# Online-to-store channel expansion strategies

Chen, Xu; Peng, Ying; Wang, Xiaojun

*DOI:*
10.1109/TEM.2023.3330374

# Online-to-store channel expansion strategies: cost, heterogeneity, and power

Xu Chen[a*], Ying Peng[a], Xiaojun Wang[b]

[a] *School of Management and Economics, University of Electronic Science and Technology of China, Chengdu, 611731, China.*

[b] *Birmingham Business School, University of Birmingham, Birmingham, B15 2TT.*

* Corresponding author: *xchenxchen@263.net (Xu Chen)*

**Abstract:** In light of the growing prevalence of online and offline mixed channels, this paper examines the channel expansion strategy of adding offline store channels to an online direct channel for a manufacturer. Based on the Nash program framework, this paper considers two alternative mixed-channel strategies, namely a conventional wholesale channel and a franchise store channel. By comparing the subgame perfect equilibrium solutions of these two mixed-channel formats to the benchmark strategy, where the manufacturer only engages in the online direct channel, we evaluate the effects of the two online-to-store mixed-channel strategies on optimal direct selling prices, the manufacturer's profits, and overall social welfare. The results indicate that a manufacturer's optimal channel expansion strategy is mainly governed by the trade-off between the extra revenue gained from the newly added offline channel and the economic loss from the direct online channel induced by channel competition. The manufacturer's negotiation power, unit production cost, and heterogeneity between the two channels are the key factors determining the relative gains and losses. In addition, our results demonstrate that both online-to-store mixed-channel strategies yield positive effects on social performance compared with the online direct channel.

**Keywords:** channel strategy; channel competition; consumer valuation; pricing; Nash bargaining game.

## 1. Introduction

As the prevalence of the Internet and mobile phones grows, e-commerce platforms have flourished and completely revolutionized the retail industry. The enormous changes in shopping and consumption habits in the past decades have facilitated the emergence of prominent e-commerce entities such as Amazon and eBay in the U.S., as well as Alibaba and JD.com in China (Yan et al., 2018; Hu et al., 2023). However, since the easing of pandemic restrictions, the expansion rate of e-commerce has been lower than that of total retail sales, and physical stores are experiencing a rebound (Ominaretail, 2023). Some companies that started their businesses online have established physical stores offline. A well-known example is Xiaomi, a smart phone manufacturer that started its business through an online direct sales channel, and in recent years the company has opened physical stores and thereby expanded its market share (Liu et al., 2020; Yan et al., 2020). Dell originated primarily as an online channel-based enterprise and subsequently added traditional retail channels to form a dual-channel operation, allowing the brand to reach a broader customer base and increase brand awareness (Qiu et al., 2021; Kim and Chun, 2018). Beyond the electronics sector, Inman, an online women's clothing brand, and Three Squirrels, a leading online snack retailer in China, have also established physical stores (Zhang and Zhang, 2020; He et al., 2022).

Physical stores provide greater value to consumers than online channels due to the experience of being able to touch, feel, and try out products (Yang et al., 2021; Chiang et al., 2003), and although operating dual channels can attract more consumers (Qiu et al., 2021), introducing a new channel will create “channel conflict”, in which the new channel cannibalizes sales through the incumbent one (Tsay and Agrawal, 2004; Zhu et al., 2020). When manufacturers choose to introduce an offline channel, they face several options. One common practice is through the wholesale channel, where the manufacturer distributes its products to a conventional retailer subsequently selling these products to consumers. For instance, some offline retailers procure personal computers from Dell and subsequently sell the products to consumers. Another popular strategy involves franchise stores, where the manufacturer authorizes franchise stores to offer their products to consumers; apart from this, the franchise store is required to pay a franchise fee to the manufacturer (Chen et al., 2018). For instance, Inman and Three Squirrels have established offline physical stores mainly by cooperating with franchisees. Hence, when a manufacturer owning an existing online channel decides to introduce an offline channel, it can adopt diverse channel expansion strategies, which are contingent upon the specific circumstances.

Generally, the Stackelberg model framework employed to investigate similar channel management problems in the literature requires supply chain members to employ complete bargaining power to ascertain the parameters with the aim of profit maximization (Matsui, 2020). Nevertheless, the real-world business landscape is characterized by a high degree of complexity, and investigating the choices made by supply chain members within a Nash bargaining framework serves to enhance the capture of generalities, thereby providing a more comprehensive depiction of the power dynamics among the members. Hence, bargaining emerges as a prevailing mechanism for profit allocation and price determination, particularly when involved parties have different bargaining power, as exemplified in the interactions between major entities such as Apple and Samsung and Macy's and Ralph Lauren (Qing et al., 2017). In addition, since consumers have different levels of channel acceptance, their purchasing decisions are often determined by the net utility derived from the respective channels (Shi et al., 2020; Liu et al., 2022). Therefore, when companies make channel selection and pricing decisions, it is important to consider consumer acceptance of the online direct channel (Luo et al., 2018).

With the backdrop that many firms with established online direct channels have later ventured into operating online and offline mixed channels, such ventures may not always lead to increased revenue in the swiftly evolving commercial landscape. Dell's expansion to offline stores has not been successful, with profits witnessing a notable decline subsequent to the adoption of the dual-channel strategy involving resellers (Xu et al., 2020). In light of increasing market competition and the desire to expand consumer reach (Xu et al., 2022), manufacturers with successful online direct channels are faced with the decision of whether to introduce offline retail channels to increase their market share. Additionally, manufacturers with a dual-channel approach face the challenge of choosing an appropriate pricing policy to maximize profits (Liu et al., 2022). They can either adopt an identical pricing strategy across online and offline channels, or price differently according to consumers' product valuations for different channels.

For example, Cavallo (2017) found that within-retailer online and offline prices are identical in about 72% of the cases studied, but with significant heterogeneity at the country and sector level, ranging from 42% in Brazil to 91% in the United Kingdom. To the best of our knowledge, most of the existing literature focuses on companies expanding from offline retail channels to online channels, and few studies have explored the expansion strategies deployed by manufacturers expanding from online to offline. This paper aims to address this gap in the literature, considering the interplay of multiple market and operational factors under the Nash program framework. To this end, we investigate the following questions.

(1) Should manufacturers with online direct channels introduce an offline channel?

(2) If so, how should the manufacturer expand the offline channel, and what is the optimal pricing policy?

(3) What impact does the additional offline channel have on the direct selling price, the manufacturer's profit, and overall social performance?

To answer these questions, we study the optimal choice between two alternative channel expansion strategies, a conventional wholesale channel (WC) and a franchise store channel (FC), and examine how introducing offline channels affects the direct selling price, the manufacturer's maximum profit, and social performance based on heterogeneous consumers' valuations of the two channels and the Nash program framework.

This paper offers several contributions to the extant literature. First, we complement the channel selection literature (Zhang et al., 2021a; Pu et al., 2020) by studying the manufacturer's optimal channel expansion strategies while transitioning from one online direct channel to a mixture of online and offline retail channels. Second, few studies have explored offline channel expansion strategies for manufacturers with direct online sales channels by incorporating the interplay of multiple market and operational factors, including channel heterogeneity, production costs, and interfirm power relationships. Under the Nash program framework, we delve into contract negotiations involving the wholesale price and franchise fee in the respective offline sales channels to analyze the manufacturer's strategy for offline channel expansion, which is different from similar studies on channel strategy (Shen et al., 2019; Zhang et al., 2021a). Finally, we derive valuable insights from our analysis that can provide useful decision support for manufacturers who intend to expand their online direct channels to online and offline mixed-channel operations. For example, while managing production costs is critical to ensuring that online sellers have sufficient profit margin to launch new offline channels, the heterogeneity between two channels moderates the financial losses from demand cannibalization triggered by an offline channel expansion strategy. Moreover, the power relationship between the channel partners plays a significant role in choosing among the different channel expansion strategies. Finally, while the financial return of a firm's decision on online-to-store channel expansion strategies is influenced by market and operational factors, channel expansion in general has a positive impact on overall social welfare.

The remainder of this paper is organized as follows. A comprehensive review of the relevant literature is presented in Section 2. Section 3 presents models and optimal solutions. Sections 4 and 5 examine the effects of the WC and FC strategies on the optimal direct selling price, the manufacturer's maximum profit, and overall social welfare by comparing the respective subgame equilibrium results with the benchmark strategy. Section 6 discusses the selection of channel expansion strategies, and Section 7 concludes the research by outlining critical findings, managerial insights, and future research directions.

## 2. Literature review

This paper is relevant to two streams of literature: (i) channel strategy and (ii) contract arrangements and the Nash bargaining game in dual-channel management.

Channel strategy has gained extensive attention over the past decades and falls into three broad categories. The first category captures the introduction of a new channel, which has attracted wide attention within the field of channel management. Most of the current studies focus on the introduction of a new direct channel by manufacturers or suppliers to add to the incumbent retail channel (Arya et al., 2007; Li et al., 2015; Yang et al., 2018; Zhang and Zhang, 2020; Ha et al., 2022). Ha et al. (2016) analyzed a manufacturer's quality strategies to effectively avoid erosion of the retailer's utility when a manufacturer opens its own direct channel. Zheng and Yu (2021) investigated the interplay between the manufacturer's decision regarding incorporating a direct channel and whether to adopt an equal pricing strategy for the online channel and traditional offline channel. Shi et al. (2023) explored the manufacturer's encroachment on a traditional retailer's channel within decentralized and centralized structures.

Additionally, some scholars looked at leading e-commerce retailers that face choices about whether to open offline channels. Zhang et al. (2017) explored optimal channel strategies by considering whether offline retailers should introduce online channels and whether online retailers should add offline channels. Wang and Ng (2020) adopted a behavior-based pricing policy to examine the competition between a new retailer and an online retailer selling identical products, where the new retailer represents an e-commerce retailer opening an offline channel. Wu et al. (2023) investigated the scenario in which a giant online retailer cooperates with offline stores to lay out chain stores and found that the fixed fee paid for the online retailer and the number of loyal customers were the key factors influencing the cooperation.

The second category is about the competition and conflict that arise from adding new channels. Although adding a new channel and offering identical products will inevitably create channel competition and may cannibalize demand in the incumbent channel (Liu et al., 2021; Xia and Niu, 2019; Shi et al., 2021), it can increase revenue and potentially yield greater profits than operating solely through a single channel (Chen et al., 2017), even achieving a win-win for retailers and manufacturers under certain conditions (Yan et al., 2018; Wan et al., 2022). Consequently, in dual-channel management, the competition and cannibalization created by the introduction of new channels are important factors for supply chain members when designing channel strategies (Zhang et al., 2021a). Xia and Niu (2019) analyzed how supplier encroachment impacts economic performance by taking service spillovers and members' power into account and showed that the cannibalization effect and channel power are critical factors. Shi et al. (2021) explored several channel structures to scrutinize whether supply chain members (i.e., a manufacturer and a retailer) should join a marketplace, and their results showed that members should balance the trade-off between the advantages of sales expansion and the disadvantages of channel cannibalization. Zhang et al. (2021b) demonstrated that the degree of channel competition and operational costs are critical factors in selecting the optimal channel strategy. Unlike these previous studies, this paper emphasizes the channel competition caused by the manufacturer introducing an offline channel and takes two alternative offline channel expansion strategies into consideration.

The third category is about multi-channel and omni-channel strategies. Adding a new offline channel alongside an incumbent online one can be included in multi-channel or omni-channel management, which has been extensively studied (Yan et al., 2018; Yan et al., 2022; Liu et al., 2020). Several e-commerce giants, such as Amazon, Alibaba, and JD.com, have opened physical stores to become multi-channel or omni-channel retailers (Wang et al., 2020; Wang and He, 2022). Many researchers scrutinize channel selection in the context of multi-channel and omni-channel management. Kim and Chun (2018) explored the two effects arising from channel conflict and found that channel strategies (i.e., multi-channel, omni-channel, and offline channel) are closely related to customer types. Liu et al. (2020) analyzed four omni-channel retail modes and found that the degree of consumer service perception significantly influences the optimal channel structure. Zhao et al. (2023) analyzed the reference effect on the two business modes (DCRS and BOPS) in a fresh food supply chain and derived the optimal channel strategy. Our paper distinguishes the abovementioned studies from these perspectives. First, this paper discusses the impact of a manufacturer's channel expansion strategy of introducing a new offline channel to form multi-channel formats and its economic and social impacts. Second, this paper focuses more on channel competition, while the above analyses mainly concentrate on the interplay between two channels.

