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SEMANTICS AND THE PLURAL CONCEPTION OF REALITY

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A CCORDING to a traditional view, reality is singular. Socrates and Plato are philosophers; each of them has the property of being a philosopher.[1] Being a philosopher is a *singular* property in that it is instantiated *separately* by Socrates and by Plato. The property of being a philosopher, like the property of being human, has the higher-order property of being instantiated. The property of being instantiated is singular too. It is instantiated separately by the property of being a philosopher and by the property of being human. If we generalize these ideas, we obtain what may be called *the singular conception of reality*. This is the view that reality encompasses entities belonging to two main categories: objects and singular properties of various orders.[2]

The singular conception of reality offers a simple picture of what there is, and it offers a simple picture of the semantics of singular predication. A basic predication of the form $S(t)$, composed of a singular term t and a singular predicate S, is true in a given interpretation of the language if and only if, relative to that interpretation, the object denoted by t instantiates the property denoted by S.

A broader conception of reality, however, has been advocated. The Romans conquered Gaul. This is not something that any Roman did separately. They conquered Gaul *jointly*. According to advocates of the broader conception of reality, conquering Gaul is a *plural* property, one that is instantiated jointly by the Romans. This broader conception of reality, which admits the existence of plural properties in addition to singular ones, has been called *the plural conception of reality* (Yi 2006).

An attractive feature of the plural conception of reality is that it offers a straightforward way of handling the semantics of plural predication. Consider an atomic formula, $P(tt)$, composed of a plural term tt and a plural predicate P. If plural properties are available, one may hold that $P(tt)$ is true in a given interpretation of the language if and

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1. Those who prefer to understand predication in terms of Fregean concepts should have no difficulty in translating claims about properties into claims about concepts throughout the article.
2. I use 'entity' as an umbrella term covering both objects and properties, and I take 'object' to mean the same as 'individual'.

only if, relative to that interpretation, the *things* denoted by *tt* jointly instantiate the plural property denoted by *P*.

Without invoking plural properties it is far from clear how one should account for plural predication. Does the singular conception have the resources to provide a satisfactory semantics for plurals? This is the *semantic challenge* to the singular conception. The apparent absence of a plausible response to the semantic challenge provides an indispensability argument for the plural conception, and it has led various authors to introduce plural properties for semantic purposes (Yi 1999, 2002, 2005, 2006; Oliver and Smiley 2001, 2004, 2005, 2006, 2013; and McKay 2006).

In this article, I argue that this semantic challenge to the singular conception can in fact be met. It follows that the adoption of the plural conception on semantic grounds alone cannot be justified. I outline an approach to the semantics of plurals which dispenses with plural properties, and I show that it fares as well as the best approach based on the plural conception. On this approach, plural terms denote singular properties and plural predicates denote singular higher-order properties. This is reminiscent of existing proposals that make use of higher-order resources to account for the semantics of plurals (see Russell 1919, 1938; Higginbotham and Schein 1989; Dummett 1991; and Lønning 1997). Despite some point of contact, however, there is a crucial difference between such proposals and the one to which I want to draw attention. To bring this difference to light I will have to take a fresh look at the semantic debate about plurals. I will clarify what is at stake between competing accounts and, on the basis of important distinctions that have been overlooked, I will provide a more accurate characterization of the territory than it is usually given. This will help elucidate the nature of the semantic challenge and open up the space for an adequate semantics of plurals which lives completely inside the singular conception.

To forestall possible misunderstandings, I should make two preliminary remarks. First, my focus is on model-theoretic semantics rather than the theory of truth. This is because it is doubtful that proper-

ties need to be invoked at all in the formal definition of truth for the language with which we are concerned (see Boolos 1984, Rayo and Uzquiano 1999, and Rayo 2006). Since there is no need to vary the interpretation of the predicates to define truth for that language, there is no need to regard predicates as denoting expressions. Thus the theory of truth has no obvious bearing on the semantic challenge and on the indispensability argument for the plural conception.

Second, my contention is that plural properties are not semantically indispensable, and thus the semantic argument in favor of plural properties fails. To secure this claim it is enough to show that there is a semantic account of plurals that satisfies the following desiderata: (a) it can be fully developed within the singular conception of reality, (b) it is semantically *equivalent* to the best account based on the plural conception, and (c) it is in good standing. Showing that such an account is *preferable* to the best account based on the plural conception is an additional task. But it is strictly more than we need to undermine the indispensability argument, and it will not be taken up here. Instead, I will argue for the more modest but still significant claim that the novel account is equivalent to the best account based on the plural conception.

1. Objections to singularism

A simple but problematic answer to the semantic challenge to the singular conception relies on the following view.

> REGIMENTATION SINGULARISM
> Plural terms and predicates are not required for the regimentation of natural language into a formal language. Singular terms and predicates suffice.

REGIMENTATION SINGULARISM is a clear ally of the singular conception. The regimentation singularist contends that plural constructions can be rendered by paraphrase in a regimenting language containing only singular expressions, and of course a semantics for such a language can be given within the singular conception. The task facing the regi-

mentation singularist is to provide a good method of paraphrase.

Two methods suggest themselves. According to the first, plural expressions can be paraphrased by means of set-theoretic expressions (Quine 1982 and Resnik 1988). According to the second, plural expressions can be paraphrased by means of mereological expressions (see Massey 1976 and Link 1983). There are familiar objections to both methods. The main one is that they yield implausible entailment relations. For instance, on both approaches a sentence such as (1) would logically entail (2).

(1) Russell and Whitehead cooperate.

(2) There is a set or there is a mereological sum.

A number of additional arguments have been marshaled against these construals of Regimentation Singularism. Consider the following sentence:

(3) There are some sets such that any set is one of them if and only if it is not self-membered.

It expresses a set-theoretic truth. The set-theoretic version of Regimentation Singularism seems bound to paraphrase (3) as (4).

(4) There is a set x such that, for any set y, y is in x if and only if y is not self-membered.

However, (4) is inconsistent. Thus a set-theoretic truth such as (3) is regimented by a logically false sentence (Boolos 1984, Lewis 1991, Higginbotham 1998, Schein 1993, Oliver and Smiley 2001).

For the mereological singularist, the natural rendering of a plural term tt is 'the sum of tt'. For instance, 'Russell and Whitehead' would be paraphrased as 'the sum of Russell and Whitehead'. [3]

Now, (5) appears to be a mereologically true claim:

(5) The sum of Russell and Whitehead is identical to the sum of the molecules of Russell and Whitehead.

But (5) commits the mereological singularist to (7) on the basis of (6) (Oliver and Smiley 2001, p. 293).

(6) Russell and Whitehead were logicians.

(7) The molecules of Russell and Whitehead were logicians.

A similar argument has been put forth by Rayo (2002). Imagine that we have some sand and that the grains of sand are grouped into piles. There is a possible scenario in which both of the following sentences are true.

(8) The piles of sand are scattered.

(9) The grains of sand are not scattered. [4]

Since the piles of sand form the same mereological sum as the grains of sand, (10) holds.

(10) The sum of the piles of sand is the sum of the grains of sand.

Given (10), the mereological singularist must implausibly regard (8) and (9) as contradictory.

Schein (1993) has pointed out that, almost ironically, the mereological version of Regimentation Singularism gets the mereological facts wrong. Define an *atom* to be an individual that does not have proper parts. The expression 'the atoms' is paraphrased by 'the sum of the atoms'. Likewise, 'the sums of atoms' is paraphrased by 'the sum of the sums of atoms'. Whatever the domain, the sum of the atoms is

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3. Since plurals must be completely eliminated in the paraphrase, 'the sum of Russell and Whitehead', which is only partially singular, should be taken to

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stand for a fully singular expression defining the relevant notion of sum, e.g. 'the object x such that anything overlaps x if and only if it overlaps Russell or it overlaps Whitehead'. For the sake of exposition, the arguments in the remainder of this section are formulated using partially singular paraphrases.
4. The scenario envisioned is one in which the grains of sand are nicely grouped into piles—hence they are not scattered—while the piles themselves are scattered.

identical to the sum of the sums of atoms, as they are both the sum of everything in the domain. So, from (11), the mereological singularist is forced to conclude (12).

(11) The atoms are exactly two.

(12) The sums of atoms are exactly two.

