g. Kalman filtering [19], Extended Kalman filtering [5], $\mathcal{H}_\infty$ filtering [18], [29], set-membership filtering [30]). Considerable effort has been devoted to the filtering problems with different conditions and performance requirements. For instance, in [17], the Kalman filtering problem with intermittent observations has been illustrated where the packet dropouts have been modeled by the general finite state Markov channel (FSMC). A novel hybrid filtering algorithm has been developed in [6] to deal with the state estimation problem for power systems where the signal is obtained from the phasor measurement units. In [25], the robust $\mathcal{H}_2$ and $\mathcal{H}_\infty$ filtering problems for linear discrete-time systems with polytopic parameter uncertainty has been studied based on a parameter-dependent Lyapunov function approach.

In this paper, we aims to develop a recursive filtering algorithm for the CBTC systems where the measurements of the train might be corrupted by noises and experience packet dropouts in the wireless communication. This is a nontrivial problem because of two challenges identified as follows: 1) how to develop an accurate dynamic model accounting for the CBTC system with packet dropouts? 2) how to develop appropriate methodology to design the recursive filter for the CBTC system with the consideration of the model description and packet dropouts? It is, therefore, the main purpose of this paper to offer satisfactory answers to the two questions. *The primary contributions of this paper are highlighted as follows. 1) The recursive filtering problem is, for the first time, investigated for CBTC systems with packet dropouts. 2) The design procedure of the filter gain is implemented in a recursive manner which is suitable for online applications.*

The rest of this paper is organized as follows. In Section II, the mathematical description of the CBTC systems with packet dropouts is proposed and the corresponding recursive filtering problem is introduced. In Section III, the desired filter gain matrix is recursively computed based on two Riccati-like difference equations. Moreover, a numerical simulation example is given in Section IV to demonstrate the effectiveness of the main results. Finally, we conclude the paper in Section V.

Notations: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The notation $X \geq Y$ ($X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite). $\mathrm{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, denote the expectation of the stochastic variable x and expectation of x conditional on y. 0 represents the zero matrix of compatible dimension. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. The shorthand $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\|x\|$ refers to the Euclidean norm of a vector x. $\|A\|$ denotes the spectral norm of the matrix A. $M^T \in \mathbb{R}^{n\times m}$ represent the transpose of the matrix $M \in \mathbb{R}^{m\times n}$.

II. Problem Formulation and Preliminaries

A. The system model

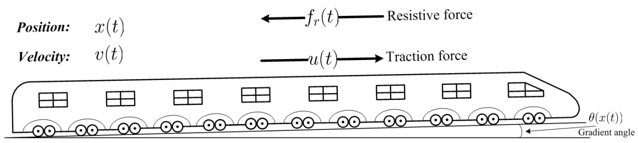

Fig. 2: Diagram of the train

As shown in [14], the dynamics of a train could be described by the following Newton's motion equations:

$$\begin{cases} \dfrac{dx(t)}{dt} = v(t) \\ M_{tr}\dfrac{dv(t)}{dt} = u(t) - f_r(t) - F_{grad}(t) \end{cases} \tag{1}$$

where $x(t)$, $v(t)$, $u(t)$ and $f_r(t)$ represent, respectively, the position, the velocity, the traction force and the resistive force of the train. M_{tr} is the effective mass of the train. $F_{grad}(t)$ is the force due to the gradient.

The force due to the gradient shows the effect of the gradient profile and gravity acceleration. In an uphill situation, the train receives a negative gravity component acceleration against the moving direction, while in a downhill situation the train receives a positive gravity component acceleration, as shown in Fig. 2. Such a force can be computed as follows:

$$F_{grad}(t) = Mg\sin(\theta(x(t))) \tag{2}$$

where $\theta(x(t))$denotes the gradient angle of the track which is dependent on the location of the train. For easy notation, we denote $\bar{\theta}(t) = \theta(x(t))$.

Remark 1: In real applications, the gradient angle of the track could always be measured in advance. Such an angle varies from one place to another and could be easily detected by track mounted balises. For technical convenience, the force due to the gradient is rewritten as follows: $F_{grad}(t) = Mg\sin(\bar{\theta}(t))$. Obviously, such a force is a known variable which is always modeled by a piecewise constant function in most of the situations.

The moment of inertia (e.g. the rotational inertia) is a measure of a body's resistance to angular acceleration, which would increase with the accelerated train mass and be transformed by gear ratio and wheel diameter [20]. For the purpose of improving the accuracy of the plant modeling, the moment of inertia should be taken into consideration in the derivation of the effective mass by using a rotary allowance. As such, the effective mass could be calculated as follows:

$$M_{tr} = (1+\lambda)M \tag{3}$$

where M is the total mass of the train (i.e. the tare mass plus passenger mass). λ denotes the rotary allowance.

