

# Optimisation of hybrid institutional incentives for cooperation in finite populations

Duong, Manh Hong; Durbac, Calina; Han, The Anh

DOI:

[10.1162/isal\\_a\\_00568](https://doi.org/10.1162/isal_a_00568)

License:

Creative Commons: Attribution (CC BY)

*Document Version*

Publisher's PDF, also known as Version of record

*Citation for published version (Harvard):*

Duong, MH, Durbac, C & Han, TA 2023, Optimisation of hybrid institutional incentives for cooperation in finite populations. in *ALIFE 2023: Ghost in the Machine: Proceedings of the 2023 Artificial Life Conference* ., 118, MIT Press, ALIFE 2023: Ghost in the Machine, Sapporo, Japan, 24/07/23. [https://doi.org/10.1162/isal\\_a\\_00568](https://doi.org/10.1162/isal_a_00568)

[Link to publication on Research at Birmingham portal](#)

## General rights

Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

- Users may freely distribute the URL that is used to identify this publication.
- Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
- User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
- Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

## Take down policy

While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact [UBIRA@lists.bham.ac.uk](mailto:UBIRA@lists.bham.ac.uk) providing details and we will remove access to the work immediately and investigate.

# Optimisation of hybrid institutional incentives for cooperation in finite populations

M. H. Duong<sup>1</sup>, C. M. Durbac<sup>1</sup>, and T. A. Han<sup>2</sup>

<sup>1</sup> School of Mathematics, University of Birmingham, B15 2TT, UK. Email: h.duong@bham.ac.uk, c.m.durbac@bham.ac.uk

<sup>2</sup> School of Computing, Engineering and Digital Technologies, Teeside University, TS1 3BX, UK. Email: T.Han@tees.ac.uk

**Introduction.** Evolutionary Game Theory has been widely used to study myriad questions in diverse disciplines like Evolutionary Biology, Ecology, Physics, Sociology and Computer Science, including the mechanisms underlying the emergence and stability of cooperation (Nowak, 2006; Perc et al., 2017; Han, 2022) and how to mitigate climate and Artificial Intelligence risks (Santos and Pacheco, 2011; Góis et al., 2019; Sun et al., 2021; Han et al., 2020). Institutional incentives, either positive (reward) and negative (punishment), are among the most important mechanisms for promoting the evolution of prosocial behaviours (Sigmund et al., 2001; Van Lange et al., 2014). In institutional incentives, an external decision-maker, such as the United Nation or NATO, has a budget to interfere in the population in order to achieve a desirable outcome, for example to ensure a desired level of cooperation. The use of institutional incentives for promoting cooperation is costly so it is important to optimise the cost while, at the same time, maintaining a level of cooperation (Ostrom, 2005; Cimpéanu et al., 2021; Wang et al., 2023; Powers et al., 2016). Several theoretical models studied how to combine institutional reward and punishment for enhancing the emergence and stability of cooperation (Chen and Perc, 2014; Góis et al., 2019; Berenji et al., 2014; Hilbe and Sigmund, 2010; Sun et al., 2021). However, little attention has been given to addressing the cost optimisation of providing incentives. Sasaki et al. (Sasaki et al., 2015) looked at a rewarding policy that switches the incentive from reward to punishment when the frequency of cooperators exceeds a certain threshold. Wang et al. (Wang et al., 2019, 2023) explored the optimal incentive that not only minimises the total cost, but also guarantees a sufficient level of cooperation in both well-mixed and structured populations via optimal control theory. This work however does not take into account various stochastic effects of evolutionary dynamics such as mutation and non-deterministic behavioural update.

In this extended abstract, starting from a finite population framework in (Han and Tran-Thanh, 2018), we summarise a recent work in (Duong et al., 2023) that provides a rigorous analysis, supported by numerical simulations, for the

problem of optimising the cost of *hybrid* incentives while sustaining a desired level of cooperation. Most relevant to this work is Duong and Han (2021) which derived analytical conditions for which a general incentive scheme can guarantee a given level of cooperation while at the same time minimising the total cost of investment. These results are highly sensitive to the intensity of selection. This work however did not study the cost-efficiency of the mixed incentive scheme, which is the focus of the present work.