We know that, if there are n atoms, the number of mereological sums over them is $2^n - 1$. Therefore, if the number of atoms is two, there are three sums of atoms, which is inconsistent with (12). (For a discussion of the prospects of the mereological approach, see Nicolas unpublished. For a large-scale defense of it, see Link 1998.)

The arguments just reviewed target Regimentation Singularism. As a result, Regimentation Pluralism has now become prominent.

Regimentation Pluralism
Plural terms and predicates are required in the regimentation of natural language into a formal language.

It is important to notice, however, that appealing to Regimentation Singularism is just one way in which the proponent of the singular conception can attempt to address the semantic challenge. The singular conception does not fall with Regimentation Singularism. Indeed, Regimentation Pluralism is compatible with what may be called Semantic Singularism.

Semantic Singularism
Once a regimenting language for plurals has been chosen, semantic interpretations can be specified adequately within the singular conception of reality.

The focus has now shifted from the level of regimentation to the level of semantics. The crucial point is that the semantic singularist may employ plural expressions in the regimenting language and does not have to regard plural predication as singular predication in disguise. Yet at the semantic level he will draw exclusively on singular resources,

characterizing the interpretations of the regimenting language within the boundaries of the singular conception. The semantic challenge can be met if there is a tenable version of Semantic Singularism. In the remainder of this article, I address the semantic challenge and show that there is a viable version of Semantic Singularism.

Those who advocate the plural conception of reality on semantic grounds reject Semantic Singularism in favor of Semantic Pluralism.

Semantic Pluralism
Specifying semantic interpretations for plural sentences requires the resources made available by the plural conception of reality.

Before turning to the semantic dispute between singularists and pluralists, I will briefly introduce a basic regimenting language for plurals in the spirit of Regimentation Pluralism. This will be the object language of the competing semantic accounts that will occupy us from section 4 onwards. By adopting it, we are leaving Regimentation Singularism and its problems behind.

2.   Regimenting plurals

As is customary in the philosophical literature, the regimenting language will be an extension of first-order logic that includes symbols for plural predicates (e.g. 'cooperate', 'are infinite', 'gather'), plural variables and quantifiers, and a distinguished relational symbol for plural membership ('being one of', 'being among'), which will be treated as logical. This will be a version of the language known as PFO+ (see Rayo 2002 and Linnebo 2003.)

We may take a predicate to have a fixed number of arguments, each of which can be exclusively singular or exclusively plural. An argument place is singular if it is occupied by singular terms, i.e. singular constants and singular variables. It is plural if it is occupied by plural variables. A predicate is said to be plural if at least one of its argument places is plural.

More specifically, the vocabulary of our language is composed by

the usual vocabulary of first-order logic plus the following symbols.

A. Plural variables: vv and vv_i for each $i \in \omega$; double variables xx, yy, zz will be used as plural variables in the metalanguage. Plural proper names as well: $aa, bb, cc, dd, \ldots$

B. Plural existential and universal quantifiers ($\exists, \forall$) binding plural variables: $\exists vv, \forall vv, \exists vv_0, \ldots$

C. Symbols for collective plural predicates of any finite arity, with or without numerical subscripts: $A, B, C, \ldots, A_1, A_2, \ldots$. For any arity n, it is convenient to allow only predicate symbols whose first m argument places ($1 \leq m \leq n$) are plural and whose remaining $n - m$ argument places are singular. A superscript enclosed in square brackets will indicate the number of plural arguments. The arity of the predicate, when marked, will precede the square brackets. For example, if $W^{2[1]}$ represents the two-place predicate 'wrote together', the sentence 'they wrote *Principia Mathematica* together' may be represented as $W^{2[1]}(vv, p)$. For the sake of readability, I will often depart from this convention and allow the order of the terms to reflect the order found in English.

D. A distinguished binary predicate $\prec$ for plural membership, taking as arguments a singular term and a plural variable.

The recursive clauses defining a well-formed formula are the obvious ones.

A central semantic phenomenon concerning plurals is the distinction between distributive and collective predicates.

Let Φ be a plural predicate (e.g. 'are philosophers') and let φ be its corresponding singular form, if it exists (e.g. 'is a philosopher'). Let us say that Φ is *distributive* if

(13) Analytically, for any things xx, $\Phi(xx)$ if and only if, for any y that is one of xx, $\varphi(y)$.

A plural predicate will be said to be *collective* if it is not distributive. For polyadic predicates the characterization is relativized to an argument

place.[5]

In the regimenting language just introduced, we can obtain the force of distributive predication without employing distributive predicates. Consider this sentence:

(14) Socrates and Plato are philosophers.

The use of the distributive 'are philosophers' can be paraphrased away via the singular 'is a philosopher' as shown in (15).

(15) Everything that is one of Socrates and Plato is a philosopher.

For simplicity, we can then expunge distributive predicates from the regimenting language.

The existence of a good method of paraphrase also makes the introduction of a symbol for phrasal conjunction unnecessary. This is shown by the following pair of sentences.

(16) Russell and Whitehead cooperate.

(17) There are some things such that Russell is one of them, Whitehead is one of them, no other thing is one of them, and they cooperate.

Finally, some additional concepts do not require a separate treatment but can be introduced as abbreviations. The plural 'are among', symbolized by $\preccurlyeq$, is defined as follows.

(18) $vv_1 \preccurlyeq vv_2 \leftrightarrow_{def} \forall v (v \prec vv_1 \rightarrow v \prec vv_2)$

Plural identity, symbolized by $\approx$, is also definable.

(19) $vv_1 \approx vv_2 \leftrightarrow_{def} (vv_1 \preccurlyeq vv_2 \land vv_2 \preccurlyeq vv_1)$

This completes the presentation of the regimenting language.

As described, the language draws a rigid distinction between singular and plural argument places: an argument place is either exclusively

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5. For further discussion of the distinction between distributive and collective predicates, see Oliver and Smiley 2013, pp. 112-18, which includes an alternative formulation of the distinction.

singular or exclusively plural. The significance of this distinction may be doubted. First, the regimentation pluralist might adopt a regimenting language containing only plural variables and recapture singular talk through it. By taking 'are among' as a primitive relation, singular variables can be paraphrased in terms of special kinds of plural variables (see Rayo 2002 and McKay 2006). Second, it might be objected that a rigid distinction does not do justice to the flexibility of predication in natural language. Although some predicates can only be combined grammatically with plural terms (e.g. 'cooperating with one another', 'gathering', 'overlapping'), other predicates—which we call *flexible*—can be combined grammatically with both singular and plural terms (e.g. 'writing *Principia Mathematica*' or 'owning a house'). The semantic implications of this phenomenon are debatable. One might hold that these predicates do not occur univocally when combined with different types of terms, perhaps emphasizing that they can be glossed differently ('writing together' vs. 'writing alone', 'owning together' vs. 'owning alone'). In contrast, one might favor the view that the predicates do occur univocally and the regimenting language should be augmented with flexible predicates (see, e.g., Yi 2005, Oliver and Smiley 2013). Some arguments in favor of flexible predicates will be mentioned in section 5. However, I will not attempt to adjudicate this dispute here. As I will note later, my main conclusion is independent of it.[6]

Clearly, a regimenting language of this sort sides with Regimen-

tation Pluralism. However, as I pointed out above, Regimentation Pluralism does not mandate any particular semantic account of plurals and is compatible with Semantic Singularism.

Now that we have settled on a basic regimenting language, we can enter the semantic debate, explore the options, and decide whether any version of Semantic Singularism is tenable.

3. Semantic approaches to plurals

Semantic singularists and semantic pluralists can agree on the general characterization of the central notion of truth relative to an interpretation for the common object language. If they admit properties in their ontology—and this is the case in which we are interested—they will hold that an atomic predication is true in a given interpretation if and only if, relative to that interpretation, the property denoted by the predicate is instantiated by whatever is denoted by the subject term.[7]

Semantic singularists and semantic pluralists can also agree on the specific characterization of truth relative to an interpretation for singular predication, since they can agree that a singular term denotes an object and a singular predicate denotes a singular property. Their disagreement lies in how to specify the denotations of plural terms, which in turn constrains what kind of properties can serve as denotations of plural predicates. As we will see in a moment, there is also some internal disagreement among semantic singularists and among semantic pluralists. Let us start by introducing the main views.