Denoting $\vec{x}(t) = \begin{bmatrix} x^T(t) & v^T(t)\end{bmatrix}^T$, the dynamics of the train can be rewritten as follows:

$$\frac{d\vec{x}(t)}{dt} = \vec{A}\vec{x}(t) + \vec{B}u(t) + \vec{E}\big(f_r(t) + F_{grad}(t)\big) \tag{4}$$

where

$$\vec{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 0 \\ \frac{1}{(1+\lambda)M} \end{bmatrix}, \quad \vec{E} = \begin{bmatrix} 0 \\ -\frac{1}{(1+\lambda)M} \end{bmatrix}.$$

In this paper, it is assumed that both the position and velocity could be easily obtained via certain sensors equipped in the train with a constant sampling period T. The measurement output of the system (4) is given by

$$\vec{y}(kT) = \tilde{C}\vec{x}(kT) + \nu(kT) \tag{5}$$

where $\vec{y}(kT)$ and $\nu(kT)$ ($k \in \mathbb{N}^+$) denote, respectively, the measurement output and the measurement noise of the k-th sampling instant. $\tilde{C} = \text{diag}\{c_1, c_2\}$ is the weight matrix of the measurement data. The constants c_1 and c_2 represent the weights of the position and velocity, respectively.

Remark 2: The measurement output is obtained from the position sensors and velocity sensors located in the train. In this paper, the units of position $x(t)$ and velocity $v(t)$ are chosen as "meter" and "meter per second" according to the SI units (international system of units), respectively. Nevertheless, the units for measuring the position and velocity in sensors might be different from the SI units. In this case, the weight matrix $\tilde{C}$ is introduced to cope with the conversion between different units.

Suppose that the traction force, the resistive force and the gradient remain unchanged between two adjacent sampling instants, i.e. $u(t) = u(kT)$, $f_r(t) = f_r(kT)$ and $\bar{\theta}(t) = \bar{\theta}(kT)$ for $kT \le t < (k+1)T$. Then, the discrete-time model of system (4) can be characterized as follows:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_1\tilde{x}(k) + \tilde{B}\tilde{u}(k) + \tilde{E}\big(\tilde{f}_r(k) + \tilde{F}_{grad}(k)\big) \\ \quad\tilde{y}(k) = \tilde{C}\tilde{x}(k) + \tilde{\nu}(k) \end{cases} \tag{6}$$

where

$$\begin{aligned} &\tilde{x}(k) = \vec{x}(kT), \quad \tilde{u}(k) = u(kT), \quad \tilde{f}_r(k) = f_r(kT), \\ &\tilde{F}_{grad}(k) = F_{grad}(kT), \quad \tilde{\nu}(k) = \nu(kT), \quad \tilde{y}(k) = \vec{y}(kT), \\ &\tilde{A}_1 = e^{\vec{A}T}, \quad \tilde{B} = \int_0^T e^{\vec{A}t}dt\vec{B}, \quad \tilde{E} = \int_0^T e^{\vec{A}t}dt\vec{E}. \end{aligned}$$

The resistive force $\tilde{f}_r(k)$ exists consistently during the train operation. Generally speaking, the resistive force can be expressed as the sum of the ramp resistance, aerodynamic drag and rolling mechanical resistance. According to the resistance model in [27], the resistive force $\tilde{f}_r(k)$ could be described as

$$\tilde{f}_r(k) = (a_0 + a_1v(kT))M + a_2Mv^2(kT) + \tilde{\omega}(k) \tag{7}$$

where the coefficients a_0, a_1 and a_2 can be obtained by experimental test. $\tilde{\omega}(k)$ is the unexpected wind force. It is assumed that $\tilde{\omega}^T(k)\tilde{\omega}(k) \le q_k$.

Note that the speed of the train could not exceed the speed limit for the purpose of safe operations. Hence, we assume that the velocity of the train belongs to a given set, i.e. $v(kT) \in$

$[0, v_{\max}]$. Then, the model of the resistive force (7) can be reformulated as follows:

$$\begin{aligned}\tilde{f}_r(k) =& (a_0 + a_1 v(kT))M + a_2 M\big(\frac{v_{\max}}{2} + \Delta v(k)\big)v(kT)\\ &+ \tilde{\omega}(k)\end{aligned} \tag{8}$$

with $|\Delta v(k)| \leq \frac{v_{\max}}{2}$. Therefore, the vector $\tilde{f}_r(k)$ can be reformulated as follows:

$$\tilde{f}_r(k) = \big(\tilde{A}_2 + \tilde{F}\Delta\tilde{A}(k)\big)\tilde{x}(k) + \tilde{\omega}(k) + \vec{f} \tag{9}$$

where

$$\tilde{A}_2 = \begin{bmatrix}0 & a_1 M + \frac{a_2 M v_{\max}}{2}\end{bmatrix}, \quad \vec{f} = a_0 M, \quad \tilde{F} = \frac{a_2 M v_{\max}}{2},$$
$$\Delta\tilde{A}(k) = \begin{bmatrix}0 & \frac{\Delta v(k)}{0.5 v_{\max}}\end{bmatrix}.$$

Obviously, the uncertainty $\Delta\tilde{A}(k)$ satisfies $\Delta\tilde{A}(k)\Delta\tilde{A}^T(k) \leq I$. Substituting (9) into (6), the system dynamics can be transformed into the following easy-to-handle formulation:

$$\begin{cases}\tilde{x}(k+1) = \big(\tilde{A} + \tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)\tilde{x}(k) + \tilde{B}\tilde{u}(k) + \tilde{E}\big(\vec{f}\\ \qquad\qquad + \tilde{F}_{grad}(k)\big) + \tilde{E}\tilde{\omega}(k)\\ \tilde{y}(k) = \tilde{C}\tilde{x}(k) + \tilde{\nu}(k)\end{cases} \tag{10}$$

where $\tilde{A} = \tilde{A}_1 + \tilde{E}\tilde{A}_2$.

