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Wilson, Alastair

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# Everettian Quantum Mechanics Without Branching Time

Alastair Wilson Monash University

#### **ABSTRACT:**

In this paper I assess the prospects for combining contemporary Everettian quantum mechanics (EQM) with branching-time semantics in the tradition of Kripke, Prior, Thomason and Belnap. I begin by outlining the salient features of 'decoherence-based' EQM, and of the 'consistent histories' formalism that is particularly apt for conceptual discussions in EQM. This formalism permits of both 'branching worlds' and 'parallel worlds' interpretations; the metaphysics of EQM is in this sense underdetermined by the physics. A prominent argument due to David Lewis [1986] supports the non-branching interpretation. Belnap et al. [2001] refer to Lewis' argument as the 'Assertion problem', and propose a pragmatic response to it. I argue that their response is unattractively ad hoc and complex, and that it prevents an Everettian who adopts branching-time semantics from making clear sense of objective probability. The upshot is that Everettians are better off without branching-time semantics. I conclude by discussing and rejecting an alternative possible motivation for branching time.

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# 1. Introduction

Over the last twenty years, Everettian quantum mechanics (for short: EQM) has been steadily gaining in popularity both amongst physicists and amongst philosophers. Its principal appeal derives from its status as the most conservative realist response to the quantum measurement problem. It is also presupposed by almost all contemporary work in quantum cosmology.

A variety of metaphors have been used to explicate EQM. Everett himself referred to 'splitting' of worlds. Others have preferred to talk of 'branching worlds', or of 'parallel worlds', or of 'relativity of value-definiteness'. The 'branching' metaphor has perhaps been the most prominent; and so it is not surprising that EQM has frequently been associated with branching-time semantics, of the sort invented by Kripke and Prior and advocated, in different ways, by Belnap, Thomason, McCall, Placek, Müller and others. What could be more appropriate for a branching ontology than a branching semantics?

Authors who recommend branching-time semantics to the Everettian include David Wallace<sup>1</sup> (Wallace [2005]), Nuel Belnap and Thomas Müller (Belnap & Müller [2010]). But adopting branching-time semantics is not compulsory for Everettians. And in fact the rationale for doing so, on closer inspection, looks decidedly murky. In this paper I argue that Everettians can and should steer clear of branching-time semantics.

I begin in section 2 with a short introduction to the salient features of EQM, and in particular to the version of EQM associated with the 'Oxford Everettians' - David Deutsch, Simon Saunders, David Wallace and Hilary Greaves. The 'consistent histories' formalism that is particularly apt for conceptual discussions in EQM is briefly set out in section 3. I argue in section 4 that it permits of both branching and non-branching interpretations: the metaphysics of EQM is in this sense underdetermined by the physics. A prominent argument from the metaphysics literature, due to David Lewis, supports the use of the non-branching interpretation. Belnap et al. [2001] refer to this argument as the 'Assertion problem' and propose a pragmatic solution to it. In section 5 I raise some objections to this response, and in section 6 I argue that the problem Lewis highlights prevents Everettians who adopt branching-time semantics from making any making clear sense of objective probability. The upshot is that Everettians are better off without branching-time semantics. Section 7 discusses and rejects an alternative way of motivating branching-time semantics; section 8 is a conclusion.

 $<sup>^{1}\,</sup>$  It should be noted that Wallace is elsewhere neutral regarding branching-time semantics.

### 2. Decoherence-based Everettian Quantum Mechanics

The basic idea behind EQM will be familiar to most readers. The theory was conceived in response to the quantum measurement problem: although transition probabilities calculated using the mathematical apparatus of quantum mechanics provide astonishingly accurate matches to the statistical results of experiments, the procedure actually used by physicists when applying the formalism (call it the *quantum algorithm*) makes essential use of the operational concept of a measurement. Everett's central idea (Everett [1955]) was that when observers are modelled correctly within the theory, as fully quantum systems, it will turn out that *from the perspective of an observer* the quantum algorithm is a reliable way to make predictions. The success of the quantum algorithm is thus explained perspectivally; no extra fundamental structure must be added to quantum theory to make sense of our observations. Or so it is claimed.

Although Everett himself never acknowledged it, a difficulty known as the *preferred basis problem* has loomed large in subsequent work on EQM. Everett's formulation of the theory presupposed a preferred decomposition of the wavefunction – essentially, a specification of a set of physically 'special' observable quantities – which seems to be unwarranted by the quantum formalism, which runs contrary to the spirit of the theory, and which undermines its claim to Lorentz covariance. An approach to quantum mechanics which needs to stipulate a preferred basis (or 'interpretation basis' as it was dubbed by Deutsch [1985]) is not the clean and conservative solution to the measurement problem that Everett seems to have thought it was.

EQM suffered for a time from the perception, encouraged by many of its advocates, that some additional ingredient had to be added to the theory to make it coherent. The most significant step towards a plausible version of EQM came when, in the early 1990s, progress in technical work on *decoherence* was applied to the preferred basis problem. Decoherence theory is a way of modelling the quantum-mechanical interactions between a system and its environment; Zurek [2003] gives an accessible introduction. All quantum systems rapidly become entangled with their environment; in decoherence models, this is represented by the rapid diminishing of the coefficients of the cross-diagonal, or 'interference' terms. Everettians interpret this effect as the suppression of interactions between components of the state of the system which correspond to distinct macroscopic states of affairs. This suppression happens extremely quickly; simple models have it occurring at the order of  $10^{-20}$  s (Zurek [2003]).

The environment-induced suppression of interference described by decoherence was first applied to the preferred basis problem by Simon Saunders (Saunders [1993], [1995]). The idea is that a preferred basis is *approximately* selected by decoherence, to a degree of approximation easily high enough to

explain the fact that superpositions of macroscopic states are unobserved and (at least for all practical purposes) unobservable. If we consider some individual component (in the decoherence basis) of the state, and focus on its forwards time-evolution, then the other components of the state have a negligible influence; it makes no significant difference to our calculated transition probabilities whether we consider the other components.

Wallace [2012] gives a comprehensive statement and a sustained defence of a decoherence-only version of EQM. Although it remains controversial whether decoherence is sufficient to resolve the preferred basis problem, there is a growing consensus that the success of EQM depends on it; 'many worlds' or 'many minds' theories which posited additional fundamental structure would not be worth the price. In this paper I will be assuming a decoherence-only version of EQM.