Competing semantics of plurals may be classified along two dimensions. The first dimension concerns how many entities are taken to be denoted by a plural term. In particular, does a plural term denote one or many entities? The second dimension of classification concerns what sort of entities are taken to be denoted by a plural term. In particular,

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6. It might be worth commenting on two additional features of this regimenting language. First, for expositional economy plural constants have been left out. Second, operators for definite descriptions have also been left out, but for a different reason. Providing an adequate semantics for plural definite descriptions presents distinctive difficulties and goes beyond the scope of this article. In one case below ('the authors of *Principia Mathematica*'), it will be convenient to assume that a Russellian analysis of the plural description is acceptable. Given that complex plural terms formed by conjoining singular constants are paraphrased away, the only plural terms contained in the language are plural variables. This parsimony, however, is not pointless. It will expedite the formulation of the semantics in the Appendix. Nothing important for our purposes hinges on these features of the regimenting language.

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7. For simplicity, I am focusing here on monadic predication. The general case is addressed in section 4 and in the Appendix.

are the denotations of plural terms objects or properties? These possibilities give rise to four views: *object singularism*, *property singularism*, *object pluralism*, and *property pluralism*.

	How many entities are denoted by a plural term?	What sort of entities are denoted by a plural term?
object singularism	one	object
property singularism	one	property
object pluralism	many	objects
property pluralism	many	properties

The last view appears to be the least attractive, since it combines controversial aspects of the other views without offering any advantage over them. So we may take object pluralism to be the only pluralist option and use 'pluralism' to mean 'object pluralism'.

Once the semantic values of plural terms have been determined, one must decide what kind of semantic value should be assigned to plural predicates. There are obvious choices in the singularist camp. On the one hand, the object singularist holds that a plural term denotes an object and will therefore take a plural predicate to denote a singular property of objects. On the other hand, the property singularist holds that a plural term denotes a singular property and will therefore take a plural predicate to denote a singular property of properties.

In the pluralist camp, there are three main views. According to the first, which may be called *untyped pluralism*, plural predicates denote plural properties construed as objects, which means that no type distinction between objects and properties is assumed (Hossack 2000 and McKay 2006). An alternative view adopted by Yi (1999, 2002, 2005, 2006) and Oliver and Smiley (2004, 2005, 2006, 2013) also takes plural predicates to denote plural properties, but it construes them as *typed*.

That is, properties are now thought of as predicable entities rather than objects. To highlight the contrast with the first view, let us call this approach *typed pluralism*. Lastly, plural predicates have been taken to denote 'superpluralities', i.e. many pluralities at once (Rayo 2006). Call this approach *superpluralism*. The chart below displays the options in the pluralist camp.

	What do plural predicates denote?
untyped pluralism	untyped properties
typed pluralism	typed properties
superpluralism	superpluralities

Since properties play no role in superpluralism, this view falls outside the main focus of this article, which is the dispute between defenders of the singular conception and advocates of the plural conception. Therefore superpluralism will be put aside here. In the remainder of this section, I will discuss some arguments that will help us narrow down our pool of candidates to two views: property singularism and typed pluralism. In the next two sections, I will compare these two views and argue that property singularism provides a satisfactory answer to the semantic challenge.

Is object singularism a satisfactory option? On the most natural construal of the view, plural terms denote (and plural quantifiers range over) non-empty sets of objects in the range of the first-order quantifiers. Accordingly, the predicate 'being one of' is interpreted as set-theoretic membership. The assumption that the sets denoted by a plural term are non-empty is needed to render sentences like the following logically false.[8]

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8. By ruling out singletons we could satisfy the further requirement that 'there are some things such that exactly one thing is among them' be logically false as well. However, this additional requirement has been thought to be misguided and will not be imposed here (see Yi 2005).

(20) There are some things such that nothing is among them.

It is important to note that object singularism avoids the objections to Regimentation Singularism discussed in section 1. Despite the fact that in any interpretation of the language the term 'Russell and White-head' denotes a set, the inference from 'Russell and Whitehead cooper-ate' to 'there is a set' is blocked. The semantics provides a simple model in which the first sentence is true while the second is false.[9] Moreover, since the object singularist embraces Regimentation Pluralism, she can do justice to the problematic sentence (3) by regimenting it as (3*) rather than (3**).

(3) There are some sets such that any set is one of them if and only if it is not self-membered.

(3*) $\exists vv \, (S(vv) \, \wedge \, \forall v \, (S(v) \, \rightarrow \, (v \prec vv \, \leftrightarrow \, v \notin v)))$

(3**) $\exists v_0 \, (S(v_0) \, \wedge \, \forall v_1 \, (S(v_1) \, \rightarrow \, (v_1 \in v_0 \, \leftrightarrow \, v_1 \notin v_1)))$

However, Regimentation Singularism forces the domain of quan-tification to be set-sized. Sentence (21) follows logically from plural comprehension applied to the formula $v = v$.[10]

(21) $\exists vv \, \forall v \, v \prec vv$

If plural terms denote non-empty sets of objects in the range of the first-order quantifiers, plural quantifiers must be taken to range over such sets. Thus the logical truth of (21) amounts to the requirement that

the things in the domain of the first-order quantifiers form a set. In the absence of a universal set, the first-order domain of quantification cannot be absolutely unrestricted. In particular, it cannot contain every set.

There is something *prima facie* unsettling about this situation. A sen-tence like (3) is intended to talk about *every* set there is, not just those contained in some appropriately large set. Object singularism cannot capture models where the domain encompasses every set. The object singularist could resort to classes to provide a domain for such models. But then the same difficulty would arise for another sentence.

(22) There are some classes such that any class is one of them if and only if it is not self-membered.

An appeal to super–classes or some other type of higher-level collec-tions would only delay the problem. Here is how Lewis (1991, p. 68) puts it.

> Whatever class-like things there may be altogether, holding none in reserve, it seems we can truly say that there are those of them that are non-self-members. Maybe the singularist replies that some mystical censor stops us from quantifying over absolutely everything without restriction.[11]

The object singularist might retort that the problem has little to do with plurals. It already arises for singular languages with the usual model-theoretic semantics, where domains are sets and thus we can never interpret a sentence like (23) as talking about every set there is.

(23) Every set is self-identical.

The inability to capture models with domains too big to form a set— the object singularist will conclude—should not cast doubt on object singularism as a semantic view about plurals.

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9. Consider, for instance, an interpretation in which (a) the extensions of the predicate 'is a set' is empty, (b) 'Russell' denotes r, (c) 'Whitehead' denotes w, and (d) 'cooperate' denotes a singular property C that is instantiated by the set $\{r, w\}$. Since $\{r, w\}$ instantiates C, 'Russell and Whitehead cooperate' is true in this interpretation, whereas 'there is a set' is false (the extension of 'is a set' is assumed to be empty).

10. The regimented version of plural comprehension is the closure of the schema (PC).

(PC) $\exists v \, \varphi(v) \, \rightarrow \, \exists vv \, \forall v \, (v \prec vv \leftrightarrow \varphi(v))$

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11. To which he adds: "Lo, he violates his own stricture in the very act of proclaiming it!"

The burden is now on the opponent of object singularism to argue that, when plurals are introduced, the inability to capture models with domains too big to form a set poses a special problem. A promising way to do that is to reflect on a logical difference between standard semantics for first-order languages and the object singularist's semantics for plurals.

A famous argument by Kreisel (1967) shows that, for a first-order language, the semantic notion of consequence and the notion of provability coincide extensionally with the intuitive notion of consequence. This turns essentially on the fact that there is a sound and complete proof system for first-order logic relative to the usual model-theoretic semantics. So, despite the fact that the semantics only appeals to models with set-sized domains, the resulting relation of semantic consequence matches the intuitive relation of logical consequence. At least in the first-order case, this should put to rest worries about the absence of models with domains too big to form a set. For the purpose of giving an extensionally correct characterization of the notion of logical consequence for a singular language, set-sized domains are enough. Can the argument be extended to plurals?

It turns out that the relation of semantic consequence sanctioned by object singularism is not compact.[12] By a standard argument, it follows that there is no effective proof system that is sound and complete for the semantics sectioned by object singularism. So Kreisel's argument is

not immediately available to the object singularist, and worries about the inability to capture models with domains too big to form a set cannot be easily put to rest.

