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# Kripke semantics for full ground references (work in progress)

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“Full ground references” means references to integers and to other references, but not functions or thunks. Game semantics for full ground references was given in [1]. This work aims to give Kripke semantics.

**Language** We fix a set  $\mathcal{S}$  of *sorts*. We extend the call-by-push-value types with a reference type  $\text{Ref}_s$  for each sort  $s$ . We then define

$$\text{full ground types } D ::= 0 \mid D + D \mid \sum_{i \in \mathbb{N}} D_i \mid 1 \mid D \times D \mid \text{Ref}_s \ (s \in \mathcal{S})$$

and fix, for each sort  $s$ , a full ground type  $D_s$ , intended to be the type of values stored in a reference cell of sort  $s$ .

A *world* is a finite set over  $\mathcal{S}$ . (Alternatively: a finite sequence of sorts.) The judgements are  $w, \Gamma \vdash^v V : A$  for values and  $w, \Gamma \vdash^c M : \underline{B}$  for computations, where  $w$  is a world and  $\Gamma$  a typing context (finite set of identifiers with an associated value type). Term syntax and operational semantics is defined as usual. Evaluation terminates because of the restriction to full ground references.

**Denotational semantics** A value type  $A$  will denote a functor from the category  $\text{Inj}$  of worlds and injections to  $\mathbf{Set}$ . Intuitively,  $\llbracket A \rrbracket w$  is a semantic domain for closed values in world  $w$ , and such values can be renamed along an injection  $w \rightarrow w'$ . In particular  $\llbracket \text{Ref}_s \rrbracket w$  is the set of  $s$ -sorted cells in  $w$ .

A  $w$ -store associates to each cell  $l$  in  $w$  a value  $w \vdash^v s_l : D_{\text{sort}(l)}$ . The semantic domain for these is

$$Sw \stackrel{\text{def}}{=} \prod_{l \in w} \llbracket D_{\text{sort}(l)} \rrbracket w$$

Although for ground references  $S$  is a functor  $\text{Inj}^{\text{op}} \rightarrow \mathbf{Set}$ , that is not so for full ground references.

Computation types have more subtle semantics. To understand it, say that a  $w$ -store  $s$  associates to each cell  $l$  in  $w$  a value  $w \vdash^v s_l : D_{\text{sort}(l)}$ . An *SC-configuration*  $\Gamma \vdash^{\text{sc}} x, s, M : \underline{B}$  consists of

- a world  $x$ —we think of cells in  $x$  as local/private, whereas those in  $w$  are global/public
- a  $w + x$ -store  $s$
- and a computation  $w + x, \Gamma \vdash^c M : \underline{B}$ .

These arise in the operational semantics of a language that has both  $w$ -many global cells and generation of local cells.

We want  $\llbracket \underline{B} \rrbracket w$  to be a semantic domain for closed SC-configurations in world  $w$ . What category should  $\llbracket \underline{B} \rrbracket$  be a functor from?

Let’s start with the ground ref setting. An *initialization*  $(i, p) : w \rightarrow w'$  consists of

- an injection  $i : w \rightarrow w'$ —we write  $\text{new}(i)$  for the cells in  $w'$  not in the range of  $i$
- and an element  $p \in \prod_{l \in \text{new}(i)} \llbracket D_{\text{sort}(l)} \rrbracket$

These form a category  $\text{Inj}$ . A *partial initialization* is defined similarly, except that  $i$  is a partial injection. The latter form a category  $\text{PInj}$  that contains both  $\text{Inj}^{\text{op}}$  and  $\text{Inj}$ , and indeed is freely generated by these subcategories modulo two equations.

A partial initialization  $i : w \rightarrow w'$  converts an SC-configuration in world  $w$  to one in world  $w'$ , by

- hiding the cells in  $w$  that are not in the domain of  $i$
- renaming each cell in the domain of  $i$  to one in the range
- creating each cell  $l \in \text{new}(i)$  with value  $p_l$ .

So a computation type  $\underline{B}$  should denote a functor  $\mathbf{PInit} \rightarrow \mathbf{Set}$ .

