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# Impact of COVID-19 on stock indices volatility

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# Impact of COVID-19 on Stock Index Volatility: Long-Memory Persistence, Structural Breaks, or Both?

### Abstract

The onset of the COVID-19 pandemic has increased volatility in financial markets, motivating researchers to investigate its impact. Some use the GARCH family of models to focus on long-memory persistence, while others use Markov chain models to better identify structural breaks and regimes. However, no study has addressed the occurrence of these two phenomena in a unified framework. Since both are important features of the data, to ignore one or the other could lead to poorly specified models. The outcome would be incorrect risk measurement, with implications for risk management, Value at risk, portfolio decisions, forecasting, and option pricing. This paper aims to fill this gap in the literature. We assemble an international dataset for 16 stock market indices in three continents over the period from August 1, 2019 to February 18, 2022, totalling 669 business days. Using R, we estimate 80 GARCH family models, 16 pure Markov-Switching models, and 900 combined GARCH/ Markov-Switching models using daily stock market log-returns. We allow for two volatility regimes (low and high). We also measure and incorporate News Impact Curves, which show how past shocks affect contemporaneous volatility. Our main finding, across estimated models, is that COVID-19 affected both long-memory persistence and volatility regimes in most markets. To describe the specific impact in each market, we report News Impact Curves. Lastly, the first wave of COVID-19 had a much greater impact on volatility than did subsequent waves linked to the emergence of new variants.

**Keywords:** COVID-19, Statistical Models, Volatility, Stock Indices

# 1 Introduction

Since the emergence of COVID-19 in December 2019, its wide-ranging effects on society have been investigated by a large scientific literature. Much of this research uses data science techniques, such as data mining, machine learning

and big data [1, 2]. Among the many impacts suffered by populations worldwide, one important shock has been an increase in the volatility of financial markets around the world. While markets have largely recovered from the initial crash in early 2020, they have remained riskier and more unpredictable than before [3].

There are two main strands in the emerging literature on COVID-19 and volatility. First, most of the early studies used standard GARCH models to show increases in volatility persistence [4–15]. These papers have much in common : (1) They applied univariate GARCH models to the first wave of COVID-19; (2) They tested a small number of models using results from previous third-party studies; (3) They looked at specific stock market indices; (4) They used dichotomous variables to capture the effect of the pandemic; or (5) They estimated models by dividing the full sample in two, namely before and during the COVID-19 crisis. These papers found that COVID-19 increased stock market volatility, that volatility is persistent (i.e., displays long memory), is time-varying and that the leverage effect is confirmed. The second wave of COVID-19 contagion and spike in cases occurred in the winter (summer) months of 2020-21 in the Northern (Southern) Hemisphere [16]. New studies examined the impact of these new waves on financial markets [17–19]. Using similar methods as the earlier literature, they found that subsequent COVID-19 waves have had less of an impact on volatility than the first wave.

The second strand of the literature has described the impact of COVID-19 as causing structural breaks in the time series of volatility. That is, there is a change in the model coefficients describing the data-generating process for volatility. For instance, [20] analyzed the S&P 500 index volatility using Markov-Switching Autoregressive models (MS-AR), while [21, 22] used Markov-Switching GARCH models (MS-GARCH) to examine the volatility of stock market indexes. These studies identified the start of the COVID-19 pandemic as an important contributor to structural breaks in volatility. However, they only looked at the first wave and did not examine subsequent waves.

In light of these results, the question becomes: How best to measure the impact of COVID-19 on stock market volatility? The papers in this literature use different models but provide little or no comparisons between them. Moreover, the estimated models are usually fairly standard and do not allow for as much flexibility as the data could require. Answering this question is fundamental[24] to stakeholders such as investors, portfolio managers, financial analysts and risk managers. Indeed, accurate volatility modeling provides investors with a better understanding of financial market dynamics. In addition, it helps investors to value assets correctly and find the best potential diversification opportunities. It also improves portfolio management practice by suggesting the most appropriate models for conditional volatility dynamics. Lastly, this can lead to improved hedging strategies by choosing the most suitable derivative instruments and correctly estimating their Value at Risk (VaR) and Expected Shortfall (ES).

The purpose of this paper is therefore (1) to determine, out of a large universe of potential models, the best one(s) to describe the impact of COVID-19 on stock market index volatility; and (2) to assess whether the impact has affected long-memory persistence, structural breaks in the process, or both. To our knowledge, this type of empirical analysis has been done for Bitcoin volatility [24–27] but not for international stock market indices, especially accounting for the effect of COVID-19's multiple waves.

