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Abstract: The split photodiode and the lateral effect photodiode are two popular detectors for measuring beam displacement. For small displacements of a Gaussian beam, which is the case of interest here, they are often seen as equivalent and used interchangeably, giving a signal proportional to the displacement. We show theoretically and experimentally that in the limit of low technical noise, where the signal to noise ratio is dominated by the shot noise of the light, the lateral effect photodiode produces a better signal to noise ratio than the split photodiode, owing to its optimum spatial detector response. This quantum advantage can be practically exploited in spite of the intrinsic thermal noise of the lateral effect photodiode.

1. Introduction

Laser beam position and displacement are useful diagnostics and measurement tools. For example in a goniometer the deflection of a laser beam, which translates into a displacement in the far field, can accurately measure the tilt of a surface on which the beam reflects. Such an arrangement is found in many designs of atomic force microscopy (AFM), where the motion of a fast oscillating cantilever is tracked while it interacts with the surface being imaged [1]. In this context, the beam deflection method has been shown to be as sensitive as interferometric methods [2].

Like any optical measurement performed with a coherent state, in practice a laser beam free from technical noise, the precision of a beam displacement measurement is ultimately limited by the shot noise of the light. Reaching the shot noise puts a stringent constraint on the level of admissible technical noise; the predominance of the shot noise nonetheless occurs naturally in applications that rely on short measurement time and/or fast beam displacement. For instance this is the case in high-performance AFM, where the cantilever oscillation frequency is high enough to be outside the low-frequency technical noise peak [3,4]. Therefore optimizing beam position measurement at the quantum limit has become of practical importance.

There are sophisticated options to improve the signal to noise ratio (SNR) limited by shot noise ($\mathrm{SNR}_{\mathrm{sn}}$) of a beam position measurement, such as reducing the quantum fluctuations of light. These have been theorized [5,6] and experimentally demonstrated [7,8] but they remain technologically challenging. Before even considering quantum engineering of light, it is worth looking at the measurement process itself. In particular, it has been shown that a very popular beam position measuring device, the split photodiode (SPD), does not perform the optimum quantum measurement [5,9] for Gaussian beams, while a properly configured homodyne detector

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can, outperforming the SPD by a factor $\pi/2$ in the SNR [9]. Unfortunately the homodyne detector is a complex optical device which requires interferometric stability. It would therefore be beneficial to reach the optimum sensitivity with direct photodetection. While it has been theoretically shown that this can be achieved for the most generic beam shape using a photodiode array of suitable spatial resolution coupled to tailored gains at the single pixel level [10,11], this remains resource intensive. In this paper, we show that the lateral effect photodiode (LEPD), another popular beam position sensing photodetector, outperforms the SPD by the same amount as the homodyne detector. We first review the quantum theory of beam position sensing for both the SPD and the LEPD. We then check experimentally that the LEPD exhibits an improved quantum-limited sensitivity when compared to the SPD. Finally we evaluate whether the quantum advantage of the LEPD persists when the technical limitations of the detector are factored in.

2. Measurement of laser beam displacement – theory

2.1. Optimum measurement

Let us consider a Gaussian beam in a coherent state traveling along the z direction, displaced by a distance θ along the x direction that is small compared to the beam radius w. At the point of detection, the transverse profile of the electric field of the undisplaced beam is

$$E(x,y) = E_0 u_0(x,y)$$

and the displaced profile is

$$E_\theta(x,y) = E(x-\theta,y) \tag{1}$$

$$\simeq E(x,y) - \theta\frac{\partial E}{\partial x} \tag{2}$$

$$\simeq E_0\left[u_0(x,y) + \frac{\theta}{w}u_1(x,y)\right], \tag{3}$$

where

$$u_0(x,y) = \sqrt{\frac{2}{\pi}}\frac{1}{w}\exp\left(-\frac{x^2+y^2}{w^2}\right) \tag{4}$$

and

$$u_1(x,y) = \sqrt{\frac{2}{\pi}}\frac{2x}{w^2}\exp\left(-\frac{x^2+y^2}{w^2}\right) \tag{5}$$

correspond to the first and second Hermite-Gauss modes along the x direction. One can see from Eq. (3) that the information about the small displacement is exclusively contained in the amount of light present in the u_1 mode. Since u_0 and u_1 are orthogonal, measuring the projection of the displaced beam on the u_1 mode, for instance with a homodyne detector featuring a matched u_1 local oscillator, yields the best quantum estimate of the displacement. That is to say it saturates the quantum Cramér-Rao bound for small-displacement measurements [12].