Turning to full ground references, an initialization  $(i, p) : w \rightarrow w'$  consists of

- an injection  $i : w \rightarrow w'$
- and an element  $p \in \prod_{l \in \text{new}(i)} \llbracket D_{\text{sort}(l)} \rrbracket(w + \text{new}(i))$

These form a category  $\mathbf{Init}$ , and  $S$  is a functor  $\mathbf{Init} \rightarrow \mathbf{Set}$ . But partial initializations are more subtle. Let's first say that a *stateful value*  $\Gamma \vdash^{\text{sv}} x, s, V : A$  consists of

- a world  $x$ —again, cells in  $x$  are local/private, whilst those in  $x$  are global/public
- for each local cell  $l$  in  $x$ , a value  $w + x \vdash^{\text{v}} s_l : D_{\text{sort}(l)}$
- and a value  $w + x \vdash^{\text{v}} V : A$ .

For any functor  $A : \mathbf{Inj} \rightarrow \mathbf{Set}$ , define

$$(\Psi A)w \stackrel{\text{def}}{=} \int^{x \in \mathbf{Init}} \prod_{l \in x} \llbracket D_{\text{sort}(l)} \rrbracket(w + x) \times \llbracket A \rrbracket(w + x)$$

so that  $(\Psi[A])w$  is a semantic domain for closed stateful values of type  $A$  in world  $w$ . If  $A$  is a constant functor, then  $(\Psi A)w \cong Aw$ . A *partial initialization*  $(i, p) : w \rightarrow w'$  consists of

- a partial injection  $i : w \rightarrow w'$
- and an element  $p \in (\Psi \prod_{l \in \text{new}(i)} \llbracket D_{\text{sort}(l)} \rrbracket)(w + \text{new}(i))$ .

These form a category  $\mathbf{PInit}$  that contains both  $\mathbf{Inj}^{\text{op}}$  and  $\mathbf{Init}$  and indeed is freely generated by these subcategories modulo two equations. A computation type  $\underline{B}$  should denote a functor  $\mathbf{PInit} \rightarrow \mathbf{Set}$ . Also, if  $A : \mathbf{Inj} \rightarrow \mathbf{Set}$  then  $\Psi A : \mathbf{PInit}^{\text{op}} \rightarrow \mathbf{Set}$ .

Semantics of judgements:

- A value  $w, \Gamma \vdash^{\text{v}} V : A$  denotes a family of functions  $\llbracket \Gamma \rrbracket(w + x) \rightarrow \llbracket A \rrbracket(w + x)$ , natural in  $x \in \mathbf{Inj}$ .
- A computation  $w, \Gamma \vdash^{\text{c}} M : \underline{B}$  denotes a family of functions  $S(w + x) \times \llbracket \Gamma \rrbracket(w + \bar{x}) \rightarrow \llbracket \underline{B} \rrbracket(w + x)$ , natural in  $x \in \mathbf{Init}$ . Here overline represents the forgetful functor  $\mathbf{Init} \rightarrow \mathbf{Inj}$ .

Semantics of types:

$$\begin{aligned} \llbracket FA \rrbracket w &\stackrel{\text{def}}{=} \int^{x \in \mathbf{Init}} S(w + x) \times \llbracket A \rrbracket(w + \bar{x}) \\ \llbracket U\underline{B} \rrbracket w &\stackrel{\text{def}}{=} \int_{x \in \mathbf{Init}} S(w + x) \rightarrow \llbracket \underline{B} \rrbracket(w + x) \\ \llbracket \prod_{i \in I} \underline{B}_i \rrbracket w &\stackrel{\text{def}}{=} \prod_{i \in I} \llbracket \underline{B}_i \rrbracket w \\ \llbracket A \rightarrow \underline{B} \rrbracket w &\stackrel{\text{def}}{=} \int_{x \in \mathbf{Init}} \llbracket A \rrbracket(w + \bar{x}) \rightarrow \llbracket \underline{B} \rrbracket(w + x) \\ \llbracket U \prod_{i \in I} (A_i \rightarrow \underline{B}_i) \rrbracket w &\cong \int_{x \in \mathbf{Init}} S(w + x) \rightarrow \prod_{i \in I} (\llbracket A_i \rrbracket(w + \bar{x}) \rightarrow \llbracket \underline{B}_i \rrbracket(w + x)) \end{aligned}$$

## References

- [1] Andrzej S. Murawski and Nikos Tzevelekos. Algorithmic games for full ground references. In Artur Czumaj, Kurt Mehlhorn, Andrew M. Pitts, and Roger Wattenhofer, editors, *ICALP (2)*, volume 7392 of *Lecture Notes in Computer Science*, pages 312–324. Springer, 2012.