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Straight thinking about groundwater recession

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Abstract

While in catchment and hillslope hydrology a more nuanced approach is now taken to streamflow recession analysis, in the context of major aquifers it is commonly still assumed that the groundwater head recession rate will take exponential form, an idea originally proposed in the 19th Century. However it is shown here that, in early times, the groundwater head recession in a major aquifer should take an almost straight line form with a rate approximately equal to the long term recharge rate divided by the aquifer storage coefficient. The length of this phase can be estimated from an analytical expression derived in the paper which depends on the aquifer diffusivity, length scale and the position of the monitoring point. A transitional phase then leads to an exponential phase after some critical time which is independent of the position of the monitoring point. Major aquifers in a state of periodic quasi-steady state are expected to have rates of groundwater flux recession which deviate little from the average rate of groundwater recharge. Where quasi-exponential groundwater declines are observed in nature, their form may be diagnostic of particular types of aquifer properties and/or boundary effects such as: proximity to drainage boundaries, variations in transmissivity with hydraulic head, storage changes due to pumping, non-equilibrium flow at a range of spatial and temporal scales and variations in specific yield with depth. Recession analysis has applicability to a range of groundwater problems and is powerful way of gaining insight into the hydrologic functioning of an aquifer.

1. Introduction

Analysis of groundwater hydrographs can yield potentially powerful insight into the hydraulic properties of an aquifer and its hydraulic functioning. Despite this, there are relatively few studies which have systematically explored the general form of groundwater head recessions for major aquifers.

Water table fluctuation observations reflect the balance of the groundwater recharge rate (q) and the net groundwater drainage rate (D) experienced by the aquifer at the monitoring location. When q is less than D a *groundwater head decline* will occur. If q is zero the groundwater hydrograph will exhibit a true *groundwater head recession*, whose rate may vary in time depending on the antecedent conditions, aquifer properties, and boundary conditions. The relative impacts of these factors on groundwater recession is the primary focus of this paper and other causes of groundwater head declines such as loading effects, barometric variations and earth tides are not considered here.

It is commonly assumed that, in the absence of groundwater recharge, a groundwater head decline will take exponential form. Superficially this seems reasonable, having in mind the conceptualisation of an aquifer as a 'linear' reservoir draining against a relatively constant boundary head such as a river: intuitively we would expect that the rate of recession will be greater for greater heads in the aquifer and decay away over time at an ever decreasing rate. This idea has a long history in the hydrological literature since at least Boussinesq (1877) who showed that both the groundwater head and also the streamflow (or baseflow) recession may be expected to take exponential form. Since then, a large body of literature has refined the understanding of baseflow recessions going well beyond the early exponential model (Polubarinova-Kochina, 1962; Lockington, 1997; Parlange, 2000; Brutsaert, 2005; Basha, 2013). Typically however, the behaviour of groundwater hydrographs is not the focus of such studies and relatively little literature explicitly addresses the question of the form of groundwater head recession. Furthermore, most detailed studies of baseflow recession which utilise the most recent understandings are applied to small, diffusive and, often, sloping hillslope environments where flows and head responses in larger aquifers are not of concern (Rupp & Selker, 2006; Troch et al, 2013). Groundwater hydrologists still typically revert to the exponential model when working in the context of major aquifers (Schwartz 2010, Domenico & Schwartz, 1998; Rousseau-Gueutin et al., 2013), since the linearization of the Boussinesq equation, which leads to such behaviour for late time, is often well justified in these cases. While the literature on groundwater head recession for large aquifers is relatively sparse, a foundational analysis was given by Rorabaugh (1960), finding that groundwater heads may indeed eventually recede exponentially. Importantly however, this only occurs after some 'critical time' which is controlled by the properties of the aquifer (see Appendix A). Furthermore, despite Rorabaugh's statement that "the question of critical time cannot be taken lightly" (Rorabaugh, 1960, p.315), most research in the intervening 50 years has ignored it and explicitly or implicitly assumed that groundwater recession will be exponential in form without due consideration of the critical time parameter, i.e. the early time behaviour is rarely considered, with the emphasis in the literature being on the late time exponential behaviour. This point is returned to in the discussion section below.

In this paper, the concept of groundwater head recession is first explored using a series of thought experiments formalised using analytical solutions to the relevant groundwater flow equations for idealised aquifers. The primary focus is on major water-table aquifers to which linearised forms of the Boussinesq equation are applicable. Observations from real aquifers are then explored to highlight the potential insight to be gained from studying deviations in recession behaviour from expectations based on ideal conditions. The objectives are (1) to test the widely held belief that groundwater head recessions should be exponential in form, (2) to see whether groundwater theory suggests a more general form of groundwater head recession for typical idealised aquifer configurations, and (3) to see what inferences can be made therefore from the form of groundwater recession observations in real aquifers regarding aquifer properties/boundary conditions where they deviate from the expected form.

To avoid confusion it should be noted that in this paper the term linear recession is taken to mean one in which the rate of change of head with respect to time is constant. This is in contrast to the concept of a hydrological 'linear store' in which the rate of change of head is linearly proportional to the head itself which, in the terminology of this paper, would be considered an exponential recession.

2. General form of groundwater recession in ideal aquifers

2.1 Governing Equations and Definitions

Let us begin by considering the case of an ideal homogeneous, horizontal aquifer bounded at one end ($x = L$) by a river assumed to be a constant head boundary and at the other ($x = 0$) by a no-flow boundary representing a flow divide (Figure 1a). Although idealised, the situation is typical of many unconfined aquifer systems. A one-dimensional Boussinesq equation of groundwater flow for an aquifer receiving homogeneous recharge can be given as follows:

$$\frac{\partial}{\partial x}\left(KH\frac{\partial H}{\partial x}\right) = S\frac{\partial H}{\partial t} - q(t) \tag{1}$$

where K is hydraulic conductivity [LT^{-1}], S is specific yield [-], $H(x,t)$ is saturated aquifer thickness [L], t is time [T], x is distance [L] and $q(t)$ is groundwater recharge [LT^{-1}].

If changes in transmissivity due to fluctuations in groundwater heads are assumed to be negligible, and generalising H to $h(x,t)$ (groundwater head above ordinary datum, [L]), Equation (1) may be linearised as follows:

$$T\frac{\partial^2 h}{\partial x^2} = S\frac{\partial h}{\partial t} - q(t) \quad (2)$$

where T is transmissivity [L^2T^{-1}].

The lateral boundary conditions are as follows:

$$\frac{\partial h(0,t)}{\partial x} = 0,\ h(L,t) = 0 \quad (3)$$

Solutions at various levels of complexity are possible depending on the applied initial conditions and form of the function governing recharge; several informative cases are described below and in the Appendices, based on the two geometries shown in Figure 1.

An important observation can be made directly from Equation 2; in the absence of any recharge (i.e. if q = 0), the *'net groundwater drainage'* flux, D [LT^{-1}] can be described by the LHS of Equation 2, i.e. $D(x,t) = T\frac{\partial^2 h}{\partial x^2}$. This is the rate of 'groundwater *flux* recession' and is equal to the rate of groundwater *head* recession multiplied by S. For understanding the nature of groundwater head recession developed in this paper, it is fundamentally important that this concept is grasped.

2.2 Phases of evolution of groundwater recession

Venetis (1971) presents an analytical solution to Equations 2&3 (Case A, Figure 1a) which includes the effect of an initial non-horizontal water table, and is thus a more realistic case than the analysis of Rorabaugh (1960). The initial condition is a steady state water table ($h(x,t) = q_c(L^2-x^2)/(2T)$) subject to a constant recharge rate, q_c. The solution for recession from this condition under subsequent conditions of zero recharge, can be shown to be:

$$h_{Ven}(x,t) = \frac{16L^2 q_c}{\pi^3 T}\sum_{m=1,3,5...}\frac{1}{m^3}\left[e^{-m^2\pi^2 Tt/4L^2 S}\sin(m\pi(L-x)/2L)\right] \quad (4)$$

For the case of an aquifer at steady state conditions, it is obvious that *if recharge suddenly ceases, at that instant, the flux recession rate must be equal to q_c*. Furthermore, because of the linearisation of Equation 1 the case of purely exponential decay will only occur once the water table has taken the form of a sinusoid (as is clear from Equation 4). The time taken for the system to show exponential decay at all points is governed by the same critical time as for the Rorabaugh (1960) solution (Appendix A).

