

Influence of unsupported sleepers on the dynamic response of a heavy haul railway embankment

Shi, Jin; Chan, Andrew; Burrow, Michael

DOI:

[10.1177/0954409713495016](https://doi.org/10.1177/0954409713495016)

License:

None: All rights reserved

Document Version

Peer reviewed version

Citation for published version (Harvard):

Shi, J, Chan, A & Burrow, M 2013, 'Influence of unsupported sleepers on the dynamic response of a heavy haul railway embankment', *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 227, no. 6, pp. 657-667. <https://doi.org/10.1177/0954409713495016>

[Link to publication on Research at Birmingham portal](#)

Publisher Rights Statement:

Checked 12/09/2016

General rights

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
- User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
- Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

Take down policy

While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact UBIRA@lists.bham.ac.uk providing details and we will remove access to the work immediately and investigate.

Influence of unsupported sleepers on dynamic responses of railroad embankment below a heavy haul railway line using simulation techniques

Jin Shi^a, A.H. Chan^b, M.P.N. Burrow^b

^aSchool of Civil Engineering, Beijing Jiaotong University, Beijing, China

^bCentre for Railway Research and Education, University of Birmingham, Birmingham, UK

Abstract: It is not very uncommon to find railway track systems with unsupported sleepers due to poor drainage and the uneven settlement of a ballasted track system. With growing trend towards heavier axle loads, unsupported sleepers will speed up track deterioration process. In order to investigate the possible implications, this paper describes the development of a dynamic finite element model using linear elastic material properties while including wheel-rail friction. The influence of unsupported sleepers on dynamic responses of vehicle, track and subgrade is studied.

Results indicate that the unsupported sleeper has important effect on ride comfort and safety, and there is a rapidly increase in the magnitude of vehicle-track dynamic responses as the number of consecutive unsupported sleepers is increased. Moreover, unsupported sleeper may induce pulverization and cracking of the ballast stone, reduce the fatigue life of the sleeper and fastener, and accelerate subgrade progressive shear failure and differential settlement. An increase in the number of unsupported sleeper amplifies the effect. Different track improvement options are suggested in order to reduce unsupported sleeper and potential track faults.

Key words: railway track, unsupported sleeper, finite element, dynamic response

1 Introduction

Poor drainage and differential settlement of subgrade may lead to some sleepers not being fully supported, or even completely unsupported, resulting in increased deflection and wheel-rail contact force. With growing trends towards heavier axle loads on railway, unsupported sleepers will speed up track deterioration process. According to [Augustin et al \(2002\)](#), they indicated that in reality a considerable number (mostly over 50%) of sleepers are badly bedded or do not touch the ballast at all.

[Grassie et al \(1985\)](#) investigated, both experimentally and numerically, the dynamic response of a track with unsupported sleepers. They concluded that in the absence of sleeper support, the sleepers are likely to crack if there are modest wheel or railhead irregularities. [Nielsen et al \(1995\)](#) put forward a discrete mathematical vehicle model of a train bogie and a linear finite element model of the railway track accounting for the discrete location of sleeper supports. Their investigations show that the dynamic loading on the sleepers neighboring an unsupported sleeper increases with increasing speed. [Lundqvist et al \(2005\)](#) presents a computer model by which the dynamic train/track interaction can be simulated. The influence of one and two unsupported sleeper on sleeper/ballast contact force and ballast settlement was investigated. [Zhang et al \(2008\)](#) investigated the effect of unsupported sleepers on the normal load of wheel/rail in detail based on a coupling dynamic model of vehicle-track. In their model, the vehicle is modelled as a multi-body system, and the track is considered as a 3-layer model with rails, sleepers, and ballast masses. The numerical results obtained indicate that the gaps between the unsupported sleepers and ballast masses have a great influence on the normal load of the wheel and the rail. [Yang et al \(2009\)](#) investigated the effect of unsupported sleeper on stress changes in subgrade by means of a two-dimensional dynamic finite-element model (FEM). [Zhu and Ahmed et al \(2010\)](#) presented analyses of a coupled vehicle-track assembly consisting of a roll plane vehicle model, a continuous track system model and an adaptive wheel-rail contact model. The dynamic response in terms of the dynamic wheel-rail interaction force due to one or multiple unsupported sleepers is studied. In paper by [Zhu et al \(2011\)](#), they considered the vehicle's running speed and the number of the unsupported sleepers, the track dynamic behavior was investigated based on vehicle-track coupling dynamics and verified by the experiments in time and frequency domains on an indoor scale wheel/rail test rig. However, little attention has been paid to the effect of unsupported sleepers on track vibration and ride safety in operation condition of heavy haul vehicle. Further, transient stress of subgrade which was analyzed is unable properly to consider subgrade failure due to repeated train passage.

In this paper, a dynamic three-dimensional finite element model using linear elastic material

properties while including wheel-rail friction was developed and used to investigate the influence of unsupported sleepers on dynamic response. The influence of unsupported sleepers on ride safety, track vibration and subgrade failure is studied.

2 Simulation model of the coupled vehicle–track dynamics

2.1 Finite element model

In order to investigate the influence of unsupported sleeper on dynamic response, a three dimensional dynamic finite element model (FEM) of the track vehicle system was built using ABAQUS explicit software (Simulia 2002). The finite element model is based on a section of railway track which are fully described in the china railway design specification (2005) [10].