The initial state $\tilde{x}(0)$ and the measurement noise $\tilde{\nu}(k)$ are mutually uncorrelated and have the following statistical properties:

$$\mathbb{E}\{\tilde{x}(0)\} = \bar{x}_0, \quad \mathbb{E}\{(\tilde{x}(0) - \bar{x}_0)(\tilde{x}(0) - \bar{x}_0)^T\} = P_{0|0},$$
$$\mathbb{E}\{\tilde{\nu}(k)\} = 0, \quad \mathbb{E}\{\tilde{\nu}(k)\tilde{\nu}^T(k)\} = R_k,$$

where $P_{0|0} > 0$ and $R_k > 0$ are known matrices with appropriate dimensions.

B. Structure of the filter

Before we introduce the structure of the filter, let us first consider the data transmission via the train-ground communication. In this paper, the communication between each wayside AP and trains in its coverage area is scheduled by the carrier sense multiple access with collision avoidance (CSMA/CA) protocol. Furthermore, an automatic repeat request (ARQ) scheme is utilized in the CSMA/CA protocol at the medium-access control layer of IEEE 802.11. As shown in [1], the packet dropout (missing measurements) phenomenon would occur in such a communication scheme. As such, the transmission model of the system is given by

$$\bar{y}(k) = \alpha(k)\tilde{y}(k) \tag{11}$$

where $\bar{y}(k)$ denotes the measurement signal received by the ZC via WLANs and $\alpha(k) \in \{0, 1\}$ is a Bernoulli distributed stochastic variable indicating whether the packet dropout phenomenon occurs at time instant k. Furthermore, we assume that $\alpha(k)$ is independent of the noise and initial state.

Assume the maximum number of retransmission times of the CSMA/CA protocol is r, and the frame error rate (FER) of the communication channel is p. Then, the probability distribution law of $\alpha(k)$ can be given by

$$\begin{cases}\text{Prob}\{\alpha(k) = 1\} = \bar{\alpha} \triangleq \sum_{j=0}^{r-1} p^j(1-p)\\ \text{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha}\end{cases} \tag{12}$$

Based on the system dynamics (10) and the transmission model (11), the recursive filter to be designed is of the following form:

$$\begin{cases}\hat{x}(k+1|k) = \tilde{A}\hat{x}(k|k) + \tilde{B}\tilde{u}(k) + \tilde{E}\big(\vec{f} + \tilde{F}_{grad}(k)\big)\\ \hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\big(\bar{y}(k+1)\\ \qquad\qquad - \alpha(k+1)\tilde{C}\hat{x}(k+1|k)\big)\end{cases} \tag{13}$$

where $\hat{x}(k|k)$ is the estimate of $\tilde{x}(k)$ at time instant k with $\hat{x}(0|0) = \bar{x}_0$, $\hat{x}(k+1|k)$ is the one-step prediction at time instant k, $K(k+1)$ is the filter gain to be determined.

Remark 3: It is worth mentioning that the identity information of each train could be transmitted to the filter simultaneously in the transmission. As such, the value of the stochastic variable $\alpha(k)$ is available to the filter.

Given the maximum duration $k_{\max}$ for the CBTC system (i.e. the maximum duration for a train covering the given distance), the objective of this paper is to design a recursive filter of the structure (13) such that, for all packet dropout phenomenon, an upper bound of the filtering error covariance is guaranteed, that is, there exists a family of positive-definite matrices $\Sigma(k+1|k+1)$ $(0 \leq k \leq k_{\max})$ satisfying

$$\begin{aligned}&\mathbb{E}\big\{(\tilde{x}(k+1) - \hat{x}(k+1|k+1))(\tilde{x}(k+1)\\ &- \hat{x}(k+1|k+1))^T\big\} \leq \Sigma(k+1|k+1).\end{aligned} \tag{14}$$

Moreover, the designed filter gain $K(k+1)$ is expected to minimize the trace of the matrix $\Sigma(k+1|k+1)$ through a recursive scheme.

Remark 4: In this paper, we aim to develop a recursive filtering algorithm for the CBTC system with packet dropouts. Obviously, the recursive filtering algorithm is implement in a discrete-time manner. For ease of analysis on the filtering performance, we transform the continuous-time system (4) into the discrete-time counterpart (6). Based on the assumption that the traction force, the resistive force and the gradient remain unchanged between two adjacent sampling instants, i.e. $u(t) = u(kT)$, $f_r(t) = f_r(kT)$ and $\bar{\theta}(t) = \bar{\theta}(kT)$ for $kT \leq t < (k+1)T$, the discrete-time system (6) is the exact discretization of the continuous-time system (4). At each sampling instant kT, the discrete-time system (6) could exactly derive the same value of (4), which implies that the results obtained for (6) still valid for the original system (4) (i.e. $\mathbb{E}\big\{(\vec{x}((k+1)T) - \hat{x}(k+1|k+1))(\vec{x}((k+1)T) - \hat{x}(k+1|k+1))^T\big\} \leq \Sigma(k+1|k+1)$ and the designed gain $K(k+1)$ could minimize the trace of $\Sigma(k+1|k+1)$).