One of the most interesting features of the multiverse which emerges from decoherence is that it is approximately defined. In particular, there is a sense in which individual histories are *indeterminate in number*. Here is how Wallace puts the point<sup>2</sup>:

There is no sense in which [chaotic] phenomena lead to a naturally discrete branching process: ... while a branching structure can be discerned in such systems it has no natural "grain". To be sure, by choosing a certain discretisation of (configuration-)space and time, a discrete branching structure will emerge, but a finer or coarser choice would also give branching. And there is no "finest" choice of branching structure: as we fine-grain our decoherent history space, we will eventually reach a point where interference between branches ceases to be negligible, but there is no precise point where this occurs. As such, the question "how many branches are there?" does not, ultimately, make sense.

#### Wallace [2010] p.67-68

Advocates of EQM use a variety of terminology to express this aspect of the theory: ascriptions of history number are 'interest-relative' (Saunders [1998] p.313), are 'arbitrary conventions' (Saunders [2010] p.12), are subject to 'some indeterminacy' (Wallace [2010] p.68), are 'not well-defined' (Greaves [2004] or '[presuppose] the existence of a piece of structure which is not in fact present in the theory' (Greaves [2007]). However we choose to express it, such imprecision has not generally been allowed for within proposed systems of branching-time

<sup>&</sup>lt;sup>2</sup> Wallace's terminology in this quote differs from my own usage, which I clarify in section 4 below. By 'branches' Wallace means those entities represented by decoherent consistent histories; and by 'branching' he means the process by which multiplicity of histories emerges.

semantics; and we might wonder whether this leads to complications in combining such systems with EQM. I will not, however, press this concern any further here.

The preferred basis problem concerns ontology. The other major difficulty faced by EQM concerns probability; it can be factored into three components.

- The Incoherence problem: what does objective probability consist in, according to Everettian quantum mechanics?
- The Quantitative problem: why should objective probability be given by the Born rule, according to Everettian quantum mechanics?
- The Epistemic problem: why is our statistical evidence evidence for quantum mechanics, according to Everettian quantum mechanics?

Although the latter two problems are somewhat incidental to the concerns of this paper, the Incoherence problem will play a central role in the argument. Branching-time semantics treats sentences about the future as *incomplete*: they are only assigned truth-values relative to particular histories. But probabilities are fundamentally probabilities *of* propositions, and in order to attach a probability to a future outcome we need to be able to make sense of the unrelativized proposition *that the outcome will occur*. Branching-time semantics does not allow us to do so.

Branching-time theorists have typically been concerned to account for qualitative modality, rather than for quantitative modality<sup>3</sup>, so the problem with probability has mostly lurked in the background. But in the Everettian case, probability takes centre stage, and the status of propositions about the future has been central to recent discussions. In section 6 I will argue that Everettians should reject branching-time semantics as part of a solution to the probability problem. Before that, some more setup is needed. The next two sections present a particularly convenient formalism for EQM, and discuss what metaphysical interpretation it should be given.

#### 3. Consistent Histories

The consistent histories formalism, developed in the 1980s by Griffiths, Omnes, Gell-Mann and Hartle, will be useful for framing the metaphysical questions I want to pursue. This formalism describes quantum processes in terms of the *Heisenberg picture*: the quantum state is represented as constant, with

<sup>&</sup>lt;sup>3</sup> There are, of course, exceptions. See e.g. Muller [2005] and Weiner & Belnap [2006]. Belnap (personal communication) is hopeful that branching space-time models may be able to accommodate probabilities in a way in which branching time models cannot.

operators representing observable quantities evolving over time. (This contrasts with the *Schrödinger picture*, according to which the operators are timeinvariant while the state – the wavefunction – evolves over time.) The Heisenberg picture allows quantum theory to be cast in the familiar and convenient Hamiltonian form; but the two pictures give the same expectation values for all observables, so the choice between them is usually regarded as having no physical or metaphysical significance.

In the consistent histories formalism, then, the set of possible alternatives for the state of any system at a time are represented by a set of orthogonal projection operators  $\widehat{P}_{\alpha}$ , summing to unity; a so-called 'resolution of the identity':

$$\begin{split} \widehat{P}_{\alpha}\widehat{P}_{\beta} &= \delta_{\alpha\beta} \ \widehat{P}_{\alpha} \\ &\sum_{\alpha} \widehat{P}_{\alpha} = 1 \end{split}$$

These two conditions ensure that the projection operators  $\hat{P}_{\alpha}$  represent an exclusive and exhaustive set of propositions, which partition the possible states of a system at a time. This is to say that the propositions they represent are pair-wise incompatible, and that for every possible state of the system there is a corresponding proposition. Such a partition is called a *coarse-graining*.

Time is likewise modelled as discrete. Given a sequence of times  $t_1 \ ; \ ... \ ;$  $t_n$ , the time-dependent projection operators  $\widehat{P}_{\alpha,n}$   $(t_n)$  are related to the  $\widehat{P}_{\alpha}$  as follows:

$$\widehat{P}_{\alpha_{n}}(t_{n}) = e^{iH(t_{n}-t_{0})} \widehat{P}_{\alpha_{n}} e^{-iH(t_{n}-t_{0})}$$

(where H is the Hamiltonian of the system,  $t_n$  labels the time,  $\alpha$  labels the history, and  $t_0 < t_1 < \ldots < \ t_{n-1}, < t_n).$ 

Quantum histories are time-ordered sequences of time-dependent projection operators:

$$C_{\underline{\alpha}} = \widehat{P}_{\alpha_{n}}(t_{n}) \, \widehat{P}_{\alpha_{n-1}}(t_{n-1}) \dots \, \widehat{P}_{\alpha_{0}}(t_{0})$$

Since each  $\hat{P}_{\alpha}$  corresponds to a proposition about the system, a history attributes some property to the system at each time  $t_n$ .

From an appropriate set S of projection operators, which is a resolution of the identity, we can generate a complete set of histories, as follows:

$$\mathbf{C}=-\ \widehat{P}_{\alpha_{1}}\left(t_{1}\right)\,,\ \widehat{P}_{\alpha_{2}}\left(t_{2}\right)\,...\ \widehat{P}_{\alpha_{n}}\left(t_{n}\right)\ "\qquad\qquad \text{where}\ \widehat{P}_{\alpha_{k}}\ \text{belongs to}\ S.$$

The set of histories  $\mathbf{C}$  represents all possible time-ordered sequences of states for the system.

