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What if the gravitational waves detected in 2015 were strongly lensed by massive galaxy clusters?

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ABSTRACT

We explore the possibility that the gravitational waves (GWs) detected in 2015 were strongly-lensed by massive galaxy clusters. We estimate that the odds on one of the GWs being strongly-lensed is 10^5:1, taking in to account the binary black hole merger rate, the gravitational optics of known cluster lenses, and the star formation history of the universe. It is therefore very unlikely, but not impossible that one of the GWs was strongly-lensed. We identify three spectroscopically confirmed cluster strong lenses within the 90% credible sky localisations of the three GWs. Moreover, the GW credible regions intersect the disk of the Milky Way, behind which undiscovered strong galaxy cluster lenses may reside. We therefore use well constrained mass models of the three clusters within the credible regions and three further example clusters to predict that half of the putative next appearances of the GWs would be detectable by LIGO, and that they would arrive at Earth within three years of first detection.

Key words: galaxies: clusters: individual Abell 1689, 1E0657−558, MACS J0416.1−2403, MACS J0140.0−0555, MACS J1311.0−0311, RCS0224−0002 — gravitational lensing: strong — gravitational waves

1 INTRODUCTION

Strong gravitational lensing by massive galaxy clusters plays an invaluable role in discovering and studying new populations of objects at high redshift (e.g. Smail, Ivison & Blain 1997; Franx et al. 1997; Smith et al. 2002; Kneib et al. 2004; Stark et al. 2007; Willis et al. 2008; Zheng et al. 2014; Bouwens et al. 2014; Atek et al. 2015; McLeod et al. 2015). Most recently, the multiply-imaged *SN Refsdal* (Kelly et al. 2015), located in a galaxy known to be at $z = 1.5$ (Smith et al. 2009), has piqued interest in observing time varying objects through massive cluster lenses (e.g. Oguri 2015; Sharon & Johnson 2015; Jauzac et al. 2016; Diego et al. 2016; Rodney et al. 2016; Treu et al. 2016).

GW sources are another exciting class of transients that can, in principle, be strongly-lensed (Takahashi & Nakamura 2003; Takahashi 2004; Seto 2004; Varvella, Angonin & Tourrenc 2004; Sereno et al. 2010, 2011; Piórkowska, Biesiada & Zhu 2013; Biesiada et al. 2014). Indeed, it is intriguing that three of the six black holes (BHs) responsible for the 2015 GW detections are inferred to have rest-frame masses of $\gtrsim 20M_{\odot}$ (Abbott et al. 2016b), which exceeds the most massive stellar mass BHs observed in the local universe (Farr et al. 2011). Whilst plausible astrophysical interpretations of these "heavy" BHs exist (Abbott et al. 2016a; Stevenson et al. 2017), it is also possible that the large detector-frame masses arise from lower mass sources at larger redshift that have been gravitationally magnified. This would cause the redshift of the GW sources to be underestimated, the BH masses to be overestimated, and raise the possibility of detecting the same object again in the future.

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The first few direct detections of GWs have stimulated numerous articles on strong-lensing of GW source, discussing the effect of lens magnification on the detectability of GWs (Dai, Venumadhav & Sigurdson 2017), forecast event rates including the effects of strong-lensing by galaxies (Ng et al. 2017), relative arrival times of GW and electromagnetic signals (Takahashi 2017), prospects for measuring the speed of GWs (Collett & Bacon 2017; Fan et al. 2017), and the impact of strong-lensing on cosmography (Baker & Trodden 2017; Liao et al. 2017).

In this letter we present the first detailed calculations of strong-lensing of GWs by known and spectroscopically confirmed cluster strong-lenses. Specifically, we consider whether it is possible that the 2015 GW detections were strongly-lensed by massive galaxy clusters, when any putative subsequent images of these events might reach Earth, and whether they would be detectable by LIGO. We explain how strong gravitational lensing modifies GW signals and estimate the probability of strong-lensing in §2, identify the candidate cluster lenses in §3, describe our lensing calculations in §4, and summarise in §5. We assume $H_0 = 70\ \mathrm{km\,s^{-1}Mpc^{-1}}$, $\Omega_{\rm M}= 0.3$ and $\Omega_\Lambda= 0.7$.