The second research stream is about contract arrangements and Nash bargaining in dual-channel management. Different contract policies exert various impacts on the channel strategies of supply chain members. The manufacturers can collaborate with the offline retailer through different contractual arrangements when adding a new channel, such as wholesale or franchise fee contracts (Choi et al., 2019). The retailer must pay a wholesale price for each product supplied by the manufacturer under the wholesale contract (Yun et al., 2023). Unlike a wholesale contract, a franchise fee contract includes the franchise fee and wholesale price (Chen et al., 2018). Shi et al. (2020) scrutinized the adoption of a wholesale contract by taking different power structures into consideration in a dual channel, while Choi et al. (2019) constructed a model for a fashion franchising supply chain that combines both online and offline channels to examine the mechanism of managing conflicts within channels. Stackelberg games, which are widely adopted to model similar channel management problems, require supply chain members to possess complete bargaining power to decide on related parameters to maximize their own profit. However, due to the interplay among members and the complex real business landscape, utilizing a bargaining framework can capture the generalities (Nash, 1950; Chen et al., 2019;

Chen et al., 2023). Recent articles have also employed the Nash bargaining framework to investigate channel management (Qing et al., 2017; Yang et al., 2018; Shen et al., 2019; Matsui, 2020; Matsui, 2022). Qing et al. (2017) adopted the Nash bargaining framework to explore the capacity allocation problem, and their results show that augmented bargaining power for the supplier corresponds to increased shared capacity and, in contrast, the manufacturer's shared capacity exhibits a negative relationship with her bargaining power. Under the bargaining framework, Yang et al. (2018) scrutinized how nonlinear pricing influences the supplier's decision on encroachment and showed that the supplier's bargaining power is a vital driver affecting profits. Considering bargaining power, Shen et al. (2019) examined a manufacturer's channel choice in the context of collaboration with either a platform retailer or a traditional retailer. Within a dual-channel framework, Matsui (2020) investigated a manufacturer's optimal decision timing to engage in wholesale price negotiations with one retailer and demonstrated that the manufacturer can attain a maximum profit by conducting wholesale price negotiations before setting the direct price. Matsui (2022) conducted a comparison to investigate whether the retailer should engage in wholesale price negotiations with the manufacturer or simply accept the manufacturer's set price. Unlike these studies, our paper utilizes the Nash program framework to investigate channel expansion strategies from the perspective of adding an offline channel to a manufacturer's existing online direct channel.

Among the studies that are most relevant to this research, Pu et al. (2020) explored how a manufacturer distributing products through a conventional retailer chooses one of three online channel strategies and analyzed the optimal pricing decision and selling mode for the online channel within the context of a dual-channel framework. Zhang and Zhang (2020) considered a supplier launching an offline channel in addition to selling products through an online platform; however, their research focused on the dynamic interaction between the retailer's need for information sharing and the supplier's channel entry through reselling and agency agreements. This paper is different from the above two studies in several aspects. First, our paper examines adding offline channels for a manufacturer operating an online direct channel with two different offline channel expansion strategies. Second, this paper explores a manufacturer's optimal offline expansion strategy incorporating market and operational factors, including customer acceptance of the online direct channel, a unit production cost, and the manufacturer's negotiation power under the Nash program framework.

# 3. The models

## 3.1. The model setup

We consider a manufacturer that operates an online direct sales channel solely and explore two online-to-store channel expansion strategies by cooperating with the retailer: conventional wholesale strategy (WC strategy) and franchise strategy (FC strategy). We illustrate the framework in Figure 1.

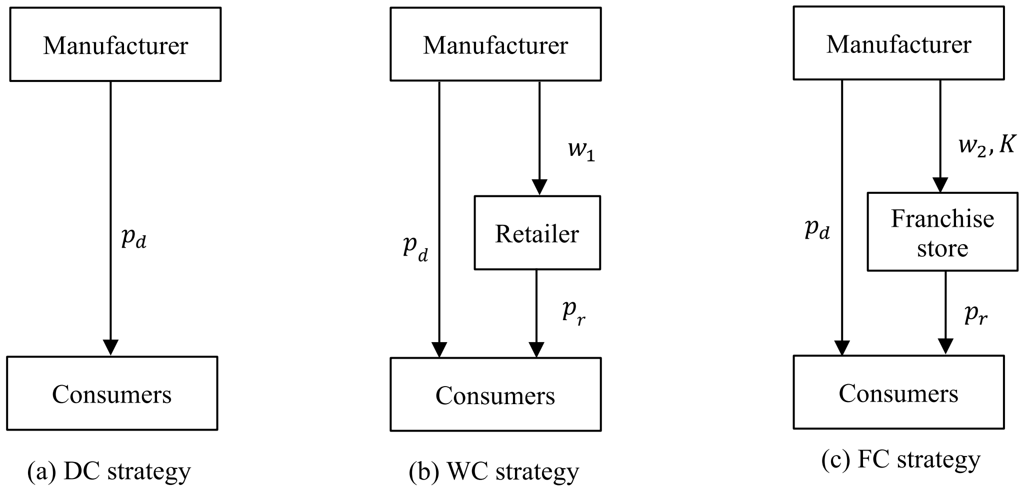


**Figure 1. The strategy framework**

We regard the online direct channel (DC strategy) as a benchmark, where the manufacturer sells products directly to consumers online. If the manufacturer opts to expand an offline channel, it faces two alternative choices: in the WC strategy, the manufacturer is required to distribute products to consumers via the retailer; additionally, in the FC strategy, the manufacturer offers products to end consumers via the franchise store, and the manufacturer receives a lump-sum franchise fee from the franchise store. For convenience, we utilize the superscript $i$ ($i = d, w, f$) to denote three strategies, in which "$d$", "$w$", and "$f$" indicate the DC strategy, WC strategy, and FC strategy, respectively. Table 1 describes the parameters and decision variables used in this paper.

**Table 1. Notations**

| Notation | Descriptions |
|---|---|
| $c$ | Unit production cost for the manufacturer |
| $w_1$, $w_2$ | Manufacturer's unit wholesale price in the WC strategy and FC strategy respectively, |

| | |
|---|---|
| | $w_1 > c$ and $w_2 > c$ |
| $p_d$, $p_r$ | Unit direct selling price for the manufacturer and unit retail price for the retailer, $p_d > c, p_r > c$, $p_r > w_1$ and $p_r > w_2$ |
| $v$ | Consumer's valuation of the product, $v \sim U[0,1]$ |
| $\eta$ | Customer acceptance of the online direct channel, $0 < \eta < 1$ |
| $\theta$ | Manufacturer's bargaining power, $0 \leq \theta \leq 1$ |
| $K$ | The franchise fee received from the franchise store ($K > 0$) |
| $q_d^i$, $q_r^i$ | The demand for the online direct channel and offline retail channel, $i = d, w, f$ |
| $\pi_m^d(p_d)$ | Profit for the manufacturer in the online direct channel strategy |
| $\pi_m^w(p_d)$, $\pi_r^w(p_r)$ | Profits for the manufacturer and retailer after introducing an offline wholesale channel (the WC strategy) |
| $\pi_m^f(p_d)$, $\pi_r^f(p_r)$ | Profits for the manufacturer and retailer after introducing an offline franchise store channel (the FC strategy) |
| $C_s^i$ | Consumer surplus for the DC, WC, and FC strategies, $i = d, w, f$ |
| $W_s^i$ | Social welfare for the DC, WC, and FC strategies, $i = d, w, f$ |

We assume that consumers' valuations of products are heterogeneous and consumer valuation $v \in [0,1]$ is a uniform distribution within the market size (consumer population) with a density of 1, which is in alignment with previous studies (Shi et al., 2020; Li et al., 2019). A product is worth $v$ if it is obtained from the offline channel; however, the valuation of a product coming from the online direct channel is presented as $\eta v$ due to the virtual description of products and customers being unable to physically touch or try them out before making a purchase (Zhao et al., 2022).

In the benchmark strategy, consumers purchase products online at a price $p_d$, so the net utility of a consumer is $u_d^d = \eta v - p_d$. Thus, the demand function for the DC strategy is $q_d^d = 1 - \frac{p_d}{\eta}$. Adding an offline retail outlet, such as a conventional retailer or a franchised store, to the existing online direct channel, transforms the manufacturer into an online and offline mixed-channel operation. One source of difference across these mixed-channel models is that different pricing policies are often adopted. For the WC strategy, consumers will make decisions based on comparing the net utility derived from the two channels: $u_d^w = \eta v - p_d$ versus $u_r^w = v - p_r$. Similarly, for the FC strategy, customers' decisions revolve around comparing the net utility obtained from online and offline channels: $u_d^f = \eta v - p_d$ versus $u_r^f = v - p_r$. Consumers with $v \in [\frac{p_d}{\eta}, \frac{(p_r - p_d)}{1-\eta}]$ tend to buy from the online direct channel, whereas those with $v \in [\frac{(p_r - p_d)}{1-\eta}, 1]$ are willing to buy from an offline franchise store. Those with $v \in [0, \frac{p_d}{\eta}]$ will not purchase from either channel. Thus, for the WC and FC strategies, the demand for the manufacturer's online direct channel is denoted as $q_d^w = \frac{p_r - p_d}{1-\eta} - \frac{p_d}{\eta}$ and $q_d^f = \frac{p_r - p_d}{1-\eta} - \frac{p_d}{\eta}$, and the demand for the offline channel is $q_r^w = 1 - \frac{p_r - p_d}{1-\eta}$ and $q_r^f = 1 - \frac{p_r - p_d}{1-\eta}$ (Yang et al., 2021; Cheng et al., 2021).

For the DC strategy, social welfare is made up of consumer surplus and the manufacturer's profit. The consumer surplus is represented as $C_s^d = \int_{\frac{p_d}{\eta}}^{1} (\eta v - p_d)\, dv$, and social welfare as $W_s^d = C_s^d + \pi_m^d(p_d)$. Similarly, for the WC and FC strategies, social welfare is comprised of consumer surplus and profits for both the retailer and the manufacturer. Hence, the consumer surplus can be represented as $C_s^w = \int_{\frac{p_d}{\eta}}^{\frac{p_r - p_d}{1-\eta}} (\eta v - p_d)\, dv + \int_{\frac{p_r - p_d}{1-\eta}}^{1} (v - p_r)\, dv$ and $C_s^f = \int_{\frac{p_d}{\eta}}^{\frac{p_r - p_d}{1-\eta}} (\eta v - p_d)\, dv + \int_{\frac{p_r - p_d}{1-\eta}}^{1} (v - p_r)\, dv$, and social welfare for the WC and FC strategies can be represented as $W_s^w = C_s^w + \pi_m^w(p_d) + \pi_r^w(p_r)$ and $W_s^f = C_s^f + \pi_m^f(p_d) + \pi_r^f(p_r)$, respectively (Li et al., 2018).

### 3.2. DC strategy

In the DC strategy, the decision sequence is presented as follows. Firstly, the manufacturer decides the unit direct selling price ($p_d$), then customer demand is realized and the manufacturer receives its revenue. Therefore, the decision process for the benchmark strategy is described as follows:

$$\max_{p_d} \pi_m^d(p_d)$$

Manufacturer's profit $\pi_m^d(p_d)$ is calculated as follows:

$$\pi_m^d(p_d) = (p_d - c)(1 - \frac{p_d}{\eta}) \tag{1}$$

### 3.3. WC strategy

In the WC strategy, the manufacturer offers products to customers online as well as via the retailer. Therefore, the decision sequence is described as follows. Firstly, the manufacturer and retailer engage in a cooperative game where they negotiate the wholesale price ($w_1$) under a Nash program framework. Secondly, the manufacturer and retailer participate in a static game where the manufacturer decides the unit direct selling price ($p_d$) and the retailer decides the unit retail price ($p_r$) independently and simultaneously. Bargaining the wholesale price is common in dual-channel studies (Qing et al., 2017; Yang et al., 2018; Matsui, 2022), and this decision sequence has been commonly adopted in previous literature (Matsui, 2020). Note that the game in this paper encompasses cooperative and noncooperative game theories, which are incorporated into the Nash program framework; thus, the Nash solution in bargaining the wholesale price can be interpreted as the subgame perfect equilibrium of a noncooperative game (Nash, 1953). Therefore, the decision process for the WC strategy is described as follows:

$$\max_{w_1>c} \pi^w(w_1) \rightarrow \begin{cases} \max\limits_{p_d} \pi_m^w(p_d) \\ \max\limits_{p_r} \pi_r^w(p_r) \end{cases}$$

Manufacturer's profit $\pi_m^w(p_d)$ is calculated as follows:

$$\pi_m^w(p_d) = (w_1 - c)(1 - \frac{p_r - p_d}{1-\eta}) + (p_d - c)\left(\frac{p_r - p_d}{1-\eta} - \frac{p_d}{\eta}\right) \tag{2}$$

The first part represents the profit generated from the offline retail channel. The second part represents the profit generated from the online direct channel.