Not all such sets of histories are on a par. The requirement of decoherence, of effective independence of the histories from one another, can be expressed in the *medium decoherence condition*<sup>4</sup>. A set **C** obeys medium decoherence if for any histories  $C_{\underline{\alpha}}$ ,  $C_{\underline{\alpha'}}$  in **C** (where  $\rho$  is a density operator representing the initial quantum state of the universe, and Tr is the matrix trace operation):

$$\operatorname{Tr}(C_{\underline{\alpha}}\rho C^{\dagger}_{\underline{\alpha}\prime}) \approx \operatorname{Tr}(C_{\underline{\alpha}}\rho C^{\dagger}_{\underline{\alpha}})\delta_{\underline{\alpha}\,\underline{\alpha}\prime}$$

A set of histories  $\mathbf{C}$  which obeys the medium decoherence condition is called a decoherent history space. If we use a partition of projection operators corresponding to centre-of-mass variables of massive particles weakly coupled to a large number of much lighter particles, then the histories in the space are approximately classical in nature. Such history spaces are called *quasi-classical realms*.

As I have set it out here, the consistent histories framework is nonrelativistic, but this is not a problem of principle. Instead of using the nonrelativistic projection operators described above, we can work in terms of a basis corresponding to field values; probabilities and the consistency conditions can then be specified in terms of path integrals. For further details, see Gell-Mann & Hartle [1990].

# 4. Overlap and Divergence

The characteristic claim of EQM is that the quantum formalism describes a genuinely existing multiplicity. What it describes a multiplicity of is disputed; this will be discussed in more detail below. But we need to get clear on some terminology straight away. I will refer to the entities represented by consistent histories as *Everett worlds*, or just as *worlds* where context allows.

'World' is often used interchangeably with 'branch' in discussions of EQM, but I prefer the former term since it avoids the risk of pre-judging the metaphysical questions addressed in this paper. I will not be presupposing that worlds overlap one another, in the sense of having parts in common at some time but not at another. To mark the difference between views which embrace partsharing of worlds and those which do not, I will adopt David Lewis' distinction<sup>5</sup> between *overlap* and *divergence*. Overlapping worlds have initial segments in

<sup>&</sup>lt;sup>4</sup> This terminology was introduced by Gell-Mann and Hartle [1990].

<sup>&</sup>lt;sup>5</sup> See Lewis [1986] p.206-7.

common; diverging worlds have distinct but qualitatively identical initial segments.

The thought that EQM might be well modelled by branching-time semantics rests on the assumption that Everett worlds overlap one another. This interpretation is widespread, but recent work on EQM has begun to call it into question. Nothing in the formalism of quantum mechanics compels us to think of consistent histories as representing branching entities rather than as representing diverging entities. Consider a set  $\mathbf{C}$  of histories which satisfy medium decoherence. It will include the following histories:

$$C_{\underline{\alpha}} = \hat{P}_{\alpha_n}(t_{n0}) \, \hat{P}_{\alpha_{n-1}}(t_{n-1}) \dots \, \hat{P}_{\alpha_1}(t_1) \, \hat{P}_{\alpha_0}(t_0)$$
$$C_{\underline{\alpha'}} = \hat{P}_{\alpha'_n}(t_n) \, \hat{P}_{\alpha'_{n-1}}(t_{n-1}) \dots \hat{P}_{\alpha'_1}(t_1) \, \hat{P}_{\alpha'_0}(t_0)$$

Let  $\mathbf{\alpha'}_{\mathbf{k}} = \mathbf{\alpha}_{\mathbf{k}}$  for all  $\mathbf{k}$  except  $\mathbf{k}=\mathbf{n}$ . Then the two histories are exactly similar up to and including the penultimate projection operator, but differ on the final projection operator – they agree at all times up to  $t_{n-1}$ , but differ at  $t_n$ . The point at issue between the diverging and branching interpretations is whether the entities represented by the projection operators  $\hat{P}_{\alpha_0} \dots \hat{P}_{\alpha_{n-1}}$  in  $C_{\underline{\alpha}}$  are numerically identical to the entities represented by the projection operators  $\hat{P}_{\alpha'_0}$  $\dots \hat{P}_{\alpha'_{n-1}}$  in  $C_{\underline{\alpha'}}$ , or whether they are (numerically distinct) qualitative duplicates. Numerically identical entities give us overlapping worlds; qualitative duplicates give us diverging worlds.

Some more terminology from metaphysics may be helpful here. According to the overlapping conception, projection operators represent *token* property-instantiations. Some particular instantiation of some particular property is then a part of many worlds, since individual projection operators appear in many different sequences of projection operators. According to the diverging conception, in contrast, projection operators represent *types* of property-instantiation<sup>6</sup>. Thus when some projection operator appears in multiple sequences of projection operators, what this entails is that the property-instantiations in the worlds represented by these sequences are of the same type, and not that there is really only one property-instantiation which is common to each world.

Although Saunders and Wallace have in previous work (e.g. Saunders & Wallace [2008]) tacitly assumed an overlapping structure of worlds, Saunders no

<sup>&</sup>lt;sup>6</sup> This is a natural way to construe the consistent histories formalism as representing a diverging ontology, but it is not the only way. For example, it is consistent with divergence that a projection operator might represent some property instantiations in an irreducibly plural manner; nominalists may find this latter picture more attractive. Of course, those with anti-metaphysical tendencies will see no real distinction here. Thanks to Laurie Paul for discussion.

longer sees this assumption as forced on us by the quantum formalism (Saunders [2010]). I agree with him that the mathematical structure of quantum mechanics does not itself decide between overlap and divergence. Nor, as far as I can see, does any aspect of physicists' own practice in applying the formalism. The type/token distinction is often elided in non-metaphysical contexts; and since the choice between overlap and divergence depends on whether we interpret projection operators as representing token property instantiations or as representing types of property-instantiation, any indecision over the token/type distinction.