By using the definition of *D* described above we can derive a simple expression for the flux recession whereby:

$$D(x,t) = \frac{4q_c}{\pi} \sum_{m=1,3,5\ldots} \frac{1}{m} \left[e^{-m^2\pi^2 Tt/4L^2 S} \sin(m\pi(L-x)/2L) \right] \quad (5)$$

Figure 2a indicates that, as expected, the rate of flux recession defined by Equation 5 is equal to the prior steady state recharge (i.e. $D/q_c \approx 1$) and remains very close to this value for significant lengths of time for moderate to low diffusivity aquifers until the change in boundary effects are felt significantly. At higher diffusivity and or closer to the constant head (drainage) boundary, the normalised recession rate reduces to an exponential rate more quickly. For example, in Figure 2, far from the drainage outlet, (Figure 2a, x/L = 0), the recession rate does not vary significantly from the steady state rate for approximately 500 d for a major (e.g. L>5000 m), moderately diffusive (T/S typically <a few thousand m^2/d) unconfined aquifer.

Figure 3 illustrates 3 distinct phases in the evolution of the groundwater recession for such an aquifer:

1. *Linear phase* - the head profile initially decays at a constant rate with the rate of groundwater flux recession almost equal to the steady state recharge applied to create the initial condition. The rate is infinitesimally smaller than the steady state recharge rate from the very beginning of the recession but will be within approximately 0.5% of the initial value while $t_{lin} < d^2S/(16T)$, with

$d = x\text{-}L$ (Figure 1), i.e. the distance away from the lateral head boundary representing a drainage outlet (see Appendix B).

2. *Transitional phase* – for $d^2S/(16T) < t < 0.15L^2S/T$, the recession rate begins to decrease much more rapidly.

3. *Exponential phase* – when the critical time is reached ($t_{crit} \approx 0.15L^2S/T$) the head profile becomes sinusoidal in shape and the rate of recession then decreases exponentially (straight line on the log-linear plot in Figure 3b). The critical time will vary with aquifer geometry and inhomogeneity and two new formulae for estimation in these cases is given in Appendix A.

Note that the length of the linear phase is dependent on the position of the value of *x* (i.e. the position of an observation point relative to a constant head boundary) but the critical time is independent of *x*, and solely controlled by the aquifer diffusivity and length scale.

2.3 Critical time versus time between recharge events

Despite the theoretical evolution of groundwater recession described above, for many, if not most aquifers, the critical time is much greater than the time between recharge events. Figure 4 shows the distribution of critical time for the case shown in Figure 1a (using Equation A3) for a range of values of hydraulic diffusivity and aquifer length scale. Unconfined aquifer transmissivity generally ranges from 10 to 1000 m^2/d (Freeze & Cherry, 1979), and specific yields are typically 0.01 to 0.2 (Kruseman & Ridder, 1990), hence the scale for *T/S* has been plotted up to 100 000 m^2/d.

It is apparent that the critical time is in the range of tens to hundreds of days for all but the most hydraulically diffusive or small aquifers. Most major (e.g. L>5000 m), moderately diffusive (*T/S* typically less than a few thousand m^2/d) unconfined aquifers will have critical times of hundreds to thousands of days. Hence, conditions under which an exponential recession can be observed is rather limited, since this requires zero recharge conditions to persist for periods of time long enough only to be generally applicable to semi-arid or arid climates.

2.4 Groundwater declines under quasi steady state conditions

On the basis of the last section, since subsequent recharge events may obscure the later phases of the groundwater head evolution, the linear phase should perhaps be the most commonly observed. However, before we can conclude this, we should note that recessions will not often begin under steady state conditions, and additional analysis is needed. Thus, we now consider the case of an aquifer in quasi-steady state conditions – this is a much more realistic scenario since, for example, many aquifers show an annual trend in water table fluctuations, superimposed on to a more slowly varying climatic signal.

If a recharge signal varies sinusoidally around an average value (q_a) as $q(t) = q_a(1 - \cos \omega t)$, with ω as the angular frequency [T^{-1}], for Case A (Figure 1a), Cuthbert (2010) showed that the amplitude (A) of oscillation of the net groundwater drainage rate, D, is given by:

$$A = \left| q_a \left(\frac{\cosh \lambda x}{\cosh \lambda L} \right) \right| \tag{6}$$

where $$\lambda^2 = \frac{i \omega S}{T} \tag{7}$$

For Case B (Figure 1b) by extending closed form solutions of the radial flow equations derived by Townley (1995), here I present an equivalent solution to Equation 6 as follows for the radial case:

$$A = \left| q_a \left(\frac{I_0(\lambda r)}{I_0(\lambda R)} \right) \right| \tag{8}$$

where I_0 is a modified Bessel function of the first kind and order 0.

Thus, for both cases, the relative variation of *D* can be calculated for a particular periodic signal, set of aquifer properties and location relative to a drainage divide.

Figure 5 indicates that for a wide range of aquifer response rates, normalised amplitude variation in *D* is minimal and can thus be assumed approximately equal to the average recharge rate. It is also important to note, contrary to the misreading of Cuthbert (2010) reported by Liang & Zhang (2012), that the above approximation holds well even in several non-idealised cases such as the non-linearised case, for non-sinusoidal recharge, for aquifers with moderately sloping bases and certain cases of spatially variable recharge as described in Cuthbert (2010).

It should be noted also that this analysis provides a way of estimating expected variations in the net groundwater drainage rate, *D*, and in many such cases these will be significantly greater than observed groundwater declines unless the recharge becomes negligible and the true rate of groundwater recession is revealed.

3. Groundwater recessions in real aquifers

3.1 Inferences based on departures from an ideal aquifer analysis

A consistent picture has emerged from the foregoing analysis that the recession exhibited by ideal aquifers will vary in form both spatially and temporally, dependent on the aquifer properties, geometry, and location of the monitoring point relative to catchment boundaries. Based on an initial conceptual model of a catchment's hydrogeology, the analytical expressions given earlier in the paper, and in the Appendices, may therefore be used to derive an expectation as to the characteristic form and timing of groundwater head recessions in different parts of the catchment in question. Where deviations from the expected behaviour are seen, these may thus be diagnostic of particular types of departure from the assumptions of the ideal model. This information may then be used to infer more detail regarding aquifer properties or boundary effects and to improve the conceptual model.

For small and/or highly hydraulically diffusive aquifers, t_{lin} may be very small and if the time between recharge events is sufficiently greater than t_{crit}, the recession would be expected to be exponential in form. Real world examples are shown by Rorabaugh (1960) and more recently in Nimmo (2010) and Cuthbert et al. (2013). For such cases, it should be noted that Rutledge (2006) tested the Rorabaugh (1960) model for some non-idealised scenarios using numerical models and showed that significant deviations from an exponential form may occur for example in cases of sloping boundaries or those with complex geometry.

Larger aquifers, those with more moderate to low diffusivities aquifers, and those experiencing prevailing quasi-steady state conditions may be expected to exhibit approximately linear recessions. However, despite the theoretical basis described above, linear recessions are rarely reported in the literature and it is therefore important to ask why this is the case, and what departures from linearity can inform us about the aquifer properties or boundary conditions of an aquifer to enable inferences to be made regarding its hydrologic functioning. Several reasons are now proposed for why non-linear effects may dominate observed groundwater declines in real systems where linear recessions may have been expected based on idealised aquifer analysis:

A. *Where the temporal variation in recharge is relatively smooth*. Where aquifers exhibit relatively smooth fluctuations in groundwater level it may be difficult to discern a true groundwater recession from a groundwater head decline during which some recharge is still occurring. The presence of thick unsaturated zones or coverings of superficial deposits (Cuthbert et al., 2009; Cuthbert et al., 2010a) will, in many cases, greatly smooth the recharge signal meaning that periods with zero recharge are very rare, at least in temperate to humid regions. For aquifers whose head variations are governed by more episodic recharge, either due to the sporadic nature of inputs from precipitation (e.g. in semi-arid to arid regions) or due to preferential flow enabling the rapid movement of water to the water table even through thick unsaturated zones (Beven & Germann, 2013; Mirus & Nimmo, 2013), there is more chance that the linear phase of recession will be observed.