III. Main results

In this section, we shall develop a unified framework to deal with the addressed filtering problem in the simultaneous presence of parameter uncertainties and packet dropout phenomenon. Before proceeding further, we are in a position to introduce the following lemmas which will be used in subsequent developments.

Lemma 1: [26] Given matrices A, H, E and F with appropriate dimensions such that $FF^T \leq I$. Let X be a symmetric positive definite matrix and γ be an arbitrary positive constant such that $\gamma^{-1}I - EXE^T > 0$. Then the following inequality holds

$$\begin{aligned}&(A+HFE)X(A+HFE)^T\\ &\leq A(X^{-1}-\gamma E^TE)^{-1}A^T+\gamma^{-1}HH^T. \end{aligned}\tag{15}$$

Lemma 2: For $0 \leq k \leq N$, suppose that $X = X^T > 0$, $\mathcal{S}_k(X) = \mathcal{S}_k^T(X) \in \mathbb{R}^{n\times n}$. If

$$\mathcal{S}_k(Y) \geq \mathcal{S}_k(X), \quad \forall X \leq Y = Y^T \tag{16}$$

then the solutions M_k and N_k to the following difference equations

$$M_{k+1} \leq \mathcal{S}_k(M_k), \quad N_{k+1} = \mathcal{S}_k(N_k), \quad M_0 = N_0 > 0 \tag{17}$$

satisfy $M_k \leq N_k$.

Proof: The proof of this lemma is performed by mathematical induction, which is divided into two steps, namely, the initial step and the inductive step.

Initial step. For $k = 0$, it can be immediately known from the condition $M_0 = N_0$ that $M_k \leq N_k$ is satisfied for $k = 0$.

Inductive step. Now that the assertion of this lemma is true for $k = 0$. Next, given that the assertion is true for $k = t$ (i.e. $M_t \leq N_t$), we aim to show that the same assertion is true for $k = t+1$ (i.e. $M_{t+1} \leq N_{t+1}$). Obviously, it follows from (16) and (17) that

$$M_{t+1} \leq \mathcal{S}_t(M_t) \leq \mathcal{S}_t(N_t) = N_{t+1}.$$

Hence, by the induction, it can be concluded that the assertion of this lemma is true for $k \geq 0$. The proof is complete. ∎

Lemma 3: [11] For matrices M, N, X and P with appropriate dimensions, the following equalities hold:

$$\begin{aligned}&\frac{\partial \mathrm{tr}\{MXN\}}{\partial X} = M^TN^T, \ \frac{\partial \mathrm{tr}\{MX^TN\}}{\partial X} = NM,\\ &\frac{\partial \mathrm{tr}\{MXNX^TL\}}{\partial X} = M^TL^TXN^T + LMXN.\end{aligned}$$

In what follows, let us consider the dynamics of the filtering error. Define the one-step prediction error as $e(k+1|k) = \tilde{x}(k+1) - \hat{x}(k+1|k)$ and the filtering error as $e(k+1|k+1) = \tilde{x}(k+1) - \hat{x}(k+1|k+1)$. Subtracting (13) from (6), we have

$$\begin{cases} e(k+1|k) = \tilde{A}e(k|k) + \tilde{E}\tilde{F}\Delta\tilde{A}(k)\tilde{x}(k) + \tilde{E}\tilde{\omega}(k)\\ e(k+1|k+1) = \big(I - \alpha(k+1)K(k+1)\tilde{C}\big)e(k+1|k)\\ \qquad\qquad - \alpha(k+1)K(k+1)\tilde{\nu}(k+1)\end{cases}\tag{18}$$

Subsequently, the following theorem can be easily accessible and therefore we omit the proof for conciseness.

Theorem 1: Let $P_{k+1|k+1} = \mathbb{E}\{e(k+1|k)e(k+1|k)^T\}$ and $P_{k+1|k} = \mathbb{E}\{e(k+1|k)e(k+1|k)^T\}$. Then, the value of $P_{k+1|k}$ could be given by

$$\begin{aligned}P_{k+1|k} =& \tilde{A}P_{k|k}\tilde{A}^T + \tilde{A}\mathbb{E}\{e(k|k)\tilde{x}^T(k)\}\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\\ &+ \tilde{E}\tilde{F}\Delta\tilde{A}(k)\mathbb{E}\{\tilde{x}(k)e^T(k|k)\}\tilde{A}^T + \tilde{E}Q_k\tilde{E}^T\\ &+ \tilde{E}\tilde{F}\Delta\tilde{A}(k)\mathbb{E}\{\tilde{x}(k)\tilde{x}^T(k)\}\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T.\end{aligned}\tag{19}$$

where $Q_k = \tilde{\omega}(k)\tilde{\omega}^T(k)$. Furthermore, the second-moment matrix $P_{k+1|k+1}$ satisfies

$$\begin{aligned}P_{k+1|k+1} =& \big(I - \bar{\alpha}K(k+1)\tilde{C}\big)P_{k+1|k}\big(I - \bar{\alpha}K(k+1)\tilde{C}\big)^T\\ &+ \bar{\alpha}(1-\bar{\alpha})K(k+1)\tilde{C}P_{k+1|k}\tilde{C}^TK^T(k+1)\\ &+ \bar{\alpha}K(k+1)R_{k+1}K^T(k+1).\end{aligned}\tag{20}$$