Property singularism sidesteps this problem. In the context of property singularism, (21) expresses the requirement that there be a property instantiated by all the objects in the range of the first-order quantifiers.

(21) $\exists vv \, \forall v \, v \prec vv$

But the requirement is clearly met, as witnessed by any universal property, say that of being self-identical. Thus accepting the possibility of absolutely unrestricted quantification, or at least quantification over all sets, gives us reasons to prefer property singularism over object singularism.[13]

Let us now turn to pluralism. The pluralist who subscribes to the plural conception takes plural terms to denote many things at once and takes plural predicates to denote plural properties. Two versions of this view were identified above, one takes properties to be objects, while the other takes them to be predicable entities of a higher type than objects.

There are at least two reasons to accept the second version of pluralism and construe properties as typed. A Russell-style argument put forth by Williamson (2003) shows that, if one accepts an intuitive requirement on what semantic interpretations there are and admits the possibility of absolutely unrestricted quantification, one cannot construe semantic interpretations as objects. Williamson's argument can be extended to show that, as long as properties are taken to be objects, one cannot construe semantic interpretations as pluralities either. But

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12. Assuming standard rather than Henkin semantics, it can be shown that there is an infinite set of sentences Γ and a sentence σ such that $\Gamma \vDash \sigma$ but for no finite $\Gamma_0 \subset \Gamma$, $\Gamma_0 \vDash \sigma$. Following Yi 2006, here is a proof sketch. Fix a binary predicate R. Let Γ be the set of sentences $\{R(c_n, c_{n+1}) \, : \, n \in \omega\}$, where $c_0, \ldots, c_n, \ldots$ are distinct singular constants. Then

$$\Gamma \vDash \exists vv \, \forall v_0 (v_0 \prec vv \rightarrow \exists v_1 (v_1 \prec vv \, \wedge \, R(v_0, v_1)))$$

However, there is no finite subset Γ_0 of Γ such that

$$\Gamma_0 \vDash \exists vv \, \forall v_0 (v_0 \prec vv \rightarrow \exists v_1 (v_1 \prec vv \, \wedge \, R(v_0, v_1)))$$

13. There is controversy over whether absolutely unrestricted quantification is possible (for arguments against the possibility of quantifying over absolutely everything see, e.g., Glanzberg 2004, Fine 2006, Glanzberg 2006, Hellman 2006, Parsons 2006, and Weir 2006). Since rejecting absolutely unrestricted quantification would remove the main difficulty with object singularism, weakening significantly the case for pluralism and for the plural conception of reality, I will assume that absolutely unrestricted quantification is possible.

this is the natural construal of the notion of an interpretation for the untyped pluralist.[14]

There is also a cardinality problem. In particular, untyped pluralism runs against a plural version of Cantor's Theorem. Speaking informally, if there is more than one object, there are more pluralities than objects. Thus, if properties are objects, there are more pluralities than properties. As a result, some intuitive interpretations of plural predicates are unavailable to the untyped pluralist. The details of these arguments against untyped pluralism are spelled out in Florio forthcoming. The important point here is that typed pluralism escapes both problems and thus appears to be the best pluralist option. Parallel arguments show that the property singularist too must construe properties as typed. This means, among other things, that both typed pluralism and property singularism have to live with the familiar expressive limitations associated with typed approaches. Notably, the notion of property employed in the semantics cannot be expressed in the object language.

To sum up, we have found reasons to reject two of the four competing semantic accounts of plurals under consideration. If we want to do justice to absolutely unrestricted quantification, or at least quantification over all sets, there is pressure to reject object singularism. Moreover, paradox and cardinality considerations undermine untyped pluralism. This leaves us with two contenders, property singularism and typed pluralism. In the next section, I will say more about them. After that, I will argue that property singularism is in good standing, defending it from a number of objections.

4. Property singularism and typed pluralism

Although it encompasses infinitely many types of entities, the hierarchical conception of reality underpinning the singularist semantics has

a simple structure: there are objects at the bottom (entities of type 0), then there are singular properties of objects (type 1), singular properties of singular properties of objects (type 2), and so on. Of course, one must countenance relational properties as well as monadic ones. This hierarchy corresponds to the hierarchy of type theory. Let us call a property of type n an nth-*order property*. A superscript will indicate the type of an entity in this hierarchy: objects will be indicated by the superscript 0 (e.g. x^0, y^0, z^0), first-order properties by the superscript 1 (e.g. x^1, y^1, z^1), and second-order properties by the superscript 2 (e.g. x^2, y^2, z^2).

The hierarchy associated with the plural conception of reality and underpinning typed pluralism is composed by the singular hierarchy outlined just above plus plural properties (indicated here by Greek letters: $\alpha, \beta, \ldots$), properties of plural properties, and so on. Fortunately, the semantic views under discussion appeal only to entities of low types. This is why we can avoid most issues concerning the exact construction and extent of the hierarchies.[15] However, some assumptions should be mentioned.

First, we may assume that genuinely plural properties are found only at the initial stage of the plural hierarchy. For our purposes, there is no need to introduce higher-order plural properties, i.e. higher-order properties instantiated jointly by many lower-level properties. Second, it is convenient to assume that a property can only be instantiated by (and can only relate) entities of lower types. But we do not require that all of its arguments be exactly of the type immediately below. In other words, we take the hierarchies to be cumulative. A related question concerns which types of entity can occupy a particular argument place. We call *exclusivity* the assumption that an argument place can be occupied by entities of one type only. Properties that satisfy exclusivity will be said to be *exclusive*, whereas properties that violate it will be said to be *flexible*. If the regimenting language is augmented with flexible

14. See McKay 2006, pp. 147-54, for a discussion of Williamson's argument from the perspective of untyped pluralism. Untyped pluralism construes semantic interpretations as pluralities in the sense that semantic interpretations are given by *some things*, i.e. ordered pairs coding the relevant semantic information.

15. The formal treatment in the Appendix makes clear which types of entities are involved in the two semantics and which semantic function they serve.

predicates (see section 2), flexible properties would naturally serve as the semantic values of those predicates.

As noted above, typed pluralism and property singualrism agree on the semantics for the singular fragment of the object language, taking a singular term to denote an object and a singular predicate to denote a singular property. For the pluralist, plural terms denote pluralities and plural predicates denote plural properties. For the property singularist, plural terms denote singular first-order properties and plural predicates denote singular second-order properties. However, the property singularist needs to impose two restrictions. The semantic values of plural terms should be confined to *non-empty* properties and those of plural predicates should be confined to second-order *extensional* properties. A second-order property is extensional just in case, if it is instantiated by a first-order property x^1, it is instantiated by any first-order property coextensive with x^1.[16] Without the extensionality requirement (24) would be consistent.

(24) $\exists vv_1 \, (C(vv_1) \, \wedge \, \exists vv_2 \, (vv_1 \approx vv_2 \, \wedge \, \neg C(vv_2)))$

But it should not be, since it regiments the following, inconsistent sentence.

(25) There are some people who cooperate but who are the same as some people who do not cooperate.

To illustrate the two semantics further, let us examine how they specify the interpretation of a basic plural predication, such as 'Plato and Aristotle disagree'. This sentence can be regimented as (26).

(26) $\exists vv \, (\forall v \, (v \prec vv \leftrightarrow (v = p \, \vee \, v = a)) \wedge \, D(vv))$

I will use subscripts to indicate the relativization of a notion to a given interpretation. For the typed pluralist, 'Plato and Aristotle disagree' is

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16. It is worth remarking that a parallel restriction would have to be introduced to extend the semantics to modal contexts. Following the majority view about the modal behavior of plurals, the additional restriction would, not surprisingly, confine the semantic values of plural terms to *modally rigid* properties.

true relative to a given interpretation I if and only if there are some things xx in the domain$_I$ such that anything is one of them just in case it is the denotation$_I$ of 'Plato' or the denotation$_I$ of 'Aristotle', and those things jointly instantiate the plural property α denoted$_I$ by 'disagree', i.e. $\alpha(xx)$.

For the property singularist, 'Plato and Aristotle disagree' is true relative to a given interpretation J if and only if there is a non-empty property x^1 such that x^1 is instantiated by the denotation$_J$ of 'Plato' and by the denotation$_J$ of 'Aristotle' (and by no other thing), and x^1 instantiates the second-order extensional property x^2 denoted$_J$ by 'disagree', i.e. $x^2(x^1)$.