Another way to perform this optimum measurement of the displacement is to use an intensity detector with a tailored detector response $g(x)$, which is defined such that an incident beam will generate the amplified photocurrent

$$I = \iint g(x)\,n(x,y)\,dxdy, \tag{6}$$

where $n(x,y)$ is the local flux of received photons. In the case of a Gaussian beam displaced by $\theta \ll w$ from the origin, Eq. (3) gives a photon flux proportional to $E_\theta^2(x,y) \simeq E_0^2(u_0^2 + 2\frac{\theta}{w}u_0u_1)$

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and the change in photocurrent becomes, to first order in θ/w,

$$S = \delta I \simeq \frac{2N\theta}{w} \iint g(x)\, u_0(x,y)\, u_1(x,y)\, dxdy, \tag{7}$$

where N is the total number of detected photons. Noticing that $u_1(x,y) = \frac{2x}{w} u_0(x,y)$, one sees that choosing $g(x) \propto x$, in essence measuring the position of the "center of mass" of the beam power, is equivalent to homodyning the u_1 component of the displaced beam. This realizes the optimum measurement, with a corresponding $\mathrm{SNR_{sn}}$ that has been shown to be [12]

$$\mathrm{SNR_{opt}} \equiv \frac{S^2}{\langle \Delta S^2 \rangle} = \frac{4N\theta^2}{w^2}, \tag{8}$$

where $\langle \Delta S^2 \rangle$ denotes the signal variance. For unity quantum efficiency, each photon converts into an elementary charge e, leading to a photocurrent i_0 such that during the observation time τ, the detected photon number is $i_0\tau/e = N$. According to Nyquist theorem, $\tau = 1/2B$, and the SNR when measuring i_0 within a bandwidth B becomes

$$\mathrm{SNR_{opt}} = \frac{2 i_0 \theta^2}{w^2 e B}. \tag{9}$$

For an imperfect quantum efficiency $\eta<1$, $\mathrm{SNR_{opt}}$ is simply scaled down by η.

2.2. Types of intensity detectors

SPDs and LEPDs are commonly used devices to measure laser beam position or motion. Both types of detectors work by detecting and integrating the optical intensity impinging on the detector, but with different detector responses. In an SPD, the detecting area is split into two or four adjacent cells that produce independent photocurrents and have identical responses. In particular, for a bi-cell SPD with a straight boundary between regions at $x = 0$, subtracting the photocurrents produces a detector response $g(x)$ proportional to sgn x provided the beam is fully contained on the detector area and is centered on the boundary line. This response does not perform the best quantum measurement and achieves an SNR of

$$\mathrm{SNR_S} = \frac{2}{\pi}\frac{4N\theta^2}{w^2} = \frac{2}{\pi}\,\mathrm{SNR_{opt}} \tag{10}$$

for a Gaussian beam and a small displacement [6,7].

The LEPD is a monolithic photodiode with a resistive layer on at least one side of the doped layers. Photo-induced charges have to travel through the resistive layer before reaching one of a set of collection electrodes. The electrical resistance between the point of creation of charge and an electrode is, to first order, proportional to the distance between the two. In a one-dimensional LEPD, of length L along the x direction, the electrodes are affixed to each end of the resistive film, at positions $x = \pm L/2$. The linear resistance of the film is homogeneous by design: $\rho = R_{\mathrm{ie}}/L$, where R_{ie} is the inter-electrode resistance of the film. For a ray of light hitting the surface at position (x, y) and generating a photocurrent i, the difference in the photocurrents i_1 and i_2 collected by the electrodes into low-impedance circuits is

$$\Delta i = i_2 - i_1 \tag{11}$$

$$= \left(\frac{1}{2} + \frac{x}{L}\right) i - \left(\frac{1}{2} - \frac{x}{L}\right) i \tag{12}$$

$$= \frac{2x}{L} i. \tag{13}$$

The photocurrent difference is therefore proportional to the distance x from the center of the detector. For a large beam, the response is the sum of the responses over the transverse profile of