The freight vehicles were idealized as a as a multi-body system consisting of a car body, bolster, frame and wheel set, with spring-dashpot suspensions between the four components as shown in Figure 1. The associated parameters of the four components are given in Tables 1. The connections of suspension system are modeled as a system of linear springs and viscous dashpots in the vertical direction. Vehicle vibration in the vertical plane only was considered. With the above assumptions, the car body is designated by vertical, pitching and rolling movements. For the frame, vertical and pitching movements are considered. For the wheel set, vertical and rolling movements are considered. So the idealized model for a wagon can be described by 18 dofs (degrees of freedom),

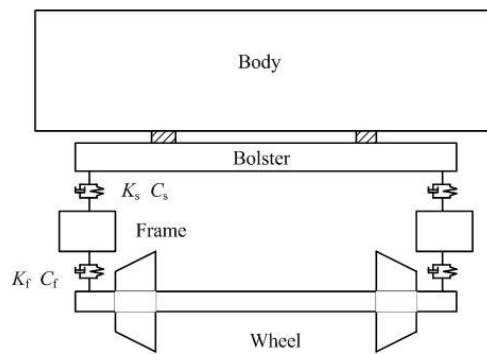


Fig.1. The cross section of freight vehicle

Table 1. Freight vehicle parameters

Mass of car body (M_c)	91400kg
Inertia of car body (J_c)	$1.33 \times 10^5 \text{ kg} \cdot \text{m}^2$
Mass of frame (M_f)	1786kg
Inertia of frame (J_b)	$420 \text{ kg} \cdot \text{m}^2$
Mass of wheel (M_f)	1257kg
Primary suspension stiffness	13MN/m
Primary suspension damping	$3 \times 10^5 \text{ Ns/m}$
Secondary suspension stiffness	4.4 MN/m

Secondary suspension damping	4×10^3 Ns/m
------------------------------	----------------------

For this study, the track and subgrade was constructed to carry 25t axle loads according to Chinese railway standards [10]. The track components consisted of 75kg/m continuously welded rail laid to a gauge of 1435mm, supported by concrete sleepers placed at a spacing of 0.6m. According the measurement in Shuohuang heavy haul line by the first author, thickness from ballast surface to foundation is about 820~870mm. The ballast thickness is about 0.6m? considering 0.2m thickness from the surface of the ballast to the underside of the sleeper. The subgrade was constructed according to China standard and consists of three distinct layers [10]: the upper layer of stiff sand gravel (approximately 0.7 m deep), and the middle layer of 2.3m deep consists of engineering fill made up of at least 50% gravel, sand and silt and is specified in the Chinese design standard to be of “Class A”. The bottom layer of 3.0m is of medium strength sand silt.

The track and subgrade was modelled as a three-dimensional dynamic system. Solid linear elastic elements, with eight nodes, were used to model the rail, sleepers, ballast and subgrade. The subgrade was modelled as three layers. Parametric properties are given Table 2. In addition, vertical stiffness and damping of fastener are 78kN/mm and 50 kN.s/m suggested by Zhai (2009) on the basis of field measurement of Daqing heavy haul line, respectively[11]. The length of model in longitudinal direction was 100m, the depth of model was set at 20m. Infinite elements were used at the boundaries of the subgrade to overcome the problem of the stress waves generated from being reflected back into the model. The finite element mesh, shown in Figure 2, comprised of a total of 124357 elements and 176268 nodes.

Table 2. Materials properties parameters in FE model

Component description	Young's modulus(MPa)	Poisson's ratio	Density(kg/m ³)
Rail	210000	0.3	7830
Sleeper	35000	0.22	2600
Ballast	180	0.27	1650
Sand gravel	180	0.3	2300
Engineering fill of class A	130	0.3	2100
Silt sand	50	0.25	1800

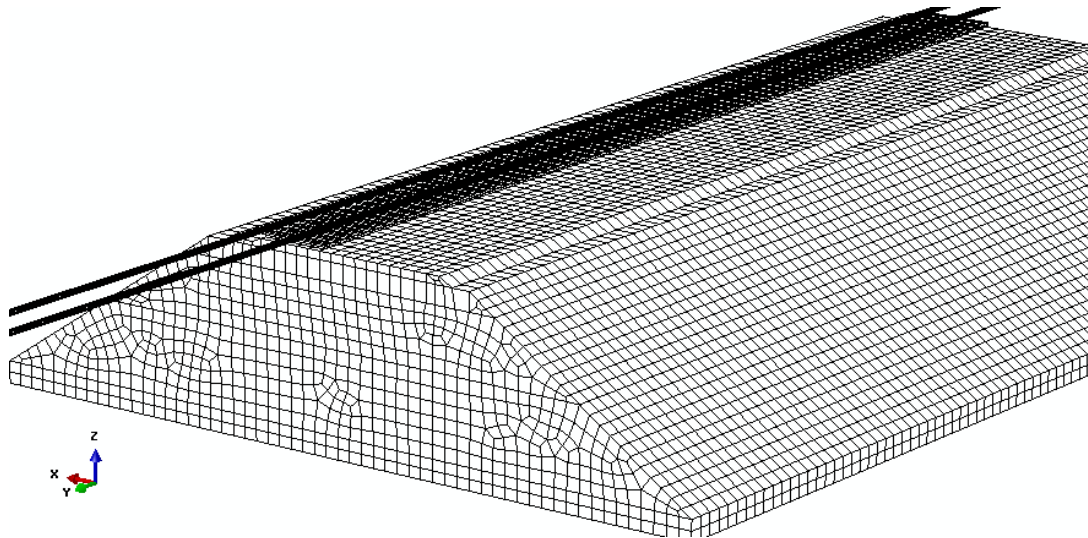


Fig. 2. FE model for dynamic analysis

In model, the contact force between wheel and rail is described by a Hertzian nonlinear contact spring with a unilateral restraint (Zhai et al.2007)[12]. With this contact algorithm, it is possible to simulate loss and recovery of contact between wheel and rail.

2.2 Effect of unsupported sleepers

At railway sites it is quite often found that isolated sections in railway tracks, where the ballast is in a poor state [3], provide weak and uneven support to the sleepers. Namely, the geometry size and density of the ballast are not uniform. Such imperfections gradually cause gaps to develop between the sleepers and the ballast. Measurements of unsupported sleepers have been performed by Li and Sun (1992) (Guangzhou Railway Corporation in China) [13]. It was found that up to 50 percent of all sleepers are more or less unsupported. The length of unsupported sleepers section was found to be in the range of 1m to 4m, The range of small gaps between sleepers and ballast were found to be between 2~4mm.