Remark 5: In Theorem 1, the recursive form of the second-moment matrix $P_{k+1|k+1}$ has been established which contains all the information contributes to the system complexities (e.g. the system parameters, the parameter uncertainty and the system noise). However, due to the simultaneous consideration of parameter uncertainty and packet dropout phenomenon, (19) and (20) are contaminated by some unknown terms such as $\mathbb{E}\{e(k|k)\tilde{x}^T(k)\}$, $\Delta\tilde{A}(k)$ and $\mathbb{E}\{\tilde{x}(k)\tilde{x}^T(k)\}$, which lead to essential difficulty in determining the accurate value of the second-moment matrix $P_{k+1|k+1}$. In the following, we shall present an alternatively way to design an appropriate filter parameter $K(k+1)$, such that an upper bound of $P_{k+1|k+1}$ is guaranteed.

Theorem 2: Consider the second-moment matrices $P_{k+1|k}$ and $P_{k+1|k+1}$ in (19)-(20), respectively. Let γ_k, ε_j $(j=1,2)$ be positive scalars. If the following two discrete-time Riccati-like difference equations:

$$\begin{aligned}\Sigma_{k+1|k}(\Sigma_{k|k}) =& q_k\tilde{E}\tilde{E}^T + (1+\varepsilon)\Big(\tilde{A}\big(\Sigma_{k|k}^{-1} - \gamma_kI\big)^{-1}\tilde{A}^T\\ &+ \gamma_k^{-1}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\Big) + (1+\varepsilon^{-1})\bar{\lambda}_{k|k}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\end{aligned}\tag{21}$$

$$\begin{aligned}\Sigma_{k+1|k+1}(\Sigma_{k+1|k}) =& \big(I - \bar{\alpha}K(k+1)\tilde{C}\big)\Sigma_{k+1|k}\big(I\\ &- \bar{\alpha}K(k+1)\tilde{C}\big)^T + \bar{\alpha}K(k+1)R_{k+1}K^T(k+1)\\ &+ \bar{\alpha}(1-\bar{\alpha})K(k+1)\tilde{C}\Sigma_{k+1|k}\tilde{C}^TK^T(k+1)\end{aligned}\tag{22}$$

with the initial condition $\Sigma_{0|0} = P_{0|0} > 0$ have positive definite solutions $\Sigma_{k+1|k}$ and $\Sigma_{k+1|k+1}$ where $\bar{\lambda}_{k|k} = \lambda_{\max}\{\hat{x}(k|k)\hat{x}^T(k|k)\}$ such that, for all $0 \leq k \leq k_{\max}$, the following constraint

$$\gamma_k^{-1}I - \Sigma_{k|k} > 0 \tag{23}$$

is satisfied, then with the filter gain K_{k+1} given by

$$K(k+1) = \bar{\alpha}\Sigma_{k+1|k}\tilde{C}^T\big(R_{k+1} + \tilde{C}\Sigma_{k+1|k}\tilde{C}^T\big)^{-1} \tag{24}$$

the matrix $\Sigma_{k+1|k+1}$ is an upper bound for $P_{k+1|k+1}$, i.e., $P_{k+1|k+1} \leq \Sigma_{k+1|k+1}$. Moreover, the filter gain K_{k+1} given by (24) minimizes the upper bound $\Sigma_{k+1|k+1}$.

Proof: To begin with, based on (19), the second-moment matrix $P_{k+1|k}$ could be rewritten as follows:

$$\begin{aligned}P_{k+1|k}(P_{k|k}) =& \tilde{A}P_{k|k}\tilde{A}^T + \tilde{A}\mathbb{E}\{e(k|k)(e(k|k)+\hat{x}(k|k))^T\}\\ &\times\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T + \tilde{E}\tilde{F}\Delta\tilde{A}(k)\mathbb{E}\{(e(k|k)+\hat{x}(k|k))\\ &\times e^T(k|k)\}\tilde{A}^T + \tilde{E}\tilde{F}\Delta\tilde{A}(k)\times\mathbb{E}\{(e(k|k)+\hat{x}(k|k))\\ &\times(e(k|k)+\hat{x}(k|k))^T\}\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T + \tilde{E}Q_k\tilde{E}^T\\ =& \tilde{A}P_{k|k}\tilde{A}^T + \tilde{A}\big(P_{k|k} + \mathbb{E}\{e(k|k)\hat{x}^T(k|k)\}\big)\Delta\tilde{A}^T(k)\tilde{F}^T\\ &\times\tilde{E}^T + \tilde{E}\tilde{F}\Delta\tilde{A}(k)\big(P_{k|k} + \mathbb{E}\{\hat{x}(k|k)e^T(k|k)\}\big)\tilde{A}^T\end{aligned}$$

$$\begin{aligned}&+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big(P_{k|k}+\mathbb{E}\{e(k|k)\hat{x}^T(k|k)\\&+\mathbb{E}\{\hat{x}(k|k)e^T(k|k)\}+\hat{x}(k|k)\hat{x}^T(k|k)\}\big)\Delta\tilde{A}^T(k)\\&\times\tilde{F}^T\tilde{E}^T+\tilde{E}Q_k\tilde{E}^T\\=&\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)P_{k|k}\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)^T+\big(\tilde{A}\\&+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)\mathbb{E}\{e(k|k)\hat{x}^T(k|k)\}\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\\&+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\mathbb{E}\{\hat{x}(k|k)e^T(k|k)\}\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)^T\\&+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\hat{x}(k|k)\hat{x}^T(k|k)\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\\&+\tilde{E}Q_k\tilde{E}^T\end{aligned}\tag{25}$$

where $Q_k=\tilde{\omega}(k)\tilde{\omega}^T(k)$.