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the beam, similar to that of Eq. (6), with a detector response $g(x) \propto x$ according to Eq. (13). As a result, the SNR for the LEPD is expected to be the largest possible one for a Gaussian beam: $\text{SNR}_\text{L} = \text{SNR}_\text{opt}$ for small displacements. To understand why $\text{SNR}_\text{L} > \text{SNR}_\text{S}$, it is important to realize that the splitting of the photocurrent between the electrodes leads to a level of common-mode rejection of the shot noise for those areas of the beam that are located toward the center of the detector. Indeed current splitting in ohmic conductors is not subject to Poisson statistics [13].

Remarkably, the signal to noise ratios derived above do not depend on the absolute displacement θ but rather on the relative displacement θ/w. This is because as long as the beam is small enough not to be clipped by the detecting surface, neither the SPD nor the LEPD has a characteristic length scale associated with its design. This means that there is no benefit with regard to the SNR_sn in (de)magnifying the beam with an optical system or even with free propagation before detection, since this does not change the relative displacement. In particular, when measuring the tilt of a surface by reflecting a beam of given size upon it, the displacement θ/w in the far field of the surface is independent from how the far field is imaged onto the detector. As a result, the choice of the beam size at the detector level can be guided solely by technical considerations.

In what follows, we are confirming experimentally that regardless of their technical limitations, the SPD and the LEPD have different quantum limits, with the LEPD having an SNR_sn that is fundamentally $\pi/2 \simeq 1.57$ times larger than that of the SPD for small displacements.

3. Experimental set-up

The goal of the experiment is to compare the SNRs of the SPD and the LEPD when measuring the oscillating small displacement of a laser beam under identical experimental conditions.

The set-up, shown in Fig. 1, consists of a laser system delivering a low-intensity-noise Gaussian beam spatially filtered with an optical fiber, a periodic deflector, a position detector recording the deflection of the beam, and a spectrum analyzer. Oscillatory deflection allows the system to operate away from the DC technical noise. Deflection is achieved with an acousto-optic modulator (AOM) driven by a frequency-modulated radio-frequency, so that the deflection angle of the first diffracted order oscillates at the modulation frequency. The SPD and LEPD are one-dimensional photodetectors with nearly identical photosensitivities and dimensions. They are placed at exactly the same position along the beam in the far field of the deflector, and are used in turn to measure the oscillation of the beam.

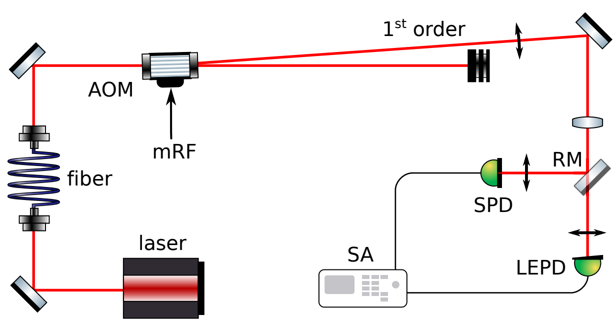

Fig. 1. Simplified experimental set-up. The frequency of the AOM is modulated so that the angle of diffraction of the first order is modulated, resulting in the beam direction or position oscillating as shown by the arrows. The removable mirror RM allows for the beam to be sent to either of the position sensing detectors. The imaging system places the far field of the AOM onto the detectors, which are carefully placed so that they intercept the beam at the same longitudinal position. SA: spectrum analyzer; mRF: modulated radio frequency.