While the choice between overlap and divergence has not been considered in any detail by physicists, philosophers of physics at least have paid it some attention. Saunders [1998] distinguished between *fatalism*, the view that the property-instantiations represented by  $\hat{P}_{\alpha_0} \dots \hat{P}_{\alpha_{n-1}}$  in  $C_{\underline{\alpha}}$  are qualitatively identical but not numerically identical to the property-instantiations represented by the projection operators  $\hat{P}_{\alpha'_0} \dots \hat{P}_{\alpha'_{n-1}}$  in  $C_{\underline{\alpha'}}$ , and *minimalism*, the view that these sets of property-instantiations are both qualitatively and numerically identical<sup>7</sup>.

It is easy to see why Saunders chose the names he did for these views. Taking the entities represented by the same projection operator as it appears in distinct histories to be numerically identical reduces the number of distinct entities we must acknowledge to a minimum. In contrast, taking the entities represented by different instances of same projection operator to be qualitatively identical but numerically distinct entails that no entity is part of more than one universe; thus once we have fixed which entity we are referring to, we have thereby picked out an entire history, and the future is in some sense 'fixed'. However, I think that the terms 'minimalism' and 'fatalism' are unhelpfully value-laden; and if Everettians combine a diverging interpretation of EQM with a Lewis-style counterpart-theoretic account of *de re* modal claims, no unsettling fatalistic consequences in fact need follow. For these reasons, I will stick with the Lewisian terminology of 'overlap' and 'divergence'.

The diverging version of EQM has a certain similarity to the proposal of Deutsch [1985]. Deutsch posited a continuously infinite set of universes corresponding to each Everett world, and interpreted objective probability in terms of our ignorance about our location amongst these universes. Like the worlds of diverging EQM, Deutsch's universes have no parts in common with one

<sup>&</sup>lt;sup>7</sup> More recently, Saunders has had a change of mind, and no longer gives much weight to the consideration he previously adduced against divergence; namely, that divergence is better suited to model an incoherent mixture of worlds rather than a superposition of worlds. See section 6 for more discussion.

another, but they do have qualitatively identical initial segments. The crucial difference between diverging EQM and Deutsch's proposal is in the way in which the parallel-worlds ontology is grounded. In Deutsch's proposal, the diverging picture arises from directly modifying the fundamental ontology of the theory. In the diverging version of EQM set out here, the diverging metaphysic arises from the interpretation of projection operators as representing types of propertyinstantiation rather than as representing token property-instantiations. Thus the proposal involves a distinctive way of aligning the physics of consistent histories with the metaphysics of macroscopic worlds.

To show that a diverging reading of the consistent histories formalism is possible is not yet to show that it is desirable. In the next section, I consider an influential argument from the metaphysics literature which tells in favour of divergence.

# 5. The 'Assertion Problem'

In his On the Plurality of Worlds (Lewis [1986]), David Lewis advocated modal realism, the thesis that alternative possible worlds exist and are of a kind with the actual world. One of the first decisions that a modal realist faces is whether to endorse overlap or divergence in the case of indeterministic worlds. Lewis came down firmly on the side of divergence. His argument is short enough to quote in full (here Lewis is using 'branching' in such a way that it entails overlap):

The trouble with branching exactly is that it conflicts with our ordinary presupposition that we have a single future. If two futures are equally mine, one with a sea fight tomorrow and one without, it is nonsense to wonder which way it will be - it will be both ways - and yet I do wonder. The theory of branching suits those who think this wondering *is* nonsense. Or those who think the wondering makes sense only if reconstrued: you have leave to wonder about the sea fight, provided that really you wonder not about what tomorrow will bring but about what today predetermines. But a modal realist who thinks in the ordinary way that it makes sense to wonder about what *the* future will bring, and who distinguishes this from wondering about what is already predetermined, will reject branching in favour of divergence.

Lewis [1986] p.207-8

It is tempting to put the argument extremely succinctly, as follows. If there are multiple futures, then the expression 'the future' suffers from presupposition failure. Definite descriptions presuppose uniqueness, but this presupposition is false in the presence of overlapping worlds. As a result, no sentences which contain the definite description 'the future' are true; but this is absurd, so there cannot be multiple futures.

This quick version of the argument is a little too quick, and there are ways for branching-time theorists to resist it. Belnap denies that 'the future' is a referring expression at all, and instead suggests that it should be given a 'quantificational' reading. The idea is that 'the future' means something like 'however things turn out' rather than meaning 'the way in which things will turn out'. I will not try to assess the extent to which this response to the quick version of the argument succeeds, because I think the quick version fails to do justice to Lewis' central point.

To get clearer on how Lewis' argument really works, it is useful to look at the context in which it occurs. In section 4.2 of Lewis [1986], he first argues against overlap of worlds as a general theory of *de re* representation. The main argument – which Lewis calls the 'argument from accidental intrinsics' – is that individuals who are parts of many worlds will have different intrinsic properties according to different worlds. This turns intrinsic properties into relations to worlds, which – Lewis claims – does too much violence to our ordinary conception of what it is to have an intrinsic property.

The passage quoted above consists in an application of this general argument against overlap of possibilia to the special case of worlds which overlap up to some time and which branch thereafter. Consider the sea-battle which may take place tomorrow, and ask whether an observer-stage X who exists now is such that there will be a sea-battle. On the overlapping conception, there can be no univocal answer; X is such that there will be a sea-battle according to one history, but such that there will be no sea-battle according to the other history. Given overlap, properties like *being such that there will be a sea-battle tomorrow* are not extrinsic monadic properties; they are relations to histories. X is future-sea-battle-related to one history and no-future-sea-battle-related to the other.

Lewis goes on to raise trouble for this replacement of monadic properties by relations to histories by considering the content of our future-directed attitudes. If I only have my future-directed properties relative to histories, then propositions about my future are likewise only true relative to histories. This means that there are no history-unrelativized propositions about the future to serve as the contents of our future-directed attitudes and illocutionary acts.

Consider first *uncertainty*. If I endorse branching-time semantics, and accept both that there will be a sea-battle relative to history  $h_1$  and that there will be no sea-battle relative to history  $h_2$ , it seems that there is nothing left for

me to be uncertain about. There is no future-specifying proposition with a truthvalue of which I can be ignorant.

Consider next *hope*. Suppose as a pacifist I hate sea-battles, and hope that there will be no sea-battle tomorrow. What is it that I am hopeful about? According to branching-time semantics, relative to  $h_1$  there will definitely be a sea-battle and relative to  $h_2$  there will definitely not be a sea-battle. There is no future-specifying proposition with a truth-value about which I can have hopes.