In the model, the gap under an unsupported sleeper is simulated by removing contact underneath the sleepers, as shown in Figure 3. According to Zhang et al (2008) [5], it was found that higher speed leads to larger normal load, and the normal load increases more rapidly with the increase of the gap size. In this paper, maximum speed of 140km/h and gap of 4mm employed for dynamic analysis was based on previous research (reference needed).

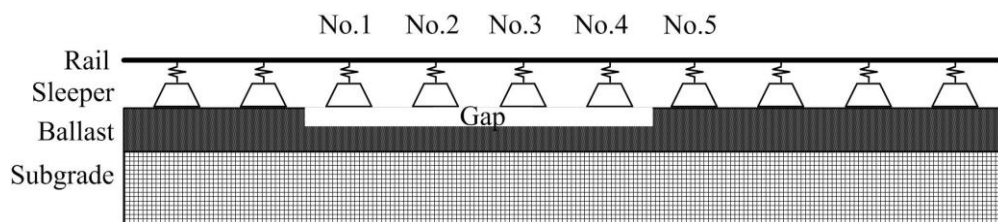


Fig. 3. Unsupported sleepers

2.3 Initial verification of model

The model is solved using the commercial finite-element program ABAQUS, which has an effective explicit solver. The time step is automatically made small enough so that high-frequency variations are well represented. The initial loading process last for 0.3s. A typical simulation of a complete vehicle passage of the 100m length track at the speed of 140km/h takes 48hours on a 2.0GHz processor on using Windows 7 operating system.

To validate the finite-element model used in the study, calculated results were compared with measured results. Vertical accelerations of the rail, sleeper and subgrade were measured by Shi et al (2010) on Shuohuang railway, an important heavy haul coal line in China. In the FE analyses, materials properties for the track system and subgrade in section 2.1 were used; the vehicle ran along the rail at a speed of 71 km/h, matching the measured train speed.

Table 3 compares the vibration levels and maximum accelerations measured in the field with those computed using the FE model for the rail, sleeper, at the bottom of the ballast, and at a depth of 2 m below the ballast bottom. From Table 3, although the computed values are slightly higher than those measured in the field, generally the results are in reasonable agreement. It is felt that the lower values determined from the field measurements may be as a result of high antecedent rainfall which occurred before the acceleration measurements were made, but after the field tests were carried out to determine the properties of the subgrade.

Table 3 Comparison of field and model vibrations at a speed of 71 km/h

	Vibration level(dB)		Max acceleration(g)	
	Field	Computed	Field	Computed
rail	153.7	155.0	23~50	40.00
sleeper	138.0	147.71	3~11	10.00
Subgrade at 0 m depth	102.5	109.6	0.3~0.8	1.0
Subgrade at 2m depth	100.5	105.0	0.4~0.6	0.40

3 Effect unsupported sleepers on response of vehicle-embankment

3.1 Effect on vehicle running

To better understand the implications of the unsupported sleepers measures on the stability and safety (in terms of derailment), simulations were performed using the FE model and compared with specifications. Each simulation was computed using the existing 250 kN axle load for the vehicle traveling at a speed of 140 km/h.

A number of railway organizations use the maximum vertical acceleration of the vehicle as a measure of stability, for example an upper limit of 0.7g is specified in Chinese standards for heavy haul railways [14]. Figure 4 shows the calculated accelerations for the four cases (with different number of consecutive unsupported sleepers) considered here. It may be seen that there is a rapidly increase in the magnitude of the accelerations for the wagon as the number of consecutive unsupported sleepers is increased, however the accelerations calculated in all cases are well below the upper limit stipulated in the Chinese standards. The value of vertical accelerations for two, three and four consecutive unsupported sleepers are respectively 35 percent, 69 percent and 138 percent higher than that for one unsupported sleeper.

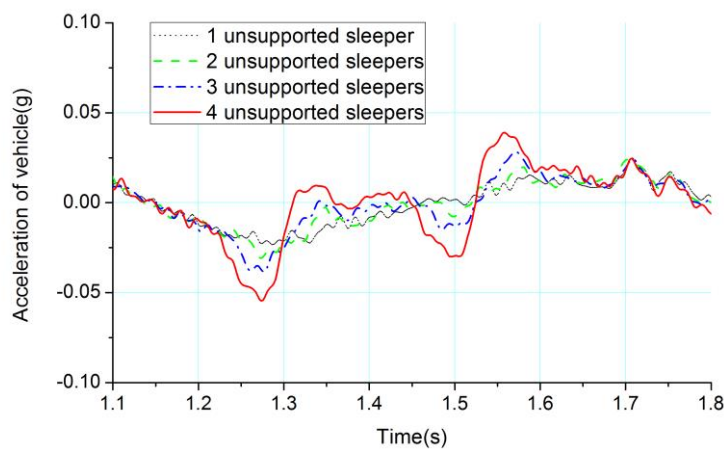


Fig.4. The vertical accelerations of the wagon

The likelihood of derailment of a train is commonly measured by computing the axle load decrement ratio (PD). It is defined as follows:

$$PD = \frac{\Delta Q}{Q_s} = \frac{Q_s - Q}{Q_s} \quad (\text{Equation 5})$$

Where, where Q_s and Q denote the static and dynamic wheel loads respectively. Since the lateral stability of the wheel is a function of the vertical load acting on it, the higher the value of PD (and therefore the lower the dynamic load) the lower the lateral stability and the greater the risk of derailment. Chinese railway standards put an upper limit of 0.6 on the PD value [14]. Figure 5 shows the computed PD values. From these it is evident that increased number of unsupported sleepers increase the risk of derailment, although the PD value in all cases is still less than that stipulated in the Chinese standards.