B. *Wells located close to drainage boundaries even in moderate to low hydraulic diffusivity catchments*. As described in the previous section, if a groundwater monitoring well is located sufficiently close to a drainage boundary, the effect of the proximity of the boundary may quickly dominate the recessional behaviour even if the hydraulic diffusivity is relatively low (Equations 5, A10).

C. *Aquifers where T varies significantly with h.* Most obviously this is the case for thin aquifers, and there is much literature devoted to finding solutions to the non-linearised Boussinesq equation (Boussinesq, 1904; Polubarinova-Kochina, 1962; Parlange et al., 2000; Brutsaert, 2005). Unfortunately, analytical solutions are not tractable for most useful applications. Perhaps less obviously, aquifers exhibiting marked variations of transmissivity or storativity with depth may show significant head dependent variations in recession rates. For example this is the case in the Chalk of NW Europe, a regionally important aquifer, whereby transmissivity and specific yield reduce with depth controlled by progressive weathering/dissolution of fractures (Ireson et al., 2009). This is thought to lead to groundwater recession rates governed, in part, by the position of the water table within the weathering profile with recession rates greatly enhanced during periods when the most permeable horizons are hydraulically active (Soley et al., 2012). In the case of lower permeability deposits where vertical rather than lateral flow dominates, such effects of vertical permeability and specific yield with depth can also be a significant factor influencing water table recessions for example in fractured glacial tills (Cuthbert et al., 2010a). Significant variations in T with h may also be likely in strongly sloping aquifers and there is a large body of literature regarding solutions to the sloping aquifer problem mainly to understand baseflow recession from hillslopes (Rupp & Selker, 2006). In aquifers with sloping bases the recession rate is related not only to the hydraulic diffusivity and length scale but also to the hydraulic advectivity which is controlled by the hydraulic diffusivity and the steepness of the slope of the aquifer (Brutsaert, 2005). Thus, deviations from the ideal groundwater head recession described above are to be expected.

D. *Where effects other than simple recharge/discharge dynamics are influenced by other factors influencing catchment storage*. Most significantly, where dynamic or spatially variable groundwater abstractions occur (either by pumping or due to natural effects such as spatially variable capillary fluxes under varying climatic conditions), the rate of groundwater recession may be significantly affected. For example this was described by Cuthbert (2010) for a case study in Shropshire, UK, whereby during a series of dry years recession rates were greatly increased due to the pumping operations of a groundwater augmentation scheme. Once the scheme was switched off again, groundwater recessions decreased once more. This principle has also been invoked by Ordens et al. (2012). Although the principles governing these effects are well understood in principle, due the inherent spatial impact of this effect exerted by the specific locations of pumping wells and their temporal dynamics, such effects may greatly complicate the interpretation of groundwater hydrographs. As a result, analysis using the analytical forms described in this paper are likely to be severely limited. In such cases, 2 or 3-D groundwater model analyses may be necessary to be able to untangle the relative contributions to groundwater recession from natural and pumping induced effects.

E. *Non-equilibrium flow at a range of scales*. Where groundwater recharge is not evenly distributed in space, the redistribution of water within both the unsaturated and saturated zones may complicate the form of groundwater recession leading to a decrease of rate with time and a quasi-exponential form. This may be envisaged at a range of spatial and temporal scales (Figure 6). Variations in local scale flow processes operating in both vertical and horizontal directions will influence the timing and magnitude of groundwater recharge. The additional complexity of inhomogeneity in the applied recharge boundary condition, both in time and space, will then influence the horizontal drainage dynamics and characteristic recession behaviour.

At a small scale this may be expected to occur under conditions of preferential flow around soil peds or 'matrix' blocks. At this scale, rapid downward flow of water via macropores or other preferential flow pathways may occur without hydraulic equilibrium occurring between such pathways and the intervening matrix materials. Thus, at the water table, an initial steep recession may be expected to occur as equilibration takes place. The author is unaware of any field data for which this mechanism has been invoked as an explanation for the form of such recession. However, several studies on soil macropores show this type of response in tensiometers (Cuthbert et al., 2013), and it is straightforward to simulate such a response using a dual domain preferential flow model.

One such simulation is shown in Figure 6a based on the dual permeability formulation of Gerke & van Genuchten (1993) implemented using Hydrus 1-D (Simunek et al., 2012). Hydrostatic initial conditions in both domains were prescribed within a 100 cm deep profile with a water table at 14 cm above the model base (datum). The upper boundary condition was an atmospheric boundary supplied with a random infiltration time series. The lower boundary condition was set to constant flux with a value of -0.05 cm/d. Standard van Genuchten-Mualem hydraulic parameters for a sandy-loam matrix (subscript m) and fracture (subscript f) domains were set as follows: ϑ_{rm}=0.05, ϑ_{sm}=0.3, α_m=0.1 cm^{-1}, n_m=1.8, K_{sm}=1 cm.d^{-1}, ϑ_{rf}=0, ϑ_{sf}=0.5, α_f=0.1 cm^{-1}, n_f=2, K_{sf}=100 000 cm.d^{-1}. Additional parameters controlling the fluid exchange were set as follows: ratio of the volumes of the fracture and total pore system, w=0.01; the geometrical shape factor, β=γ=a=1; the effective hydraulic conductivity of the fracture-matrix interface, K_{sa}=0.01 cm.d^{-1} (see Simunek et al. (2003), for a detailed description of these parameters). Figure 6a is the resultant time series of head at the base of the soil profile.

At an intermediate scale, an example is described in more detail for the Ugandan case below, and illustrated in Figure 6b.

At a larger scale, dynamic groundwater mounding under losing streams due to so called ‘indirect recharge’ (Healy, 2010) can also lead to nonlinear forms of groundwater recession. For example, in a disequilibrium flow process at a larger length scale, initial groundwater declines following ephemeral streamflow events are typically very steep, decaying at a decreasing rate as the groundwater mound beneath the stream recedes, spreading out across the catchment (Figure 6c). A number of analytical solutions are available in the literature for describing the transient evolution of such a groundwater mound (e.g. Abdulrazzak &Morel-Seytoux, 1983). At later times following a recharge event the groundwater recessions take a linear form.

Thus, across a great range of spatial scales, any processes that focus recharge preferentially may cause groundwater hydrograph recessions to be characterised by an initially steep decline due to the re-equilibration of local groundwater mounding followed by a more linear form governed by the larger scale groundwater flow system.

F. *Shallow water table conditions*. Where water tables are shallow enough, even if the aquifer materials are homogeneously permeable, the form of recession may become nonlinear for at least two reasons. First, since the available storage (i.e. the specific yield) increases with depth to water table (Childs, 1960), the rate of recession may be steeper at early times until the water table is sufficiently lower than the ground surface. Second, in such shallow water table cases, evapotranspiration is also likely to drive upwards flow which will also lead to non-linearity in the observed water table declines, with faster recessions expected at earlier times (and therefore for smaller depths to water table) due to greater upward capillary flux.

G. *Transience in specific yield.* In most aquifers, drainage does not occur instantaneously; the drainage rate is dependent on the hydraulic properties of the aquifer and the depth to water table

(Nachabe, 2002; Acharya et al., 2012). Thus, the concept of a time independent specific yield is of limited use in such contexts. Unsaturated zone theory would suggest that following a sharp water table rise, early time recession may be faster than that at later times due to the decrease in hydraulic conductivity with lowering moisture content in the zone above the capillary fringe as it progressively drains. However, most recharge pulses are significantly smoothed during passage through the unsaturated zone such that this transient effect may in practice be hard to observe unless the water table is very shallow. In such cases, the effect may be hard to separate from the effect noted above regarding the variation of specific yield with depth to water table.

3.2 A worked example from Uganda

A brief worked example is now given in order to demonstrate that linear recession behaviour is actually observable in real systems, since it is not often reported in the literature. The example also illustrates how observed departures of recession behaviour based on ideal aquifer analysis can lead to refinement of a hydrological conceptual model.

Figure 7 shows a 10 year groundwater monitoring record from Soroti, Uganda, including several extended periods of negligible rainfall. Groundwater flows from a topographic high on a ridgeline, through weathered and fractured basement rocks, discharging mostly via evaporation in a valley wetland. The detailed hydrogeological background is given by Cuthbert & Tindimugaya (2010), and based on the findings of that paper, the values of t_{lin} and t_{crit} are estimated to be around 44 d and 420 d respectively. This suggests that the recessions observed during dry periods which last up to 2 months over the monitored period should be approximately linear in form. Furthermore, the system appears to be in a quasi-steady state; groundwater head fluctuations show an annual signal superimposed on an approximately 3 yearly cycle. Using Equation 6 for periods of 1 and 3 years, the variation in the recession rate from the average recharge rate would be expected to be approximately just 10% and 25% respectively.