Similarly, the retailer's profit $\pi_r^w(p_r)$ is calculated as follows:

$$\pi_r^w(p_r) = (p_r - w_1)(1 - \frac{p_r - p_d}{1-\eta}) \tag{3}$$

Assuming the manufacturer's negotiation/bargaining power is $\theta$, the negotiation process of the wholesale price in the WC strategy is as follows:

$$\max_{w_1>c} \pi^w(w_1) = \max_{w_1>c} [\pi_m^w(w_1) - \pi_m^d]^\theta [\pi_r^w(w_1)]^{1-\theta} \tag{4}$$

**3.4. FC strategy**

In the FC strategy, the manufacturer can sell products to customers directly online as well as through the franchise store. The decision sequence is represented as follows. Firstly, the manufacturer and franchise store engage in a cooperative game where they negotiate the wholesale price ($w_2$) and franchise fee ($K$) simultaneously under a Nash program framework. Secondly, the manufacturer and franchise store participate in a static game where the manufacturer decides a unit direct selling price ($p_d$) and the franchise store determines the unit retail price ($p_r$) independently and simultaneously. Therefore, the decision process for the FC strategy is described as follows:

$$\max_{w_2>c, K>0} \pi^f(w_2, K) \rightarrow \begin{cases} \max\limits_{p_d} \pi_m^f(p_d) \\ \max\limits_{p_r} \pi_r^f(p_r) \end{cases}$$

Manufacturer's profit $\pi_m^f(p_d)$ is calculated as follows:

$$\pi_m^f(p_d) = (w_2 - c)\left(1 - \frac{p_r - p_d}{1-\eta}\right) + (p_d - c)\left(\frac{p_r - p_d}{1-\eta} - \frac{p_d}{\eta}\right) + K \quad (5)$$

The first part indicates the profit from the offline retail channel. The second part is the profit from the online direct channel. The third part represents the franchise fee.

Similarly, the profit of the franchise store $\pi_r^f(p_r)$ is calculated as follows:

$$\pi_r^f(p_r) = (p_r - w_2)\left(1 - \frac{p_r - p_d}{1-\eta}\right) - K \quad (6)$$

The process of negotiating the wholesale price and franchise fee for the FC strategy is as follows:

$$\max_{w_2>c, K>0} \pi^f(w_2, K) = \max_{w_2>c, K>0} [\pi_m^f(w_2, K) - \pi_m^d]^\theta [\pi_r^f(w_2, K)]^{1-\theta} \quad (7)$$

Table 2 lists the manufacturer's optimal wholesale price ($w_1^i$ and $w_2^i$) and unit direct selling price ($p_d^i$), the retailer's optimal unit retail price ($p_r^i$), and the optimal quantities for the two channels ($q_d^i$, $q_r^i$) and the franchise fee ($K^i$). The appendix provides the derivation process for these optimal solutions.

**Table 2. Optimal solutions for the three strategies**

| Strategies | DC strategy | WC strategy | FC strategy |
|---|---|---|---|
| | $(i = d)$ | $(i = w)$ | $(i = f)$ |
| ${w_1}^i$ | / | $\frac{c}{2} + \frac{16 + 2\eta^2 + 8\theta + 2\eta\theta - \eta^2\theta - (4-\eta)T_a}{4(8+\eta)}$ | / |
| ${w_2}^i$ | / | / | $\frac{c}{2} + \frac{\eta(8+\eta)}{8+10\eta}$ |
| $p_d^i$ | $\frac{c}{2} + \frac{\eta}{2}$ | $\frac{c}{2} + \frac{20\eta - 2\eta^2 + (6\eta + 3\eta^2)\theta - 3\eta T_a}{4(8+\eta)}$ | $\frac{c}{2} + \frac{\eta(2+7\eta)}{8+10\eta}$ |
| $p_r^i$ | / | $\frac{c}{2} + \frac{24 - 4\eta - 2\eta^2 + (4 + 4\eta + \eta^2)\theta - (2+\eta)T_a}{4(8+\eta)}$ | $\frac{c}{2} + \frac{4 + 6\eta - \eta^2}{8+10\eta}$ |
| $q_d^i$ | $\frac{\eta - c}{2\eta}$ | $\frac{\eta(2+\eta)(2-\theta) - 2c(8+\eta) + \eta T_a}{4\eta(8+\eta)}$ | $\frac{\eta(2+\eta) - c(4+5\eta)}{2\eta(4+5\eta)}$ |

| | | | |
|---|---|---|---|
| $q_r^i$ | / | $\frac{(2+\eta)(2-\theta)+T_a}{2(8+\eta)}$ | $\frac{2+\eta}{4+5\eta}$ |
| $K^i$ | / | / | $\frac{(1-\eta)(-5\eta-4\eta^2+4\theta+9\eta\theta+5\eta^2\theta)}{(4+5\eta)^2}$ |

where $T_a = \sqrt{16(1-\eta)(1-\theta)+(2+\eta)^2\theta^2}$.

## 4. Effects of the WC strategy

We compare the subgame perfect equilibrium solutions in the DC and WC strategies to analyze the effects of introducing a conventional wholesale channel on the optimal direct selling price, the manufacturer's maximum profit, and social welfare.

### 4.1. Effect on optimal direct selling price

First, regarding its effect on the optimal direct selling price, the following lemma is derived.

**Lemma 1:** *If* $0 < c \le c^C$ *and* $0 \le \theta < \frac{2}{3}$, *or* $c^C < c < c^B$ *and* $0 \le \theta < \theta^A$, *then* $p_d^w < p_d^d$; *if* $0 < c \le c^A$ *and* $\frac{2}{3} \le \theta \le 1$, *or* $c^A < c < c^C$ *and* $\frac{2}{3} \le \theta < \theta^A$, *then* $p_d^w \ge p_d^d$.[1]

Lemma 1 shows that introducing a conventional wholesale channel can drive the direct selling price up or down, depending on the manufacturer's unit production cost ($c$), negotiation power ($\theta$), and associated critical thresholds. More specifically, when the manufacturer possesses low and medium negotiation power ($0 \le \theta < \frac{2}{3}$ or $0 \le \theta < \theta^A$), then introducing a conventional wholesale channel will result in more intense channel competition. In this situation, fierce competition between the two channels brings down the direct selling price and can therefore benefit consumers. By contrast, if the manufacturer's negotiation power increases to a higher range ($\frac{2}{3} \le \theta \le 1$ or $\frac{2}{3} \le \theta < \theta^A$), the manufacturer tends to gain more marginal profit by setting a higher direct selling price, thereby harming customers.

### 4.2. Effect on manufacturer's maximum profit

Next, we compare the manufacturer's maximum profits in the DC and WC strategies and derive the following proposition:

**Proposition 1:**

*(1) If* $0 < c \le c^A$ *and* $0 \le \theta \le 1$, *or* $c^A < c < c^B$ *and* $0 \le \theta < \theta^A$, *then the WC is*

[1] $c^A = \frac{2\eta+\eta^2}{8+\eta}$, $c^B = \frac{2\eta+\eta^2+2\eta\sqrt{1-\eta}}{8+\eta}$, $c^C = \frac{\eta}{3}$, $\theta^A = \frac{-8c^2+4c\eta-c^2\eta+2c\eta^2-\eta^3}{2c\eta+c\eta^2-\eta^3}$

*the better strategy for the manufacturer.*

*(2) If* $c^A < c < c^B$ *and* $\theta^A \leq \theta \leq 1$, *or* $c^B \leq c < c^D$ *and* $0 \leq \theta \leq 1$, *then the DC is the better strategy for the manufacturer.*[2]

Proposition 1 indicates that the decision as to whether the manufacturer ought to add a conventional wholesale channel to the existing online direct channel is determined by customer acceptance ($\eta$), unit production cost ($c$), and negotiation power ($\theta$) for the manufacturer. The result is further illustrated in Figure 2, in which **Regions I** and **II** outline the circumstances dictating the manufacturer's decision to either add or forgo a conventional wholesale channel apart from the existing online channel.

Interestingly, the unit production cost is a main factor. Specifically, if the unit production cost is within the low range ($0 < c \leq c^A$), then the manufacturer's negotiation power does not influence the selection of channel expansion strategies. Intuitively, a low unit production cost ($c$) enables the manufacturer to attain greater marginal profit, and adding a conventional wholesale channel brings the manufacturer a new revenue stream. In contrast, when the unit production cost increases to a high level ($c^B \leq c < c^D$), the manufacturer should not introduce a conventional wholesale channel as it may cannibalize the market demand for the online direct channel; furthermore, double marginalization in a conventional wholesale channel negatively impacts profitability compared to a single company in the direct online channel. Additionally, in the scenario of a high unit production cost, the financial loss from demand cannibalization and double marginalization outweighs the financial benefit of an extra revenue source from introducing the conventional wholesale channel. Hence, the manufacturer will obtain a greater economic outcome by only operating the online direct sales channel. These results are consistent with industrial practices; for example, Xiaomi initially only operated the online direct channel and started to sell through offline retail stores in recent years after improving their operational efficiency and reducing the production cost of smartphones (Liu et al., 2020).

[2] $c^D = \eta$.

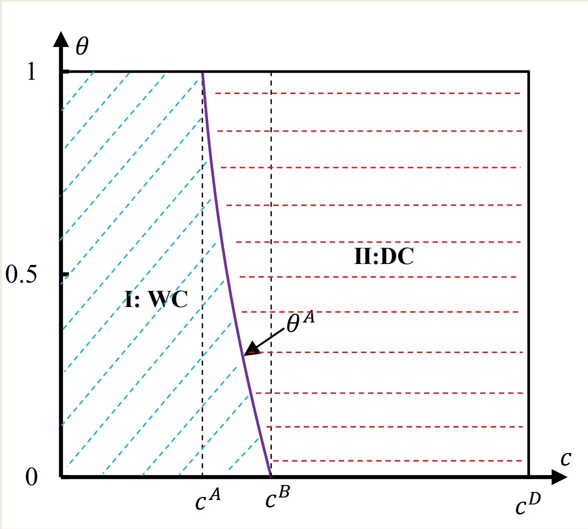


**Figure 2. Effect of WC strategy on manufacturer's profit**

However, the manufacturer's negotiation power starts to influence the channel expansion decision if the unit production cost falls within a medium range ($c^A < c < c^B$). For example, if the manufacturer's negotiation power drops below the threshold, $\theta^A$, then it is financially better off under the WC strategy; conversely, the DC strategy delivers greater profit when the manufacturer's negotiation power exceeds $\theta^A$. This phenomenon arises because a rise in the retailer's power in negotiating the wholesale price mitigates the negative effect generated by double marginalization. Consequently, to make the WC strategy a viable online-to-store channel expansion strategy, the requirement for the minimum unit production cost is relaxed to $c^B$ when the downstream retailer has the dominant power ($\theta = 0$) in the negotiation process of the wholesale price; and the unit production cost is restricted to $c^A$ when the upstream manufacturer has the dominant power ($\theta = 1$)in the negotiation process of the wholesale price. Our results support the view taken in previous literature (Xu et al., 2012) that customer acceptance of the online direct channel as well as related cost parameters (i.e., unit production cost and logistics cost) are key factors in determining whether the manufacturer should introduce new channels.

To delve deeper into the effect of the WC strategy on the manufacturer, we explore the influence of customer acceptance of the online direct channel ($\eta$) on the manufacturer's channel selection decision and obtain the following corollary.

**Corollary 1:** *The likelihood that the WC is the better strategy decreases in* $\eta$*; the likelihood that the DC is the better strategy increases in* $\eta$.

This shows that customer acceptance of the online direct channel ($\eta$) exerts great influence on the manufacturer's channel decision. Here, as customer acceptance increases, the heterogeneity between two channels reduces, which intensifies channel competition. The negative effect created by channel competition on the manufacturer's economic gains will rise further with an increase in customer acceptance of the online direct channel, and therefore the likelihood that WC becomes a viable channel strategy decreases in $\eta$.

### 4.3. Effect on social welfare

Now, we assess the effect of introducing a conventional wholesale channel on overall social welfare and obtain the following corollary.

**Corollary 2:** $W_s^w > W_s^d$.

It is evident from Corollary 2 that introducing a conventional wholesale channel can always benefit society. Intuitively, adding an offline channel, such as a conventional wholesale channel, will intensify price competition between the two channels. The decrease in retail prices due to intensified competition increases consumer surplus. Although, as discussed earlier, the WC strategy does not guarantee a rise in the manufacturer's profit, the rise in consumer surplus as well as the retailer's profit will improve the social welfare of the WC strategy compared to the DC strategy.