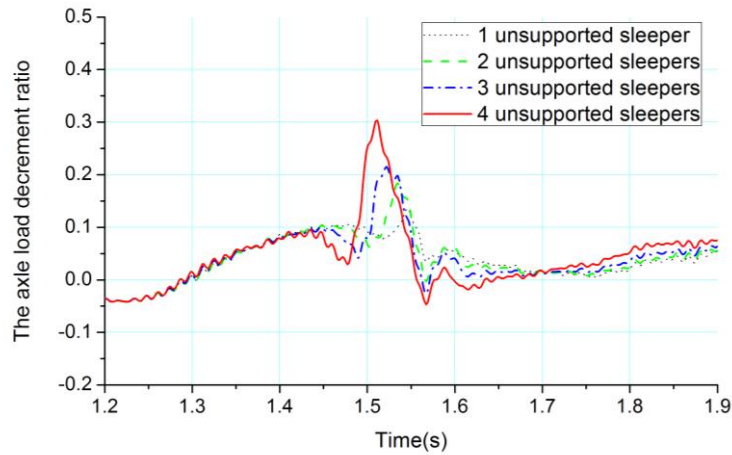


Fig.5. The axle load decrement ratio for the wagon

The wheel/rail contact force is a parameter used to assess the likelihood of excess deterioration wheel or rail deterioration and work by British Rail suggests that its maximum allowable value should be 250kN [15]. From the analysis described herein, computed values are shown in Figure 6 which may indicate that excessive wear at the wheel / rail interface may occur on the transition zone between lower stiffness tracks with unsupported sleepers to normal track. It can be seen that the magnitude of contact force for unsupported sleepers are less than that for normal track, as the wheel falls off the lower stiffness support and as a result compressed area of interface between wheel and rail decrease suddenly.

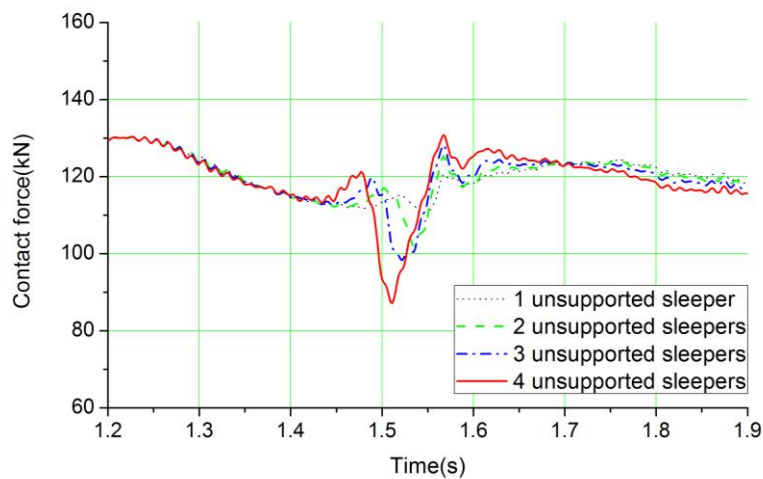


Fig.6. Contact force of wheel

3.2 Effect on track vibration

In this section, the effect of unsupported sleepers on track vibration was investigated.

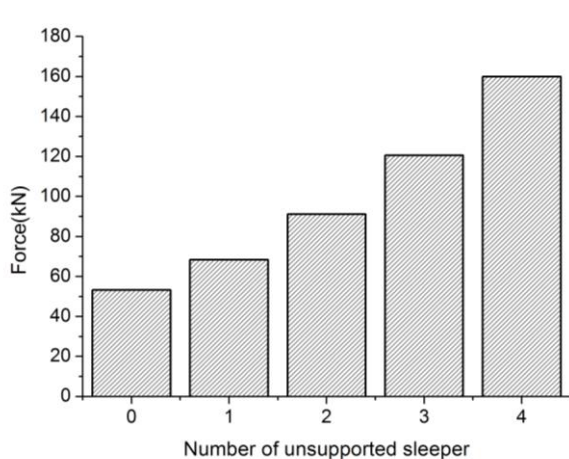
The sleeper next to the unsupported sleeper will be most exposed to the increased dynamic force.

Fig 7(a) shows the maximum reaction force of fastener at sleeper 5. There is a rapid increase in reaction force as the number of consecutive unsupported sleepers increases from one to four. The value of the reaction force for 4 unsupported sleepers is about 220% higher than that for normal track. It can be seen that the unsupported sleepers amplify reaction force of fastener which would result in deterioration of fatigue damage of fastener.

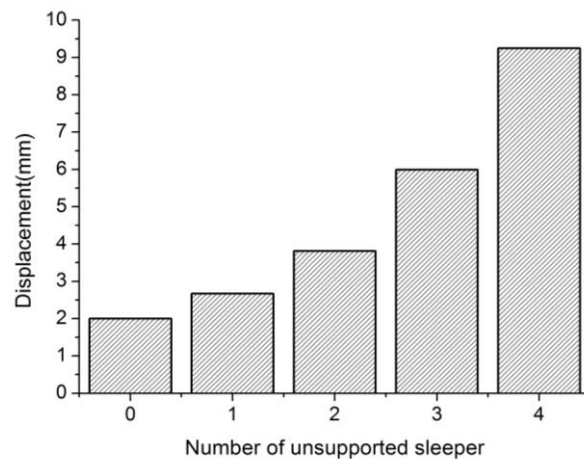
The displacement of sleeper and rail on unsupported sleeper zone are higher than that of normal track due to low stiffness. Fig 7(b) shows the maximum rail displacement in the middle of unsupported sleeper zone. The fluctuating amplitude increases when the number of unsupported sleeper increases. The abrupt change of rail displacement on unsupported sleeper zone will have undesirable effect on riding comfort and safety.

Fig 7(c) shows the maximum displacement of sleeper 3. As expected, an increase in sleeper displacement is observed as number of unsupported sleeper increases. When the number of unsupported sleeper reaches 3 or 4, all combinations of gap sizes will result in contact between the unsupported sleepers and the ballast with a gap of 4mm. Moving sleepers in the track may, of course, induce pulverization and cracking of the ballast stone and reduce the fatigue life of the sleeper.