As expected, long periods of linear recessions are observed as shown for 5 dry periods in Figure 7. Also, the range of gradients of the recessions observed, accounting for the likely error in the daily manual dip measurements, is consistent with the variations predicted by calculations based on Equation 6. However, at early times following recharge, an initially steep groundwater decline occurs before the recession becomes linear. This warrants further explanation.

Most of the mechanisms, A-G, described above can be ruled out in this case; as has been argued by Cuthbert & Tindimugaya (2010), the most likely explanation is that a localised focussing of infiltration occurs through preferential pathways within the lateritic regolith which overlies the weathered basement aquifer in this location (Figure 6b). Thus, following recharge, an initially steep groundwater decline occurs while the local groundwater mounds equilibrate across the aquifer. After this time, the recession exhibits an almost exactly linear form for periods of up to two months until the next recharge event causes a slowing of the groundwater decline or an increase in head (Figure 7b).

Thus, the form of the groundwater recession has, in this case, been useful in inferring the mechanism of groundwater recharge in this location.

4. Discussion

It has been shown in this paper that groundwater head recession in an idealised major aquifer may evolve from being initially linear to eventually exponential in form. This raises the important question as to why previous literature has predominantly focussed on the exponential phase. I propose that this may be for a number of reasons. First, the literature describing groundwater recession from a hydraulic perspective generally report case studies based on small and highly diffusive aquifers where t_{crit} is small in any case (Rorabaugh, 1960; Venetis 1969, 1971; Olin, 1992; Crosbie, 2005; Rutledge, 2006; Park & Parker, 2008; Jie et al., 2011; Liang & Zhang, 2012). Venetis

(1969) even explicitly states that t_{crit} will be less than one month most of the time, but without giving any justification for that assertion, and Venetis (1971) suggests "experience often shows that this [i.e. critical time is reached] occurs after the first week". Rutledge (2006) notes that departures from the exponential form will occur prior to the critical time but does not go further to present a range of critical times for typical aquifer conditions. Second, the popularity, simplicity and intuitively appealing idea of aquifers acting as 'linear stores' has become standard modelling practice in both hydrogeology (e.g. Schoeller, 1959; Gehrels & Gieske, 2003) and hydrology (e.g. Nash, 1959). This has, I suggest, also strengthened the perception that groundwater recessions should be generally exponential in form.

Clearly, from the above analysis, the form of a groundwater recession may be complex and governed by a series of contributory factors at a range of flow scales. Nevertheless, their analysis may yield insight into the nature of the aquifer, its boundary conditions, and other aspects of its hydrological behaviour. The insights gained from the preceding analysis lead to a number of other practical implications for groundwater science as follows.

A. *Groundwater recharge estimation*. With a better understanding of the variation of the underlying net groundwater drainage rate, Cuthbert (2010) proposed an improved time series approach for estimating recharge even for smoothly varying water tables. This was based on the approximation that in many instances the underlying net groundwater drainage rate will be approximately equal to the average recharge rate (q_a). Extending this idea to the case of observable groundwater recession, should recharge cease for a period in such a case, the groundwater may exhibit a linear recession for a significantly long period of time. This gives a very straightforward way of estimating groundwater recharge from a linear recession whereby $q_a = S\dfrac{\partial h}{\partial t}$.

This may be of particular use in water scarce areas where groundwater recessions can be clearly observed during periods of zero rainfall/recharge. This can help bring necessary improvements in the understanding of the impact of climate variability on groundwater recharge (Holman et al., 2012) as has been recently shown by Taylor et al. (2013).

2. *Master Recession Curve (MRC) analysis*. Due to the critical time concept, the nature of the net groundwater drainage rate is often obscured by the onset of the next groundwater recharge event. Thus in many instances attempts to use techniques such as MRC (Heppner & Nimmo, 2005; Delin et al., 2007; Heppner et al., 2007) for semi-automated groundwater hydrograph analysis are therefore highly problematic. It is self-evident that a decline in groundwater heads (in the absence of pumping or other effects other than recharge and drainage) does not necessarily mean an absence of recharge. Thus to generalise the recessional characteristics using a series of groundwater declines which may or may not themselves be subject to recharge could be highly misleading and great care is needed in the use of such an analysis.

3. *Choosing appropriate lower boundary conditions for 1-D unsaturated zone modelling*. The preceding discussion helps inform the choice of a suitable lower boundary condition for 1-D unsaturated zone models, a source of debate since at least Freeze (1969). Such models are often used for recharge estimation and contaminant (e.g. pesticide, nitrate) transport modelling in the soil zone. Commonly, a free drainage boundary condition is used rather than modelling the whole unsaturated profile to the water table, but the sensitivity to the choice of the lower boundary condition seems rarely to be tested. Given some estimation of the aquifer length scales and hydraulic properties, analytical approximations for the expected groundwater recessional characteristics may be made using the type of equations described above helping to inform the appropriate choice of the lower boundary condition to apply to such a model. For example, in the case of moderate to low diffusivity aquifers, the use of a constant flux condition may actually be a better choice than free drainage or constant head boundary conditions.

4. *Baseflow recession analysis*. The analysis carried out above for groundwater head fluctuations is of obvious relevance to the question of baseflow recession and surface water hydrograph separation. River stage variations which are not relevant to the variation of groundwater recession for most cases (due to damping of the small time frequency events to within short distances of the stream), will be of much greater relevance to the variation of baseflow in time. The conceptual and mathematical development necessary for a rigorous analysis of this issue is not within the scope of this paper. However, it is noted that for all but the most highly diffusive idealised aquifers the variation of regional groundwater discharge to such boundaries will hardly vary on an 'event' basis and short timescale groundwater contributions to streamflow will be dominated by local flow influences from near stream heterogeneity, bank storage effects and shallow subsurface flow contributions (Cuthbert et al., 2010b). This is demonstrated usefully for the problem of periodically varying recharge/discharge by Erskine and Pappaiannou (1997). There is a massive literature devoted to baseflow analysis (e.g. Dewandel et al., 2003; Brutseart, 2005; Troch et al., 2013).

5. Conclusion

This paper has explored the controls on the form of groundwater recession in both idealised and real aquifers. A general form for groundwater recession has been suggested for idealised aquifers based on developments of existing analytical solutions to linearised Boussinesq equations, and some new solutions have been presented. It has been demonstrated how consideration of the form of groundwater recession may lead to insights regarding the hydrologic functioning of an aquifer and also has practical applicability to a range of problems in groundwater science. The following are concluded, with respect to the objectives set out in the introduction:

1. Although an intuitively attractive idea, and one that is easily applied in hydrological models, the exponential phase is just one special case of the general form of recession expected for an idealised aquifer.

2. Groundwater recessions in ideal aquifers are expected to evolve from an initial linear decrease of head with time, through a transitional phase, to eventually show an exponential decrease. New analytical formulae have been presented which relate the timescales of each phase to the aquifer properties.

3. For many major aquifers in which recharge events occur more frequently than t_{crit}, the observable groundwater recession rate may more often be expected to have a linear form, with the flux recession rate approximately equal to the long term recharge.

4. Expectations made using ideal aquifer conceptualisations may be unrealistic in some contexts. Thus, departures from a straight line recessional form may also be diagnostic of particular types of aquifer properties and/or boundary effects, such as proximity to drainage boundaries, variations in transmissivity with hydraulic head, storage changes due to pumping, non-equilibrium flow at a range of spatial and temporal scales and variations in specific yield with depth.

5. Recessions in real aquifers are likely to be governed by flow systems at different scales that may be superimposed on one another. Where this leads to complex recessional forms one mechanism must be disentangled from another during interpretation.