## 5. Effects of the FC strategy

This section compares the subgame perfect equilibrium solutions in the DC and FC strategies to analyze the effects of adding a franchise store channel on the optimal direct selling price, the manufacturer's maximum profit, and social welfare.

### 5.1. Effect on optimal direct selling price

Regarding the FC strategy's effect on the optimal direct selling price, the following lemma is derived.

**Lemma 2**: $p_d^f < p_d^d$.

Lemma 2 implies that the unit direct selling price is consistently lower in the FC strategy than in the DC strategy. This result is in contrast to that of the WC strategy, where the direct selling price can go up or down depending upon the manufacturer's unit production cost ($c$), negotiation power ($\theta$), and associated critical thresholds. This can be attributed to two main causes. First, introducing a franchise store as an offline channel intensifies channel competition and brings down the direct selling price, which is similar to the WC strategy; second, the negative impact of double marginalization is neglected more in the FC strategy than in the WC strategy because the payment of a franchise fee enables channel coordination to alleviate the double marginalization problem (Xu et al., 2017). A combination of these two factors contributes to a reduction in the direct selling price in the FC strategy, which is beneficial to consumers.

### 5.2. Effect on manufacturer's maximum profit

Next, we obtain the following proposition by analyzing the FC strategy's effect on the maximum profit of the manufacturer.

**Proposition 2:**

*(1) If* $0 < c < c^E$ *and* $\theta^B < \theta \leq 1$*, then the FC is the better strategy for the manufacturer.*

*(2) If* $0 < c < c^E$ *and* $0 \leq \theta \leq \theta^B$*, or* $c^E \leq c < c^D$ *and* $0 \leq \theta \leq 1$*, then the DC is the better strategy for the manufacturer.*[3]

Proposition 2 highlights the fact that the manufacturer's decision to expand into an offline franchise channel is mainly contingent upon the unit production cost ($c$) and negotiation power ($\theta$), as illustrated in Figure 3.

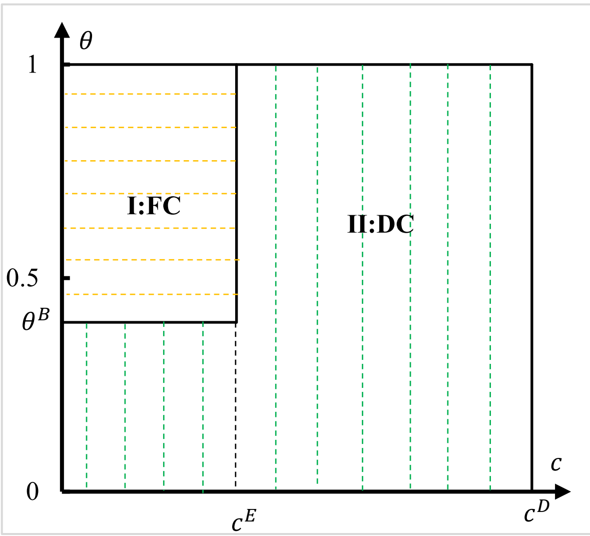


**Figure 3. Effect of FC strategy on manufacturer's profit**

[3] $c^E = \frac{2\eta+\eta^2}{4+5\eta}$ and $\theta^B = \frac{5\eta+4\eta^2}{4+9\eta+5\eta^2}$.

Specifically, if the unit production cost surpasses a critical threshold, $c^E$, the manufacturer should not add a franchise store to the existing online direct channel and should rather focus on the online direct channel only, as specified in **Region II**, since the loss created by demand cannibalization outweighs the benefit of the added revenue source from the offline channel. Similar to the previous section, only the manufacturer's low unit production cost can enable it to expand into an offline channel. The manufacturer's negotiation power also significantly influences the manufacturer's channel expansion strategy, which means that the FC strategy becomes a favorable choice for the manufacturer only when the unit production cost falls within the low range ($0 < c < c^E$) and the negotiation power is higher than the critical threshold, $\theta^B$ (**Region I)**. This is because a low production cost can create a greater profit margin for the manufacturer. Additionally, the manufacturer's superior negotiation power enables it to charge a high fixed franchise fee and further increase the total profit. This finding aligns with Shen et al. (2019), Chen et al. (2020), and Chen et al. (2022), all of whom argue that the power dynamics between manufacturer and retailer in the contract negotiation will affect the pricing policies.

Next, we scrutinize the effect of customer acceptance ($\eta$) on the selection of channel expansion strategies and derive the following corollary.

**Corollary 3**: *The likelihood that the FC is the better strategy decreases in* $\eta$*; the likelihood that the DC is the better strategy increases in* $\eta$.

This corollary implies that the selection of online-to-store channel expansion strategies is influenced by customer acceptance ($\eta$). Corollary 3 shows that as customer acceptance of the online channel increases, the level of homogeneity between the two kinds of channels increases. Therefore, the manufacturer is reluctant to introduce offline channels to avoid fierce channel competition. Combining Corollaries 1 and 3, the likelihood that the introduction of an offline channel will improve the manufacturer's economic performance decreases with customer acceptance of the online direct channel ($\eta$).

### 5.3. Effect on social welfare

Regarding the effect of introducing the FC strategy on social welfare, the following corollary is obtained.

**Corollary 4:** $W_s^f > W_s^d$.

It is clear from Corollary 4 that adding a franchise store channel to the existing online channel will improve social welfare. Like the effect of introducing a conventional wholesale channel, the addition of an offline franchise store will stimulate channel competition, which is beneficial to consumers. Furthermore, online-to-store channel expansion enables the manufacturer to expand its market share and thereby increase revenues. Combining Corollaries 2 and 4, adding an offline channel to the manufacturer's existing online direct channel always benefits society overall.

## 6. Selection of channel expansion strategies

In the subsequent analysis, we discuss the selection decision among alternative online-to-store channel expansion strategies. We derive the following proposition through a thorough comparison of the manufacturer's maximum profits in the DC, WC, and FC strategies.

**Proposition 3:**

*(1) When* $0 < c < c^E$ *and* $0 \leq \theta \leq 1$, *or* $c^E \leq c < c^B$ *and* $0 \leq \theta < \theta^A$, *then the manufacturer should adopt an online-to-store expansion strategy. Specifically, (i) if* $0 < c < c^E$ *and* $\theta^B < \theta \leq 1$, *then the FC strategy is the optimal strategy for the manufacturer; (ii) if* $0 < c < c^E$ *and* $0 \leq \theta \leq \theta^B$, *or* $c^E \leq c < c^B$ *and* $0 \leq \theta < \theta^A$, *then the WC strategy is the optimal strategy for the manufacturer.*

*(2) When* $c^E \leq c < c^B$ *and* $\theta^A \leq \theta \leq 1$, *or* $c^B \leq c < c^D$ *and* $0 \leq \theta \leq 1$, *then the manufacturer should solely operate an online channel and the DC strategy is the optimal strategy for the manufacturer.*

Proposition 3 shows that the optimal decision for online-to-store channel expansion strategies is determined by customer acceptance of the online direct channel ($\eta$), the unit production cost ($c$), the manufacturer's negotiation power ($\theta$), and their relationships with the thresholds ($\theta^A$, $\theta^B$, $c^B$, $c^D$ and $c^E$). The results are depicted in the three decision regions shown in Figure 4.

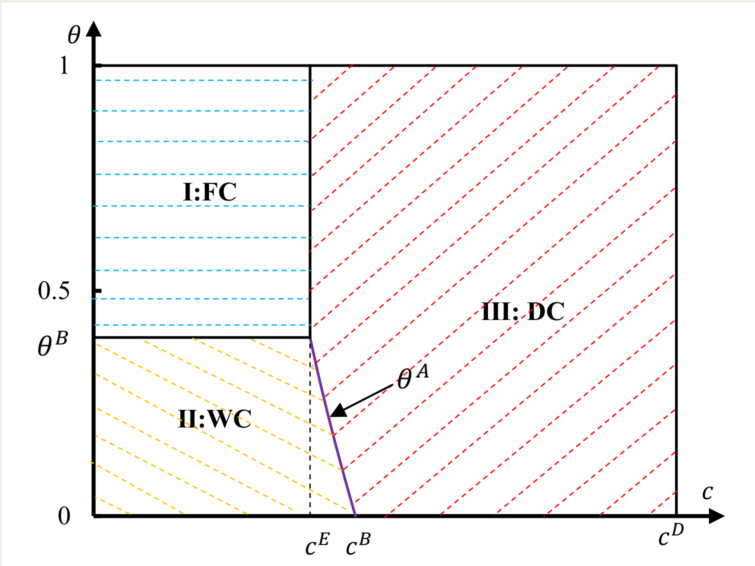


**Figure 4. Selection of channel strategies**

Figure 4 clearly demonstrates that the unit production cost is the most critical factor in determining the selection of online-to-store channel expansion strategies. More specifically, it is more profitable to operate an online direct sales channel only when the unit production cost exceeds the critical threshold, $c^B$, as the profit from the new offline channel will not offset the economic loss caused by the cannibalization of the existing online channel. As a reduction in unit production cost, the manufacturer will shift to expanding an offline channel. If the manufacturer's unit production cost falls within the range of $c^E \leq c < c^B$, the WC strategy comes into consideration as the increased profit margin enables it to gain greater economic benefit from the extra revenue than the loss of channel cannibalization. However, it requires the retailer to have superior negotiation power in the wholesale contract negotiation with the manufacturer ($\theta < \theta^A$) as the retailer's channel leadership enables it to alleviate the double marginalization in the WC strategy. A further decrease in the unit production cost ($c \leq c^E$) will completely swing the decision in favor of introducing a conventional wholesale channel or a franchise store due to increased profit margins. Intriguingly, the power relationship in negotiating the franchise fee or wholesale price—more specifically, the relationship between the manufacturer's bargaining power and a threshold $\theta^B$—is the deciding factor when choosing between the FC and WC strategies. The wholesale channel strategy generates a greater profit than the franchise store channel if the retailer holds greater negotiation power ($\theta \leq \theta^B$). Conversely, the franchise store channel delivers the best economic performance with a rise in the manufacturer's negotiation power ($\theta > \theta^B$) as outlined in **Region I**. Our findings align with industrial practices. For instance, in addition to Xiaomi expanding the sales channel to traditional retail stores in recent years, Inman, the Tao brand, has also begun to deploy offline channels by rolling out offline franchise stores.

Next, we analyze the wholesale price in different strategies and get the following corollary.

**Corollary 5:** $w_2^f < w_1^w$.

Corollary 5 shows that introducing a franchise channel results in a reduction in the wholesale price when compared to the WC strategy. The underlying explanation is that a manufacturer with relatively high negotiation power ($\theta^B < \theta \leq 1$) can charge the franchise store a franchise fee when the manufacturer adds a franchise store to the existing online sales channel. Hence, the franchise fee can drive a reduction in the wholesale price, prompting the retailer to subsequently lower its retail price, thereby increasing market share and generating more profit for the manufacturer.

## 7. Conclusions

In this paper, we explore whether a manufacturer operating an online direct channel should introduce offline channels, considering the heterogeneity of consumers' evaluations of products from online and offline channels. By comparing the subgame perfect equilibriums of the WC and FC strategies to the benchmark strategy of operating one online channel only, this paper examines the underlying economic principles that govern the manufacturer's decisions on channel expansion strategies. This study also examines the impacts of introducing offline retail channels on consumers and social welfare. Through analysis, the following key results are obtained.

First, the manufacturer's choice of channel selection strategies is contingent upon the trade-off between the financial gain from extra revenue sources created by expanding an offline channel and the loss arising from the existing online channel triggered by channel competition. The trade-off between the gains and losses from the alternative channel expansion strategies is influenced by a combination of key factors, including customer acceptance of the online direct channel ($\eta$), the manufacturer's negotiation power ($\theta$), and its unit production cost ($c$). Specifically, unit production cost dominates whether the manufacturer should introduce an offline channel. In particular, the addition of an offline channel can deliver economic improvements when the unit production cost falls within a sufficiently low range to ensure that there is a sufficient profit margin to guarantee that the gain can offset the loss from channel competition. Moreover, the power relationship in the contract negotiation between the manufacturer and its channel partner (i.e., the retailer or franchise store) exerts significant influence in choosing between the three channel expansion strategies. Being dominant in the negotiations enables the manufacturer to benefit most from the FC strategy, but when the retailer dominates, the wholesale channel becomes the best choice due to the mitigation of double marginalization. Second, the loss from the existing online channel due to cannibalization by an offline channel is contingent on customer acceptance of the online direct channel. Regardless of the cooperative format in which the manufacturer engages with the retailer (i.e., either a conventional retailer or a franchise store), the manufacturer should adopt a mixed-channel operation when the heterogeneity of the two channels increases. Finally, our analysis also shows that online-to-store channel expansion strategies generally have a positive impact on consumers and society overall because the online and offline mixed channels can intensify price competition.