This paper considers a volatility modelling framework that allows for long memory (e.g., a hyperbolic rather than geometric rate of decay), different potential distributions for innovations (errors), and a Markov-switching framework to capture regime changes in volatility. To this end, we tested 80 univariate models from the GARCH family as well as 916 Markov-Switching models for two regimes. We estimate these models using time series data for 16 of the most important stock market indices in the world, representing more than two-thirds of the global market capitalization (in value).

The main contribution of this paper is to significantly expand upon the number of models tested on stock indices worldwide, which helps shed new light on the models' ability to explain in-sample volatility (e.g., tracking) as well as downside risk measures (VaR and ES) for major stock indices before and during the COVID-19 period.

Lastly, the objectives of this paper also align with the concept of Big Data Analytics and Data Mining techniques [28–31]. We apply algorithms for nonlinear regression models to the analysis of a considerable amount of data acquisition and processing. Moreover, the empirical approach uses statistical models for knowledge discovery and data visualization (e.g., News Impact Curves from GARCH models) in order to enhance the stakeholders' decision-making, problem-solving and data learning outcomes concerning stock index volatility.

# 2 Theoretical Support

## 2.1 Nested GARCH family

Since the seminal works of [32] and [33], a large family of models has grown to to explain the stylized facts established by the financial econometrics literature on economic and financial time series, such as volatility clustering, asymmetry, long memory and fat tails.

The GARCH family of models is the leading econometric framework to model volatility because it provides a parsimonious approximation of the process generating asset return volatility dynamics [34–36]. While this family has grown considerably, [37] proposed a model encompassing some of the most useful GARCH models (ALLGARCH). This model can be represented as in function (1). As for the number of lags, we have chosen the GARCH(1,1) specification because it is the most common and because research finds that additional lags typically do not improve performance [38].

$$\frac{\sigma_t^\lambda - 1}{\lambda} = \omega + \alpha \sigma_{t-1}^\lambda \left[ |Z_{t-1} - \eta_2| - \eta_1 (Z_{t-1} - \eta_2) \right]^\delta + \beta \left( \frac{\sigma_{t-1}^\lambda - 1}{\lambda} \right) \quad (1)$$

Equation (1) is a Box-Cox transformation of the conditional volatility $\sigma_t$, where $\omega$ is the intercept and $\alpha$ and $\beta$ are the persistence of the standardized lagged shocks $Z_{t-1}$ and the conditional volatility, respectively. Moreover, $\lambda$ determines the shape of the function and $\delta$ transforms the absolute value function. For the latter, $\eta_1$ and $\eta_2$ control the rotations and shifts.

**Table 1**  Possible GARCH models nested in the ALLGARCH model

| Coefficient constraints | Resulting equation | Effects on NIC | References |
|---|---|---|---|
| $\lambda = \delta = 2$; $\eta_1 = \eta_2 = 0$; $\beta = 0$ | ARCH | No shift/rotation | [32] |
| $\lambda = \delta = 2$; $\eta_1 = \eta_2 = 0$ | GARCH | No shift/rotation | [33] |
| $\lambda = \delta$; $\eta_1 = \eta_2 = 0$ | NGARCH | No shift/rotation | [39] |
| $\lambda = \delta = 2$; $\eta_1 = 0$ | NAGARCH | Shift | [40] |
| $\lambda = 0$; $\delta = 1$; $\eta_2 = 0$ | EGARCH | Rotation | [41] |
| $\lambda = \delta = 2$; $\eta_2 = 0$ | GJR-GARCH | Rotation | [42] |
| $\lambda = \delta$; $\eta_2 = 0$; $|\eta_1| \leq 1$ | APARCH | Rotation | [43] |
| $\lambda = \delta = 1$; $\eta_2 = 0$; $|\eta_1| \leq 1$ | TGARCH | Rotation | [44] |
| $\lambda = \delta = 1$; $|\eta_1| \leq 1$ | AVGARCH | Shift and Rotation | [45] |
| $\lambda = \delta$ | ALLGARCH | Shift and Rotation | [37] |

Source: authors following the framework in [37].