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The detailed description of the set-up can be found in the Supplementary section 1. It makes explicit the steps that were taken to ensure that the shot-noise-limited SNRs of the detectors can be compared. To this effect, the following conditions must be fulfilled. Firstly, since we are interested in the shot-noise-limited SNR, both the technical noise on the beam position and the electronic noise on the detectors must ideally be negligible compared to the shot noise of the light. In our case the electronic noise is lower than the shot noise but not negligible, and is therefore subtracted from the measurements. Secondly, all the parameters that appear in the expressions for the SNRs in Eqs. (8) and (10) must be equal for both detectors. In particular the laser power, which translates into the total number of detected photons N, must be kept constant throughout the experiment. The laser beam is subject to slow power drifts of 10% maximum, which is much less than the difference of SNR between the detectors. We nonetheless measure and correct for these small fluctuations in order to obtain a more accurate ratio of the SNRs. Equally, the two photodiodes should have the same quantum efficiencies and therefore the same photosensitivities. In our case these differ by 5%. Again, this is much smaller than the effect we want to evidence but we nonetheless correct for this difference. Finally, the beam position must be modulated without spurious effects such as further deformation of the beam.

Our SPD and LEPD have a common cathode arrangement, which means that the photocurrents of both anodes must be transamplified before being subtracted. Supplementary section 1 gives the details and limitations of the approach. The photodiodes need to be reversed-biased to reduce their capacitance and increase their speed. As will be seen later, this is a potential issue for the LEPD.

4. Experimental results

In the following, we use an optical power of 150 μW and a displacement frequency of 200 kHz, for which the shot noise is 12 dB and 4.2 dB above the electronic noise floor for the SPD and the LEPD, respectively. Figure 2 shows examples of noise spectra obtained when oscillating the beam position on the detectors. The SNR of the beam displacement amplitude is obtained by subtracting the electronic noise from the spectrum, then evaluating the noise and the signal separately. The noise is the average of the side noise around the modulation peak, i.e. over a

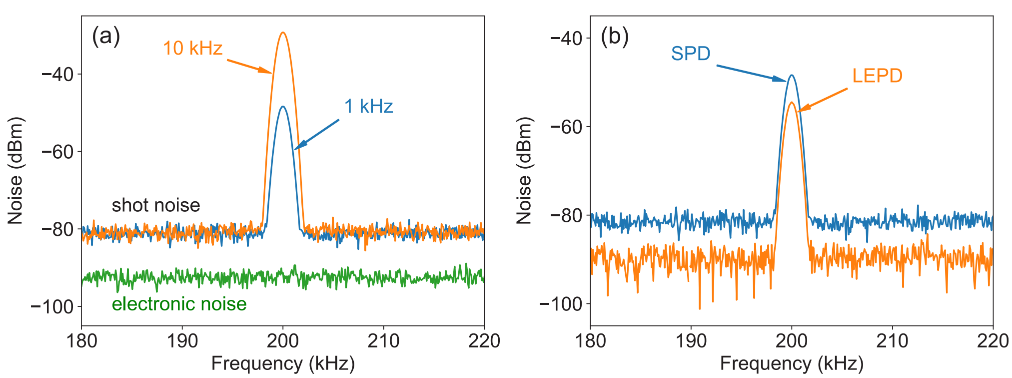

Fig. 2. (a) Spectra obtained with the SPD for a frequency modulation of the AOM driving RF of 200 kHz and frequency modulation depths of 1 kHz (blue) and 10 kHz (orange). The shot noise is given by the constant noise around the peak, while the signal is the height of the peak relative to the noise. The resolution bandwidth is 1 kHz and the video bandwidth 30 Hz. (b) Same spectra obtained for the SPD (blue) and the LEPD (orange) at a modulation depth of 1 kHz, after electronic noise subtraction. The LEPD has a modulation peak visibility, i.e height above the noise level, which is larger than that of the SPD, hence a better SNR.

small range of frequencies above and below the modulation frequency. The signal is the power difference between the maximum of the peak and the average side noise level.

Figure 3 shows the ratio of SNRs obtained for the SPD and LEPD used in identical conditions, for a range of oscillation amplitudes. The error bars indicate the statistical fluctuations in the noise measurement. Since the expected difference in SNR is purely geometric, the ratio of SNRs should be constant, which is what we observe within the statistical fluctuations. Averaging that ratio over the oscillation amplitudes gives a SNR ratio of 1.53 ± 0.015. The small 3% discrepancy with the theoretical 1.57 value may be due to systematic deviations from the nominal quantum efficiencies of the detectors or the ideal Gaussian shape of the diffracted beam. It also reflects the accuracy with which we can center the beams onto the detectors.