This paper offers some valuable managerial insights. While operating dual-channel management can increase revenue streams, it inevitably introduces channel competition and may not always result in higher profit compared to exclusively operating an online channel. The primary focus of this paper is to explore the strategic decision of whether the manufacturer currently operating solely through an online channel should consider expanding into an offline channel and the corresponding optimal pricing policies under different conditions. The decision of whether to choose an online-to-store expansion strategy hinges predominately on the trade-off between the gains from adding an offline channel and the potential losses from channel competition. This trade-off is primarily contingent on the interplay among several factors. Specifically, a manufacturer with a low unit production cost should change its channel management from solely operating an online channel to implementing an online-to-store expansion strategy. In practice, Xiaomi, renowned for its cost-effectiveness approach, has shifted from exclusively operating a purely online direct channel to incorporating offline channels, thus broadening its consumer reach (Yan et al., 2020). Furthermore, as the manufacturer’s negotiation power strengthens, the FC strategy emerges the optimal strategy when expanding an offline channel. Conversely, as the manufacturer’s negotiation power weakens, the shift from the FC strategy to the WC strategy becomes advisable due to the alleviation of double marginalization. Meanwhile, with the increase in homogeneity between the two channels, maintaining the exclusive operation of an online channel become a preferable strategy for the manufacturer to mitigate competition. Our comprehensive analysis of these online-to-store expansion strategies provides useful guidance to firms contemplating such strategic transitions.

This paper has a few limitations, and addressing them leads to several promising extensions to this paper. First, this paper adopts a common linear demand function to consider demand substitution between channels. One potential future extension is to consider stochastic demand, which can provide a thorough exploration of how demand uncertainty affects the results. Second, we assume that the manufacturer only considers adding one offline retail channel when, in practice, it may expand its sales operation to several offline channels. Thus, one future extension would consider multiple offline retail channels simultaneously, which will create competition between multiple channels. Finally, customer acceptance of the online direct channel serves to characterize the difference between online and offline channels, thereby influencing the manufacturer’s decision regarding channel expansion. One potential extension is to incorporate specific factors that affect customer acceptance and their impact on the channel expansion decision.

## Declaration of interest statement

No potential conflict of interest was reported by the authors.

## References

Arya, A., Mittendorf, B., & Sappington, D. E. M. (2007). The bright side of supplier encroachment. *Marketing Science*, 26(5), 651-659.

Cavallo, A. (2017). Are online and offline prices similar? Evidence from large multi-channel retailers. *American Economic Review*, 107(1), 283-303.

Chen, X., Wang, X. and Gong, K. (2020). The effect of bidimensional power structure on supply chain decisions and performance. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 50(3), 1095-1110.

Chen X., Zhang, H., Zhang, M., & Chen, J. (2017). Optimal decisions in a retailer Stackelberg supply chain. *International Journal of Production Economics*, 187, 260-270.

Chen X., Wang X., & Xia Y. (2019). Production coopetition strategies for competing manufacturers that produce partially substitutable products. *Production and Operations Management*, 28(6): 1446-1464.

Chen X., Wang X., & Xia Y. (2022). Low-carbon technology transfer between rival firms under cap-and-trade policies. *IISE Transactions*, 54(2): 105-121.

Chen X., Wang X., Zhang D., & Yang M. (2023). Selling models for platforms under service sensitive demand. *OMEGA-International Journal of Management Science*. 115: 102794

Chen, Y., Chung, S. H., & Guo, S. (2018). Franchising contracts in fashion supply chain operations: models, practices, and real case study. *Annals of Operations Research*, 291, 83-128.

Cheng, R., Duan, Y., Zhang, J., & Ke, H. (2021). Impacts of store-brand introduction on a multiple-echelon supply chain. *European Journal of Operational Research*, 292, 652-662.

Chiang, W. Y. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science*, 49(1), 1-20.

Choi, T. M., Chen, Y., & Chung, S. H. (2019). Online-offline fashion franchising supply chains without channel conflicts: Choices on postponement and contracts. I*nternational Journal of Production Economics*, 215, 174-184.

Ha, A., Long, X., & Nasiry, J. (2016). Quality in supply chain encroachment. *Manufacturing and Service Operations Management*, 18(2), 280-298.

Ha, A. Y., Luo, H., & Shang, W. (2022). Supplier encroachment, information sharing, and channel structure in online retail platforms. *Production and Operations Management*, 31(3), 1235-1251.

He, P., He, Y., Tang, X., Ma, S., & Xu, H. (2022). Channel encroachment and logistics integration strategies in an e-commerce platform service supply chain. *International Journal of Production Economics*, 244, 108368.

Hu, H., Lyu, G., Cheng, T. C. E., & Zhang, Y. (2023). Reselling, consignment or In-Marketplace selling Mode? Competitive strategy for E-tailers to counteract online third-party marketplaces. *Transportation Research Part E: Logistics and Transportation Review*, 176, 103190.

Kim, J. C., & Chun, S. H. (2018). Cannibalization and competition effects on a manufacturer's retail channel strategies: Implications on an omni-channel business model. *Decision Support Systems*, 109, 5-14.

Li, K. J. (2019). Status goods and vertical line extensions. *Production and Operations Management*, 28(1), 103-120.

Li, S., Cheng, H. K., & Yong, J. (2018). Optimal distribution strategy for enterprise software: Retail, saas, or dual channel? *Production & Operations Management*, 27(11), 1928-1939.

Li, T., Xie, J., & Zhao, X. (2015). Supplier encroachment in competitive supply chains. *International Journal of Production Economics*, 165, 120-131.

Liu, B., Guan, X., & Wang, Y. (2021). Supplier encroachment with multiple retailers. *Production and Operations Management*, 30(10), 3523-3539.

Liu, C., Lee, C. K. M., & Zhang, L. L. (2022). Pricing strategy in a dual-channel supply chain with overconfident consumers. *Computers & Industrial Engineering*, 172, 108515.

Liu, L., Feng, L., Xu, B., & Deng, W. (2020). Operation strategies for an omni-channel supply chain: Who is better off taking on the online channel and offline service? *Electronic Commerce Research and Applications*, 39, 100918.

Luo, Z., Chen, X., & Kai, M. (2018). The effect of customer value and power structure on retail supply chain product choice and pricing decisions. *Omega*, 77, 115-126.

Matsui, K. (2020). Optimal bargaining timing of a wholesale price for a manufacturer with a retailer in a dual-channel supply chain. *European Journal of Operational Research*, 287(1), 225-236.

Matsui, K. (2022). Should a retailer bargain over a wholesale price with a manufacturer using a dual-channel supply chain? *European Journal of Operational Research*, 300(3), 1050-1066.

Nash, J. (1950). The bargaining problem. *Econometrica: Journal of the Econometric Society*, 18 (2), 155-162.

Nash, J. (1953). Two-person cooperative games. *Econometrica*: *Journal of the Econometric Society,* 21, 128-140.

Ominaretail. (2023). A new shopping era: 2023 trends and how brick-and-mortar can capitalise on the current e-commerce slump. https://www.omniaretail.com/blog/a-new-shopping-era-2023-trends-and-how-brick-and-mortar-can-capitalise-on-the-current-e-commerce-slump.

Qing, Q., Deng, T., Wang, H. (2017). Capacity allocation under downstream competition and bargaining.

*European Journal of Operational Research*, 261(1), 97-107.

Qiu, R., Hou, L., Sun, Y., Sun, M., & Sun, Y. (2021). Joint pricing, ordering and order fulfillment decisions for a dual-channel supply chain with demand uncertainties: A distribution-free approach. *Computers & Industrial Engineering*, 160, 107546.

Pu, X., Sun, S., & Shao, J. (2020). Direct selling, reselling, or agency selling? Manufacturer's online distribution strategies and their impact. *International Journal of Electronic Commerce*, 24(2), 232-254.

Shen, Y., Willems, S. P., & Dai, Y. (2019). Channel selection and contracting in the presence of a retail platform. *Production and Operations Management*, 28(5), 1173-1185.

Shi, D., Wang, M., & Li, X. (2021). Strategic introduction of marketplace platform and its impacts on supply chain. *International Journal of Production Economics*, 242, 108300.

Shi, S., Sun, J., & Cheng, T. (2020). Wholesale or drop-shipping: Contract choices of the online retailer and the manufacturer in a dual-channel supply chain. *International Journal of Production Economics*, 226, 107618.

Shi, S., Wang, C., Cheng, T. C. E., & Liu, S. (2023). Manufacturer encroachment with an e-commerce division. *Production and Operations Management*, 32(6), 2002-2019.

Tsay, A. A., & Agrawal, N. (2004). Channel conflict and coordination in the e-commerce age. *Production and Operations Management*, 13(1), 93-110.

Wan, X., Chen, J., & Chen, B. (2022). The manufacturer's encroachment strategy in the presence of the retailer's in-store service. *International Journal of Production Research*. https://doi.org/10.1080/00207543.2022.2147238.

Wang, J., & He S. (2022). Optimal decisions of modularity, prices and return policy in a dual-channel supply chain under mass customization. *Transportation Research Part E: Logistics and Transportation Review*, 160, 102675.

Wang, R., Nan, G., Chen, L. and Li, M., (2020). Channel integration choices and pricing strategies for competing dual-channel retailers. *IEEE Transactions on Engineering Management*, 69(5), 2260-2274.

Wang, X., & Ng, C. T. (2020). New retail versus traditional retail in e-commerce: channel establishment, price competition, and consumer recognition. *Annals of Operations Research*, 291, 921-937.

Wu, Jie., Lu, W., & Ji, X. (2023). The interactions between upstream quality disclosure and downstream entry. *European Journal of Operational Research*, 309(2), 545-559.

Xia, J., & Niu, W. (2019). Adding clicks to bricks: An analysis of supplier encroachment under service spillovers. *Electronic Commerce Research and Applications*, 37, 100876.

Xu, H., Liu, Z., & Zhang, S. (2012). A strategic analysis of dual-channel supply chain design with price and delivery lead time considerations. *International Journal of Production Economics*, 139(2), 654-663.

Xu, J., You, X., Li, Y., & Lu, J. (2022). Channel and pricing strategy of competing retailers in a two-stage supply chain. *IEEE Transactions on Engineering Management*, 69(6), 2882-2896.

Xu, X., He, P., Xu, H., & Zhang, Q. (2017). Supply chain coordination with green technology under cap-and-trade regulation. *International Journal of Production Economics*, 183, 433-442.

Xu, L., Wei, J., Choi, T.M. and Wang, W. (2020). Managing online channel and optimization in supply chain systems with different channel leaderships. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *51*(11), 7177-7190.

Yan, B., Chen, Y., & Liu, S. (2020). Dynamic channel choice of mobile phone companies by considering the time preference of consumers. *Computers & Industrial Engineering*, 145, 106493.

Yan, Y., Zhao, R., & Liu, Z. (2018). Strategic introduction of the marketplace channel under spillovers from online to offline sales. *European Journal of Operational Research*, 267(1), 65-77.

Yan, S., Archibald, T. W., Han, X., & Bian, Y. (2022). Whether to adopt "buy online and return to store" strategy in a competitive market? *European Journal of Operational Research*, 301(3), 974-986.

Yang, H., Luo, J., & Zhang, Q. (2018). Supplier encroachment under nonlinear pricing with imperfect substitutes: Bargaining power versus revenue-sharing. *European Journal of Operational Research*, 267(3), 1089-1101.

Yang, W., Zhang, J., & Yan, H. (2021). Impacts of online consumer reviews on a dual-channel supply chain. *Omega*, 101(1), 102266.

Yun, X., Liu, H., Li, Y., & Lai, K. K. (2023). Contract design under asymmetric demand information for sustainable supply chain practices. *Annals of Operations Research*, 324, 1429-1459.

Zhang, C., Li, Y., & Ma, Y. (2021a). Direct selling, agent selling, or dual-format selling: Electronic channel configuration considering channel competition and platform service. *Computers & Industrial Engineering*, 157, 107368.

Zhang P., He, Y., & Shi, C. V. (2017). Retailer's channel structure choice: Online channel, offline channel, or dual channels? *International Journal of Production Economics*, 191, 37-50.