Table 1 shows ten possible GARCH model specifications that can be obtained from equation (1), depending on the specific constraints on the $\lambda, \delta, \eta_1$ and $\eta_2$ coefficients. Note that the ARCH model is a special case of the GARCH model when $\beta$ is zero. The table also summarizes how each GARCH specification yields particular effects on the News Impact Curve (NIC), essentially showing how past shocks affect present volatility. Depending on the model, the effects can be to shift (small shocks) or rotate (large shocks) the NIC [40, 46]. Thus, NIC is an invaluable graphical tool to display how volatility incorporates new information (shocks).

$$\max \ln (Z, \sigma) = \max \sum_{t=1}^{T} \ln \left( \frac{1}{\sigma_t} f(Z_t) \right) \quad (2)$$

We obtain the coefficients of equation (1) by maximizing the log-likelihood function (2), where $Z_t = \epsilon_t / \sigma_t$ is the shock $\epsilon_t$ standardized by $\sigma_t$. For flexibility, we allow $f(Z_t)$ to be drawn from a broad set of univariate probability distributions, as seen in Table 2.

As a result, the specifications in Tables 1 and 2 can be combined to provide 80 different model specifications to investigate long memory effects in stock index volatility.

**Table 2**  Probability Distributions Functions (PDF's) for $Z_t$

| Function | Equation |
| --- | --- |
| Normal (norm) | $f(Z_t) = \dfrac{e^{-0.5Z_t^2}}{\sqrt{2\pi}}$ |
| Student-t (std) | $f(Z_t, \nu) = \dfrac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(\nu-2)\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{Z_t^2}{\nu-2}\right)^{\frac{-(\nu+1)}{2}}$ |
| Generalized Error Distribution (ged) | $f(Z_t, \nu) = \dfrac{\nu e^{-0.5(\lvert\kappa Z_t\rvert)^\nu}}{\kappa 2^{(1+1/\nu)}\Gamma(1/\nu)};\ \kappa = \sqrt{\dfrac{2^{(-2/\nu)}\Gamma(1/\nu)}{\Gamma(3/\nu)}}$ |
| Skewed Normal (snorm) | See [47], equation 1 |
| Skewed Student-t (sstd) | See [47], equation 1 |
| Skewed ged (sged) | See [47], equation 1 |
| Generalized Hyperbolic (ghyp) | See [48] |
| Johnson's reparametrized SU (jsu) | See [49] |

Source: summary by the authors.

## 2.2 Markov-Switching models

The emerging popularity and widespread use of GARCH models to estimate conditional volatility in financial markets soon led to questions about the degree of persistence in the volatility process. That is, how much does past volatility explain the present? Early empirical results suggested that persistence in GARCH models was high. This led [50] to confirm that GARCH model coefficients can vary over time, thus explaining high persistence. To capture this, the literature has considered structural breaks: following large-scale events in the market, the structure of the time series process changes significantly. For instance, the market may switch from a low-volatility to a high-volatility environment. Locating the structural break becomes a fundamental question [51].

A useful framework to deal with this issue is the pure Markov-Switching model (MSwM) of variance proposed by [52]. To describe this model, first consider the log-return (shocks) $y_t$ of any financial asset at time $t$, with mean zero and no autocorrelation. This model can be represented by the following set of equations under the assumption of two volatility regimes:

$$y_t = N(0, \sigma_t^2) \tag{3}$$
$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} \tag{4}$$
$$\sigma_1^2 < \sigma_2^2 \tag{5}$$
$$S_{kt} = 1, if\ S_t = k;\ \text{otherwise}, S_{kt} = 0, k = 1, 2 \tag{6}$$
$$p(S_t = 1 | S_{t-1} = 1) = p_{11};\ p(S_t = 2 | S_{t-1} = 1) = 1 - p_{11} \tag{7}$$

$$p(S_t = 2|S_{t-1} = 2) = p_{22}; \ p(S_t = 1|S_{t-1} = 2) = 1 - p_{22} \qquad (8)$$

$$\max \ln(y, \theta) = \max \sum_{t=1}^{T} \sum_{i=1}^{2} \ln \left[ \frac{p_{ii}}{\sqrt{2\pi\sigma_i^2}} \exp \left( \frac{-(y_t - \mu_i)^2}{2\sigma_i^2} \right) \right] \qquad (9)$$

In this model, variance is persistent: earlier values continue to affect later values. This occurs because there is a change in the variance regime, which is defined by the latent variable $S_t$, a first-order ergodic homogeneous Markov chain with transition probabilities described by equations (6) to (8). For instance, $p_{12}$ is the transition probability of going from state 1 (low volatility) to state 2 (high volatility). While a larger number of distinct volatility regimes could be considered, this is generally not necessary to obtain well-performing models in financial time series. In this setting, parsimony is preferable. $P$'s transition probability matrix is constructed using $p_{11}$ and $p_{22}$ to calculate the 1-step ahead regime. To estimate the coefficient vector $\theta \equiv \left( \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p_{11}, p_{22} \right)$ we maximize the log-likelihood equation (9) using numerical methods. Finally, note that variance is constant within each regime when using this approach.