Acknowledgements

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Appendix A: Critical time formulae

Case A: Homogeneous

Rorabaugh (1960) studied the case of an initially horizontal water table receiving a pulse of recharge resulting in an instantaneous water table rise of magnitude h_0 at time t_0, followed by zero recharge thereafter. The analytical solution for the evolution of head through time was given as follows:

$$h_{Rora}(x,t) = h_0 (1/L) \sum_{m=1}^{\infty} \left[e^{-m^2 \pi^2 Tt/4L^2 S} (2L/m\pi)(1-\cos m\pi)\sin(m\pi(L-x)/2L) \right] \quad \text{(A1)}$$

Alternative forms of the solution can be found, and one example is developed in Appendix B. Rorabaugh (1960) went on to show, using a graphical method, that after some critical time, t_{crit} [T], the recession rate of the groundwater head at any point in the aquifer is governed by an exponential decay whereby:

$$h = h_0 (4/\pi) e^{-\pi^2 Tt/4L^2 S} \sin(\pi(L-x)/2L) \quad \text{(A2)}$$

$$t_{crit} \approx 0.15 \frac{L^2 S}{T} \quad \text{(A3)}$$

Thus, once the critical time has passed, theoretically, the aquifer parameters may be estimated by observing the rate of decay of the groundwater head.

Case B: Inhomogeneous

It can also be shown that an identical analysis holds for an inhomogeneous aquifer. For example, Kuiper (1972) considers the case identical to Figure 1a, but with transmissivity decreasing linearly away from the head boundary (at x = L) where it has a value of T_0, to a value of zero at the drainage divide (x = 0). The solution is as follows (with terms consistent to those used above):

$$h(x,t) = h_0\left[1-2\sum_{m=1}^{\infty}\left[(J_1(\alpha_n)\alpha_n)^{-1}J_0(\alpha_n(1-(L-x)/L)^{0.5})\exp(-\alpha_n^2 T_0 t/(4L^2S))\right]\right] \quad \text{(A4)}$$

where J_0 and J_1 are Bessel functions of the first kind and order 0 and 1, respectively, and α_n is the nth root of J_0.

By applying the graphical analysis that Rorabaugh (1960) carried out for the homogeneous case to Kuiper's solution it is shown in Figure A1 that the recessions also become exponential after some critical time for the inhomogeneous case, but with:

$$t_{crit} \approx 0.75\frac{L^2S}{T_0} \quad \text{(A5)}$$

Case C: Radial flow

This analysis also holds true for diverging flow fields such as the radial flow Case B sketched in Figure 1b. The initial condition is again a steady state water table (in this case $h(x,t)$ = $q_c(R^2$-$r^2)/(4T)$) for a constant recharge rate, q_c. The solution for recession from this initial condition under subsequent conditions of zero recharge, using terms consistent with the preceding discussion can be shown to be (Bruggeman 1999, Bakker et al. 2007):

$$h(r,t) = \frac{2q_c R^2}{T} \sum_{m=0..}^{\infty} \left[\frac{J_0(\alpha_n r/R)}{\alpha_n{}^3 J_1(\alpha_n)} e^{-\alpha_n{}^2 Tt/R^2 S} \right] \qquad \text{(A6)}$$

As for the linear 1-D case, this function gives an exponential decay after a critical time related to the aquifer diffusivity and length scale. Again, by applying a graphical method, it is shown using Figure A2 that:

$$t_{crit} \approx 0.15 \frac{R^2 S}{T} \qquad \text{(A7)}$$

These formulae should provide a useful extension of Rorabaugh's original analysis for a wider range of cases for estimating the critical time.

Appendix B: Deriving an approximate expression for the length of the linear recession phase, t_{lin}

As discussed in Appendix A, the problem considered by Rorabaugh (1960) was for a sudden increase in head (h_0) across an entire aquifer due to recharge, with an initially horizontal water table. With reference to Case A in Figure 1a, this is equivalent to the case of an instantaneous decrease in head by an amount h_0 at $x = L$. Solutions can be found that are expressed as an infinite sum of sines as in Equation A1. Alternatively the problem can be approached by first considering the solution for an instantaneous change in head at one end of a semi-infinite aquifer (at $x = L$) adapted from the heat flow literature (Carslaw & Jaeger, 1959, p.59) as follows:

$$h(x,t) = h_0 erf\left((L-x)\sqrt{\frac{S}{4Tt}} \right) \qquad \text{(A8)}$$

Next, applying the method of images to deal with the groundwater divide (no flow boundary at $x = 0$), the complete solution becomes:

$$h(x,t) = h_0 \sum_{n=0..}^{\infty} \left[erf\left((2nL + (-1)^n (L-x))\sqrt{\frac{S}{4Tt}} \right) - erf\left((2(n+1)L + (-1)^{(n+2)} (L-x))\sqrt{\frac{S}{4Tt}} \right) \right]$$

(A9)

With all terms defined previously in the paper. This solution is equivalent to Equation A1 and other permutations of solutions to the same problem found in the literature (e.g. Rushton 2003, Equation 2.31).

Each image boundary makes a smaller and smaller contribution to the combined solution. For early times, less than t_{crit} = 0.15 L^2S/T, just using the first term in the summation (identical to Equation A8) gives a very good approximation of the exact solution, with the error varying from <7% at $x = 0$ to zero at $x = L$.

The time it will take for a change in head at $x = L$ to cause a significant change in head (say, 0.5%) at a distance d from the constant head boundary (i.e. $d = L-x$) can now be directly found from Equation A8. Rearranging for $h/h_0 \geq 0.995$ yields:

$$t_{lin} \leq \frac{d^2 S}{16T} \qquad \text{(A10)}$$

Furthermore, comparing Equations 5 and Equation A1 it can easily be shown that $D/q_c = h_{Rora}/h_0$; that is to say that the recession after an instantaneous rise in head on a horizontal water table normalised to the applied head increment is identical to the rate of flux recession of a water table starting at steady state conditions, normalised to the initial flux recession rate (i.e. equal to the steady state recharge rate).

Thus Equation A10 may be applied to estimate the length of the linear recession phase exhibited by an ideal aquifer subject to zero recharge starting from an initial steady state condition. In this case it expresses the time at which the flux recession rate has decreased from the steady state recharge rate by more than 0.5%.