Zhang, S., & Zhang, J. (2020). Agency selling or reselling: E-tailer information sharing with supplier offline entry. *European Journal of Operational Research*, 280(1), 134-151.

Zhang, T., Feng, X., & Wang, N. (2021b). Manufacturer encroachment and product assortment under vertical differentiation. *European Journal of Operational Research*, 293(1), 120-132.

Zhao, C., Wang, X., Xiao, Y. and Sheng, J. (2022). Effects of online reviews and competition on quality and pricing strategies. *Production and Operations Management*, 31(10), 3840-3858.

Zhao, Y., Li, Y., Yao, Q., & Guan X. (2023). Dual-channel retailing strategy vs. omni-channel buy-online-and-pick-up-in-store behaviors with reference freshness effect. *Inernational Journal of Production Economics*, 263, 108967.

Zheng, S., & Yu, Y. (2021). Manufacturer encroachment with equal pricing strategy. *Transportation Research Part E: Logistics and Transportation Review*, 152, 102346.

Zhu, B., Wen, B., Ji, S., & Qiu, R. (2020). Coordinating a dual-channel supply chain with conditional value-at-risk under uncertainties of yield and demand. *Computers & Industrial Engineering*, 139, 106181.

# Appendix

**Derivation of Table 2**

**(1) DC strategy:** From (1), we obtain $\frac{d^2\pi_m^d(p_d)}{dp_d^2} = -\frac{2}{\eta} < 0$, so $\pi_m^d(p_d)$ is a concave function of $p_d$. Let $\frac{d\pi_m^d(p_d)}{dp_d} = 0$, so $p_d^d = \frac{c}{2} + \frac{\eta}{2}$. Recall that $q_d^d = 1 - \frac{p_d}{\eta}$, we obtain $q_d^d = \frac{\eta - c}{2\eta}$.

**(2) WC strategy:** From (2), we obtain $\frac{d^2\pi_m^w(p_d)}{dp_d^2} = \frac{2}{(-1+\eta)\eta} < 0$, so $\pi_m^w(p_d)$ is a concave function of $p_d$. Similarly, from (3), we obtain $\frac{d^2\pi_r^w(p_r)}{dp_r^2} = -\frac{2}{1-\eta} < 0$, so $\pi_r^w(p_r)$ is a concave function of $p_r$. $\frac{d\pi_m^w(p_d)}{dp_d} = \frac{d\pi_r^w(p_r)}{dp_r} = 0$ shows that $p_d(w_1) = \frac{2c(-1+\eta)+\eta(-1-3w_1+\eta)}{-4+\eta}$ and $p_r(w_1) = \frac{2+c-2\eta-c\eta+w_1(2+\eta)}{4-\eta}$. From (4), we can obtain

$$ln\pi^w(w_1) = \theta\, ln[\pi_m^w(w_1) - \pi_m^d] + (1-\theta) ln\, [\pi_r^w(w_1)]$$

$$s.t. w_1 - c > 0$$

We define $f_1(w_1) = \theta\, ln[\pi_m^w(w_1) - \pi_m^d] + (1-\theta)\, ln[\pi_r^w(w_1)]$, $g_1(w_1) = w_1 - c$. So, we can obtain $L_1(w_1) = f_1(w_1) + \mu_1 g_1(w_1)$. Apply KKT conditions, we obtain

$$\begin{cases} \frac{dL_1(w_1)}{dw_1} = 0 \\ \mu_1 g_1(w_1) = 0 \end{cases} \quad \text{(a)}$$

$$\mu_1 \geq 0$$

where $\mu_1$ is Lagrange multiplier. Replace $p_d(w_1)$ and $p_r(w_1)$ in (a), we can obtain $\frac{dL_1(w_1)}{dw_1} = \theta\left(\frac{c^2(8+\eta)+4w^2(8+\eta)-4w(8+\eta^2)+\eta(12-4\eta+\eta^2)+2c\left(8+\eta^2-2w(8+\eta)\right)}{4(4-\eta)^2}\right)^{-1}\left(\frac{-4c(8+\eta)+8w(8+\eta)-4(8+\eta^2)}{4(4-\eta)^2}\right) + (1-\theta)\left(\frac{(2+c-2w)^2(1-\eta)}{(-4+\eta)^2}\right)^{-1}\left(\frac{4(2+c-2w)(-1+\eta)}{(-4+\eta)^2}\right) + \mu_1 = 0$, and $\mu_1(w_1 - c) = 0$. Thus, we can obtain three group roots: $w_{11} = c$ and $\mu_{11} = (c^3(8+\eta) - 2\eta(12-4\eta+\eta^2) - 2c^2(16+\eta+\eta^2) + c(32+12\eta+\eta^3))^{-1}(-4(-16c+12\eta-4\eta^2-2c\eta^2+\eta^3+c^2(8+\eta)) - 4(16-8c-12\eta-2c\eta+6\eta^2+c\eta^2-\eta^3)\theta)$; $w_{12} = \frac{c}{2} + \frac{16+2\eta^2+8\theta+2\eta\theta-\eta^2\theta-(4-\eta)T_a}{4(8+\eta)}$ and $\mu_{12} = 0$; and $w_{13} = \frac{c}{2} + \frac{16+2\eta^2+8\theta+2\eta\theta-\eta^2\theta+(4-\eta)T_a}{4(8+\eta)}$ and $\mu_{13} = 0$ , where $T_a = \sqrt{16(1-\eta)(1-\theta)+(2+\eta)^2\theta^2}$. $w_{11} = c$ does not meet the constraint $(w_1 - c > 0)$, so we omit $(w_{11}, \mu_{11})$. $\pi_m^w(w_{13}) - \pi_m^d < 0$, therefore, we omit $(w_{13}, \mu_{13})$. According to the second derivative of $\pi^w(w_1)$, we can obtain $\left.\frac{d^2\pi^w(w_1)}{dw_1^2}\right|_{w_1=w_{12}} < 0$, so $\pi^w(w_{12})$ is a concave function of $w_{12}$, thus $w_1^w = w_{12} = \frac{c}{2} + \frac{16+2\eta^2+8\theta+2\eta\theta-\eta^2\theta-(4-\eta)T_a}{4(8+\eta)}$. Replace $w_1^w$ into $p_d(w_1)$ and $p_r(w_1)$, we obtain

$p_d^w = \frac{c}{2} + \frac{20\eta - 2\eta^2 + (6\eta + 3\eta^2)\theta - 3\eta T_a}{4(8+\eta)}$ and $p_r^w = \frac{c}{2} + \frac{24 - 4\eta - 2\eta^2 + (4 + 4\eta + \eta^2)\theta - (2+\eta)T_a}{4(8+\eta)}$. Recall that $q_d^w = (\frac{p_r - p_d}{1-\eta} - \frac{p_d}{\eta})$ and $q_r^w = 1 - \frac{p_r - p_d}{1-\eta}$, so we obtain $q_d^w = \frac{\eta(2+\eta)(2-\theta) - 2c(8+\eta) + \eta T_a}{4\eta(8+\eta)}$ and $q_r^w = \frac{(2+\eta)(2-\theta) + T_a}{2(8+\eta)}$, where $T_a = \sqrt{16(1-\eta)(1-\theta) + (2+\eta)^2\theta^2}$.

**(3) FC strategy:** From (5), we obtain $\frac{d^2\pi_m^f(p_d)}{dp_d^2} = \frac{2}{(-1+\eta)\eta} < 0$, so $\pi_m^f(p_d)$ is a concave function of $p_d$. Similarly, from (6), we obtain $\frac{d^2\pi_r^f(p_r)}{dp_r^2} = -\frac{2}{1-\eta} < 0$, so $\pi_r^f(p_r)$ is a concave function of $p_r$. $\frac{d\pi_m^f(p_d)}{dp_d} = \frac{d\pi_r^f(p_r)}{dp_r} = 0$ shows that $p_d(w_2) = \frac{2c(-1+\eta) + \eta(-1 - 3w_2 + \eta)}{-4+\eta}$ and $p_r(w_2) = \frac{2 + c - 2\eta - c\eta + w_2(2+\eta)}{4-\eta}$. From (7), we obtain

$$ln\pi^f(w_2, K) = \theta\, ln[\pi_m^f(w_2, K) - \pi_m^d] + (1-\theta) ln\pi_r^f(w_2, K)$$

$$s.t. \begin{cases} w_2 - c > 0 \\ K > 0 \end{cases}$$

We define $f_2(w_2, K) = \theta\, ln[\pi_m^f(w_2, K) - \pi_m^d] + (1-\theta) ln\pi_r^f(w_2, K)$, $g_2(w_2) = w_2 - c$ and $g_3(K) = K$. So, we can obtain $L_2(w_2, K) = f_2(w_2, K) + \mu_2 g_2(w_2) + \mu_3 g_3(K)$. Apply KKT conditions, we obtain

$$\begin{cases} \frac{\partial L_2(w_2, K)}{\partial w_2} = 0 \\ \frac{\partial L_2(w_2, K)}{\partial K} = 0 \\ \mu_2 g_2(w_2) = 0 \\ \mu_3 g_3(K) = 0 \end{cases} \quad \text{(b)}$$

$$\mu_j \geq 0, j = 2,3$$

where $\mu_i$ is the Lagrange multiplier. Replace $p_d(w_2)$ and $p_r(w_2)$ into (b), we obtain five group roots: $w_{21} = c$ and $K_1 = 0$; $w_{22} = c$ and $K_2 = \frac{\left(-16 + 28\eta - 4\eta^2 + \eta^3 - 2c\eta(8+\eta) + c^2(4+5\eta)\right)\theta + 16c - 8c^2 - 12\eta - c^2\eta + 4\eta^2 + 2c\eta^2 - \eta^3}{4(4-\eta)^2}$; $w_{23} = \frac{4c + 8\eta + 5c\eta + \eta^2}{2(4+5\eta)}$ and $K_3 = \frac{(1-\eta)(5\eta + 4\eta^2 - 4\theta - 9\eta\theta - 5\eta^2\theta)}{(4+5\eta)^2}$; $w_{24} = \frac{c}{2} + \frac{16 + 2\eta^2 + 8\theta + 2\eta\theta - \eta^2\theta - (4-\eta)T_a}{4(8+\eta)}$ and $K_4 = 0$; $w_{25} = \frac{c}{2} + \frac{16 + 2\eta^2 + 8\theta + 2\eta\theta - \eta^2\theta + (4-\eta)T_a}{4(8+\eta)}$ and $K_5 = 0$.

Recall that $q_d^d = \frac{-c+\eta}{2\eta} > 0$, we can get $0 < c < \eta$, we discuss these five group roots $(w_{21}, K_1, \mu_{21}, \mu_{31})$, $(w_{22}, K_2, \mu_{22}, \mu_{32})$, $(w_{23}, K_3, \mu_{23}, \mu_{33})$, $(w_{24}, K_4, \mu_{24}, \mu_{34})$ and $(w_{25}, K_5, \mu_{25}, \mu_{35})$. $w_{21} = c$ and $w_{22} = c$ don't meet the constraint $(w_2 - c > 0)$, so we omit $(w_{21}, K_1, \mu_{21}, \mu_{31})$ and $(w_{22}, K_2, \mu_{22}, \mu_{32})$. For $(w_{24}, K_4, \mu_{24}, \mu_{34})$ and $(w_{25}, K_5, \mu_{25}, \mu_{35})$, $K_4 = K_5 = 0$ does not meet the constraint $(K > 0)$, so we omit $(w_{24}, K_4, \mu_{24}, \mu_{34})$ and $(w_{25}, K_5, \mu_{25}, \mu_{35})$.