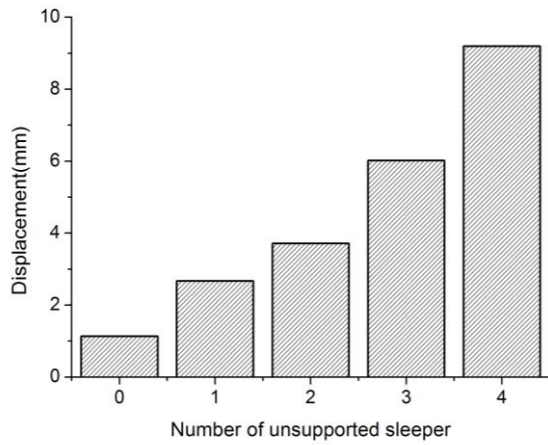
The maximum acceleration of rail in the middle of unsupported sleeper zone and sleeper 3 are given in Fig 7(d) and 7(e), respectively. An increase in number of unsupported sleeper is seen to have minimal effects on the vibration acceleration.



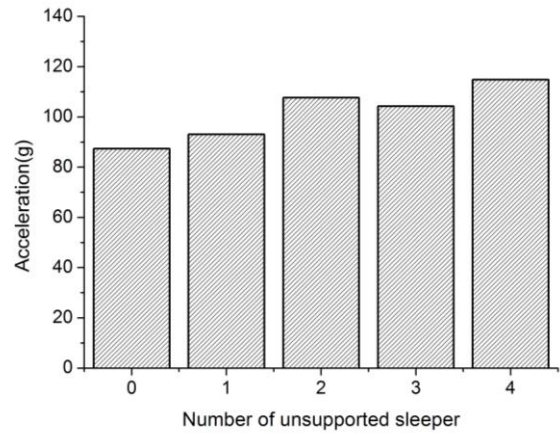
(a)



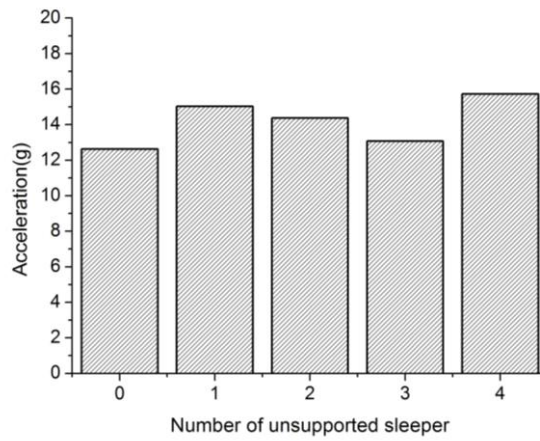
(b)



(c)



(d)



(e)

Fig.7.The effect of unsupported sleeper on track vibration : (a) reaction force of fastener ;(b) rail displacement; (c) sleeper displacement (d) rail acceleration (e) sleeper acceleration

3.3 Effect on subgrade stress

In this section, the effect of unsupported sleepers on subgrade stress, cumulative strain and design life were investigated.

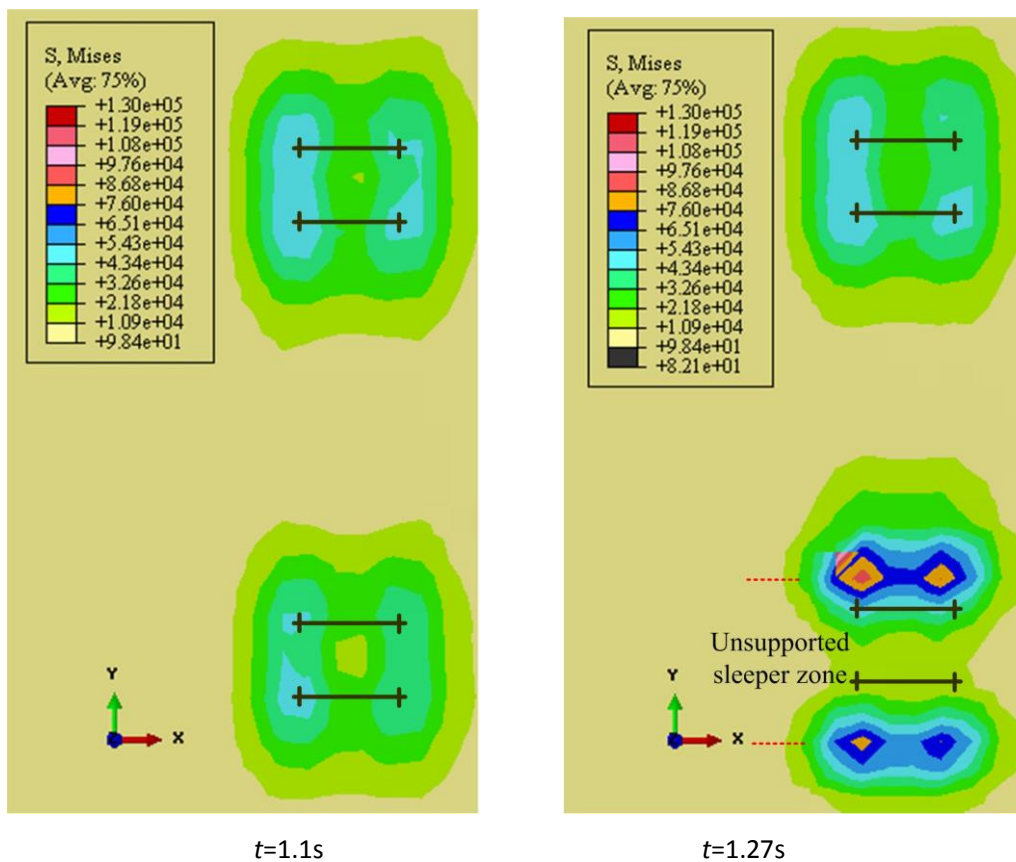
In three-dimensional stress state, the deviator stress σ_d can be determined using the following equation:

$$\sigma_d = \frac{\sqrt{\left[(\sigma'_x - \sigma'_y)^2 + (\sigma'_y - \sigma'_z)^2 + (\sigma'_z - \sigma'_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{1/2}}}{2} \quad (2)$$

where σ'_x, σ'_y and σ'_z are the normal compressive effective stresses in the x, y and z directions respectively, and τ_{xy}, τ_{yz} and τ_{zx} are the shear stresses in the xy, yz and zx planes respectively.