2 STRONG LENSING OF GRAVITATIONAL WAVES

The 2015 GW sources have been interpreted, assuming no gravitational lensing, as lying at redshifts of $z \sim 0.1 - 0.2$ (Abbott et al. 2016b). Relaxing this assumption, we write the amplitude of the strain signal detected at Earth due to a GW as $A \propto \sqrt{|\mu|}/D_{\rm L}$, where $|\mu|$ is the gravitational magnification (hereafter μ), and $D_{\rm L}$ is the luminosity distance to the GW source (Wang, Stebbins & Turner 1996). Writing the luminosity distance inferred assuming $\mu = 1$ as $D_{{\rm L},\mu=1}$, a threshold magnification of $\mu_{\rm thresh}(z) = [D_{\rm L}(z)/D_{{\rm L},\mu=1}]^2 \sim 5{-}1000$ is therefore required to interpret the 2015 GW sources as being at redshifts similar to those of strongly-lensed galaxies behind massive galaxy clusters, i.e. $z \sim 0.4 - 2$ (Fig. 1). The increased redshift imposed by $\mu > 1$ would reduce the rest-frame masses of the compact objects responsible for the GW by a factor of $(1 + z)$.

To interpret GW150914 or GW151226 (both at $z \simeq 0.09$, assuming $\mu = 1$) as actually being at $z = 1$ requires $\mu \geqslant \mu_{\rm thresh} \simeq 260$; this would reduce the source frame BH masses by a factor of $\gtrsim 1.8$. Similarly, to interpret LVT151012 (at $z = 0.2$ assuming $\mu = 1$) as actually being at $z = 1$ requires $\mu \geqslant \mu_{\rm thresh} \simeq 46$, and implies a source frame BH mass reduction by a factor of $\gtrsim 1.7$. Typical strongly-lensed galaxies suffer gravitational magnifications of $\mu \sim 10 - 30$ (e.g. Richard et al. 2010), i.e. at the lower end of the range discussed above. The high magnifications implied by the large values of $\mu_{\rm thresh}$ are physically possible (Ng et al. 2017), because the physical region from which GWs emerge is ~ 100km in size. It is therefore possible for a GW source to be very closely aligned with the caustic of a gravitational lens, and thus achieve a high magnification value. This is not the case for a galaxy with a typical size of $\sim 1 - 10$ kpc.

If a GW source is strongly-lensed, then an odd number of light paths connect the source with the observer, and the arrival time at Earth along these paths is described

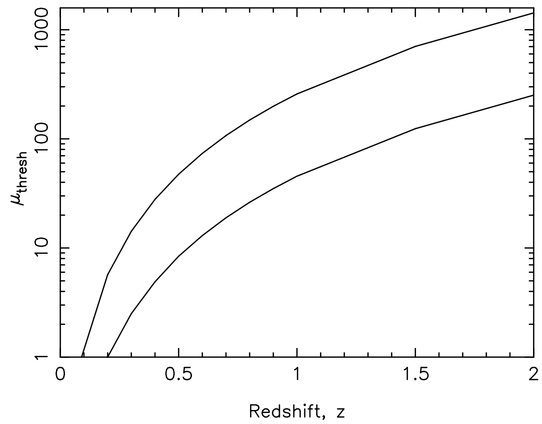

Figure 1. Gravitational magnification ($\mu_{\rm thresh}$) required to modify the inferred luminosity distance to GW150914, GW151226 (both upper curve) and LVT151012 (lower curve) as a function of redshift. The lower curve is also relevant to GW170104.

by the Fermat surface. Therefore, a strongly-lensed (hereafter multiply-imaged) GW source should arrive at Earth on multiple occasions. For completeness, we note that gravitational lensing does not alter the frequency of the GW signal, as is intuitively clear from considering the achromaticity of the source to image plane transformation (e.g. Schneider, Ehlers & Falco 1992).

