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Poster presentation

Realistic activity propagation for mean field models of human cortex

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Inspired by the great density of neurons in cortex (about 10^6 per macrocolumn), continuum mean field models (CMFMs) treat the cortex as one continuous neural medium. Interactions between neurons thus become flows of activity spreading in this medium. The most popular propagation model [1] is derived by a two-fold ansatz: a pulse of activity will on one hand spread isotropically with a conduction velocity c, and on the other hand its amplitude will decay exponentially with a distance scale σ. This ansatz reflects action potentials traveling with constant speed through axons and that the number of synapses axons form falls roughly exponentially with distance. However, a pulse in a CMFM means a mass of 10^5 to 10^6 neurons pulsing *together*. It is well known that axons can vary greatly in conduction speed, depending on their myelination and diameter. Hence it is natural to expect that such a mass has a broad conduction velocity distribution $f(v)$, rather than a singular one $f(v) = \delta(v\text{-}c)$. Furthermore, one still has to expand for large wavelengths in order to derive a manageable partial differential equation (PDE) – a damped wave equation as it turns out. If one calculates the velocity distribution of this approximation, one sees that the Dirac delta peak has softened only towards lower velocities, leaving an unnatural distribution with a hard velocity cut-off. We hence propose a new propagation PDE:

$$\left[\frac{\partial}{\partial t} + \frac{c}{2\sigma}\left(1 - \sigma^2 \bar{\nabla}^2 \right) \right]^n \Phi(\bar{x}, t) = \frac{N^\alpha c^n}{2^n \sigma^n} S(\bar{x}, t),$$

where Φ is the activity being propagated, S is a local source (e.g., a firing rate function), N^α the total number of connections, c is the conduction velocity and σ the decay parameter, and $n > 0$ will usually be chosen as an integer.

This new PDE has the following advantages:

For $n > 1$ the connectivity implied by the PDE remains finite for small distances.

The PDE has a smooth velocity distribution, the shape of which depends only on n and c.

The velocity distribution for $n = 3$ and $c = 14.35$ m/s fits human data [2] well. This value of c implies a mean conduction velocity of 7.87 m/s. The distribution for the damped wave equation fits this data very badly; see Figure 1.

There is no switch between finite wave number Hopf-Turing and Hopf bifurcations as is the case for the damped wave equation. Self-sustained oscillations can emerge as spatial patterns with arbitrarily small wavenumber.
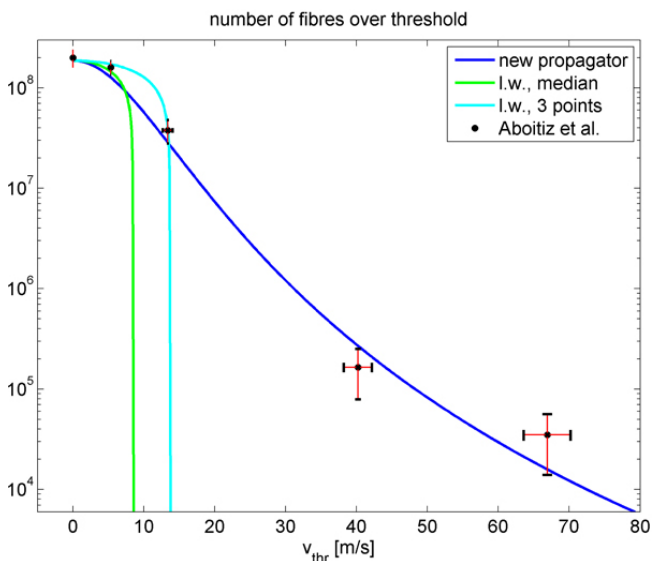
Figure 1
Number of fibres faster than a velocity threshold.
Aboitiz et al. data [2] compared to a fit with the new PDE
(blue), and with the damped wave equation matching the new
PDE's median speed (green) or fit to the first three data
points (cyan, a fit to all points is here not feasible due to
severe functional mismatch).

We have investigated the bifurcations of this propagator
both analytically and numerically on large grids. We have
also compared to more detailed data from the rat [3], but
find that they do not scale to human in a simple manner.
Our new propagation PDE provides now for CMFMs the
best match to activity conduction in humans, and can be
easily adjusted as more human data become available
later.

References
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3. Partadiredja G, *et al.*: **The number, size, and type of axons in rat subcortical white matter on left and right sides: A stereological, ultrastructural study.** *J Neurocytol* 2003, **32**:1165-1179.