Notice that the following elementary inequality

$$\Big(\varepsilon^{0.5}a-\varepsilon^{-0.5}b\Big)\Big(\varepsilon^{0.5}a-\varepsilon^{-0.5}b\Big)^T\geq 0\tag{26}$$

yields

$$ab^T+ba^T\leq\varepsilon aa^T+\varepsilon^{-1}bb^T\tag{27}$$

where $\varepsilon>0$ is a scalar. Then, it can be concluded that

$$\begin{aligned}&\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)\mathbb{E}\{e(k|k)\hat{x}^T(k|k)\}\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\\&+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\mathbb{E}\{\hat{x}(k|k)e^T(k|k)\}\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)^T\\\leq&\,\varepsilon\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)P_{k|k}\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)^T\\&+\varepsilon^{-1}\tilde{E}\tilde{F}\Delta\tilde{A}(k)\hat{x}(k|k)\hat{x}^T(k|k)\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\end{aligned}\tag{28}$$

Based on (28) and Lemma 1, we have the following inequality:

$$\begin{aligned}&P_{k+1|k}(P_{k|k})\\&\leq(1+\varepsilon)\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)P_{k|k}\big(\tilde{A}+\tilde{E}\tilde{F}\Delta\tilde{A}(k)\big)^T\\&\quad+(1+\varepsilon^{-1})\tilde{E}\tilde{F}\Delta\tilde{A}(k)\hat{x}(k|k)\hat{x}^T(k|k)\Delta\tilde{A}^T(k)\tilde{F}^T\tilde{E}^T\\&\quad+\tilde{E}Q_k\tilde{E}^T\\&\leq(1+\varepsilon)\Big(\tilde{A}\big(P_{k|k}^{-1}-\gamma_kI\big)^{-1}\tilde{A}^T+\gamma_k^{-1}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\Big)\\&\quad+(1+\varepsilon^{-1})\bar{\lambda}_{k|k}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T+\tilde{E}Q_k\tilde{E}^T.\end{aligned}\tag{29}$$

Noticing that $Q_k=\tilde{\omega}(k)\tilde{\omega}^T(k)\leq\mathrm{tr}\{\tilde{\omega}(k)\tilde{\omega}^T(k)\}I\leq\tilde{\omega}^T(k)\tilde{\omega}(k)I\leq q_kI$, we have

$$\begin{aligned}&P_{k+1|k}(P_{k|k})\\&\leq(1+\varepsilon)\Big(\tilde{A}\big(P_{k|k}^{-1}-\gamma_kI\big)^{-1}\tilde{A}^T+\gamma_k^{-1}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\Big)\\&\quad+(1+\varepsilon^{-1})\bar{\lambda}_{k|k}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T+q_k\tilde{E}\tilde{E}^T.\end{aligned}\tag{30}$$

Next, according to (29), we continue to rewrite $\Sigma_{k+1|k}$ as the function of $\Sigma_{k|k}$ as follows:

$$\begin{aligned}\Sigma_{k+1|k}(\Sigma_{k|k})=&\,q_k\tilde{E}\tilde{E}^T+(1+\varepsilon)\Big(\tilde{A}\big(\Sigma_{k|k}^{-1}-\gamma_kI\big)^{-1}\tilde{A}^T\\&+\gamma_k^{-1}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\Big)+(1+\varepsilon^{-1})\bar{\lambda}_{k|k}\tilde{E}\tilde{F}\tilde{F}^T\tilde{E}^T\end{aligned}\tag{31}$$

Obviously, the matrix functions $\Sigma_{k+1|k}(\cdot)$ and $\Sigma_{k+1|k+1}(\cdot)$ satisfies

$$\begin{aligned}\Sigma_{k+1|k}(Y)\geq\Sigma_{k+1|k}(X),\quad&\forall X^T=X\leq Y=Y^T\\\Sigma_{k+1|k+1}(\bar{Y})\geq\Sigma_{k+1|k+1}(\bar{X}),\quad&\forall\bar{X}^T=\bar{X}\leq\bar{Y}=\bar{Y}^T\end{aligned}$$

which implies that for all $X^T=X\leq Y=Y^T$, we have

$$\Sigma_{k+1|k+1}\big(\Sigma_{k+1|k}(Y)\big)\geq\Sigma_{k+1|k+1}\big(\Sigma_{k+1|k}(X)\big).$$

By applying Lemma 2, it is easy to see that $P_{k+1|k+1}\leq\Sigma_{k+1|k+1}$.