To calculate the order of magnitude probability that one of the 2015 GW events was multiply-imaged, we begin by selecting an example spectroscopically confirmed strong-lensing cluster close to the peak of the redshift distribution of such systems, namely 1E 0657−558 at $z = 0.296$ (also known as the "Bullet cluster"). We use the detailed parametric mass model of this cluster (Paraficz et al. 2016) to compute the co-moving volume, $V_{\rm thresh}$, behind this cluster that is gravitationally magnified by $\mu > \mu_{\rm thresh}$ in a series of redshift slices. We find that $f_{\rm thresh} = V_{\rm thresh}/V_{\rm total}$ rises steeply at $z = 0.4$ just behind the cluster, peaks at $z \simeq 0.6$, and by $z = 1$ is a factor of 3 lower than the peak, where $V_{\rm total}$ is the total comoving volume of the universe in each redshift slice. For simplicity, we therefore adopt $p_{\rm geom} = N_{\rm SL} \int_{0.4}^{1} f_{\rm thresh}\, dz \simeq 10^{-10}$ as our estimate, based purely on gravitational lens geometry, of the probability that one of the 2015 GW events was multiply-imaged by a massive cluster, where $N_{\rm SL} \simeq 100$ is the number of known spectroscopically confirmed strong lensing clusters (§3).

If one of the 2015 GW events was multiply-imaged by a massive galaxy cluster, and actually lies at $0.4 \lesssim z \lesssim 1$, then our order of magnitude calculations imply that there are of order $p_{\rm geom}^{-1} = 10^{10}$ binary black hole (BBH) mergers per year in that redshift range. For comparison, assuming that none of the events were lensed, the comoving binary black hole merger rate at $z \lesssim 0.2$ that was inferred from the 2015 GW detections and updated following GW170104 is $12 - 213\,\mathrm{Gpc^{-3}\,yr^{-1}}$ (Abbott et al. 2016b,e,d; Abbott et al. 2017); adopting a rate of $R = 50\,\mathrm{Gpc^{-3}\,yr^{-1}}$ at all redshifts would imply $\sim 4\times10^3$ BBH mergers per year at $0.4 \leqslant z \leqslant 1$. The rate inferred from $p_{\rm geom}$ exceeds this rate by a factor of $\sim 10^6$. If a merger rate near that inferred from $p_{\rm geom}$ persists over a broad range of redshifts it would imply ~ 2 BBH mergers per solar mass of star formation (folding in the star formation history of the universe from Madau & Dickinson 2014), which is impossible.

It is therefore clear that if one of the 2015 GW events was multiply-imaged by a massive galaxy cluster, we must have been lucky to detect this rare event. We estimate the "luck factor" of the multiple-imaging hypothesis, given the star-formation history of the universe, constraints on the BBH merger rate in the local universe, and the lens geometry encoded in $p_{\rm geom}$. We impose a log-flat prior on the number of mergers per year Λ with $z \in [0.4, 1]$ for $\Lambda \in [10^3, 10^{12}]$, and assume one of the 2015 GW detections lies in this redshift range and is multiply-imaged with $\mu \geqslant \mu_{\rm thresh}$ by a cluster at $z = 0.3$. Based on these assumptions, the posterior probability that the BBH merger rate at $0.4 \leqslant z \leqslant 1$ is smaller than $24,000$ (roughly the upper limit from the $z = 0$ rate that would be inferred from the remaining LIGO detections) is $\sim 2 \times 10^{-6}$. However, if one of the GW detections were multiply-imaged, then a more likely scenario is that BBH merger rate more closely tracks the star formation history of the universe (Madau & Dickinson 2014). An upper limit of $300\,{\rm Gpc}^{-3}\,{\rm yr}^{-1}$ at redshift zero would then correspond to $\sim 10^5$ BBH mergers per year for $0.4 \leqslant z \leqslant 1$. The posterior probability that the rate is smaller than this, assuming one lensed GW detection, is therefore $\sim 10^{-5}$, i.e. the mutiple-imaging hypothesis requires us to be lucky at one part in 10^5, equivalent to $\sim 4\sigma$. It is therefore very unlikely, but not impossible that one of the 2015 GWs was multiply-imaged. We also note that our calculations ignore previously undiscovered strong cluster lenses and all strong group and galaxy lenses. Therefore, our estimate should be regarded as a lower limit.