Typed pluralism and property singularism characterize the notion of truth relative to an interpretation differently. But do they sanction different logics for plurals? The answer is negative: they are model-theoretically equivalent. As proved in the Appendix, property singularism yields the same relation of logical consequence for the regimenting language as does typed pluralism. Property singularism relies on resources that are deemed legitimate by the proponent of the plural conception of reality, who countenances both singular and plural properties. The result then shows that plural resources are semantically dispensable, even by the lights of the proponent of the plural conception. If property singularism is in good standing, we should conclude that the semantic challenge to the singular conception has been met.

5. Is property singularism in good standing?

The idea of using properties in connection with plurals is not new. It may already be found in Russell, and it has been suggested repeatedly since then (see Russell 1919, 1938; Higginbotham and Schein 1989; Dummett 1991; and Lønning 1997). However, these proposals differ from property singularism in a critical way. They pursue forms of REGIMENTATION SINGULARISM, employing the language of higher-order logic in the regimentation. As a result, they face a number of objections (see Yi 1999, 2005; Oliver and Smiley 2001, 2013). In contrast, property singularism rejects REGIMENTATION SINGULARISM and

takes plural predication at face value. It invokes properties only in the interpretation of the regimenting language.

Suppose that we had tried to use higher-order resources in the regimentation, paraphrasing plural terms as higher-order ones. As pointed out by Yi (1999, 2005), this would have had unpalatable consequences for the theory of properties. Consider this valid inference:

(27)

> Russell and Whitehead cooperate
> Russell and Whitehead are the authors of *Principia Mathematica*
> _____
> The authors of *Principia Mathematica* cooperate

According to the proposal under consideration, (27) would be regimented along the lines of (28).

(28)

> $$\exists X (\forall x \, (Xx \leftrightarrow (x = r \; \lor \; x = w)) \land C(X))$$
> $$\exists X (\forall Y \, (A(Y, p) \; \leftrightarrow \; Y = X) \land \forall x \, (Xx \; \leftrightarrow \; (x = r \; \lor \; x = w)))$$
> $$\overline{\exists X (\forall Y \, (A(Y, p) \leftrightarrow Y = X) \land C(X))}$$

But (28) is invalid. To be rendered valid it would have to be supplemented with the assumption that cooperating is an extensional second-order property.

(29) $\forall X \, \forall Y((C(X) \land \forall x \, (Xx \leftrightarrow Yx)) \rightarrow C(Y))$

Since the original inference is logically valid as it stands, i.e. without additional premises, (29) must be taken to be a logical truth. That is, one must assume that it is a matter of logic that cooperating is an extensional property. Parallel arguments involving other collective predicates in the language show that the assumption must extend to a great deal of plural properties. Yi concludes:

it is one thing to hold the extensional conception, quite another

to hold, more implausibly, that the truth of the conception rests on logic alone. [...] One cannot meet the objections [...] under the assumption that the property indicated by "*COOPERATE*" is one that Russell calls *extensional* (that is, a second-order property instantiated by any first-order property coextensive with one that instantiates it). This does not help unless the assumption holds by logic [...]. (Yi 2005, p. 475; see also Yi 1999, p. 173)

Be that as it may, property singularism is immune from this difficulty. As it would be easy to verify, property singularism validates (27) without having to regard the argument as enthymematic. This turns essentially on the extensionality requirement built into the semantics.

The requirement that plural predicates be interpreted by second-order extensional properties salvages the validity of (27), but is it acceptable? The property singularist does not have to embrace an extensional conception of properties, let alone embrace it as logically true. On semantic grounds, he just has to confine the semantic values of plural predicates to extensional properties. As it happens, there is no shortage of such properties. It follows from the principle of second-order comprehension that any second-order property can be *extensionalized*. In symbols, for any z^2:

(30) $\exists x^2 \, \forall y^1 \, (x^2(y^1) \leftrightarrow \exists z^1 (z^2(z^1) \land \forall x^0 (y^1(x^0) \leftrightarrow z^1(x^0))))$

So the problem is not whether extensional properties are available. Rather, it is whether one may impose a restriction on the semantic values of plural predicates that excludes non-extensional properties. But restrictions of this sort are ubiquitous in semantics and, as far as I can see, there is nothing extraordinary about the one at hand.

Below I would like to discuss four possible objections to property singularism. They concern respectively flexible predicates, the use of properties as semantic values of plural terms, Boolos's plural interpretation of second-order logic, and the use of irrelevant entities in semantics.

The first objection is based on some *prima facie* evidence for flexible

predicates. Consider these two sentences:

(31) (a) Russell and Whitehead wrote a book.
    (b) Wittgenstein wrote a book.

From them, it seems that one may infer (32).

(32) Russell and Whitehead wrote a book and Wittgenstein did too.

The possibility of coordination displayed in (32) gives some support to the claim that the predicate 'wrote a book' occurs univocally in (31a) and (31b) and thus should be assigned the same semantic value in both sentences. It might be thought that this threatens the property singularist, as he would interpret 'wrote a book' as a second-order property in (31a) and as a first-order property in (31b). However, this would be a problem only if it is assumed that all properties are exclusive.

 Does the predicate occur univocally in (31a) and (31b)? A number of other tests could be used to argue in favor of univocity. For example, one can seemingly quantify over both predicates.

(33) Wittgenstein did something that Russell and Whitehead did.

Moreover, the predicate 'wrote a book' can be used with disjunctive noun phrases with a plural and singular component, such as 'two famous British logicians or Wittgenstein':

(34) Two famous British logicians or Wittgenstein wrote a book.

Furthermore, both 'Russell and Whitehead (did)' and 'Wittgenstein (did)' would be appropriate answers to the question: 'Who wrote a book?'.[17]

 Trying to decide whether univocity holds, and thus whether the regimenting language should encompass flexible predicates, would take us too far afield. Fortunately, there is no need to resolve this issue here. The important point is that univocity can be upheld simply by rejecting

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17. Thanks to Ben Caplan and Alex Oliver here. For discussion, see Rumfitt 2005 as well as Oliver and Smiley 2013, chapter 4.

exclusivity and embracing flexible properties in the semantics. This is quite natural for the typed pluralist who can imitate what happens at the level of regimentation. But it is an option for the property singularist too. He would have to allow for flexible second-order properties that can take both objects and first-order properties as an argument. Though this is not common in standard presentations of type theory, it is consistent and it amounts to removing what Gödel has described as a 'superfluous restriction' of type theory (Gödel 1933). An alternative, less attractive response is of course to raise the type of singular terms to match those of plural ones. In any case, flexible predicates cut no ice in the dispute between the pluralist and the property singularist. For simplicity, the technical result in the Appendix is formulated in and for a language without flexible predicates, but I indicate how it can be extended to a language that contains them.

 Another objection to property singularism concerns the use of properties as semantic values of plural terms. One might think that this commits the property singularist to the implausible view that if Russell and Whitehead wrote *Principia*, then a property wrote *Principia*. But that would be mistaken: property singularism does not incur any consequence of this sort. To begin with, the result proved in the Appendix implies that the inference from 'Russell and Whitehead wrote *Principia*' to 'a property wrote *Principia*' is valid according to property singularism if and only if it is valid according to typed pluralism. However, it would be straightforward to verify that both semantics make the inference invalid. Once the distinction between SEMANTIC SINGULARISM and REGIMENTATION SINGULARISM has been properly recognized, it should be easier to see why this objection fails. The central point is that property singularism appeals to properties in the semantics, not in the regimentation.

 The third objection is based on Boolos's plural interpretation of monadic second-order logic. Boolos (1984, 1985) has famously pointed out that monadic second-order logic can be interpreted in a fragment of our plural regimenting language not involving plural predicates. This might suggest that plural resources are prior or better understood

than higher-order ones. From this perspective, property singularism is wrong-headed, as it interprets plurals by means of higher-order resources. However, there are obvious obstacles in extending Boolos's results to full second-order logic or higher-order logics (see, e.g., Lewis 1991, Rayo and Yablo 2001). Moreover, if the regimenting language contains collective plural predicates, the resort to higher-order resources in the metalanguage is compelling (see Williamson 2003, Yi 2006, and the discussion in section 3 above). This is acknowledged by the typed pluralist who, like the property singularist, has to make use of such resources in the semantics (see Florio forthcoming). So both the property singularist and the typed pluralist have to forgo the benefits of Boolos's program.