Next, we are ready to show that the filter gain given by (24) is the optimal in the sense that it minimizes the upper bound $\Sigma_{k+1|k+1}$. Taking the partial derivative of $\Sigma_{k+1|k+1}$ with respect to K_{k+1} and letting the derivative be zero, we have

$$\begin{aligned}&\frac{\partial\mathrm{tr}\{\Sigma_{k+1|k+1}\}}{\partial K_{k+1}}\\&=-2\bar{\alpha}\Sigma_{k+1|k}\tilde{C}^T+2K_{k+1}(R_{k+1}+\tilde{C}\Sigma_{k+1|k}\tilde{C}^T)=0\end{aligned}\tag{32}$$

Based on the above equation, the optimal filter gain K_{k+1} can be determined as

$$K(k+1)=\bar{\alpha}\Sigma_{k+1|k}\tilde{C}^T\big(R_{k+1}+\tilde{C}\Sigma_{k+1|k}\tilde{C}^T\big)^{-1}\tag{33}$$

which is identical to (24). It is clear that the filter gain given by (33) is optimal that minimizes the upper bound $\Sigma_{k+1|k+1}$ for the second-moment matrix $P_{k+1|k+1}$. This completes the proof. ■

Remark 6: So far, we have studied the dynamics of the CBTC systems and then developed a robust Kalman filter to minimize the upper bound of the second-moment matrix about the filtering error. In our main results, all the important aspects are dealt with in a unified yet effective framework. The proposed filter is designed in terms of the solutions of two Riccati-like difference equations, which is recursive and therefore suitable for online applications. Furthermore, note that the scalar γ_k is involved in the discrete Riccati-like difference equation (21). In the implementation, the value of γ_k could be adjusted at each time instant to guarantee the inequality (23) so as to help enhance the solvability of the filtering algorithm proposed in this paper. It is worth mentioning that, in most of the existing CBTC systems, the information about the position and velocity is always derived by direct measurements. The Zone-Controller would receive such information via the network-based communication directly, which would lead to significant error between the true values and the received data (especially when the the transmitted data is missing due to the packet dropouts phenomenon). To the best of our knowledge, this paper represents the first attempt to study the filtering problem for CBTC systems with packet dropouts. Based on our developed recursive filtering, we could generate an accurate estimates of the position and velocity for the train.

IV. SIMULATION RESULTS AND DISCUSSIONS

In order to verify the effectiveness of the proposed filtering strategy, a series of simulations studies are carried out. The parameters are given in the following Table. In the following simulation, the corresponding data of the train is selected from a real railway system.

The rotary allowance of the train is 0.16 (i.e. $\lambda=0.16$). The initial location and velocity of the train is 1333.8031 and 0, respectively (i.e. $x(0)=0.16$ and $v(0)=0$). The gradient angle of the track is given as follows:

Assume the frame error rate (FER) of the communication channel p is 0.5, and the maximum number of retransmission

TABLE I: Parameters of the high speed train

Symbol	Value	Unit
M	288×10^3	kg
v_{max}	22.23	m/s
a_0	3.4818036×10^3	None
a_1	144.9154	None
a_2	8.5212	None
T	1	s

TABLE II: Gradient angle of the track

Gradient angle	Location
-3	$[0, 1706)$
3	$[1706, 2372)$
-3	$[2372, 2479)$
-9	$[2479, 2619)$

times of the CSMA/CA protocol is 3. Hence, the probability distribution law of $\alpha(k)$ is given by the following equality:

$$\begin{cases} \text{Prob}\{\alpha(k) = 1\} = \bar{\alpha} \triangleq \sum_{j=0}^{r-1} p^j (1-p) = 0.875 \\ \text{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha} = 0.125 \end{cases}$$

Furthermore, the speed limit of the train is $v_{\max} = 22.23$. Then, based on the main results of this paper, we can derive the discrete-time mathematical description of the train according to (6). Fig. 3 shows the position trajectories of the discrete-time mathematical description and the measured location data derived from the experiment. Fig. 4 shows the velocity trajectories of the discrete-time model and the measured velocity data derived from the experiment. Note that the measurement data of the train is obtained from the subway experiment. We assumed that such data is precise enough (e.g. there is no measurement noise and packet dropouts). It can be found that that our discrete-time model could track the measurement data of the experiment well. There exists certain tracking error between the model and measurement data. In what follows, let us show that our developed filtering algorithm could achieve better tracking performance than direct measurements.

In the following simulation, an external measurement noise is added to the measurement data of the experiment (e.g. $R_k = \text{diag}\{1, 0.01\}$). Furthermore, we assume that $q_k = 0.49$. Based on the derived discrete-time mathematical description and the "revised" measurement data (with measurement noise and packet dropouts), we could design the recursive filter of the form (13). By applying Theorem 2, we could achieve the desired filter parameter recursively. The simulation results are shown in Figs. 5-6.