Properly understood, the Lewisian argument against overlap of worlds does not depend on any thesis about the reference of the expression 'the future'. Rather, it depends on the thesis that we can have attitudes and perform speech acts with history-unrelativized future-directed propositional contents. Wondering whether p, and hoping that p, are normally modelled as attitudes that agents have to the proposition p; we make this explicit when we refer to these states as 'propositional attitudes'. But branching-time semantics does not deliver any history-unrelativized propositions that can serve as the content of propositional attitudes concerning the future. The branching-time theorist either must deny that we can coherently wonder about the future when we know the full range of open possibilities, or must deny that wondering about the future involves wondering about the truth-value of any proposition. Neither option seems much good.

Branching-time theorists, of course, are well aware of the Lewisian argument. The most prominent response to it is that of Belnap *et al.* [2001], who refer to the argument as the 'Assertion problem' (despite their recognition that 'an analogous line of thought would apply to attempts to evaluate other attitude and performative verbs, such as "believe," "wonder" and "predict".<sup>8</sup>) Belnap *et al.* grant that future contingent sentences evaluated without relativization to a history-of-evaluation parameter lack a truth value, and accordingly they accept that no clear sense can be made of what it is to assert or to believe or to hope or to fear that future contingent sentences are true at the moment of utterance. Accordingly, they attempt to account for our future-directed thought and talk in pragmatic terms, without ascribing it propositional content in the usual way.

What this means is best illustrated by the example of assertion, the only case which Belnap *et al.* discuss in any detail. They propose that to assert a future-directed sentence which lacks a truth-value at the moment of assertion is to undertake later to be *vindicated* in histories relative to which the sentence would have expressed a true proposition, and later to be *impugned* in histories relative to which the sentence would have expressed a false proposition. Assertion of future contingent sentences is thus construed as a speech act

<sup>&</sup>lt;sup>8</sup> Belnap *et al.* [2001], p.172.

directed towards future truth and falsity, rather than towards present truth and falsity about the future.

The more general idea is this: to adopt a particular future-directed attitude or to make a future-directed speech act is to make a conditional commitment. It is to prescribe a particular range of present-directed attitudes to your various different possible successors. Assertions of unrelativized future contingents are for Belnap *et al.* relevantly like ascriptions of moral properties are for non-cognitivists: at least at the time of utterance they are not truth-apt, and so they can only be evaluated for various sorts of appropriateness.

In at least two ways, the pragmatic account results in a loss of theoretical unity and simplicity. The first of these ways is that a different characterization is needed for each possible attitude. If to assert is to undertake to be either vindicated or impugned, then maybe to hope is to undertake to be either relieved or disappointed; to suspect is to undertake to be either confirmed or surprised, and to desire is to undertake to be either pleased or displeased. Generalizing the strategy recommended by Belnap *et al.*, the plan is to identify, for each attitude one can take to future contingents, a 'positive' state and a 'negative' state; and then to characterize the in question attitude *via* those states.

This procedure raises a number of questions. It is not straightforward to come up with simple characterizations for all of the possible attitudes one could have to future contingent propositions. What about, for example, to imagine, or to intend? Even in the case of assertion, where candidate positive and negative states are explicitly provided by Belnap *et al.*, or in the case of – for example – hope, where it seems relatively easy to provide such candidates, we might worry about whether the resulting characterizations are correct and exhaustive. Constructing a pragmatic analogue of what Churchland [1981] calls 'folk psychology' seems to be a highly non-trivial exercise, and it seems unwise for the branching-time theorist to rest the adequacy of their view on the prospects for providing a pragmatic theory of this sort.

Even if pragmatic characterizations could be produced of all the various propositional attitudes as they apply to future contingent propositions, and if we could somehow be assured of their adequacy, it would remain the case that the resulting picture is complex and disunified. Belnap *et al.* show no inclination to extend their pragmatic treatment of attitudes to future contingent propositions to cover attitudes to propositions which are not future contingents. If we do not extend the treatment in this way, then it looks like we invoke a dualism in our treatment of the attitudes which is not linguistically or psychologically marked. Hoping that your football team wins the match tomorrow does not *seem* to be a fundamentally different sort of thing from hoping that they won the match yesterday, in a circumstance where you are unaware of the result. But if we do extend the pragmatic treatment to propositional attitudes in general, then we sharply reduce the scope and power of our semantic theory.

Regardless of how seriously we take these worries about their pragmatic solution to the Assertion problem, it is important to note that rather than rebutting Lewis' main criticism, Belnap *et al.* have conceded the point and sought to soften the blow. Since according to branching-time semantics there are no true non-history-relativized propositions about the future, *a fortiori* there are no true non-history-relativized propositions about the future that can serve as the contents of our future-directed thought and talk. Belnap *et al.* attempt to show that this situation does not undermine our practice of assertion by giving it a special-purpose pragmatic treatment; and perhaps the treatment can be extended to cover certain other fragments of our future-directed thought and talk. However, in the next section I will argue that the pragmatic treatment cannot be applied to the attitude of epistemic uncertainty without serious consequences for the interpretation of probability.

# 6. Motivating Branching-Time Semantics via Physics

All of the groundwork is now in place to properly assess the motivations for combining EQM with branching-time semantics. In this section I will first argue that the physics of EQM provides only weak and defeasible support for an overlapping ontology of worlds. I will then argue that the overlapping ontology, because it does not allow for future-directed attitudes to have straightforward propositional content, prevents us from ascribing objective probabilities to propositions about the future. Since probabilities are crucial both to the predictions of quantum mechanics and to the evidence which supports it, this provides strong grounds for preferring diverging EQM to overlapping EQM. Branching-time semantics then drops out of the Everettian picture altogether.

The motivation for combining EQM with branching-time semantics is set forth clearly by Bacciagaluppi [2002], by Wallace [2005], by Saunders and Wallace [2008] and by Belnap and Müller [2010]; it rests on the reading of EQM as a theory with an ontology of overlapping worlds, and on the (quite reasonable) assumption that a semantic theory should not posit semantic distinctions without a metaphysical difference<sup>9</sup>. I have no quarrel with this latter assumption, but I reject the premise that EQM must be read as a theory with an overlapping ontology. As I argued in section 4, both diverging and overlapping interpretations of the consistent histories formalism are possible, and the physics itself does not distinguish between them.