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Figures

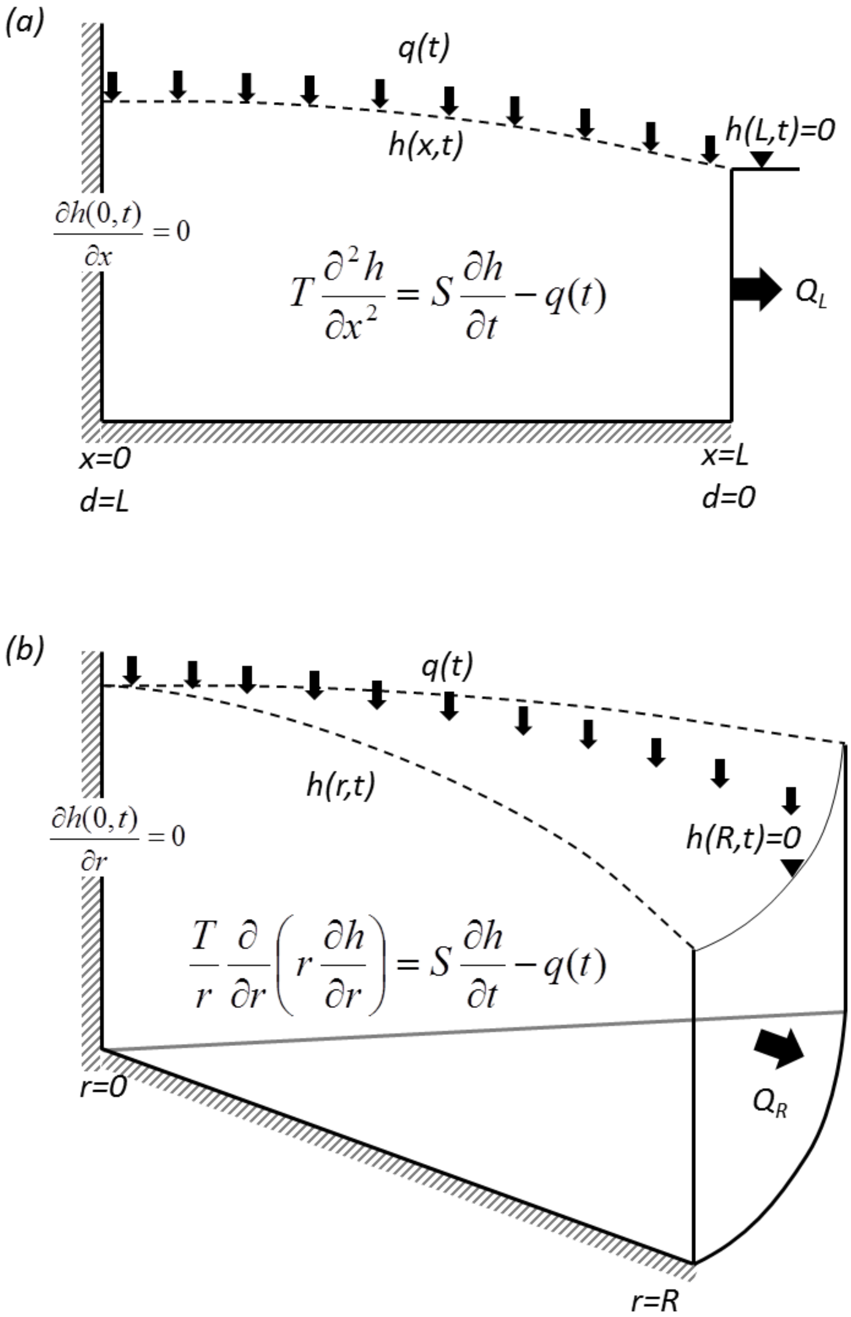

Figure 1. Idealised aquifers used for analytical derivations. (a) Case A – 1-D flow (b) Case B – radial 1-D flow. In each case the governing equation and boundary conditions are given; the initial conditions are described in the text for particular solutions of interest.

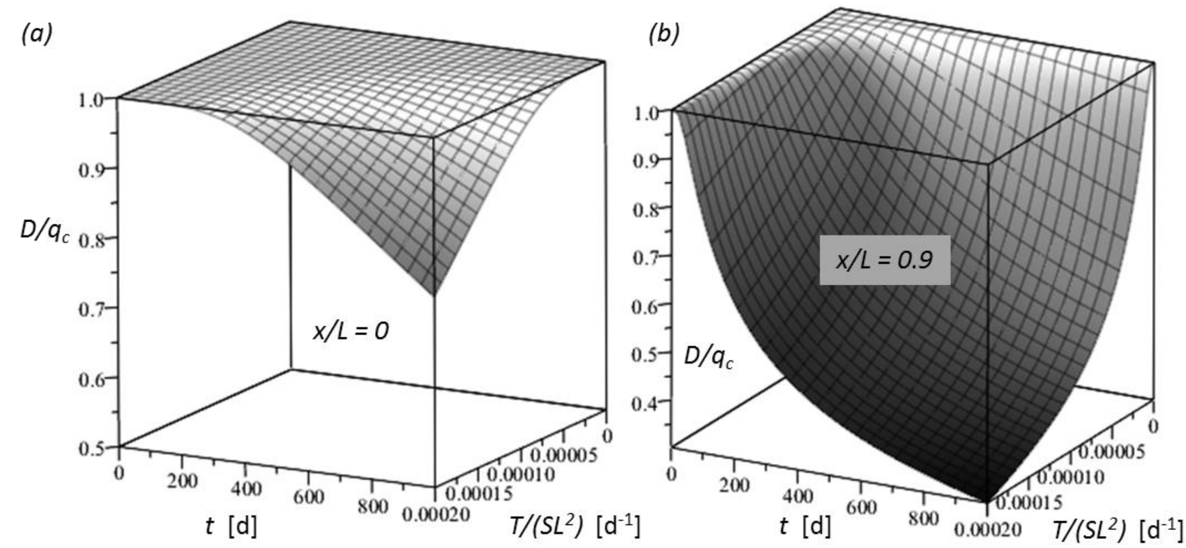

Figure 2. Groundwater recession rates following cessation of steady state recharge conditions (normalised against the steady state recharge rate) for a range of aquifer diffusivity, length scales and timescales and for (a) x/L = 0 and (b) x/L = 0.9, using Equation 5.

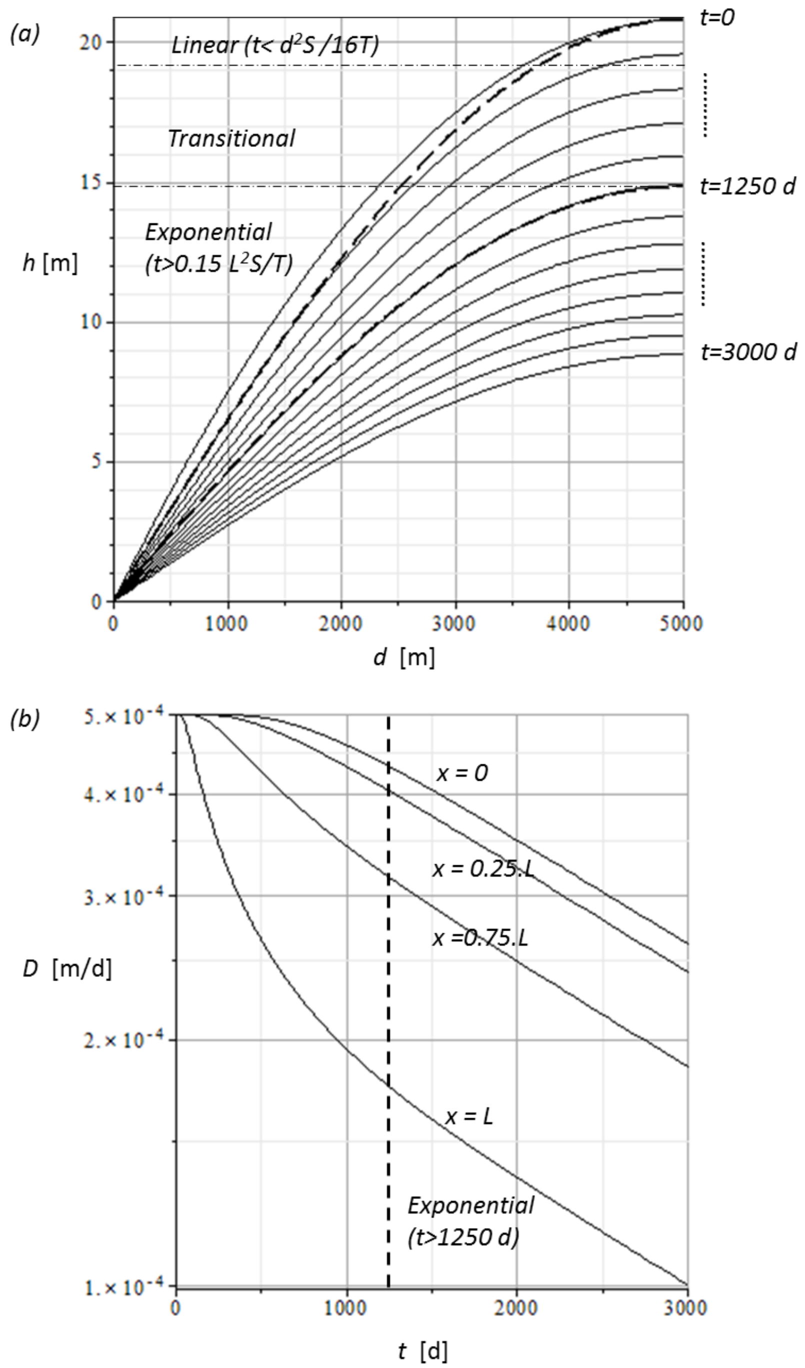

Figure 3. (a) Head profiles decaying from steady state conditions according to Equation (4), plotted at intervals of 250 d. Timing of linear phase is defined for $x = 0$ (i.e. $d = L$). Aquifer properties are $T = 300$ m^2/d, $S = 0.1$, $L = 5000$ m, $q_c = 5 \times 10^{-4}$ m/d. Bold dashed lines are sinusoidal curves. (b) Recession rates against time using Equation (5) for the same aquifer properties as in (a) for a range of values of *x*. Critical time for this aquifer is *approx*. 1250 d.