**Models and Methods.** We consider a well-mixed, finite population of  $N$  self-regarding individuals or players, who interact with each other using one of the following cooperation dilemmas, the Donation Game (DG) and the Public Goods Game (PGG). We adopt here the finite population dynamics with the Fermi strategy update rule (Traulsen and Nowak, 2006), stating that a player  $X$  with fitness  $f_X$  adopts the strategy of another player  $Y$  with fitness  $f_Y$  with a probability given by,  $P_{X,Y} = (1 + e^{-\beta(f_Y - f_X)})^{-1}$ , where  $\beta$  represents the intensity of selection. To reward a cooperator (respectively, punish a defector), the institution has to pay an amount  $\theta/a$  (resp.,  $\theta/b$ ) so that the cooperator's (defector's) payoff increases (decreases) by  $\theta$ , where  $a, b > 0$  are constants representing the efficiency ratios of providing the corresponding incentive. The population dynamics are modelled using an absorbing Markov chain consisting of  $(N + 1)$  states,  $\{S_0, \dots, S_N\}$ , where  $S_i$  represents a population with  $i$  C players.  $S_0$  and  $S_N$  are absorbing states. Let  $U = \{u_{ij}\}_{i,j=1}^{N-1}$  denote the transition matrix between the  $N-1$  transient states,  $\{S_1, \dots, S_{N-1}\}$ . The entries  $n_{ij}$  of the so-called fundamental matrix  $\mathcal{N} = (n_{ij})_{i,j=1}^{N-1} = (I - U)^{-1}$  of the absorbing Markov chain gives the expected number of times the population is in the state  $S_j$  if it is started in the transient state  $S_i$  (Kemeny, 1976). As a mutant can randomly occur either at  $S_0$  or  $S_N$ , the expected number of visits at state  $S_i$  is:  $\frac{1}{2}(n_{1i} + n_{N-1,i})$ . Assuming that we desire to obtain at least an  $\omega \in [0, 1]$  fraction of cooperation, then  $\theta$  needs to satisfy the following lower bound (Han and Tran-Thanh, 2018):

$$\theta \geq \theta_0 = \frac{1}{(N-1)\beta} \log \left( \frac{\omega}{1-\omega} \right) - \delta.$$

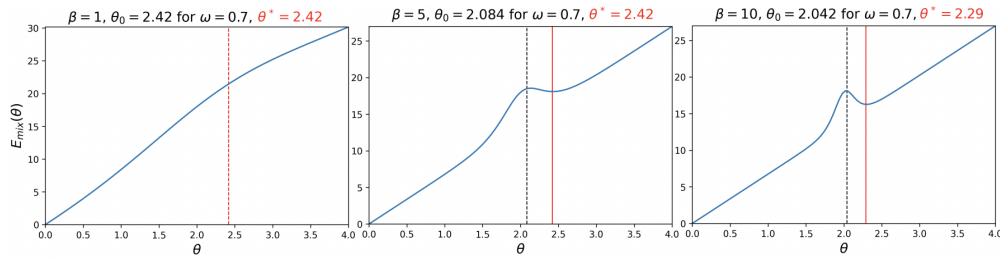


Figure 1: Finding optimal  $\theta$ , denoted by  $\theta^*$  (represented by the red line in the figures), that minimises  $E_{mix}$  while ensuring a minimum level of cooperation  $\omega$ , where  $N = 3$  for DG with  $B = 2, c = 1$ . The critical threshold value for the strength of selection is  $\beta^* = 3.67$ . The numerical results obtained are in accordance with our theoretical results: for  $\beta < \beta^*$ , the cost function increases, while for  $\beta > \beta^*$  it is not monotonic.

**Optimisation problem.** The expected total cost of interference for hybrid institutional incentives is

$$E_{mix}(\theta) = \frac{\theta}{2} \sum_{j=1}^{N-1} (n_{1j} + n_{N-1,j}) \min\left(\frac{j}{a}, \frac{N-j}{b}\right).$$

In summary, we obtain the following cost-optimisation problem of institutional incentives in stochastic finite populations:  $\min_{\theta \geq \theta_0} E_{mix}(\theta)$ .

**Main results.** The main results of (Duong et al., 2023) can be summarised as follows.

1. It is always more cost efficient to use the mixed incentive approach than a separate incentive, reward or punishment,

$$E_{mix} \leq \min\{E_r, E_p\}.$$

If  $\frac{b}{a} \leq \frac{1}{N-1}$ , then  $E_{mix}(\theta) = E_r(\theta)$ . If  $\frac{b}{a} \geq N-1$ , then  $E_{mix}(\theta) = E_p(\theta)$ .