The last objection I would like to address is the charge that property singularism introduces irrelevant entities in the semantics. The mere possibility of interpreting plural terms as standing for properties—the objection goes—does not show that this is in fact how they should be interpreted. Consider the parallel case of first-order languages. In light of familiar metatheoretic results, we know that countable models suffice to characterize the relation of first-order consequence. This means that a semantics for such languages can assume that the quantifiers range only over natural numbers. That is, from the semantic point of view, 'we can ... think of all our objects as natural numbers' (Quine 1968, p. 207): we can think of singular terms as standing for natural numbers and we can think of predicates as standing for sets of natural numbers. Quine asked: 'May we not thus settle for an all-purpose Pythagorean ontology outright?' Of course, the mere possibility of interpreting the object language by means of entities of a certain kind does not show that such entities encompass all there is. What this possibility does show, however, is that the introduction of entities of other kinds cannot be justified on purely semantic grounds. Going back to the case of plurals, the possibility of interpreting plural terms as singular properties (and plural predicates as second-order properties) shows that semantics alone cannot sustain the plural conception of reality. This shifts the debate to metaphysics.

It might be that there are strong metaphysical considerations in favor of the plural conception. It seems to me, though, that there is no clear basis for optimism. A proper discussion of the metaphysical debate cannot be articulated here but, in closing, I would like to make at least some brief general remarks. An important caveat has to do with expressibility problems of the kind gestured at above. If quantification over absolutely everything is possible, then there is pressure to assume a type distinction between objects and properties. Properties and facts become higher-order entities that can be expressed in the expanded language used in the metatheory but not in the object language. That means that ordinary talk about these metaphysical entities has to be taken with a grain of salt, as it will ultimately have to be recast in higher-order terms. This is the same grain of salt Frege was hoping his reader would not begrudge.

Can the plural conception be vindicated on metaphysical grounds? Since the singular conception is compatible with a wide array of views about what reality is like, a metaphysical case for the plural conception requires the rejection of a number of alternative ways of specifying what there ultimately is. A prime example is Lewis's Humean Supervenience (see Lewis 1986, pp. vii-xv, Lewis 1994, and Weatherson forthcoming). On this view, all contingent facts can be characterized in terms of the distribution of perfectly natural, intrinsic properties of space-time points or occupants of points, where these properties are all singular. If plural properties are absent from the characterization of contingent facts, it is doubtful that they will have to be invoked in the characterization of non-contingent facts. This indicates that the proponent of the plural conception cannot expect an easy victory on the metaphysical front.

It should be stressed that accepting the particular reduction suggested by Lewis's Humean Supervience is not compulsory. To undermine the metaphysical case for the plural conception, it is enough that there be *one* way of reducing plural properties and facts. In several paradigm cases, it is relatively straightforward to see how the first step of a reduction might go. For instance, the fact that the natural numbers

are infinite could be reduced to the fact that the property of being a natural number is equinumerous with a proper subproperty of itself. The fact that two lines are parallel could be reduced to the fact that their distance is, in the appropriate sense, always the same. The fact that two individuals are shipmates could be reduced to the fact that they sail on the same boat. Other cases will no doubt prove more difficult, but nothing indicates that any difficulty would be insurmountable and that all reductionist strategies are bound to fail. In the end, the details of the reductionist project will depend on one's metaphysical outlook. What is important to keep in mind, however, is that in carrying out the reduction one does not seek to provide a singular paraphrase of plural sentences. As we have seen, there is little hope for that. What one seeks to do is to articulate a view of what reality is like in which plural properties play no fundamental role. Once this task is disentangled from the other, the prospects of success will certainly look more favorable.

6. Conclusion

Semantic considerations about plurals offer an indispensability argument in favor of the plural conception of reality. Plurals do indeed pose a semantic challenge for those who subscribe to the singular conception. Traditional responses to the challenge have relied on Regimentation Singularism, which is problematic. However, adopting a regimenting language that distinguishes between syntactically singular and syntactically plural expressions is compatible with the view that the interpretations for this language can be specified adequately within the boundaries of the singular conception.

Four competing semantic accounts of plurals were examined: object singularism, property singularism, untyped pluralism, and typed pluralism. If one admits the possibility of quantification over absolutely everything, property singularism and typed pluralism appear to be the most promising views. Typed pluralism invokes plural properties, whereas property singularism does not. Nonetheless, as proved in the Appendix, the two accounts are model-theoretically equivalent, delivering the same relation of logical consequence for the common regi-

menting language.

As I have argued, property singularism is in good standing as a semantic account of plurals. Since it employs resources that are deemed legitimate by those who embrace the plural conception, the equivalence between property singularism and typed pluralism is significant. It shows that, even by the lights of the proponent of the plural conception, plural properties are semantically dispensable. Of course, this does not exclude that there might be pressure in favor of the plural conception arising from other domains and in particular from metaphysics. Although this is doubtful, the point remains that the singular conception of reality is rich enough to provide a viable semantics for plurals. Semantic considerations alone do not give us reasons to abandon it.[18]

Appendix

After introducing some preliminary notions (Appendix A), I provide a more rigorous presentation of typed pluralism (Appendix B1) and property singularism (Appendix B2). Under minimal assumptions about the relationship between pluralities and properties, I then prove

that the two approaches yield the same relation of logical consequence for the common regimenting language (Appendix C).

Appendix A: Preliminary notions

The notation for entities of various types was introduced in section 4. Some abbreviations will be convenient:

$$x^1 \sqsubseteq y^1 \leftrightarrow_{def} \forall z^0 (x^1(z^0) \rightarrow y^1(z^0))$$

$$xx \trianglelefteq x^1 \leftrightarrow_{def} \forall z^0 (z^0 \prec xx \rightarrow x^1(z^0))$$

$$xx \equiv x^1 \leftrightarrow_{def} \forall z^0 (z^0 \prec xx \leftrightarrow x^1(z^0))$$

$$\alpha \bowtie^{n[m]} x^2 \leftrightarrow_{def} \forall xx_1, \ldots, xx_m \, \forall x_1^1, \ldots, x_m^1 \, \forall x_1^0, \ldots, x_{n-m}^0$$
$$((\alpha(xx_1, \ldots, xx_m, x_1^0, \ldots, x_{n-m}^0) \, \& \, xx_1 \equiv x_1^1 \, \& \, \ldots \, \& \, xx_m \equiv x_m^1) \rightarrow$$
$$x^2(x_1^1, \ldots, x_m^1, x_1^0, \ldots, x_{n-m}^0))$$

$$\alpha \ltimes^{n[m]} x^2 \leftrightarrow_{def} \forall x_1^1, \ldots, x_m^1 \, \forall xx_1, \ldots, xx_m \, \forall x_1^0, \ldots, x_{n-m}^0$$
$$((x^2(x_1^1, \ldots, x_m^1, x_1^0, \ldots, x_{n-m}^0) \, \& \, xx_1 \equiv x_1^1 \, \& \, \ldots \, \& \, xx_m \equiv x_m^1) \rightarrow$$
$$\alpha(xx_1, \ldots, xx_m, x_1^0, \ldots, x_{n-m}^0))$$

$$\alpha \bowtie^{n[m]} x^2 \leftrightarrow_{def} \alpha \bowtie^{n[m]} x^2 \, \& \, \alpha \ltimes^{n[m]} x^2$$

Let us define the basic semantic notions. First, we have relations of interpretation (I) and variable assignment (A) between constants or variables of the regimenting language and entities populating the universe of the metatheory. Since interpretations and assignments are functional, we may use the square bracket notation $[\![s]\!]_R$ to indicate the unique entity or entities assigned to s by R. The typed pluralist and the property singularist agree on the semantics of the singular fragment of the language. They disagree on the semantics of the plural fragment. So interpretations of singular constants (I_0) and interpretations of singular predicates (I_1), as well as assignments for singular variables (A_0),

will be the same for both accounts. These semantic relations are characterized by the following conditions on I_0, I_1, and A_0, relative to a domain of quantification given by a singular property d^1.[19]

1. For any singular constant c, there is x^0 such that $[\![c]\!]_{I_0} = x^0$ and $d^1(x^0)$.
2. For any singular predicate S, there is x^1 such that $[\![S]\!]_{I_1} = x^1$ and, for every $z_1^0, \ldots, z_n^0$, if $x^1(z_1^0, \ldots, z_n^0)$, then $d^1(z_1^0), \ldots, d^1(z_n^0)$.
3. For any singular variable v, there is x^0 such that $[\![v]\!]_{A_0} = x^0$ and $d^1(x^0)$. A variant $A_0(v/y^0)$ of A_0 with respect to v is defined in the usual way.