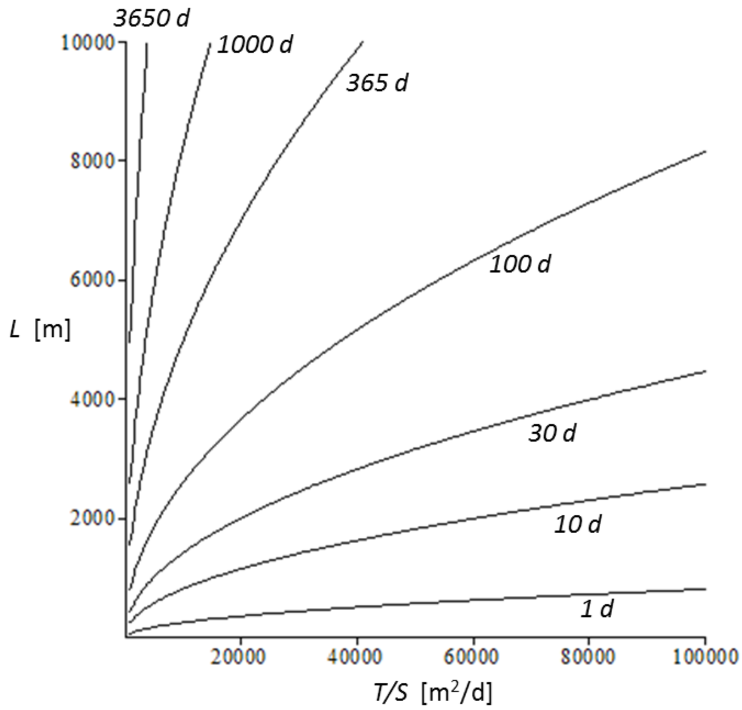

Figure 4. Contours of critical time for combinations of aquifer length (*L*) and diffusivity (*T/S*).

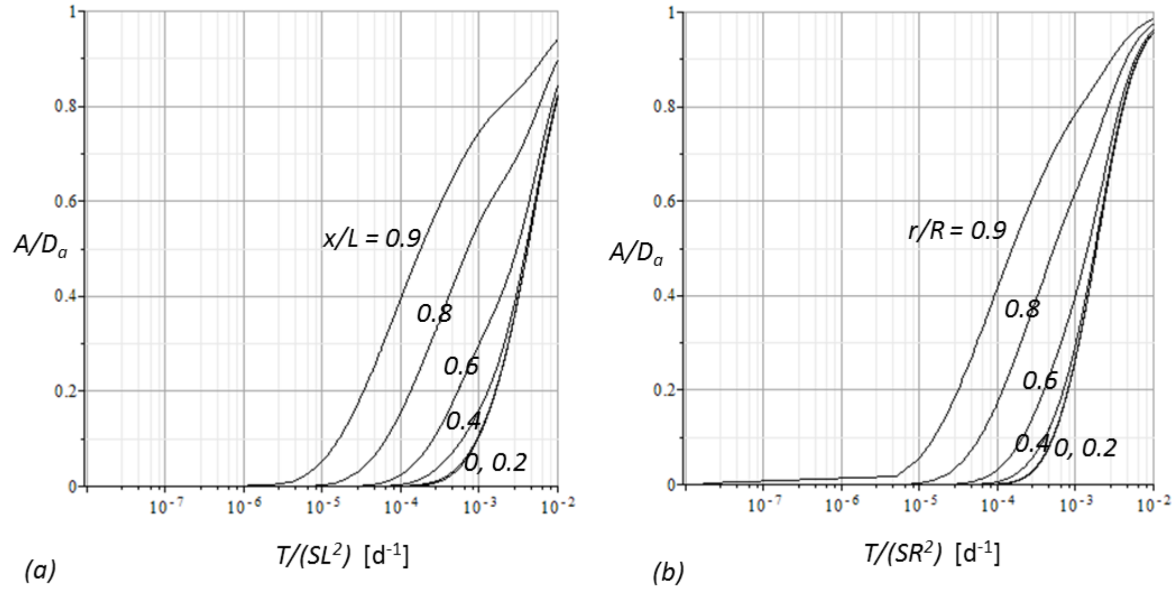

Figure 5. Variation of amplitude (A) of the net groundwater drainage rate (D), normalised to the average value of D, under sinusoidal conditions with an annual period for a variety of aquifer length scales (x/L or r/R) and diffusivities for (a) a 1-D aquifer of length L and (b) a radially symmetric aquifer of radius R. Values of A/D_0 close to zero indicate little variation in the net groundwater drainage rate.

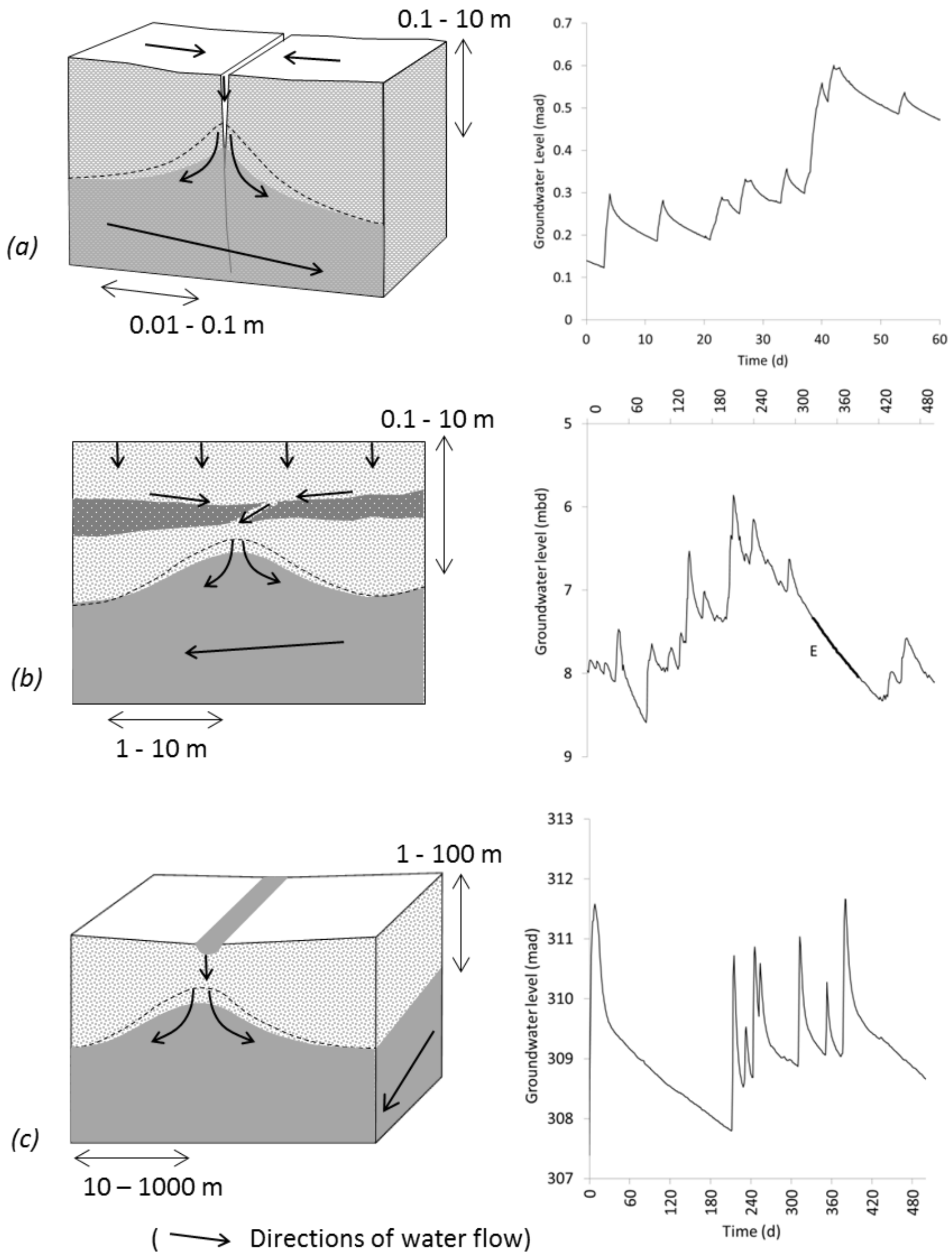

Figure 6. Conceptual model of the influence of non-equilibrium flow on groundwater recession:

(a) small scale simulation of preferential flow through macroporous soil to a shallow water table using a dual permeability model – see text for parameters and model set-up (b) intermediate scale localised recharge conditions hypothesised to generate the groundwater hydrograph presented for Soroti, Uganda. The labelled linear recession "E" refers forwards to Figure 7. Localised focussing of recharge is envisaged through heterogeneous lateritic layers (c) larger scale process of transient indirect recharge from a losing stream illustrated with data from Maules Creek, Australia. In all cases, local mounding due to non-equilibrium flow causes an initially steep groundwater recession which transitions to a background straight line form governed by a larger scale groundwater flow system recession.