 $<sup>^9</sup>$  This is essentially the assumption, in the terminology of Bigelow [1988], that truth depends on being  $% 10^{-1}$  .

It would be possible to accept that a diverging interpretation of the consistent histories formalism is available, but to nevertheless maintain that the physics provides us with reason to prefer overlap. That is, it could be claimed that while divergence is *compatible* with EQM, it fails to do justice to the spirit of the theory. Simon Saunders has used this style of argument in a number of papers from the 1990s. Here is one example:

The universal state, with its unitary evolution, is a single object in its own right. We may take various cross-sections through this object, and consider relations among various of its parts, but the totality of cross-sections and relations does not exist as something over and above the original. I take it that this is the guiding inspiration of the relational approach, and the core concept of the physics.

Saunders [2010]

Saunders no longer takes this 'guiding inspiration' to be tension with divergence, since diverging worlds – despite having no *mereological* overlap – can be combined via quantum superposition to recover the universal state. In Saunders [2010], he argues that divergence is the more serviceable metaphysics, the one which best meshes ordinary language and thought with the mathematics of quantum mechanics, and that it is, *ipso facto*, the correct metaphysics<sup>10</sup>:

...the suspicion is that whether worlds in EQM diverge or overlap is underdetermined by the mathematics. One can use either picture; they are better or worse adapted to different purposes. If so, it is pretty clear which is the right one for making sense of [future-directed] uncertainty.

Saunders [2010]

Saunders diagnoses the presumption of overlap as historical accident, deriving from Everett's use of the term 'branching' to set out his theory and from the prevalence of the Schrödinger picture in conceptual discussions<sup>11</sup>. It is

<sup>&</sup>lt;sup>10</sup> Saunders in this paper goes to some lengths to show that no hidden inconsistency lurks in the diverging picture. In particular, he sets out a mereology for Heisenbergpicture vectors which is such that macroscopic objects represented by orthogonal vectors are merologically non-overlapping. This account is valuable as an existenceproof; though whether we are inclined to adopt Saunders' particular mereological picture will depend on what we say about the nature of the representation relation between Heisenberg-picture vectors and macroscopic objects. If a one-one correspondence is assumed, then Saunders's vector mereology is apt; if instead we allow that vectors correspond one-many to macroscopic objects then other, simpler vector mereologies may be apt. Whether the needed ideological structure is best located in the parthood relation or in the representation relation is a metaphysical question that will here be left open; either of these options can underwrite a diverging metaphysics of Everett worlds.

 $<sup>^{\</sup>rm 11}$  Saunders [2010], p.200.

plausible that considerations of parsimony have also played a role. There is after all a real sense in which overlap is more ontologically minimal than divergence – it involves fewer property-instantiations in  $total^{12}$ . But while I recognise the appeal of the overlapping conception, its extra parsimony must be offset against the difficulties that it generates for thought and talk about the future.

It should be clear that Lewis' argument against overlap applies to the worlds of EQM in exactly the same way as it applies to the worlds of Lewisian modal realism. In each case both a diverging metaphysic and an overlapping metaphysic are possible; in each case ordinary thought and talk about the future appears to conflict with the overlapping metaphysic. It is therefore surprising that while Lewis' argument has been highly influential in the metaphysics of modality, it has until recently had little impact on the debate over EQM.

In fact, I think the Lewisian argument is potentially even more damaging to overlapping EQM than it is to overlapping modal realism, because of the centrality of questions about objective probability to quantum mechanics. Many critics of EQM, including Loewer [1996] and Price [2010], have focused their objections around the 'Incoherence problem' described in section 2: if we know for certain that all possible outcomes will be realized, how can we coherently assign probabilities to outcomes? The problem is often put in terms of the unavailability of a notion of 'objective uncertainty'. For an agent who knows that quantum mechanics is correct, and who knows the complete current quantum state, what is there left to be uncertain about? And if there is nothing to be uncertain about, what room is there for probabilities other than 1 or 0?

At the root of the Incoherence problem is the requirement that objective probabilities should attach to propositions. The probability of some future outcome is the probability of the proposition that the outcome will occur; hence, assigning probabilities to a range of distinct future outcomes requires assigning probabilities to a range of mutually exclusive propositions about the future. But given an overlapping ontology, there are no appropriate mutually exclusive propositions about the future. To paraphrase Lewis, given overlapping EQM it is nonsense to wonder which outcome of a measurement will occur – both outcomes will occur. Consequently, it is nonsensical to assign objective probabilities other than 1 to either outcome.

When about to perform a spin measurement, I cannot assign a 50% probability to seeing spin-up and a 50% probability to seeing spin-down: I am both in a spin-up world and in a spin-down world, so I am certain to see spin-up

<sup>&</sup>lt;sup>12</sup> The kind of parsimony in question is quantitative parsimony: see Nolan [1997] for a clear discussion. David Lewis famously recognised no presumption whatsoever in favour of quantitative parsimony: see Lewis [1973], p.87.

and I am certain to see spin-down. The future-directed attitudes which Lewis argues are unavailable in an overlapping ontology, and which Belnap *et al.* concede cannot be represented in the branching-time framework, seem to be indispensable for grounding objective probability in the Everettian picture.

Lewis' own response to the problems that overlap generates for futuredirected attitudes is to reject overlap and to embrace divergence in the context of modal realism. I propose the same response in the context of EQM. If we adopt a diverging version of EQM, then objective uncertainty can be thought of as *self-locating* uncertainty: uncertainty about which world we are a part of. Such uncertainty will persist even when we know the laws of nature and the quantum state in complete detail: knowledge of the state guarantees that there are many Everett worlds which are indiscernible from ours up to the present time and which have different futures, and we have no way of knowing which of these worlds we are in.

The interpretation of objective uncertainty as self-locating uncertainty is unavailable given an overlapping ontology. If I am currently located in a history segment which is both part of a spin-up world *and* part of a spin-down world, then I know all there is to know about my location. There is no further selflocating fact of which I am uncertain that could comprise a subject-matter for objective probabilities.