According to typed pluralism, interpretations of plural predicates (I_2) assign a plural property to each plural predicate. Variable assignments for plural variables (A_1) assign some things in the domain to each plural variable. That is, relative to a domain d^1, we have the following conditions on I_2 and A_1.

4. For any plural predicate $P^{n[m]}$, there is α such that $[\![P^{n[m]}]\!]_{I_2} = \alpha$ and, for every $zz_1, \ldots, zz_m, z_1^0, \ldots, z_{n-m}^0$, if $\alpha(zz_1, \ldots, zz_m, z_1^0, \ldots, z_{n-m}^0)$, then $zz_1 \trianglelefteq d^1, \ldots, zz_m \trianglelefteq d^1$ and $d^1(z_1^0), \ldots, d^1(z_{n-m}^0)$.
5. For any plural variable vv, there are xx such that $[\![vv]\!]_{A_1} \approx xx$ and $xx \trianglelefteq d^1$. A variant $A_1(vv/yy)$ of A_1 with respect to vv is also defined in the usual.

Next we have counterparts of these semantic relations for the property singularist, i.e. interpretations of plural predicates (I_2^*) assigning

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19. For the typed pluralist, domains of quantifications are more naturally given by some *things*, those constituting the domain. We could accommodate this option but defining domains in terms of singular properties simplifies the exposition. That does not do violence to typed pluralism, since the typed pluralist accepts the existence of singular properties.

a second-order extensional property to each plural predicate, and variable assignments for plural variables (A_1^*) assigning a non-empty singular property to each plural variable.

4^*. For any plural predicate $P^{n[m]}$, there is an extensional x^2 such that $[\![P^{n[m]}]\!]_{I_2^*} = x^2$ and, for every $z_1^1, \ldots, z_m^1, z_1^0, \ldots, z_{n-m}^0$, if $x^2(z_1^1, \ldots, z_m^1, z_1^0, \ldots, z_{n-m}^0)$, then $z_1^1 \sqsubseteq d^1, \ldots, z_m^1 \sqsubseteq d^1$ and $d^1(z_1^0), \ldots, d^1(z_{n-m}^0)$.

5^*. For any plural variable vv, there is a non-empty x^1 such that $[\![vv]\!]_{A_1^*} = x^1$ and $x^1 \sqsubseteq d^1$. A variant $A_1^*(vv/y^1)$ of A_1^* with respect to vv must satisfy the condition that y^1 be non-empty.

An additional abbreviation will be useful. For any singular term t,

$$[\![t]\!]_{A_0/I_0} = \begin{cases} [\![t]\!]_{A_0} & \text{if } t \text{ is a variable,} \\ [\![t]\!]_{I_0} & \text{if } t \text{ is a constant.} \end{cases}$$

We are now ready to formulate the two semantics.

Appendix B1: Typed pluralism

The notion of satisfaction according to the typed pluralist, $\mathbf{Sat}_{TP}(\varphi, I_0, I_1, I_2, A_1, A_2)$ (rewritten as $I_0, I_1, I_2 \vDash_{TP} \varphi\ [A_0, A_1]$), is implicitly defined by the following clauses relative to interpretations and assignments all defined on the same domain d^1.

(i) If φ is of the form $t = s$, $I_0, I_1, I_2 \vDash_{TP} \varphi\ [A_0, A_1]$ if and only if $[\![t]\!]_{A_0/I_0} = [\![s]\!]_{A_0/I_0}$.

(ii) If φ is of the form $t \prec vv$, $I_0, I_1, I_2 \vDash_{TP} \varphi\ [A_0, A_1]$ if and only $[\![t]\!]_{A_0/I_0} \prec [\![vv]\!]_{A_1}$.

(iiia) If φ is of the form $S^n(t_1, \ldots, t_n)$, $I_0, I_1, I_2 \vDash_{TP} S^n(t_1, \ldots, t_n)\ [A_0, A_1]$ if and only if

$$[\![S^n]\!]_{I_1}([\![t_1]\!]_{A_0/I_0}, \ldots, [\![t_n]\!]_{A_0/I_0}).$$

(iiib) If φ is of the form $P^{n[m]}(vv_1, \ldots, vv_m, t_1, \ldots, t_{n-m})$,

$$I_0, I_1, I_2 \vDash_{TP} P^{n[m]}(vv_1, \ldots, vv_m, t_1, \ldots, t_{n-m})\ [A_0, A_1] \quad \text{if and only if}$$
$$[\![P^{n[m]}]\!]_{I_2}([\![vv_1]\!]_{A_1}, \ldots, [\![vv_m]\!]_{A_1}, [\![t_1]\!]_{A_0/I_0}, \ldots, [\![t_{n-m}]\!]_{A_0/I_0}).$$

(iv) If φ is of the form $\exists v\ \psi$, $I_0, I_1, I_2 \vDash_{TP} \varphi\ [A_0, A_1]$ if and only if, for some x^0 such that $d^1(x^0)$, $I_0, I_1, I_2 \vDash_{TP} \psi\ [A_0(v/x^0), A_1]$.

(v) If φ is of the form $\exists vv\ \psi$, $I_0, I_1, I_2 \vDash_{TP} \varphi\ [A_0, A_1]$ if and only if, for some xx such that $xx \unlhd d^1$, $I_0, I_1, I_2 \vDash_{TP} \psi\ [A_0, A_1(vv/xx)]$.

(vi-xi) The clauses for negation and for the binary connectives are the obvious ones.

The last step is to characterize the notions of logical consequence and logical truth. For any set of sentences Γ and any sentence σ, σ is a logical consequence of Γ according to typed pluralism ($\Gamma \vDash_{TP} \sigma$) just in case for any interpretations I_0, I_1, I_2 and assignments A_0, A_1, if $I_0, I_1, I_2 \vDash_{TP} \gamma\ [A_0, A_1]$ for any member γ of Γ, then $I_0, I_1, I_2 \vDash_{TP} \sigma\ [A_0, A_1]$.

Appendix B2: Property singularism

The notion of satisfaction according to the property singularist, $\mathbf{Sat}_{PS}(\varphi, I_0, I_1, I_2^*, A_1, A_2^*)$ (rewritten as $I_0, I_1, I_2^* \vDash_{PS} \varphi\ [A_0, A_1^*]$), is also given an implicit definition with respect to interpretations and assignments defined on the same domain d^1. The satisfaction clauses for the singular fragment of the language are the same as those of typed pluralism. The clauses for the plural fragment of the language are as follows.

(ii*) If φ is of the form $t \prec vv$, $I_0, I_1, I_2^* \vDash_{PS} \varphi\ [A_0, A_1^*]$ if and only if $[\![vv]\!]_{A_1^*}([\![t]\!]_{A_0/I_0})$.

    - 17 -

(iiib*) If φ is of the form $P^{n[m]}(vv_1, \ldots, vv_n, t_1, \ldots, t_{n-m})$,

$$I_0, I_1, I_2^* \vDash_{\text{PS}} \varphi \, [A_0, A_1^*] \quad \text{if and only if}$$
$$[\![P^{n[m]}]\!]_{I_2}([\![vv_1]\!]_{A_1^*}, \ldots, [\![vv_m]\!]_{A_1^*}, [\![t_1]\!]_{A_0/I_0}, \ldots, [\![t_{n-m}]\!]_{A_0/I_0}).$$

(v*) If φ is $\exists vv \, \psi$, $I_0, I_1, I_2^* \vDash_{\text{PS}} \varphi \, [A_0, A_1^*]$ if and only if, for some non-empty $x^1 \sqsubseteq d^1$, $I_0, I_1, I_2^* \vDash_{\text{PS}} \psi \, [A_0, A_1^*(vv/x^1)]$.