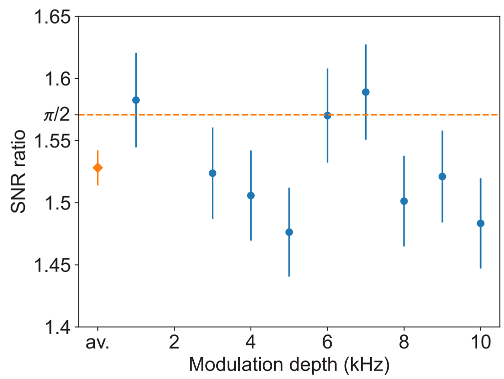

Fig. 3. Ratio of the SNRs of the two detectors for a range of modulation depths of the AOM driving radio frequency. The first data point (orange diamond) is the average of all the other data points. The error bars show plus and minus the standard error on the mean. The dashed line indicates the theoretical value of the ratio, which is independent of the modulation depth.

We make no attempt at directly showing that the quantum noise performance of the LEPD is at the quantum Cramér-Rao bound for a Gaussian beam, as this would entail a knowledge of the measurement chain, in particular the exact shape of the band-pass filter of the spectrum analyzer, which is not accessible to us at the level of accuracy which is necessary to reach a meaningful conclusion. This uncertainty does not affect the comparison between the detectors.

5. LEPD saturation

Once classical effects such as electronic noise are removed, the data shows unequivocally that the LEPD performs a better quantum measurement of the small displacement of a Gaussian beam than the SPD. The question remains whether the quantum noise can be made the main contributor to the noise so that this advantage can be carried over to a realistic experimental scenario. For instance in our experiment, without electronic noise subtraction, the raw, and therefore operationally usable, SNR ratio is 1.01 ± 0.01, which means that there is no advantage in using the LEPD. We here look at whether the quantum advantage of the LEPD could be exploited without being impeded by sources of noise.

The SPD is not fundamentally limited by technical noise, as the electronic noise of the transamplifier can be made smaller than the shot noise of the light by improving the electronics. The working principle of the LEPD, however, relies on the ohmic behavior of the resistive film and is necessarily subject to the associated Johnson noise. In the differential arrangement, where the photocurrents from both anodes are transamplified, then subtracted, the inter-electrode resistance R_{ie} contributes four times the equivalent Johnson noise power since it gets measured by both arms of the amplifier in an anticorrelated fashion before subtraction. This gives a current noise power spectral density of $\langle i_J^2 \rangle = 16 k_B T / R_{ie}$, where T is the temperature and k_B is the Boltzmann constant.

At a given temperature, the Johnson noise is constant whereas the shot noise is proportional to the optical power. As a result, there is a value of the optical power, or equivalently the photocurrent, above which the Johnson noise is negligible compared to the shot noise. This value of the photocurrent, however, may not be accessible due to a saturation effect in the LEPD. Indeed, the photocurrent induces a voltage drop across the resistive film, which reduces the reverse bias voltage V applied to the photodiode, particularly at the center of the chip. At the point where the bias is canceled, the detector ceases to function correctly. There is therefore a trade-off between the (quantum) noise performance of the detector and the saturation of the bias voltage. It is shown in the Supplementary section 2 that the detail of the trade-off depends on the size of the beam, and that for a typical beam size of $w = L/2.5$, for which the beam fills up the detector, the shot noise to Johnson noise ratio is

$$\frac{\langle i_{sn}^2 \rangle}{\langle i_J^2 \rangle} \simeq 0.12 \frac{Ve}{k_B T}. \tag{14}$$

One sees that the shot noise can be made linearly larger than the Johnson noise by increasing the voltage drop, with a crossover voltage $V = V_0 \simeq 0.22$ V at room temperature $T = 300$ K. Given that the voltage drop is only limited by the reverse bias voltage applied to the detector, making the shot noise the dominant noise is largely feasible in view of the maximum admissible reverse bias voltage, which is typically a hundred times larger than V_0. It is therefore expected that a transimpedance amplifier that would be largely limited by the shot noise could be built, for which the improvement on the sensitivity of the LEPD compared to the SPD would be fully realized.