We might try to recover an indexical notion of objective uncertainty by combining overlap with a particular metaphysics of personal identity<sup>13</sup>. The trick would be to think of self-location not as a matter of the location of an instantaneous time-slice, or stage, but instead as a matter of the location of a whole spatio-temporal continuant. If such a continuant performs an experiment, then it overlaps either a spin-up outcome or a spin-down outcome, but not both. However, Wilson [2010] argues that this proposal breaks down when it is applied to objective uncertainty about events which occur after our own deaths: as a result, it is not capable of grounding probabilities for future outcomes in full generality.

Once we abandon overlap for divergence, then no macroscopic object or event is part of more than one world, and so no analogous problem arises for the self-locating conception of objective uncertainty. When we talk about the future, on the diverging picture, we are talking about the future of our world only. There are true propositions, not relativized to any history, about our world's future; and it is these which can provide propositional contents for our attitudes

<sup>&</sup>lt;sup>13</sup> Although aspects of the discussion in Saunders and Wallace [2008] suggest that this is the proposal they have in mind, in fact the main proposal made in that paper is more complex; it identifies macroscopic objects with ordered pairs of parts of worlds and whole worlds. See Wilson [2010] for discussion.

and for our illocutionary acts. Given divergence, we can treat objective uncertainty as indexical uncertainty in full generality, without having to commit to any contentious metaphysics of personal identity.

Divergence does not solve the incoherence problem in EQM all by itself. Other fragments of theory – in particular, appropriate treatments of actuality (as indexical) and of chance-bearing propositions (as sets of worlds) – are also required. And even when the Incoherence problem has been successfully addressed, the Quantitative problem and the Epistemic problem may still continue to trouble us. In Wilson [forthcoming] I develop a more detailed account of objective probability in diverging EQM. Here I have only aimed to establish that overlap clashes with the indexical conception of objective uncertainty in a way in which divergence does not.

As the metaphysics of EQM is underdetermined by the physics, we are free to adopt whichever metaphysics is of most help in making sense of objective probability. Although an overlapping metaphysic may in some sense be the simpler or more parsimonious way of reading ontological conclusions off from the consistent histories formalism, in my view this additional parsimony is outweighed by the problems that overlap generates for objective probability. And if Everettians reject overlap, they have no need for branching-time semantics.

# 7. Motivating Branching-Time Semantics via Common Sense

Of course, branching-time semantics has not historically been motivated by any particular piece of physics. Its pioneers, in particular Saul Kripke and Arthur Prior, seem to have been motivated to capture something about our position as agents with ineliminably limited capacities for accessing and processing information, and hence with ineliminably limited capabilities for predicting the future. This is parlayed into the thought that indeterminism is a *real* phenomenon, and that our metaphysics must incorporate it as a basic element. Belnap *et al.* sum up their motivation for developing branching-time semantics as follows:

...we try never to forget the central constraining thought: There is neither action nor agency nor doings without real choices, choices that find their place not merely in the agent's mind, but within the (indeterminist) causal order of our world. To see to it that Q, an agent must make a real choice among objectively incompatible future alternatives. When we say that an event may have many possible but incompatible outcomes, we thereby come down on the side of "hard" indeterminism as against determinism.

Belnap *et al.* [2001] p. xi

I would reconstruct this line of thought as follows.

- (A) It is a platitude about human beings and our place in the world that we make choices.
- (B) The choices we make are real choices; they are not illusory.
- (C) It is analytic of the notion of a real choice that there be multiple genuinely possible and objectively incompatible future alternatives amongst which the choice distinguishes.
- (D) The existence of genuinely possible and objectively incompatible future alternatives requires that the present moment be part of multiple distinct histories.
- (E) In a world in which the present moment is part of multiple distinct histories, branching-time semantics is the only semantic theory which avoids drawing distinctions without a difference, while still being expressively adequate.

The place where I want to object to this line of thought is premise (D). Compatibilists about alternative possibilities will maintain that all that is needed for there to presently be genuine alternative possibilities is for there to presently be possibilities which match the segment of the actual world up to the present time, but which differ from it afterwards. Belnap *et al.* refer to this sort of compatibilist picture as *soft indeterminism*, and contrast it unfavourably with their own *hard indeterminism*. But it is not clear in what the hardness of their position consists, or why we should care about this distinction, or why we should take common sense to support hard indeterminism over soft indeterminism.

One way to sharpen up this latter concern is to cast it as an indiscriminability argument. If we grant (D), then we lose any reason to believe (B). Branching-time theorists typically allow for determinism as at least a logical possibility, and allow that there could be a deterministic world with an initial segment which exactly duplicates the segment of history that the branching-time theorist takes to correspond to actual past history. Agents located in this deterministic world would have exactly the same common-sense intuitions, and exactly the same phenomenology, as agents in the actual world; but their conclusions that their own world was hard-indeterministic would be false. This, it might be argued, casts doubt on the motivation from 'common sense' for believing that the actual world is hard-indeterministic.

The problem with indiscriminability arguments of this sort is that they can too easily be assimilated to sceptical arguments more generally. If a deceiving demon is at least a logical possibility, then parity of reasoning with the previous argument casts doubt on the conclusion that an external world exists and is much like we take it to be. So perhaps the kind of indiscriminability argument just given need worry the hard-indeterminist. The analogy between hard-indeterminism-scepticism and externalworld-scepticism is not, of course, precise. Perhaps some disanalogy between the cases can be used to uphold hard-indeterminism-scepticism without ushering in external-world-scepticism. A plausible diagnosis is that the external-world realist has an account of how we gain reliable information about the external world *if all goes well* – sense perception – while the branching-time theorist who embraces hard indeterminism can give no equivalent account of how we gain reliable information about other branches. But even when bolstered by this diagnosis, it is not clear that the indiscriminability argument against hard indeterminism is compelling.

There is, however, a more powerful argument in the vicinity. The case against hard indeterminism is best posed not as an epistemological argument – that, were hard indeterminism correct, we could not know that it was correct – but as a methodological argument – that it could never be rational to posit hard indeterminism rather than soft indeterminism. This methodological argument combines the premise that a theory incorporating hard indeterminism is more complex than a theory incorporating soft indeterminism with the premise that hard indeterminism provides no additional explanatory power, and concludes that we cannot have any good reason to believe in hard indeterminism.

In contrasting epistemological arguments against the knowability of some posited ingredient of a metaphysical worldview with methodological arguments against positing that ingredient, I follow John Hawthorne (Hawthorne [2002]). With Hawthorne, I take the epistemological style of argument to be generally inconclusive but the methodological style of argument to be potentially compelling.