Figure 8 shows stress field of subgrade top surface when a vehicle is approaching and passing

over unsupported sleeper zone. A change in the stress pattern as vehicle approaches and passes over the void areas is clearly shown, stress concentrations around unsupported sleepers is observed. If sleepers are unsupported, the neighboring sleepers must carry a high load leading to locally increased stresses in the subgrade. Fig 9 shows the calculated deviator stress for the five cases at different depth of subgrade. The effect of unsupported sleepers, in terms of deviator stress, was particularly evident at depth of 0~3m below ballast. The value of deviator stress for one, two, three and four unsupported sleepers are 9 percent, 32 percent, 71 percent and 136 percent higher than that for normal track at top of subgrade respectively. From these it is evident that unsupported sleeper will accelerate deterioration at the top of the subgrade.



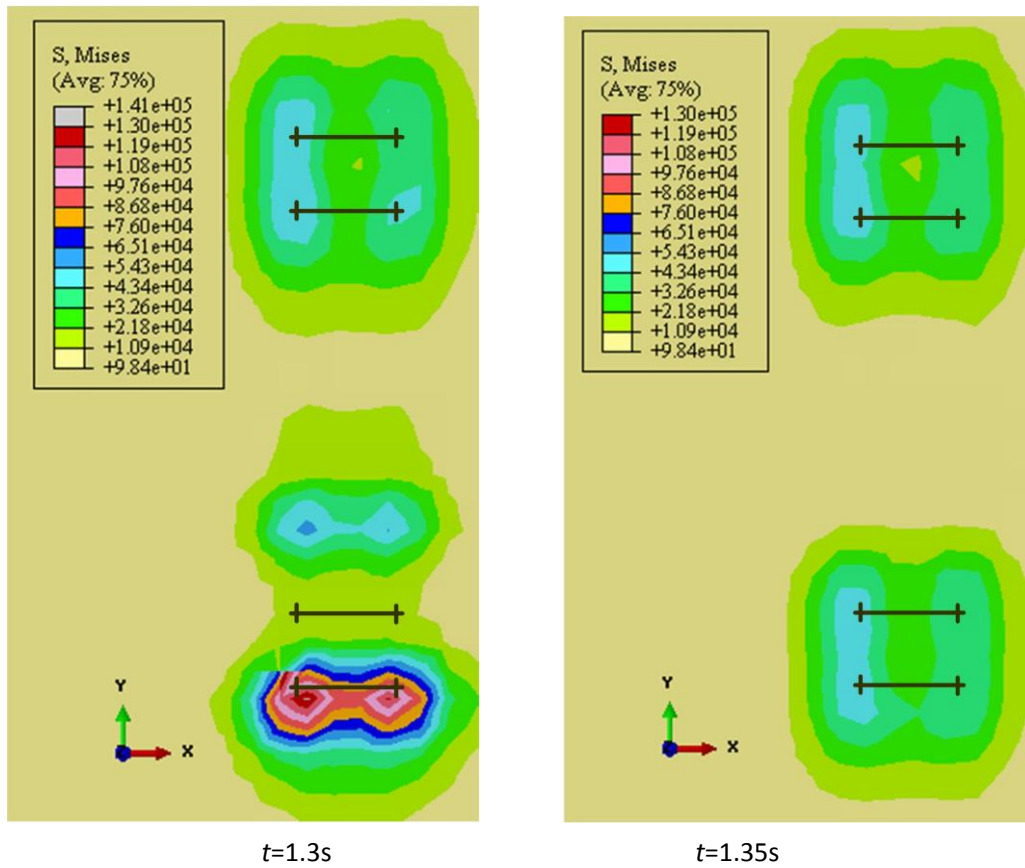


Fig.8. Induced stress field of subgrade top surface by vehicle passing over unsupported sleepers zone

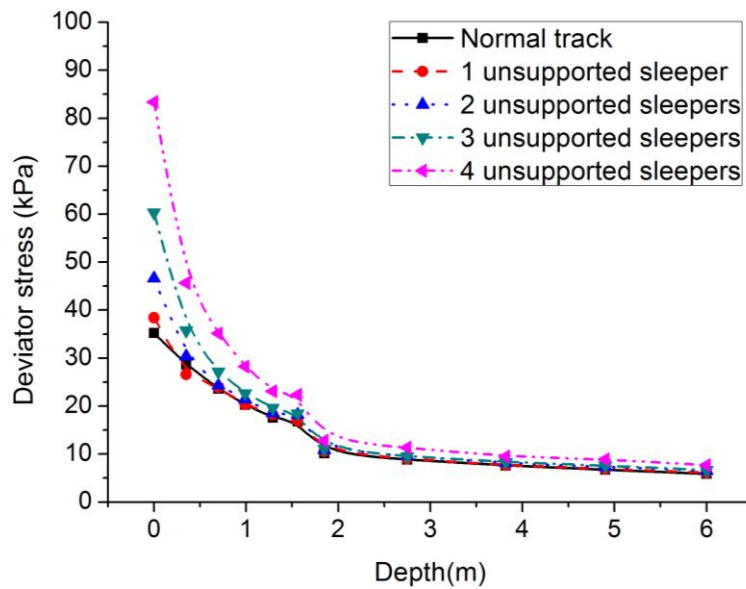


Fig.9. the effect of unsupported sleepers on deviator stress of subgrade

Progressive shear failure and excessive plastic deformation (ballast pocket) in the subgrade under truck passage occur mainly in embankment comprised of fine-grained soils and grained soils and

can be related to subgrade cumulative plastic strain as represented by following equation (Li et al.1994, Dong et al.2010) [16][17]

$$\varepsilon_p (\%) = a \left(\frac{\sigma_d}{\sigma_s} \right)^m N^b \quad (1)$$

Where ε_p is cumulative soil plastic strain, N is the number of repeated stress applications, σ_d is soil deviator stress caused by train dynamic load, σ_s is soil compressive strength, a , m , b are parameters dependent on the soil type.