3 THE CLUSTER LENSES

We assembled a list of 130 spectroscopically confirmed strong cluster lenses from the literature, drawing mainly on *HST* studies of X-ray selected clusters, and strong-lensing clusters from the Sloan Digital Sky Survey (e.g. Smith et al. 2005, 2009; Limousin et al. 2007, 2012; Richard et al. 2010; Christensen et al. 2012; Oguri et al. 2012; Jauzac et al. 2015; Umetsu et al. 2016; Paraficz et al. 2016). None of these cluster strong lenses are located within the 90% credible sky localisation of GW150914; two are located within the 90% credible region of GW151226 – MACS J0140.0−0555 at $z = 0.451$, and MACS J1311.0−0310 at $z = 0.398$; and one within the 90% credible region of LVT151012 – RCS0224−0002 at $z = 0.773$. We note that there are no clusters in common between the sky localisations of GW151226 and LVT151012, and none of them lie in or near the intersection of the sky localisations of these two events (see for example, Figure 6 of Abbott et al. 2016b). Detailed mass models are available for all three clusters (Christensen et al. 2012; Ho, Ebeling & Richard 2012; Smit et al. 2017).

The sky localisations of all three GW events intersect the disk of the Milky Way. Unfortunately, severe dust extinction and stellar obscuration make it very difficult to find clusters at low galactic latitude, let alone identify whether any of them are strong-lenses. Despite some clusters being identified at low galactic latitude (e.g. Ebeling, Mullis & Tully 2002; Kocevski et al. 2007), searches for strong-lensing clusters have concentrated on high latitudes ($|b| > 20^\circ$). It is therefore possible that an unknown massive galaxy cluster at low galactic latitude strongly-lensed one or more of the GW events. To explore this possibility we also select three well-studied powerful cluster lenses, for which detailed mass models are available: Abell 1689 ($z = 0.183$; Limousin et al. 2007), 1E 0657−558 ($z = 0.296$; also known as the "Bullet Cluster"; Paraficz et al. 2016), and MACS J0416.1−2403 ($z = 0.398$; Jauzac et al. 2014).

The detection of GW170104 was announced during the latter stages of preparing this letter (Abbott et al. 2017). We identified two cluster lenses within the 90% credible region of this detection, neither of which alter the conclusions and discussion presented here.

4 TIME DELAY AND MAGNIFICATION CALCULATIONS

We estimate the arrival times and magnifications of putative future appearances of the 2015 GW events due to the six cluster lenses discussed in §3. The detailed mass models referred to above are all constrained by spectroscopically confirmed strongly-lensed galaxies, thus breaking the redshift space degeneracies. The mass distribution of each cluster core was modelled as a superposition of mass components that represent the large-scale cluster mass distribution, and the cluster galaxies, and optimized using the publicly available LENSTOOL software (Jullo et al. 2007).

Starting from these models, we identify the sky locations in the $z_{\rm S} = 1$ and $z_{\rm S} = 1.5$ source planes of each cluster that are magnified by $\mu > \mu_{\rm thresh}$. Then we ray traced these sky locations through the relevant lens models to obtain the respective image positions, $\vec{\theta}$. Given the large magnification values, all of these sky locations are multiply-imaged. We then measured the gravitational potential at the image positions, $\phi(\vec{\theta})$. The arrival-time surface for a light ray emitted by a lensed source, at the source-plane position $\vec{\beta}$, traversing the cluster lens at the image-plane position $\vec{\theta}$, is given by:

$$\tau(\vec{\theta}, \vec{\beta}) = \frac{1 + z_{\rm L}}{c} \frac{D_{\rm OL} D_{\rm OS}}{D_{\rm LS}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \phi(\vec{\theta}) \right] \qquad (1)$$

where, c is the speed of light in vacuum, $z_{\rm L}$ is the redshift of the cluster lens, $D_{\rm OL}$, $D_{\rm OS}$, and $D_{\rm LS}$ are the observer-lens, observer-source, and lens-source angular diameter distances respectively, and $\phi(\vec{\theta})$ represents the projected cluster gravitational potential (Schneider 1985). These calculations were performed following the analytic procedure described by Jauzac et al. (2016).