The final reason I want to adduce for rejecting the motivation from common-sense for branching-time semantics is that it is in tension with the motivation from the physics discussed in section 6. The claim that EQM fits naturally with branching-time semantics could only provide a reason to adopt branching time for those who aspire to let physical theory guide metaphysical theory. But it is a distinctive feature of this 'naturalistic' methodology that our common-sense intuitions about the world are treated as only a very weak and defeasible source of evidence in metaphysics. Conversely, those practitioners of 'analytic metaphysics' who regard the contingencies of actual physics as irrelevant to their concerns will be indifferent to any argument from EQM to the correctness of branching-time semantics. The motivation from physics and the motivation for common-sense are not then, as we might have thought, complementary; to the extent that we are moved by one, we are likely to be unmoved by the other.

## 8. Conclusion

The project of accounting for language use in Everett worlds looks like a paradigmatic application for branching-time semantics; but appearances are deceptive. A diverging version of EQM can treat the semantics of future contingent propositions as continuous with the semantics of other descriptive propositions, can preserve a bivalent classical logic in full generality, and can help to ground a non-eliminative conception of objective probabilities. If this is correct, then the 'parallel worlds' metaphor is vindicated as an intuitive gloss on EQM, and Everettians are better off without branching time

This conclusion casts doubt on whether there could be any good motivation for adopting branching time semantics. My own view is that branching-time theorists have not done enough to persuade us that indeterminism is the sort of phenomenon which is best treated by adding complications to classical logic and semantics<sup>14</sup>.

<sup>&</sup>lt;sup>14</sup> An early version of this paper was presented at the BIRTHA Branching Time and Indeterminacy Conference held at the University of Bristol in August 2010. It has benefited from helpful conversations with Andrew Bacon, Cian Dorr, Antony Eagle, Rohan French, Toby Handfield, John Hawthorne, Alex Malpass, Thomas M ller, Laurie Paul, David Wallace and Robert Williams, and from the comments of two anonymous referees. Particular thanks to Simon Saunders and to Nuel Belnap for providing extensive comments on multiple drafts, and for enthusiasm and encouragement.

# References

Bacciagaluppi, G. [2002]. 'Remarks on Space-Time and Locality in Everett's Interpretation', in *Modality, Probability, and Bell's Theorems*. NATO Science Series.

Belnap, N. & Müller, T. [2010]. 'Branching with uncertain semantics'. British Journal for Philosophy of Science, 61:3, pp. 681-696.

Belnap, N.; Perloff, M; Xu, M. [2001]. *Facing the Future: Agents and Choices in our Indeterministic World*. Oxford University Press.

Bigelow, J. [1988]. The Reality of Numbers. Oxford University Press.

Deutsch, D. [1985]. 'Quantum Theory as a Universal Physical Theory'. International Journal for Theoretical Physics 24:1-41.

Everett III, H. [1955]. *Wave Mechanics Without Probability*. Draft of Princeton doctoral dissertation.

Churchland, P. [1981]. 'Eliminative Materialism and the Propositional Attitudes'. *Journal of Philosophy* 78, pp. 67-90.

Gell-Mann, M. & Hartle, J. [1990]. 'Quantum Mechanics in the Light of Quantum Cosmology'. *Complexity, Entropy and the Physics of Information*, ed. W. Zurek. Addison-Wesley.

Greaves, H. [2004]. 'Understanding Deutsch's Probability in a Deterministic Universe'. *Studies in History and Philosophy of Modern Physics* 35:3, pp. 423-456.

Greaves, H. [2007]. 'Probability in the Everett Interpretation'. *Philosophy Compass* 2 (1):109-128.

Hawthorne, J. [2002] 'Causal Structuralism'. *Philosophical Perspectives* 15, pp. 361-378.

Lewis, D. [1973]. Counterfactuals. Blackwell.

Lewis, D. [1986]. On the Plurality of Worlds. Blackwell.

Loewer, B. [1996]. 'Comment on Lockwood', British Journal for the Philosophy of Science, 47:2, pp. 229-232.

Müller, T. [2005]. 'Probability Theory and Causation: A Branching Space– Times Analysis'. *British Journal for the Philosophy of Science* 56, pp. 487-520.

Nolan, D. [1997]. 'Quantitative Parsimony', British Journal for Philosophy of Science 48, pp. 329-343.

Price, H [2010]. 'Decisions, decisions, decisions: Can Savage salvage Everettian probability?' in Saunders *et al.* [2010].

Saunders, S. [1993]. 'Decoherence, Relative States, and Evolutionary Adaptation', *Foundations of Physics* 23, pp. 1553-1585.

Saunders, S. [1995] 'Time, Quantum Mechanics, and Decoherence'. *Synthese* 102:2, pp.235-266.

Saunders [1998]. 'Time, Quantum Mechanics, and Probability'. *Synthese* 114:3, pp. 373-404.

Saunders, S.; Barrett, J.; Kent, A.; Wallace, D. [2010]. *Many Worlds? Everett, Quantum Theory, and Reality.* Oxford University Press.

Saunders, S. [2010]. 'Chance in the Everett Interpretation', in Saunders *et al.* [2010].

Saunders, S. & Wallace, D. [2008]. 'Branching and Uncertainty'. *British Journal for Philosophy of Science* 59:3, pp. 293-305.

Weiner, M. & Belnap, N. [2006]. 'How Causal Probabilities Might Fit into Our Objectively Indeterministic World'. *Synthese* 149, pp. 1-36.

Wallace, D. [2005]. 'Language use in a branching universe'. Available online at <u>http://philsci-archive.pitt.edu/2554/</u>

Wallace, D. [2010]. 'Decoherence and Ontology', in Saunders et al. [2010].

Wallace, D. [2012]. *The Emergent Multiverse: Quantum Theory According* to the Everett Interpretation. Oxford University Press.

Wilson, A. [2010]. 'Macroscopic Ontology in Everettian Quantum Mechanics'. *Philosophical Quarterly* 61:243, pp. 363-382.

Wilson, A. [forthcoming]. 'Objective Probability in Everettian Quantum Mechanics'. To appear in *British Journal for Philosophy of Science*.

Zurek, W. [2003]. 'Decoherence and the transition from quantum to classical – REVISITED'. Available online at <u>http://arxiv.org/abs/quant-ph/0306072</u>