The material parameters (a , m , b) used are given in Table 4. The material parameters of gravel and engineering fill were estimated from testing results of cyclic loads measured on Kongshan test section of Beijing-Shanghai railway in China (Dong et al.2010)[17]. The material parameter of silt sand were determined according to Li et al (1994)[16].

Table 4. Values of soil parameters for various soil type in this paper

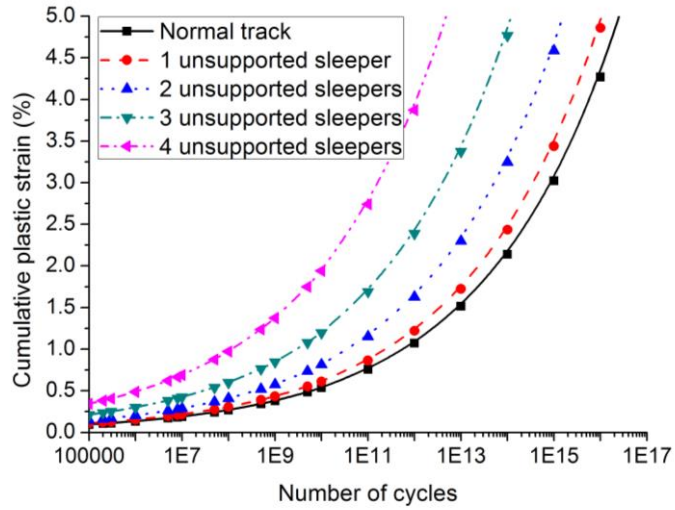
Soil type	a	B	m	σ_s (Mpa)	Note
Gravel	0.52	0.15	1.49	350	Dong et al.2010
Engineering Filler of class A	0.85	0.14	1.16	200	Dong et al.2010
Silt sand	0.64	0.10	1.7	100	Li and Selig(1996)

The design approach used to prevent subgrade progressive shear failure is to limit the total cumulative plastic strain at the embankment foundation surface below an allowable level for the period. This criterion is thus expressed by following equation:

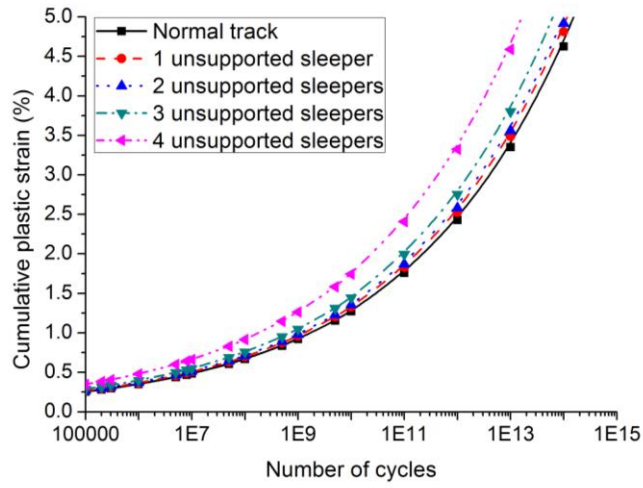
$$\varepsilon_p \leq \varepsilon_{pa} \quad (4)$$

Where ε_{pa} is allowable plastic strain at the foundation surface for design period. Limiting value of plastic strain of $\varepsilon_{pa} = 2\%$ suggested by Li and Selig(1998)[18].

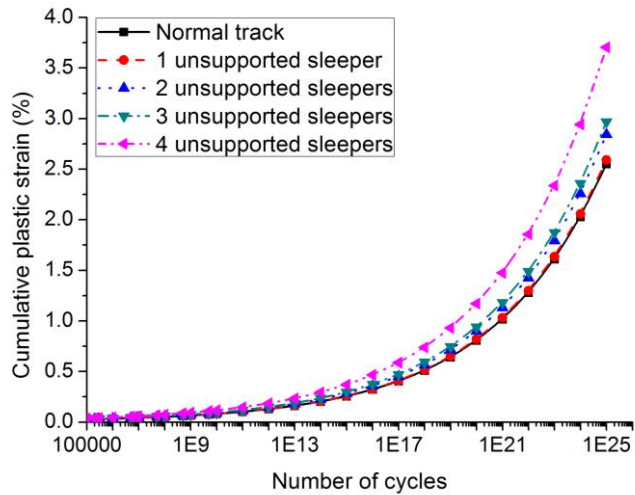
Figure 10 shows the calculated cumulative plastic strain for unsupported sleepers at different depth of foundation. As with the cumulative plastic strain of subgrade, there is a gradual increase in plastic strain as the number of unsupported sleepers increases from one to four. According to the strain criteria, the number of cycles for subgrade failure is summarized in Figure 11. It can be seen that there is a rapidly decrease in expected subgrade life which was determined by first layer that fails as the number of unsupported sleepers is increased. The life of subgrade failure for one, two, three and four unsupported sleepers are 25 percent, 35 percent, 60 percent and 95 percent less than that for normal track respectively. This implies, unsupported sleeper may accelerate subgrade progressive shear failure and differential settlement, increasing the number of unsupported sleepers amplifies the effect.



(a)



(b)



(c)

Fig.10.the effect of unsupported sleeper on subgrade failure:(a) cumulative plastic strain at gravel layer;(b)

cumulative plastic strain at filler layer; (c) cumulative plastic strain at silt layer

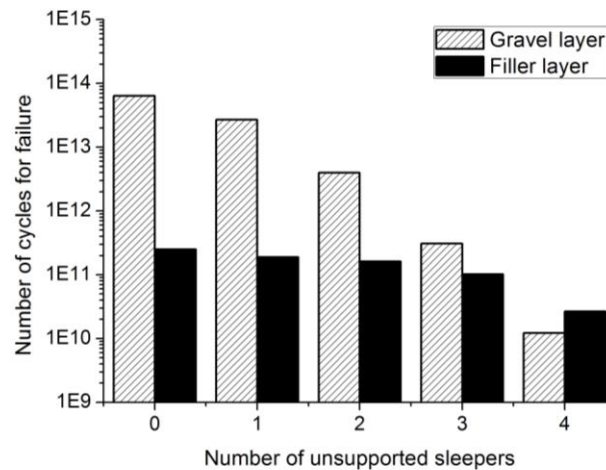


Fig.11.The number of cycles for subgrade failure

4 Summary and conclusions

In order to consider the effect of the number of consecutive unsupported sleepers, a dynamic three-dimensional finite element model using linear elastic material properties while including wheel-rail friction was developed and used to investigate the influence of unsupported sleepers on dynamic response. The influence of unsupported sleepers on ride safety, track vibration and subgrade failure is studied. The following conclusions have been drawn from the calculations:

1) There is a rapidly increase in the magnitude of the vehicle accelerations and the axle load decrement ratio for the wagon as the number of unsupported sleepers is increased. A preliminary investigation showed that the value for the commonly used measures of ride comfort and safety would remain within specified limits, provided that the speed of the vehicle is not increased unduly.