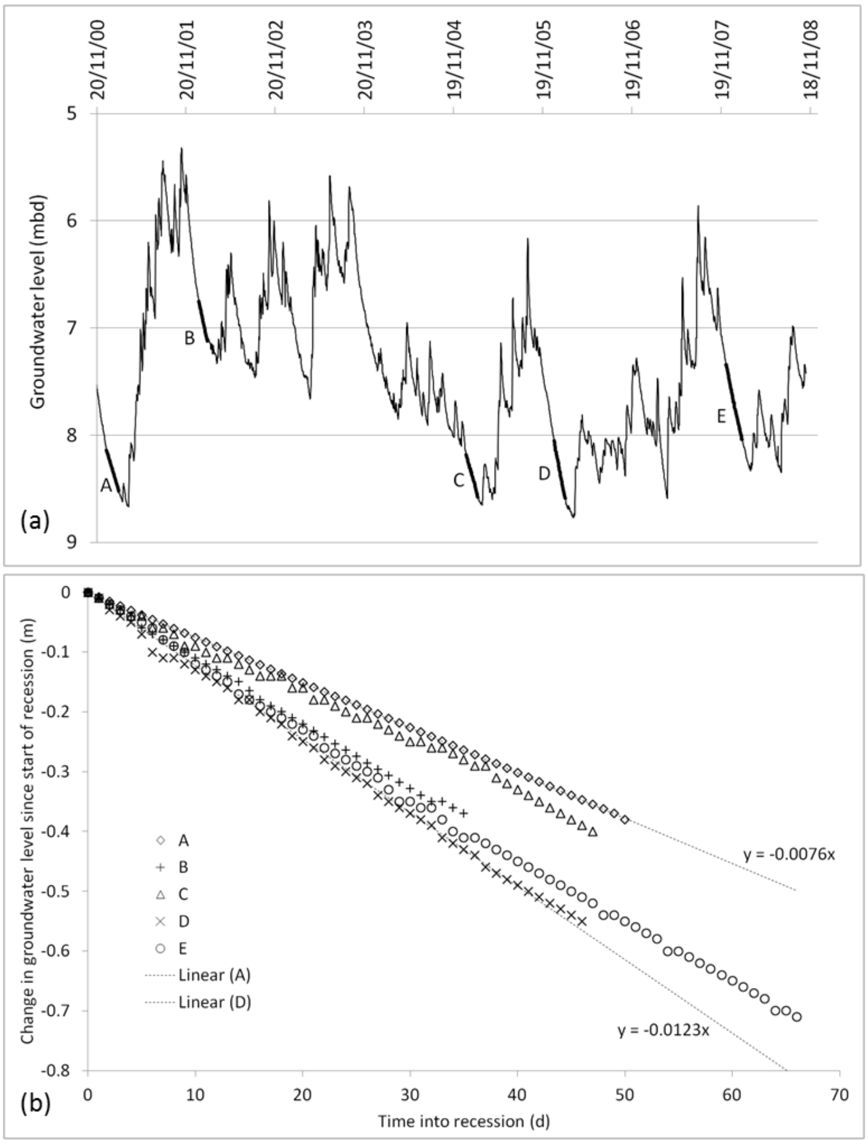

Figure 7. (a) Examples of straight line recessions (bold sections A-E) from Soroti, Uganda (Cuthbert & Tindimugaya, 2010) (b) change in groundwater head since the start of the recession for each section A-E.

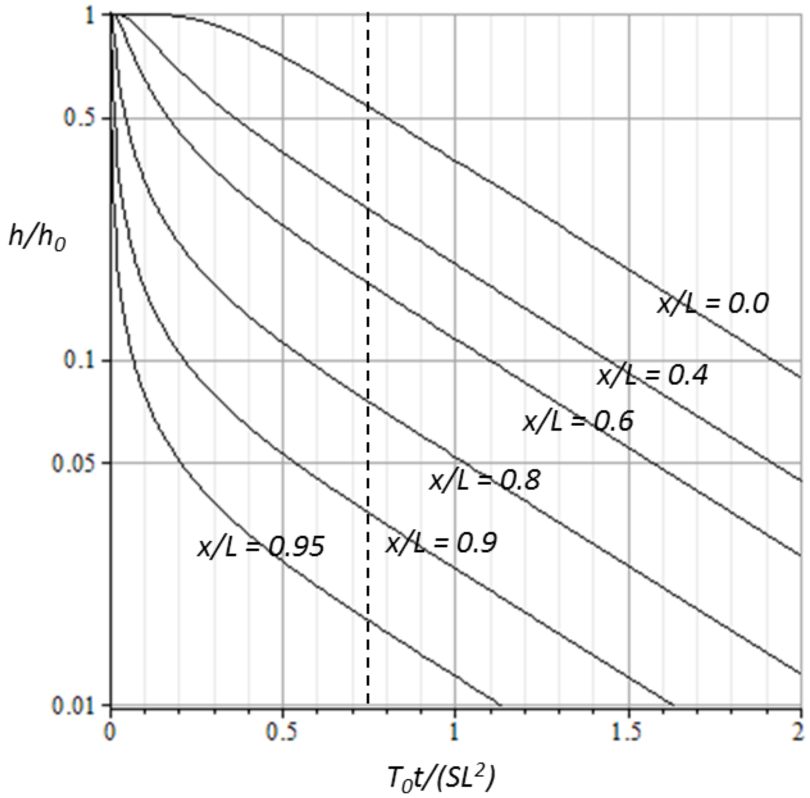

Figure A1. Normalised head recessions using Equation A4 for an inhomogeneous aquifer, indicating that the recessions become exponential (straight line on the semi-log plot) at $T_0t/(L^2S) \approx 0.75$, leading to Equation A5.

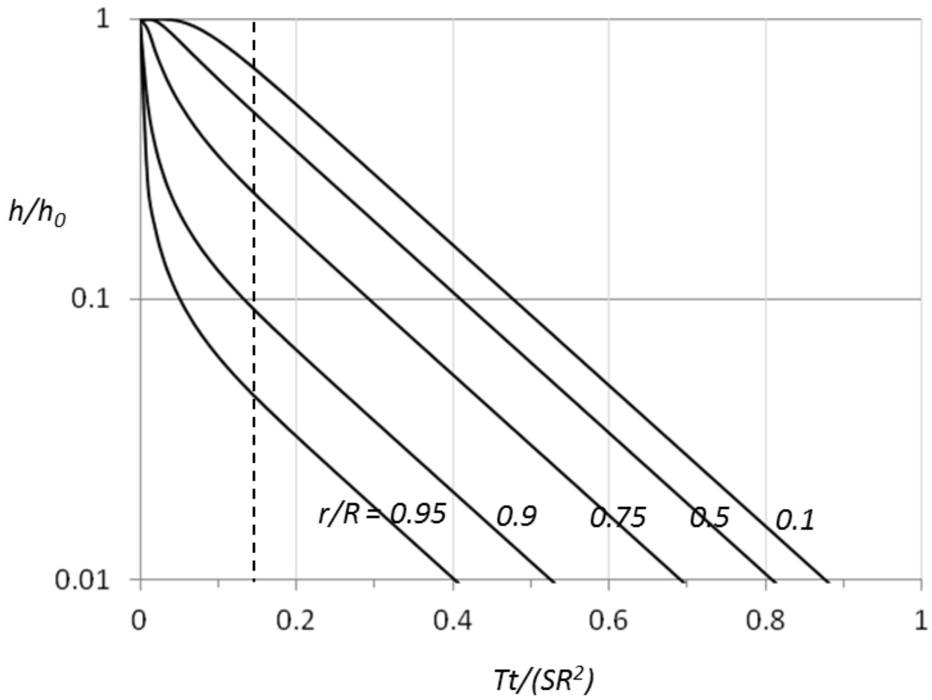

Figure A2. Normalised head recessions using Equation A6 for radial flow, indicating that the recessions become exponential (straight line on the semi-log plot) at $Tt/(R^2S) \approx 0.15$, leading to Equation A7.