The notions of logical consequence ($\Gamma \vDash_{\text{PS}} \sigma$) and logical truth according to property singularism are then characterized in the obvious way by quantifying over interpretations.

Appendix C: The equivalence

For any two interpretations I_2 and I_2^* and for any two assignments A_1 and A_1^* all defined on the same domain, the notation $R(I_2, I_2^*; A_1, A_1^*)$ will abbreviate the following: for any plural predicate of the language $P^{n[m]}$, $[\![P^{n[m]}]\!]_{I_2} \bowtie^{n[m]} [\![P^{n[m]}]\!]_{I_2^*}$ and, for any plural variable vv, $[\![vv]\!]_{A_1} \equiv [\![vv]\!]_{A_1^*}$.

Here are the assumptions used to prove the main result below. They sanction a correspondence in extension between pluralities and non-empty singular first-order properties, and between plural properties and singular second-order ones.

(1) $\forall xx \, \exists x^1 \, xx \equiv x^1$
(2) $\forall x^1 (\exists x^0 \, x^1(x^0) \rightarrow \exists xx \, xx \equiv x^1)$
(3) For every n, m such that $n \geq m \geq 1$, $\forall \alpha \, \exists x^2 \, \alpha \bowtie^{n[m]} x^2$
(4) For every n, m such that $n \geq m \geq 1$, $\forall x^2 \, \exists \alpha \, \alpha \bowtie^{n[m]} x^2$

These assumptions are certainly problematic if properties are construed as objects. In that case, a plural version of Cantor's Theorem would bar the assumptions. But once properties are typed, the obstacle is removed.

Lemma 1. *For any formula φ, interpretations I_2 and I_2^*, and assignments A_1 and A_1^*, if $R(I_2, I_2^*; A_1, A_1^*)$, then for every I_0, I_1 and A_0,*

$$I_0, I_1, I_2 \vDash_{\text{TP}} \varphi \, [A_0, A_1] \quad \text{if and only if} \quad I_0, I_1, I_2^* \vDash_{\text{PS}} \varphi \, [A_0, A_1^*].$$

Proof. By induction on the complexity of φ. If φ is singular, the claim is trivial. If φ is of the form $t \prec vv$, then:

$$\begin{aligned}
&I_0, I_1, I_2 \vDash_{\text{TP}} \varphi \, [A_0, A_1] &\Leftrightarrow \\
&[\![t]\!]_{A_0/I_0} \prec [\![vv]\!]_{A_1} &\Leftrightarrow \quad (\text{since } R(I_2, I_2^*; A_1, A_1^*), [\![vv]\!]_{A_1} \equiv [\![vv]\!]_{A_1^*}) \\
&[\![vv]\!]_{A_1^*}([\![t]\!]_{A_0/I_0}) &\Leftrightarrow \\
&I_0, I_1, I_2^* \vDash_{\text{PS}} \varphi \, [A_0, A_1^*]
\end{aligned}$$

If φ is of the form $P^{n[m]}(vv_1, \ldots, vv_m, t_1, \ldots, t_{n-m})$, then:

$$\begin{aligned}
&I_0, I_1, I_2 \vDash_{\text{TP}} P^{n[m]}(vv_1, \ldots, vv_m, t_1, \ldots, t_{n-m}) \, [A_0, A_1] &\Leftrightarrow \\
&[\![P^{n[m]}]\!]_{I_2}([\![vv_1]\!]_{A_1}, \ldots, [\![vv_m]\!]_{A_1}, [\![t_1]\!]_{A_0/I_0}, \ldots, [\![t_{n-m}]\!]_{A_0/I_0}) &\Leftrightarrow \\
&\quad (\text{since } R(I_2, I_2^*; A_1, A_1^*), [\![vv_i]\!]_{A_1} \equiv [\![vv_i]\!]_{A_1^*} \text{ and } [\![P^{n[m]}]\!]_{I_2} \bowtie^{n[m]} [\![P^{n[m]}]\!]_{I_2^*}) \\
&[\![P^{n[m]}]\!]_{I_2^*}([\![vv_1]\!]_{A_1^*}, \ldots, [\![vv_m]\!]_{A_1^*}, [\![t_1]\!]_{A_0/I_0}, \ldots, [\![t_{n-m}]\!]_{A_0/I_0}) &\Leftrightarrow \\
&I_0, I_1, I_2^* \vDash_{\text{PS}} P^{n[m]}(vv_1, \ldots, vv_m, t_1, \ldots, t_{n-m}) \, [A_0, A_1^*]
\end{aligned}$$

Finally, if φ is of the form $\exists vv \, \psi$ and the claim holds for ψ, then:

$$\begin{aligned}
&I_0, I_1, I_2 \vDash_{\text{TP}} \exists vv \, \psi \, [A_0, A_1] &\Leftrightarrow \\
&\text{for some } xx \trianglelefteq d^1, I_0, I_1, I_2 \vDash_{\text{TP}} \psi \, [A_0, A_1(vv/xx)] &\Leftrightarrow \\
&&(\text{by (1) and (2)}) \\
&\text{for some non-empty } x^1 \sqsubseteq d^1, I_0, I_1, I_2^* \vDash_{\text{PS}} \psi \, [A_0, A_1^*(vv/x^1)] &\Leftrightarrow \\
&I_0, I_1, I_2^* \vDash_{\text{PS}} \exists vv \, \psi \, [A_0, A_1^*]
\end{aligned}$$

The other clauses are straightforward. So this completes the induction.
$\square$

On the basis of (1)–(4), the following is clear.

Lemma 2. *For any I_2 and A_1 defined on the same domain, there are I_2^* and A_1^* such that $R(I_2, I_2^*; A_1, A_1^*)$. Conversely, for any I_2^* and A_1^* defined on the*

same domain, there are I_2 and A_1 such that $R(I_2, I_2^*; A_1, A_1^*)$.

Now we can prove the result we are after.

Proposition. *For any sentence σ and any set of sentences Γ,*

$$\Gamma \vDash_{\text{TP}} \sigma \text{ if and only if } \Gamma \vDash_{\text{PS}} \sigma.$$

Proof. Suppose that $\Gamma \vDash_{\text{TP}} \sigma$. We want to show that $\Gamma \vDash_{\text{PS}} \sigma$. Let $I_0, I_1, I_2^*, A_0, A_1^*$ be arbitrary interpretations and assignments such that $I_0, I_1, I_2^* \vDash_{\text{PS}} \gamma \, [A_0, A_1^*]$ for every member γ of Γ. By Lemma 2, $R(I_2, I_2^*; A_1, A_1^*)$ for some I_2 and A_1. By Lemma 1, $I_0, I_1, I_2 \vDash_{\text{TP}} \gamma \, [A_0, A_1]$ for every member γ of Γ. But $\Gamma \vDash_{\text{TP}} \sigma$, hence $I_0, I_1, I_2 \vDash_{\text{TP}} \sigma \, [A_0, A_1]$. Therefore, by Lemma 1 again, $I_0, I_1, I_2^* \vDash_{\text{PS}} \sigma \, [A_0, A_1^*]$. Since $I_0, I_1, I_2^*, A_0, A_1^*$ are arbitrary, $\Gamma \vDash_{\text{PS}} \sigma$. The other direction is proved similarly. $\square$

A final point: the main result has been formulated for a regimenting language without flexible predicates, but it can be extended to a language that contains them. The details would be vexing, so I will just state the assumptions needed to prove the result for a language with monadic flexible predicates. In parallel with (3) and (4) above, we need to assume a correspondence between flexible plural properties and flexible singular ones, i.e.

(3*) $\forall \alpha \, \exists x^2 \, \forall xx \, \forall x^1 \, \forall x^0 ((\alpha(xx) \,\&\, xx \equiv x^1 \to x^2(x^1)) \,\&$
$$(\alpha(x^0) \to x^2(x^0)))$$

(4*) $\forall x^2 \, \exists \alpha \, \forall xx \, \forall x^1 \, \forall x^0 ((x^2(x^1) \,\&\, xx \equiv x^1 \to \alpha(xx)) \,\&$
$$(x^2(x^0) \to \alpha(x^0)))$$

The polyadic case is a straightforward extension of the monadic one.

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