2) The fluctuating amplitude of rail displacement and sleepers on unsupported sleeper zone increases when the number of unsupported sleeper increases. When the number of unsupported sleeper reaches 3 or 4, all combinations of gap sizes will result in contact between the unsupported sleepers and the ballast with a gap of 4mm. Moving sleepers in the track may induce pulverization and cracking of the ballast stone and reduce the fatigue life of the sleeper.

3) The sleeper next to the unsupported sleeper will be the most exposed to the increase in dynamic force. The value of the reaction force for 4 consecutive unsupported sleepers is about 220% higher than that for normal track. It can be seen that the unsupported sleepers amplify reaction force of fastener resulting in deterioration of fatigue damage of fastener.

4) If sleepers are unsupported, the neighboring sleepers must carry a high load leading to locally increased stresses in subgrade. The life of subgrade failure for one, two, three and four unsupported

sleepers are 25 percent, 35 percent, 60 percent and 95 percent less than that for normal track respectively. This implies, unsupported sleeper may accelerate subgrade progressive shear failure and differential settlement, increasing the number of unsupported sleeper amplifies the effect.

It is suggested that track maintenance should seriously take into account the reduction in support due to unsupported sleeper and potential track faults resulted. Different track improvements should be considered: increase thickness of ballast, adding washer between sleeper and rail, use the stoneblower machine to improve ballast.

5 Acknowledgements

The work described in this paper has been supported by the National Natural Science Foundation of China (51008018). The computations described in this paper were performed using the University of Birmingham's BlueBEAR HPC service, which was purchased through HEFCE SRIF-3 funds.

References

- 1 **Augustin,S., Gudehus,G., Huber G., and Schünemann,A.** Numerical model and laboratory tests on settlement of ballast track. *System Dynamics and long-term behaviour of railway vehicles, track and subgrade*(Eds K. Popp and W. Schiehlen), 2003,pp.317-336 (Springer Verlag, Berlin).
- 2 **Grassie, S. L.,and Cox, S. J.** Dynamic response of railway track with unsupported sleepers. *Proc. Instn Mech. Engrs, Part D: Transport Engineering*, 1985, 199(D2), 123-135.
- 3 **Nielsen,J.C.O., and Igeland, A.** Vertical dynamic interaction between train and track- influence of wheel and track imperfections. *J. Sound Vibrat.*, 1995,187(5), 825-839.
- 4 **Lundqvist,A., and T. Dahlberg,T.** Load impact on railway track due to unsupported sleepers. *Proc. Instn Mech. Engrs, Part F: Transport Engineering*, 2005, 219 (F1),67-77.
- 5 **Zhang,S.G., Xiao,X.B.,Wen,Z.F., and Jin,X.S.**Effect of unsupported sleepers on wheel/rail normal load. *Soil Dynamics and Earthquake Engineering*, 2008,28: 662-673
- 6 **Yang, L.A., Powrie, W., and Priest, J.A.** Dynamic stress analysis of a ballasted railway track bed during train passage. *Journal of Geotechnical and Geoenvironmental Engineering*, 2009, 135(5), 680–689.
- 7 **Zhu,J.J.,Ahmed,A.K.W.,Rakheja,S., and Khajepour,A.**Development of a vehicle–track model assembly and numerical method for simulation of wheel–rail dynamic interaction due to unsupported sleepers.*Vehicle System Dynamics*,2010,48(12) , 1535-1552.
- 8 **Zhu,J.Y.,Zhang,Y.**Investigation on the track structure dynamic characteristics of with hanging

- sleepers. *China Railway Science*, 2011, 32(3), 8-15.
- 9 **Hibbitt, K.** *ABAQUS/Explicit User's Manual (version 10.0)*. Hibbitt, Karlsson & Sorensen, Inc., Pawtucket, RI, 2002.
- 10 **TB10001-2005**. Design Standards for Railway Embankments. China railway press, 2005.
- 11 **Zhai, W.M.** *Vehicle-track coupling dynamics*. 3rd ed. Beijing: China Railway Publishing House, 2009.
- 12 **Zhai, W.M., Wang, K.Y., and Lin, J.H.** Modeling and experiment of railway ballast vibrations. *J. Sound Vibr*, 2007, 270(4), 673–683.
- 13 **Li, Z.F., and Sun, J.G.** Maintenance and cause of unsupported sleeper. *China Railway Building*, 1992, 2, 15-17.
- 14 **Xia, H.** *Dynamic interactions between vehicles and structures*. Beijing: Science Press, 2002.
- 15 **Clark, R.A., Dean, P.A., Elkins, J.A., and Newtons, S.G.** An investigation into the dynamic effects of railway vehicle running on corrugated rails. *Journal of Mechanical Engineering Science*, 1982, 24(2), 65-76.
- 16 **Li, D. and Selig, E.T.** Resilient modulus for fine-grained. subgrade soils. *Journal of Geotechnical Engineering*, ASCE, 1994, 120(6), 939-957.
- 17 **Dong, L. and Zhao, C.G.** Method for the dynamic response of subgrade subjected to high-speed moving load. *Engineering Mechanics*, 2005, 25(11), 231-240.
- 18 **Li, D. and Selig, E.T.** Method for railroad track foundation design. I: Development. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 1998, 124(4), 316-322.