2. (Infinite population limits)

$$\lim_{N \rightarrow +\infty} \frac{E_{mix}(\theta)}{\frac{N^2\theta}{2} H_{a,b}} = \begin{cases} 1 + e^{-\beta|\theta-c|} & \text{for DG,} \\ 1 + e^{-\beta|\theta-c(1-\frac{c}{a})|} & \text{for PGG.} \end{cases}$$

3. (Weak selection limit):  $\lim_{\beta \rightarrow 0} E_{mix}(\theta) = \theta N^2 H_{N,a,b}$ .

4. (Strong selection limits)

$$\lim_{\beta \rightarrow +\infty} E_{mix}(\theta) = \begin{cases} \frac{N^2\theta}{2} \left( H_{N,a,b} + \frac{1}{a(N-1)} \right), & \text{for } \theta < -\delta, \\ \frac{NA}{2} \left[ 2NH_{N,a,b} + \frac{1}{a(N-1)} + \frac{1}{b(N-1)} \right] \\ - \frac{\min(2/a, (N-2)/b)}{2(N-2)} - \frac{\min((N-1)/a, 1/b)}{N-1}, & \\ \text{for } \theta = -\delta, \\ \frac{N^2\theta}{2} \left[ H_{N,a,b} + \frac{1}{b(N-1)} \right] & \text{for } \theta > -\delta. \end{cases}$$

5. (Growth of the cost function) The cost function satisfies

$$\begin{aligned} \frac{N^2\theta}{2} \left( H_{N,a,b} + \frac{1}{\max(a,b)(N-1)} \right) &\leq E_{mix}(\theta) \\ &\leq N(N-1)\theta \left( H_{N,a,b} + \frac{1}{\min(a,b) \lfloor \frac{N-1}{2} \rfloor} \right). \end{aligned}$$

6. (Phase transition phenomena and behaviour under the threshold) There exists a threshold value  $\beta^*$  such that  $\theta \mapsto E_{mix}(\theta)$  is non-decreasing for all  $\beta \leq \beta^*$  and it is non-monotonic when  $\beta > \beta^*$ . As a consequence, for  $\beta \leq \beta^*$

$$\min_{\theta \geq \theta_0} E_{mix}(\theta) = E_{mix}(\theta_0).$$

7. (Behaviour above the threshold value) For  $\beta > \beta^*$ , the number of changes of sign of  $E'_{mix}(\theta)$  is at least two for all  $N$  and there exists an  $N_0$  such that the number of changes is exactly 2 for  $N \leq N_0$ . As a consequence, for  $N \leq N_0$ , there exist  $\theta_1 < \theta_2$  such that for  $\beta > \beta^*$ ,  $E_{mix}(\theta)$  is increasing when  $\theta < \theta_1$ , decreasing when  $\theta_1 < \theta < \theta_2$ , and increasing when  $\theta > \theta_2$ . Thus, for  $N \leq N_0$ ,

$$\min_{\theta \geq \theta_0} E_{mix}(\theta) = \min\{E_{mix}(\theta_0), E_{mix}(\theta_2)\}.$$

Figure 1 demonstrates the behaviour of the cost function for providing incentives in different regimes of intensities of selection  $\beta$ , the phase transitions that occur when  $\beta$  is sufficiently large, as well as the accuracy of the theoretical prediction of the optimal incentive cost  $\theta^*$ .

**Summary and Outlook.** We have summarised a recent theoretical analysis of the problem of optimising the cost of hybrid institutional incentives while guaranteeing a minimum amount of cooperation, in stochastic finite populations. In this context, institutional approaches have been widely adopted to study biological and artificial life systems (Andras et al., 2018; Jones et al., 2013; Smaldino and Lubell, 2014; Andras, 2020; Powers et al., 2016). This analysis provides new, fundamental insights into a cost-efficient design of institution-based solutions for promoting pro-social behaviours in social and artificial systems.