The distribution of time delay between the first arrival of an image that satisfies $\mu > \mu_{\rm thresh}$ and the next arrival of an image from the same source location, $\Delta t_{\rm arrival}$, spans a fraction of a day to ~ 3 years (Figure 2). We classify these "next images" as being detectable if they are magnified by $\mu \geqslant \mu_{\rm detect,2} = \mu_{\rm thresh}/\alpha^2$ where $\alpha = 25/8$, $13/8$, and $9.7/8$ for GW150914, GW151226, and LVT151012 respectively, i.e. the ratio of the SNR at which each was detected in 2015 and the minimum SNR required of a detection by LIGO Abbott et al. (2016c). The fraction of the next images that would be detectable by LIGO is $\sim 40 - 60\%$ – i.e. roughly half. Note that in Figure 2 we show results for $z_{\rm S} = 1$ for the clusters at $z_{\rm L} < 0.5$, to be consistent with the calculations

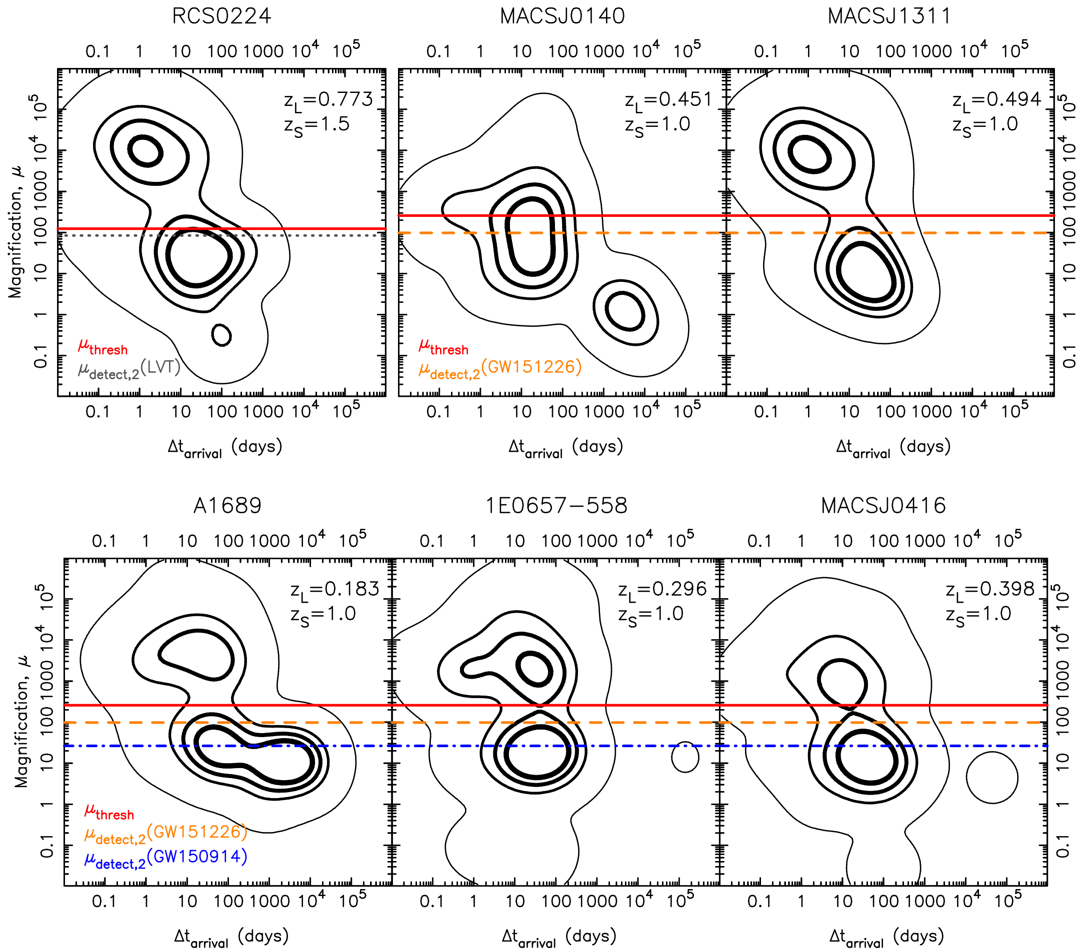

Figure 2. Magnification and time delay suffered by the next appearance of a GW event that has been multiply-imaged and magnified by $\mu > \mu_{\rm thresh}$. Upper — The three clusters found within the 90% credible regions of LVT151012 and GW151226, as indicated in the panels. Lower — Three example strong lensing clusters; considers the putative next appearance of GW150914 and GW151226. The value of $\mu_{\rm thresh}$ relevant to the published redshifts of the respective GW events and the respective values of $z_{\rm S}$ is shown as the red solid line. The grey dotted, orange dashed, and blue dashed-dotted lines show the minimum magnification factor that a subsequent image would require in order to be detectable by LIGO at SNR > 8. Contours enclose 25%, 50%, 75%, and 99% of the probability density.

described in §2. The equivalent plots for $z_{\rm S} = 1.5$ are very similar and do not change the conclusions. We show the plot for $z_{\rm S} = 1.5$ for RCS0224−0002, because the cross-section of this high-redshift cluster to a source at $z_{\rm S} = 1$ is tiny.

5 SUMMARY AND DISCUSSION