The other approach we consider consists of Markov-Switching GARCH Models (MS-GARCH), where variance is time-varying in each regime. Here, there is persistence in shocks as well as in regime changes persistence. The general MS-GARCH expression is [53, 54]:

$$y_t | (S_t = k, I_{t-1}) \sim D(0, h_{k,t}, \xi_k) \qquad (10)$$

$$h_{k,t} \equiv h(y_{t-1}, h_{k,t-1}, \theta_k) \qquad (11)$$

where $D(0, h_{k,t}, \xi_k)$ is a density (PDF) with a zero mean, time-varying variance $h_{k,t}$ and additional shape coefficients (see Table 2) contained in the vector $\xi_k$. Similar to MSwM, the latent variable $S_t$ is also a first-order ergodic homogeneous Markov chain defined on the discrete space $k = 1, 2$, with $p_{11}$ and $p_{22}$ belonging to the transition probability matrix $\mathbf{P}$. The information set available as of period $t - 1$ is represented by the vector $I_{t-1} \equiv (y_{t-i}, i > 0)$, and $h_{k,t}$ is the variance of $y_t$ conditional on the realization of $S_t$ and $I_{t-1}$ as described by a GARCH family model. This conditional variance $h_{k,t}$ is defined by equation (11), with a regime-dependent vector of coefficients $\theta_k$ that was previously described in equation (1) and in Table 1 (different GARCH specifications).

$$\max \ln L(\psi|I) \equiv \max \sum_{t-1}^{T} \ln f(y_t|\psi, I_{t-1}) \qquad (12)$$

Lastly, let $\psi \equiv (\theta_1, \theta_2, \xi_1, \xi_2, P)$ be the vector of model coefficients for two regimes. These are estimated by maximizing the log-likelihood equation (12) using numerical methods, where $f(y_t|\psi, I_{t-1})$ is the conditional density of $y_t$ given the vector of coefficients $\psi$ and past observations contained in $I_{t-1}$.

# 3 Data and Methods

We collected our main data from Yahoo! Finance. The sample period runs from August $1^{st}$ 2019 to February $18^{th}$ 2022. The reason why the sample begins in 2019 is to allow for a pre-COVID-19 "normal" period, which we use to estimate the volatility models and obtain "baseline" results prior to the impact of COVID-19. With the up-to-date series, we can estimate new models that account for the impact of COVID-19.

This data sample consists of 669 daily observations of the stock market level for each of the 16 stock indices over three continents. In total, there are 10,704 observations. For this task, we used the *pdfetch* R library [55]. As shown in Table 3, these stock indices account for nearly 70% of global stock market capitalization (by value). Using a spline interpolation procedure from the *imputeTS* R library, we filled the missing values in these time series [56]. Finally, we computed the daily price log-returns for all stock indices and report their descriptive statistics in Table 4.

**Table 3** Selected stock markets and their corresponding stock indices from Yahoo! Finance

| Continent | Region | Symbol | Index Name | Index Symbol |
|-----------|--------|--------|------------|--------------|
| Europe | England | UK | FTSE 100 | FTSE |
| | Eurozone | Euro | EURO STOXX 50 | STOXX50E |
| | Norway | NOR | Oslo Exchange All Share Index | OSEAX |
| | Germany | DEU | DAX Performance Index | GDAXI |
| | Spain | ESP | IBEX 35 | IBEX |
| | Swiss | CHE | Swiss Stock Market Index | SSMI |
| Asia | Australia | AUS | All Ordinaries | AORD |
| | China | CHN | Shanghai Composite Index | SSEC |
| | Hong Kong | HK | Hang Seng Index | HSI |
| | India | IND | NIFTY 50 | NSEI |
| | Korea | KOR | KOSPI Composite Index | KS11 |
| | Japan | JPN | Nikkei 225 | N225 |
| America | Brazil | BRA | BVSP BOVESPA Index | BVSP |
| | Canada | CAN | S&P/TSX Composite Index | GSPTSE |
| | United States | US | S&P 500 | GSPC |
| | Mexico | MEX | IPC Mexico | MXX |

Source: Liu et al.[57].