In order to show the improvements of the tracking performance compared with direct measurements. We shall make a simulation comparison on the *statistical variances*. Due to the existence of the packet dropout phenomenon, sometimes the received measurement data would be zero when $\alpha(k) = 0$. A reasonable scheme is to "compensate" the missing data by some easy-to-implement algorithm. In this simulation, we

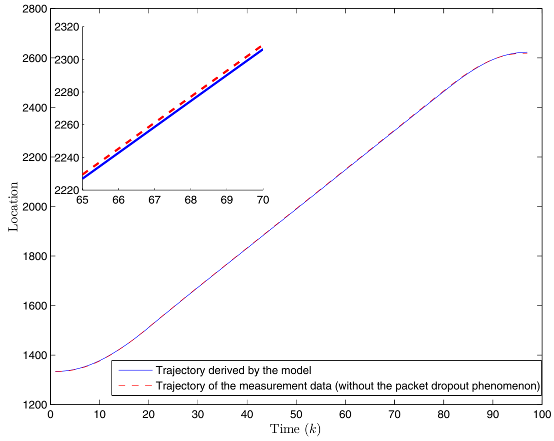

Fig. 3: The position simulation trajectories (discrete-time model and measurement data from experiment(without packet dropouts))

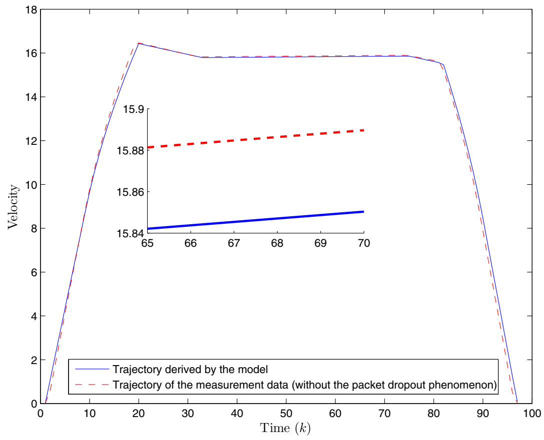

Fig. 4: The velocity simulation trajectories (discrete-time model and measurement data from experiment(without packet dropouts))

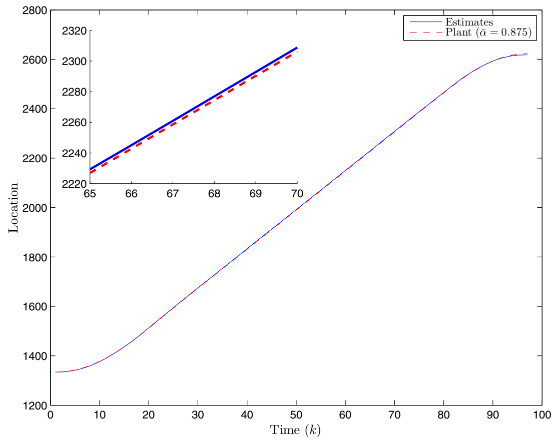

Fig. 5: The position tracking trajectories

adopt the well-known First-Order Hold (FOH) scheme to generate the corresponding data if the measurement data is missing. The corresponding statistical variances of our pro-

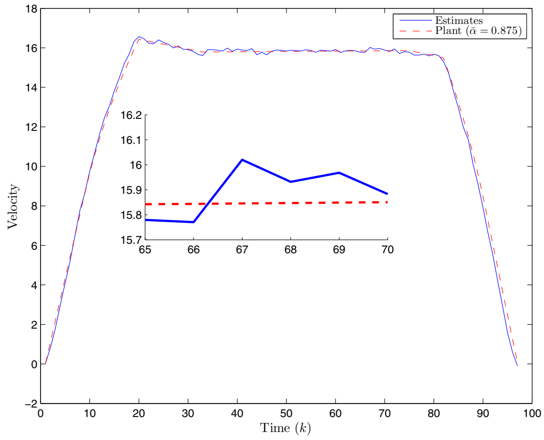

Fig. 6: The velocity tracking trajectories

posed filtering scheme and the direct measurements with FOH scheme is given in the following table. It can be observed from Tab. III that our proposed recursive filtering scheme could significantly improve the accuracy on the obtained information of position and velocity.

TABLE III: Comparison on the statistical variances

	Direct measurements with FOH scheme	Recursive filtering scheme
Statistical variance (location)	8.4972	5.1516
Statistical variance (velocity)	0.3226	0.0518

V. Conclusion

This paper has addressed the recursive filtering problem for the Communication-based Train Control (CBTC) systems. The dynamics of the train has been first modeled by a continuous-time system and then reformulated as a discrete-time system. Due to the nature of wireless communication, a Bernoulli distributed sequence has been employed to characterize the packet dropouts of the train-ground communication. A recursive filter has been developed to generate the estimates of the train position and velocity for CBTC systems subject to the measurement noise and packet dropouts. By solving two Riccati-like difference equations, the desired filter gain has been calculated in a recursive form suitable for online applications. The derived recursive filter has ensure that there exists an upper bound for the second-moment matrix about the filtering error. Furthermore, the designed filter parameter could minimize such an upper bound. An illustrative example has been adopted to demonstrate the effectiveness of the proposed filter design method.

Further research topics include the extension of the main results to 1) recursive filtering problem for CBTC systems with quantization effects; and 2) $\mathcal{H}_\infty$ filtering problem for CBTC systems.

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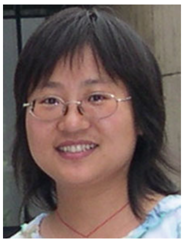

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