The degeneracy between gravitational magnification (μ) and luminosity distance (Wang, Stebbins & Turner 1996) causes the luminosity distance to a GW source to be revised upwards by a factor of $\sqrt{\mu}$ if it is gravitationally magnified, and the inferred source frame masses of the compact objects to be revised down by a factor of $(1 + z)$. This is interesting because some of the early GW detections appear to come from heavy black holes (Abbott et al. 2016a; Stevenson et al. 2017), and gravitational lensing by massive galaxy clusters have been central to the first detection of distant galaxy populations, most notably at sub-mm wavelengths (Smail, Ivison & Blain 1997; Ivison et al. 1998).

We show that the odds on a single GW source at a redshift of $0.4 < z < 1$ being multiply-imaged by a massive galaxy cluster, and magnified sufficiently to be detectable by LIGO in 2015 are $\sim 10^5 : 1$. This calculation takes in to account the gravitational optics of clusters with sky positions consistent with the GW detections, the local BBH merger rate, and the star formation history of the universe. It is therefore unlikely, but not impossible that one of the 2015 GWs was multiply-imaged. Our calculations ignore previously undiscovered strong cluster lenses and all strong-lenses less massive than clusters of galaxies. We therefore consider our estimate of the odds to be conservative.

We have identified 0, 2, and 1 spectroscopically confirmed cluster strong lenses within the 90% credible sky localisations of GW150914, GW151226, and LVT151012 respectively. We also note that a significant fraction of all three localisations intersect the disk of the Mily Way, raising the possibility that a previous undetected strong cluster lenses might have lensed the GW events. We therefore used detailed mass models of these three strong cluster lenses, and three further examples, to calculate the magnifications and time delays suffered by sources at plausible background redshifts. We find that, if the GW events have been strongly lensed by a cluster, then 40−60% of the next images to arrive at Earth will be detectable by LIGO, and they are expected to arrive at Earth within 3 years of the original detections. Unambiguous identification of a strongly-lensed GW event would require the identification of electromagnetic counterparts, in order to localise the GW to a strong-lensing cluster core. This will require prompt follow up observations of galaxy cluster cores within future GW sky localisations.

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