For $(w_{22}, K_3, \mu_{23}, \mu_{33})$, $w_{22} - c = \frac{-4c+8\eta-5c\eta+\eta^2}{2(4+5\eta)} > 0$, $K_3 = \frac{(1-\eta)(-5\eta-4\eta^2+4\theta+9\eta\theta+5\eta^2\theta)}{(4+5\eta)^2}$, $\mu_{23} = 0$ and $\mu_{33} = 0$. According to second partial derivative of $\pi^f(w_2, K)$, we can get $\frac{\partial^2 \pi^f(w_2,K)}{\partial w_2^2}|_{w_2=w_{23},K=K_3} = \frac{16(-1+\eta)(2+\eta)^2-2(1+\eta)(4+5\eta)^2\theta+2(1+\eta)(4+5\eta)^2\theta^2}{(-4+\eta)^2(1-\eta)(1+\eta)^2(1-\theta)\theta} < 0$, $\frac{\partial^2 \pi^f(w_2,K)}{\partial K^2}|_{w_2=w_{23},K=K_3} = \frac{(4+5\eta)^2}{(-1+\eta^2)^2(-1+\theta)\theta} < 0$ and the Hessian matrix is $\begin{vmatrix} \frac{\partial^2 \pi^f(w_2,K)}{\partial w_2^2}|_{w_2=w_{23},K=K_3} & \frac{\partial^2 \pi^f(w_2,K)}{\partial w_2 \partial K}|_{w_2=w_{23},K=K_3} \\ \frac{\partial^2 \pi^f(w_2,K)}{\partial K \partial w_2}|_{w_2=w_{23},K=K_3} & \frac{\partial^2 \pi^f(w_2,K)}{\partial K^2}|_{w_2=w_{23},K=K_3} \end{vmatrix} = \frac{2(4+5\eta)^4}{(-4+\eta)^2(1-\eta^2)^3(1-\theta)\theta} > 0$. Then $\pi^f(w_2, K)$ is jointly concave in $w_2$ and $K$. So $(w_{23}, K_3, \mu_{23}, \mu_{33})$ is the optimal solution for FC strategy. Therefore, $w_2^f = w_{23} = \frac{4c+8\eta+5c\eta+\eta^2}{2(4+5\eta)} = \frac{c}{2} + \frac{\eta(8+\eta)}{8+10\eta}$ and $K^f = K_3 = \frac{(1-\eta)(-5\eta-4\eta^2+4\theta+9\eta\theta+5\eta^2\theta)}{(4+5\eta)^2}$. For $K^f = \frac{(1-\eta)(-5\eta-4\eta^2+4\theta+9\eta\theta+5\eta^2\theta)}{(4+5\eta)^2}$, $4 + 9\eta + 5\eta^2 > 0$ means that $(-5\eta - 4\eta^2 + 4\theta + 9\eta\theta + 5\eta^2\theta)$ is increases in $\theta$, and we can get a positive root $\theta^B = \frac{\eta(5+4\eta)}{(1+\eta)(4+5\eta)} < \frac{1}{2}$. Thus, when $\theta^B < \theta \le 1$, then $K^f > 0$ and there exists FC strategy. Therefore, $w_2^f = \frac{c}{2} + \frac{\eta(8+\eta)}{8+10\eta}$ and $K^f = \frac{(1-\eta)(-5\eta-4\eta^2+4\theta+9\eta\theta+5\eta^2\theta)}{(4+5\eta)^2}$.

Replace $w_2^f$ into $p_d(w_2)$ and $p_r(w_2)$, we obtain $p_d^f = \frac{c}{2} + \frac{\eta(2+7\eta)}{8+10\eta}$ and $p_r^f = \frac{c}{2} + \frac{4+6\eta-\eta^2}{8+10\eta}$. Recall that $q_d^f = \frac{p_r-p_d}{1-\eta} - \frac{p_d}{\eta}$ and $q_r^f = 1 - \frac{p_r-p_d}{1-\eta}$, so we obtain $q_d^f = \frac{\eta(2+\eta)-c(4+5\eta)}{2\eta(4+5\eta)}$ and $q_r^f = \frac{2+\eta}{4+5\eta}$.

**Proof of lemma 1**

From Table 2, we obtain $p_d^w - p_d^d = \frac{\eta(4-4\eta+(6+3\eta)\theta-3T_a)}{4(8+\eta)}$, $4 - 4\eta + (6+3\eta)\theta > 0$. $(4 - 4\eta + (6 + 3\eta)\theta)^2 - (3T_a)^2 = 8(1-\eta)(8+\eta)(-2+3\theta)$, so if $\theta < \frac{2}{3}$, then $(4 - 4\eta + (6 + 3\eta)\theta)^2 - (3T_a)^2 < 0$ and $p_d^w < p_d^d$; if $\theta > \frac{2}{3}$, then $(4 - 4\eta + (6 + 3\eta)\theta)^2 - (3T_a)^2 > 0$ and $p_d^w > p_d^d$. For WC strategy, the feasible region should satisfy $q_d^w > 0$, that is $0 < c \le c^A$ and $0 \le \theta \le 1$, or $c^A < c < c^B$ and $0 \le \theta < \theta^A$, where $c^A = \frac{2\eta+\eta^2}{8+\eta}$, $c^B = \frac{2\eta+\eta^2+2\eta\sqrt{1-\eta}}{8+\eta}$ and $\theta^A = \frac{-8c^2+4c\eta-c^2\eta+2c\eta^2-\eta^3}{2c\eta+c\eta^2-\eta^3}$. So $\theta^A - \frac{2}{3} = \frac{F_1(c)}{3\eta(2c+c\eta-\eta^2)}$, where $F_1(c) = c^2(-24-3\eta) - \eta^3 + c(8\eta + 4\eta^2)$, $-24 - 3\eta < 0$ means that $F_1(c)$ is a concave function of $c$, and the discriminant of $F_1(c)$ is $\Delta = 4(-4+\eta)^2\eta^2 > 0$, there are two roots for $F_1(c)$: $c^C = \frac{\eta}{3} > c^A$ and $c_1 = \frac{\eta^2}{8+\eta} < c^A$. Therefore, if $c^A < c < c^C$, then $\theta^A > \frac{2}{3}$; if $c^C < c < c^B$, then $\theta^A < \frac{2}{3}$.

In summary, if $0 < c \le c^C$ and $0 \le \theta < \frac{2}{3}$, or $c^C < c < c^B$ and $0 \le \theta < \theta^A$, then $p_d^w < p_d^d$; if $0 < c \le c^A$ and $\frac{2}{3} \le \theta \le 1$, or $c^A < c < c^C$ and $\frac{2}{3} \le \theta < \theta^A$, then $p_d^w \ge p_d^d$.

**Proof of Proposition 1**

From Table 2, (1), (2) and (3), for the online direct channel strategy, the feasible region should satisfy $p_d^d - c > 0$ and $q_d^d > 0$, that is $0 < \eta < 1$ and $0 < c < c^D$, where $c^D = \eta$. Similarly, for the introducing offline wholesale strategy, the feasible region should satisfy $p_d^w - c > 0$, $w_1^w - c > 0$, $p_r^w - w_1^w > 0$, $q_d^w > 0$, $q_r^w > 0$ and $\pi_m^w - \pi_m^d > 0$, that is $0 < c \le c^A$ and $0 \le \theta \le 1$, or $c^A < c < c^B$ and $0 \le \theta < \theta^A$, where $c^A = \frac{2\eta+\eta^2}{8+\eta}$, $c^B = \frac{2\eta+\eta^2+2\eta\sqrt{1-\eta}}{8+\eta}$ and $\theta^A = \frac{-8c^2+4c\eta-c^2\eta+2c\eta^2-\eta^3}{2c\eta+c\eta^2-\eta^3}$.

(1) Based on above conditions, if $c^A < c < c^B$ and $\theta^A \le \theta \le 1$, or $c^B \le c < c^D$ and $0 \le \theta \le 1$, there is only online direct channel.

(2) Based on above conditions, if $0 < c \le c^A$ and $0 \le \theta \le 1$, or $c^A < c < c^B$ and $0 \le \theta < \theta^A$, then $\pi_m^w > \pi_m^d$.

In summary, (1) if $0 < c \le c^A$ and $0 \le \theta \le 1$, or $c^A < c < c^B$ and $0 \le \theta < \theta^A$, then the WC is the better strategy for the manufacturer. (2) If $c^A < c < c^B$ and $\theta^A \le \theta \le 1$, or $c^B \le c < c^D$ and $0 \le \theta \le 1$, then the DC is the better strategy for the manufacturer.

**Proof of Corollary 1**

From Table 2, the likelihood that WC is the better strategy for the manufacturer can be presented as $P^w = \frac{c^A+\int_{c^A}^{c^B}\theta^A dc}{c^D} = \frac{1}{(2+\eta)^3(8+\eta)}[(2+\eta)(4+8\sqrt{1-\eta}-8\eta\sqrt{1-\eta}+14\eta+8\eta^2+\eta^3)+4\eta(-8+7\eta+\eta^2)ln(\frac{4\eta-4\eta^2}{8+\eta})-4\eta(-8+7\eta+\eta^2)ln(\frac{4\eta-4\eta^2+4\eta\sqrt{1-\eta}+2\eta^2\sqrt{1-\eta}}{8+\eta})]$, then we obtain $\frac{dP^w}{d\eta} = \frac{1}{(2+\eta)^4(8+\eta)^2(2+2\sqrt{1-\eta}-2\eta+\eta\sqrt{1-\eta})}(2\sqrt{1-\eta})[(2+\eta)(-184-184\sqrt{1-\eta}+800\eta-36\eta\sqrt{1-\eta}+114\eta^2+48\eta^2\sqrt{1-\eta}-2\eta^3+10\eta^3\sqrt{1-\eta}+\eta^4)-2(8+\eta)^2(2+2\sqrt{1-\eta}+\eta)(2-6\eta+\eta^2)ln\left(\frac{4\eta-4\eta^2}{8+\eta}\right)+2(8+\eta)^2(2+2\sqrt{1-\eta}+\eta)(2-6\eta+\eta^2)ln\left(\frac{4\eta+4\eta\sqrt{1-\eta}-4\eta^2+2\eta^2\sqrt{1-\eta}}{8+\eta}\right)] < 0$, so $P^w$ decreases in $\eta$. Thus, the likelihood that DC is better strategy is $P_1^d = 1 - P^w = \frac{1}{(2+\eta)^3(8+\eta)}[2(2+\eta)(14+11\eta+2\eta^2-4\sqrt{1-\eta}+4\eta\sqrt{1-\eta})-4\eta(-8+7\eta+\eta^2)ln(\frac{4\eta-4\eta^2}{8+\eta})+4\eta(-8+7\eta+\eta^2)ln(\frac{4\eta+4\eta\sqrt{1-\eta}-4\eta^2+2\eta^2\sqrt{1-\eta}}{8+\eta})]$, $\frac{dP_1^d}{d\eta} = \frac{2\sqrt{1-\eta}}{(2+\eta)^4(8+\eta)^2(2-2\eta+2\sqrt{1-\eta}+\eta\sqrt{1-\eta})}[(2+\eta)(184+184\sqrt{1-\eta}-800\eta+36\eta\sqrt{1-\eta}-114\eta^2-48\eta^2\sqrt{1-\eta}+2\eta^3-10\eta^3\sqrt{1-\eta}-\eta^4)+2(8+\eta)^2(2+2\sqrt{1-\eta}+\eta)(2-6\eta+\eta^2)ln(\frac{4\eta-4\eta^2}{8+\eta})-2(8+\eta)^2(2+2\sqrt{1-\eta}+\eta)(2-6\eta+\eta^2)ln(\frac{4\eta-4\eta^2+4\eta\sqrt{1-\eta}+2\eta^2\sqrt{1-\eta}}{8+\eta})] > 0$, then $P_1^d$ increases in $\eta$.