## References

- Andras, P. (2020). Composition of games as a model for the evolution of social institutions. In *Artificial Life Conference Proceedings*, pages 171–179. MIT Press.
- Andras, P., Esterle, L., Guckert, M., Han, T. A., Lewis, P. R., Milanovic, K., Payne, T., Perret, C., Pitt, J., Powers, S. T., et al. (2018). Trusting intelligent machines: Deepening trust within socio-technical systems. *IEEE Technology and Society Magazine*, 37(4):76–83.
- Berenji, B., Chou, T., and D’Orsogna, M. R. (2014). Recidivism and rehabilitation of criminal offenders: A carrot and stick evolutionary game. *PLoS One*, 9(1):e85531.
- Chen, X. and Perc, M. (2014). Optimal distribution of incentives for public cooperation in heterogeneous interaction environments. *Frontiers in behavioral neuroscience*, 8:248.
- Cimpeanu, T., Perret, C., and Han, T. A. (2021). Cost-efficient interventions for promoting fairness in the ultimatum game. *Knowledge-Based Systems*, 233:107545.
- Duong, M. H., Durbac, C. M., and Han, T. A. (2023). Cost optimization of hybrid institutional incentives for promoting cooperation in finite populations. *Pre-print: arXiv:2212.08823*.
- Duong, M. H. and Han, T. A. (2021). Cost efficiency of institutional incentives for promoting cooperation in finite populations. *Proceedings of the Royal Society A*, 477(2254):20210568.
- Góis, A. R., Santos, F. P., Pacheco, J. M., and Santos, F. C. (2019). Reward and punishment in climate change dilemmas. *Sci. Rep.*, 9(1):1–9.
- Han, T. A. (2022). Emergent behaviours in multi-agent systems with evolutionary game theory. *AI Communications*, 35(4):327 – 337.
- Han, T. A., Pereira, L. M., Santos, F. C., and Lenaerts, T. (2020). To Regulate or Not: A Social Dynamics Analysis of an Idealised AI Race. *Journal of Artificial Intelligence Research*, 69:881–921.
- Han, T. A. and Tran-Thanh, L. (2018). Cost-effective external interference for promoting the evolution of cooperation. *Scientific reports*, 8(1):1–9.
- Hilbe, C. and Sigmund, K. (2010). Incentives and opportunism: from the carrot to the stick. *Proceedings of the Royal Society B: Biological Sciences*, 277(1693):2427–2433.
- Jones, A. J., Artikis, A., and Pitt, J. (2013). The design of intelligent socio-technical systems. *Artificial Intelligence Review*, 39(1):5–20.
- Kemeny, J. (1976). Perspectives on the micro-macro distinction. *The Sociological Review*, 24(4):731–752.
- Nowak, M. A. (2006). Five rules for the evolution of cooperation. *Science*, 314(5805):1560–1563.
- Ostrom, E. (2005). *Understanding institutional diversity*. Princeton university press.
- Perc, M., Jordan, J. J., Rand, D. G., Wang, Z., Boccaletti, S., and Szolnoki, A. (2017). Statistical physics of human cooperation. *Physics Reports*, 687:1–51.
- Powers, S. T., Van Schaik, C. P., and Lehmann, L. (2016). How institutions shaped the last major evolutionary transition to large-scale human societies. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 371(1687):20150098.
- Santos, F. and Pacheco, J. M. (2011). Risk of collective failure provides an escape from the tragedy of the commons. *Proceedings of the National Academy of Sciences*, 108(26):10421–10425.
- Sasaki, T., Chen, X., Brännström, Å., and Dieckmann, U. (2015). First carrot, then stick: How the adaptive hybridization of incentives promotes cooperation. *Journal of the Royal Society Interface*, 12:20140935.
- Sigmund, K., Hauert, C., and Nowak, M. A. (2001). Reward and punishment. *Proceedings of the National Academy of Sciences*, 98(19):10757–10762.
- Smaldino, P. E. and Lubell, M. (2014). Institutions and cooperation in an ecology of games. *Artificial life*, 20(2):207–221.
- Sun, W., Liu, L., Chen, X., Szolnoki, A., and Vasconcelos, V. V. (2021). Combination of institutional incentives for cooperative governance of risky commons. *Iscience*, 24(8):102844.
- Traulsen, A. and Nowak, M. A. (2006). Evolution of cooperation by multilevel selection. *Proceedings of the National Academy of Sciences*, 103(29):10952–10955.
- Van Lange, P. A., Rockenbach, B., and Yamagishi, T. (2014). *Reward and punishment in social dilemmas*. Oxford University Press.
- Wang, S., Chen, X., and Szolnoki, A. (2019). Exploring optimal institutional incentives for public cooperation. *Communications in Nonlinear Science and Numerical Simulation*, 79:104914.
- Wang, S., Chen, X., Xiao, Z., Szolnoki, A., and Vasconcelos, V. V. (2023). Optimization of institutional incentives for cooperation in structured populations. *Journal of The Royal Society Interface*, 20(199):20220653.