6. Conclusion

We have shown that the LEPD has a SNR which is intrinsically larger than that of the SPD by a factor of $\pi/2$, and that this advantage can be practically leveraged in those applications where the technical noise on the beam position is smaller than the shot noise. In the context of shot-noise-limited fast AFM, the frequency response of the detector would need to be extended to the tens of megahertz band, and there are already commercially available such LEPDs. Generally, our analysis shows that since the noise performance does not depend on w or R_{ie}, there is scope for adjusting the size and the inter-electrode resistance of the detector in order to fulfill both the speed and optical power requirements of state-of-the-art AFM.

These results can easily be generalized to the two-dimensional case due to the separability of the x and y directions in a Gaussian beam. Four-quadrant SPD and two-dimensional LEPD are readily available.

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Disclosures. The authors declare no conflicts of interest.

Data Availability. Data underlying the results presented in this paper are available in Ref. [14].

Supplemental document. See Supplement 1 for supporting content.

References

1. G. Meyer and N. M. Amer, "'Novel optical approach to atomic force microscopy," Appl. Phys. Lett. **53**(12), 1045–1047 (1988).
2. C. A. J. Putman, B. G. De Grooth, N. F. Van Hulst, and J. Greve, "A detailed analysis of the optical beam deflection technique for use in atomic force microscopy," J. Appl. Phys. **72**(1), 6–12 (1992).
3. D. P. E. Smith, "Limits of force microscopy," Rev. Sci. Instrum. **66**(5), 3191–3195 (1995).
4. T. Fukuma, M. Kimura, K. Kobayashi, K. Matsushige, and H. Yamada, "Development of low noise cantilever deflection sensor for multienvironment frequency-modulation atomic force microscopy," Rev. Sci. Instrum. **76**(5), 053704 (2005).

Optics EXPRESS

5. C. Fabre, J. B. Fouet, and A. Maître, "Quantum limits in the measurement of very small displacements in optical images," Opt. Lett. **25**(1), 76–78 (2000).
6. S. Barnett, C. Fabre, and A. Maître, "Ultimate quantum limits for resolution of beam displacements," Eur. Phys. J. D **22**(3), 513–519 (2003).
7. N. Treps, N. Grosse, W. P. Bowen, C. Fabre, H.-A. Bachor, and P. K. Lam, "A Quantum Laser Pointer," Science **301**(5635), 940–943 (2003).
8. R. C. Pooser and B. Lawrie, "Ultrasensitive measurement of microcantilever displacement below the shot-noise limit," Optica **2**(5), 393–399 (2015).
9. M. T. L. Hsu, V. Delaubert, P. K. Lam, and W. P. Bowen, "Optimal optical measurement of small displacements," J. Opt. B: Quantum Semiclassical Opt. **6**(12), 495–501 (2004).
10. N. Treps, V. Delaubert, A. Maître, J. M. Courty, and C. Fabre, "Quantum noise in multipixel image processing," Phys. Rev. A **71**(1), 013820 (2005).
11. H.-A. Bachor, V. Delaubert, C. C. Harb, M. T. L. Hsu, P. K. Lam, C. Fabre, and N. Treps, "Spatial quantum effects with continuous-wave laser beams," J. Mod. Opt. **53**(5-6), 597–611 (2006).
12. H. Sun, K. Liu, Z. Liu, P. Guo, J. Zhang, and J. Gao, "Small-displacement measurements using high-order Hermite-Gauss modes," Appl. Phys. Lett. **104**(12), 121908 (2014).
13. H.-A. Bachor and T. C. Ralph, *A Guide to Experiments in Quantum Optics* (WILEY-VCH, 2004, pp. 111, 181), 2nd ed.
14. E. Fradgley and V. Boyer, "Research data supporting 'Quantum limits of position-sensitive photodiodes'," University of Birmingham (2022). https://doi.org/10.25500/edata.bham.00000860.