**Proof of Corollary 2**

From Table 2, we compare $W_s^w$ and $W_s^d$, $W_s^w = C_s^w + \pi_m^w(p_d) + \pi_r^w(p_r) = \frac{1}{16\eta(8+\eta)^2}\{H_1(\theta) + \eta[48 - 48c + 12\eta - 6c\eta - 6\eta^2 + (8 + 14\eta + 5\eta^2)\theta]T_a\}$, where $H_1(\theta) = -\eta(2+\eta)^2(4+5\eta)\theta^2 + (-64\eta + 96c\eta - 64\eta^2 + 60c\eta^2 - 40\eta^3 + 6c\eta^3 + 6\eta^4)\theta + 6c^2(8+\eta)^2 - 4c\eta(176 + 62\eta + 5\eta^2) + 2\eta(96 + 104\eta + 44\eta^2 - \eta^3)$, $W_s^d = C_s^d + \pi_m^d(p_d) = \frac{3(c-\eta)^2}{8\eta}$. Thus, $W_s^w - W_s^d = \frac{H_2(\theta)+[(8+14\eta+5\eta^2)\theta-6(-8+8c-2\eta+c\eta+\eta^2)]T_a}{16(8+\eta)^2}$, where $H_2(\theta) = -(2+\eta)^2(4+5\eta)\theta^2 + (-64 + 96c - 64\eta + 60c\eta - 40\eta^2 + 6c\eta^2 + 6\eta^3)\theta + 8(1-\eta)(24 + 2\eta + \eta^2 + 8c + \eta c)$, and $[(8 + 14\eta + 5\eta^2)\theta - 6(-8 + 8c - 2\eta + c\eta + \eta^2)]T_a > 0$. $-(2+\eta)^2(4+5\eta) < 0$ shows that $H_2(\theta)$ is a concave function of $\theta$ and the discriminant of $H_2(\theta)$: $\Delta = 32(1-\eta)(2+\eta)^2(4+5\eta)(24 + 2\eta + \eta^2 + 8c + \eta c) + [64 + 64\eta + 40\eta^2 - 6\eta^3 - 6c(16 + 10\eta + \eta^2)]^2 > 0$. There are two roots for $H_2(\theta)$: $\theta_1 = \frac{-32-32\eta-20\eta^2+3\eta^3+3c(16+10\eta+\eta^2)+T_b}{(2+\eta)^2(4+5\eta)} > 0$ and $\theta_2 = \frac{-32-32\eta-20\eta^2+3\eta^3+3c(16+10\eta+\eta^2)-T_b}{(2+\eta)^2(4+5\eta)} < 0$, where $T_b = (8+\eta)[512 + 704\eta - 32\eta^2 - 320\eta^3 - 104\eta^4 - 31\eta^5 + 9c^2(2+\eta)^2(8+\eta) - 2c(128 + 208\eta + 264\eta^2 + 118\eta^3 + 11\eta^4)]$. $\theta_1 - 1 = \frac{-48-68\eta-44\eta^2-2\eta^3+3c(16+10\eta+\eta^2)+T_b}{(2+\eta)^2(4+5\eta)}$, if $0 < \eta \le \eta_1 \approx 0.369$ and $0 < c < \eta$, or $\eta_1 < \eta \le \eta_2 \approx 0.6748$ and $c_2 < c < \eta$, then $\theta_1 > 1$ and $H_2(\theta) > 0$ and $W_s^w > W_s^d$, where $c_2 = \frac{112-276\eta-72\eta^2-7\eta^3}{-160-4\eta+2\eta^2}$; if $\eta_1 < \eta \le \eta_2$ and $0 < c < c_2$, or $\eta_2 < \eta < 1$ and $0 < c \le c^A$, then $\theta_1 < 1$. So if $\eta_1 < \eta \le \eta_2$ and $0 < c < c_2$ and $0 \le \theta < \theta_1$, or $\eta_2 < \eta < 1$ and $0 < c < c^D$ and $0 \le \theta < \theta_1$, then $H_2(\theta) > 0$ and $W_s^w > W_s^d$; if $\eta_1 < \eta \le \eta_2$ and $0 < c < c_2$ and $\theta_1 < \theta \le 1$, or $\eta_2 < \eta < 1$ and $0 < c < c^D$ and $\theta_1 < \theta \le 1$, then $H_2(\theta) < 0$, $\{[(8 + 14\eta + 5\eta^2)\theta - 6(-8 + 8c - 2\eta + c\eta + \eta^2)]T_a\}^2 - [H_2(\theta)]^2 > 0$, so $W_s^w > W_s^d$.

In summary, $W_s^w > W_s^d$.

**Proof of lemma 2**

From Table 2, we obtain $p_d^f - p_d^d = \frac{(-1+\eta)\eta}{4+5\eta} < 0$, so $p_d^f < p_d^d$.

**Proof of Proposition 2**

From Table 2, (1), (5) and (6), for the DC strategy, the feasible region should satisfy $p_d^d - c > 0$ and $q_d^d > 0$, that is $0 < \eta < 1$ and $0 < c < c^D$, where $c^D = \eta$. Similarly, for the FC strategy, the feasible region should satisfy $p_d^f - c > 0$, $w_2^f - c > 0$, $p_r^f - w_2^f > 0$, $q_d^f > 0$, $q_r^f > 0$, $\pi_m^f - \pi_m^d > 0$, $\pi_m^f > 0$ and $\pi_r^f > 0$, that is $0 < c < c^E$ and $\theta^B < \theta \le 1$,where $c^E = \frac{2\eta+\eta^2}{4+5\eta} < c^D$ and $\theta^B = \frac{5\eta+4\eta^2}{4+9\eta+5\eta^2}$.

(1) Based on above conditions, if $0 < c < c^E$ and $0 \le \theta \le \theta^B$, or $c^E \le c < c^D$ and $\theta^B \le \theta \le 1$,

there is only online direct channel.

(2) Based on above conditions, if $0 < c < c^E$ and $\theta^B < \theta \le 1$, then $\pi_m^f > \pi_m^d$.

Therefore, (1) if $0 < c < c^E$ and $\theta^B < \theta \le 1$, then the FC is the better strategy for the manufacturer. (2) If $0 < c < c^E$ and $0 \le \theta \le \theta^B$ or $c^E \le c < c^D$ and $0 \le \theta \le 1$, then the DC is the better strategy for the manufacturer.

**Proof of Corollary 3**

From Table 2, the likelihood that FC is the better strategy for the manufacturer is $P^f = \frac{(1-\theta^B)\times c^E}{c^E} = \frac{(2+\eta)^2}{(1+\eta)(4+5\eta)}$, $\frac{dP^f}{d\eta} = -\frac{(2+\eta)(10+11\eta)}{(1+\eta)^2(4+5\eta)^2} < 0$, then $P^f$ decreases in $\eta$. Thus, the likelihood that DC is better strategy is $P_2^d = 1 - P^f = \frac{8+44\eta+59\eta^2+24\eta^3}{(1+\eta)(4+5\eta)^2}$, $\frac{dP_2^d}{d\eta} = \frac{64+132\eta+84\eta^2+17\eta^3}{(1+\eta)^2(4+5\eta)^3} > 0$, then $P_2^d$ increases in $\eta$.

**Proof of Corollary 4**

From Table 2, we compare $W_s^f$ and $W_s^d$, $W_s^f = C_s^f + \pi_m^f(p_d) + \pi_r^f(p_r) = \frac{3\eta(2+\eta)^2+3c^2(4+5\eta)-2c\eta(14+13\eta)}{8\eta(4+5\eta)}$ and $W_s^d = C_s^d + \pi_m^d(p_d) = \frac{3(c-\eta)^2}{8\eta}$. Thus, $W_s^f - W_s^d = \frac{(1-\eta)(3-c+3\eta)}{8+10\eta} > 0$, then $W_s^f > W_s^d$.

**Proof of Proposition 3**

From proposition 1 and proposition 2, for the DC strategy, the feasible region should satisfy and $0 < c < c^D$; for the WC strategy, the feasible region should satisfy $0 < c \le c^A$ and $0 \le \theta \le 1$, or $c^A < c < c^B$ and $0 \le \theta < \theta^A$; for the FC model, the feasible region should satisfy $0 < c < c^E$ and $\theta^B < \theta \le 1$, where $c^A = \frac{2\eta+\eta^2}{8+\eta}$, $c^B = \frac{2\eta+\eta^2+2\eta\sqrt{1-\eta}}{8+\eta}$, $c^D = \eta$, $c^E = \frac{2\eta+\eta^2}{4+5\eta}$, $\theta^A = \frac{-8c^2+4c\eta-c^2\eta+2c\eta^2-\eta^3}{2c\eta+c\eta^2-\eta^3}$ and $\theta^B = \frac{5\eta+4\eta^2}{4+9\eta+5\eta^2} < \frac{1}{2}$.

$\theta^A - \theta^B = \frac{F_2(c)}{\eta(2c+c\eta-\eta^2)(4+9\eta+5\eta^2)}$, where $F_2(c) = c^2(-32-76\eta-49\eta^2-5\eta^3) + c(16\eta+34\eta^2+25\eta^3+6\eta^4) - 4\eta^3 - 4\eta^4 - \eta^5$.$(-32-76\eta-49\eta^2-5\eta^3) < 0$ means that $F_2(c)$ is a concave function, and $\Delta = \eta^2(16+18\eta-3\eta^2-4\eta^3)^2 > 0$ implies there are two roots for $F_2(c)$: $c^E = \frac{2\eta+\eta^2}{4+5\eta}$ and $c_3 = \frac{\eta^2(2+\eta)}{8+9\eta+\eta^2} < c^A$. Thus, if $c^A < c < c^E$, then $\theta^A > \theta^B$; if $c = c^E$, then $\theta^A = \theta^B$.

① Based on above conditions, if $0 < c < c^E$ and $0 \le \theta \le \theta^B$, or $c^E \le c < c^B$ and $0 \le \theta < \theta^A$, then there is only WC strategy, so $\pi_m^w > \{\pi_m^f, \pi_m^d\}$.

② Based on above conditions, if $c^E \le c < c^B$ and $\theta^A \le \theta \le 1$, or $c^B \le c < c^D$ and $0 \le \theta \le 1$, there is only DC strategy, so $\pi_m^d > \{\pi_m^f, \pi_m^w\}$.

③ Based on above conditions, if $c^A < c < c^E$ and $\theta^A \le \theta \le 1$, then there is only FC model, so $\pi_m^f > \{\pi_m^w, \pi_m^d\}$.

④ When $0 < c \le c^A$ and $\theta^B < \theta \le 1$, or $c^A < c < c^E$ and $\theta^B < \theta < \theta^A$, there exists FC strategy and WC strategy, we get $\pi_m^f - \pi_m^w = \frac{(2+\eta)\theta[16-8\eta-8\eta^2+8\theta+14\eta\theta+5\eta^2\theta-(4+5\eta)T_a]}{8(8+\eta)(4+5\eta)}$, for $16 - 8\eta - 8\eta^2 + 8\theta + 14\eta\theta + 5\eta^2\theta > 0$, so $(16 - 8\eta - 8\eta^2 + 8\theta + 14\eta\theta + 5\eta^2\theta)^2 - [(4+5\eta)T_a]^2 = 16(1-\eta)(8+\eta)H_3(\theta)$, where $H_3(\theta) = -5\eta - 4\eta^2 + (4 + 9\eta + 5\eta^2)\theta$. $(4 + 9\eta + 5\eta^2) > 0$ shows that $H_3(\theta)$ increases in $\theta$, there is one root for $H_3(\theta)$: $\theta^B = \frac{5\eta+4\eta^2}{4+9\eta+5\eta^2} < \frac{1}{2}$, so if $\theta^B < \theta \le 1$ or $\theta^B < \theta < \theta^A$, then $H_3(\theta) > 0$ and $\pi_m^f > \pi_m^w$.

In summary, (1) when $0 < c < c^E$ and $0 \le \theta \le 1$, or $c^E \le c < c^B$ and $0 \le \theta < \theta^A$, then the manufacturer should adopt an online-to-offline expansion strategy. Specifically, (i) if $0 < c < c^E$ and $\theta^B < \theta \le 1$, then the FC strategy is the optimal strategy for the manufacturer; (ii) if $0 < c < c^E$ and $0 \le \theta \le \theta^B$, or $c^E \le c < c^B$ and $0 \le \theta < \theta^A$, then the WC strategy is the optimal strategy for the manufacturer. (2) When $c^E \le c < c^B$ and $\theta^A \le \theta \le 1$, or $c^B \le c < c^D$ and $0 \le \theta \le 1$, then the manufacturer should solely operate an online channel and the DC strategy is the optimal strategy for the manufacturer.

**Proof of Corollary 5**

From Table 2, we compare $w_2^f$ and $w_1^w$, then $w_2^f - w_1^w = \frac{1}{4(8+\eta)(4+5\eta)}[H_4(\theta) + (16 + 16\eta - 5\eta^2)T_a]$, where $H_4(\theta) = -64 + 48\eta + 24\eta^2 - 8\eta^3 + (-32 - 48\eta - 6\eta^2 + 5\eta^3)\theta$, $(-32 - 48\eta - 6\eta^2 + 5\eta^3) < 0$ means that $H_4(\theta)$ decreases in $\theta$ and there is a root for $H_4(\theta)$: $\theta_3 = \frac{8(-1+\eta)}{4+5\eta} < 0$, so $H_4(\theta) < 0$. $[(16 + 16\eta - 5\eta^2)T_a]^2 - H_4^2(\theta) = 16(4-\eta)^2(1-\eta)(8+\eta)H_5(\theta)$, where $H_5(\theta) = 5\eta + 4\eta^2 + (-4 - 9\eta - 5\eta^2)\theta$. $(-4 - 9\eta - 5\eta^2) < 0$ implies $H_5(\theta)$ decreases in $\theta$ and there is one root for $H_5(\theta)$: $\theta^B = \frac{5\eta+4\eta^2}{4+9\eta+5\eta^2}$. If $0 \le \theta < \theta^B$, then $H_5(\theta) > 0$; if $\theta^B < \theta \le 1$, then $H_5(\theta) < 0$. Recall that the feasible region for FC strategy is $0 < c < c^E$ and $\theta^B < \theta \le 1$, so $w_2^f < w_1^w$.