Combinations of the equations presented in Tables 1 and 2 lead to a total of 80 models. We estimated the coefficients of these models using the *rugarch* R library [58]. Moreover, the pure Markov-Switching models described by equations (3) – (9) generate a total of 16 models, whose coefficients were estimated using the *MSwM* R library [59]. Lastly, the coefficients of the models described by equations (10) – (12) were estimated using the *MSGARCH* R library [60]. For this last library, we selected the ARCH, GARCH, TGARCH, EGARCH, GJR-GARCH models and the Normal, Student-t, and GED PDFs (as well as their asymmetric versions). With the starting point being the estimation of a two-regime model, the outcome was the generation of 900 models

resulting from different combinations. The algorithm written for this analysis is available in the supplementary materials, for researchers aiming to reproduce the results presented below.

# 4 Results and discussion

## 4.1 Descriptive statistics

Table 4 presents descriptive statistics for price log-returns in each of the 16 stock market indices. The indices are ranked in descending order according to their 1% empirical VaR. Overall, these results indicate that (1) means are usually positive and very close to zero; (2) standard deviations (SD) are larger than means; and return distributions are (3) left-skewed and (4) leptokurtic. Next, we apply Jarque-Bera normality and Phillips-Perron unit root tests. For each test, the null hypothesis is rejected for all stock market indices (p-value < 0.01). Therefore, price log-returns for all indices are stationary with innovations than are not distributed as Normal.

**Table 4**  Descriptive statistics for price log-returns in 16 stock market indices

| Index | Mean(%) | SD(%) | Skewness | Kurtosis | VaR 5% | VaR 1% | Min(%) | Max(%) |
|---|---|---|---|---|---|---|---|---|
| BVSP | 0.0158 | 1.96 | -1.65 | 22.14 | -2.45 | -5.44 | -15.99 | 13.02 |
| GSPC | 0.0590 | 1.50 | -1.08 | 20.03 | -2.02 | -4.52 | -12.77 | 8.97 |
| GSPTSE | 0.0385 | 1.38 | -1.93 | 37.18 | -1.42 | -4.39 | -13.18 | 11.29 |
| GDAXI | 0.0329 | 1.48 | -1.04 | 18.18 | -2.10 | -4.25 | -13.05 | 10.41 |
| STOXX50E | 0.0296 | 1.46 | -1.42 | 18.29 | -2.12 | -4.24 | -13.24 | 8.83 |
| AORD | 0.0131 | 1.29 | -1.49 | 15.38 | -1.97 | -4.24 | -10.01 | 6.35 |
| NSEI | 0.0681 | 1.42 | -2.02 | 22.17 | -1.85 | -4.21 | -13.90 | 6.41 |
| KS11 | 0.0461 | 1.31 | -0.19 | 10.96 | -1.93 | -3.95 | -8.77 | 8.25 |
| FTSE | -0.0009 | 1.31 | -1.26 | 17.41 | -2.00 | -3.88 | -11.51 | 8.67 |
| OSEAX | 0.0463 | 1.29 | -1.53 | 14.16 | -1.89 | -3.86 | -9.83 | 5.84 |
| HSI | -0.0159 | 1.27 | -0.31 | 4.88 | -2.14 | -3.85 | -5.72 | 4.92 |
| IBEX | -0.0062 | 1.53 | -1.58 | 21.52 | -2.21 | -3.75 | -15.15 | 8.23 |
| N225 | 0.0345 | 1.28 | 0.06 | 7.91 | -2.06 | -3.60 | -6.27 | 7.73 |
| MXX | 0.0400 | 1.18 | -0.57 | 6.41 | -1.78 | -3.51 | -6.64 | 4.18 |
| SSMI | 0.0312 | 1.09 | -1.43 | 19.42 | -1.62 | -3.34 | -10.13 | 6.78 |
| SSEC | 0.0263 | 0.99 | -0.17 | 6.00 | -1.61 | -2.73 | -4.60 | 5.55 |

Source: computed by the authors from Yahoo! Finance data.

It is worth pointing out here that if we look at risk measures across stock indices, the BVSP (São Paulo, Brazil) is the stock index with the highest values (e.g., greatest risk) whether expressed as standard deviation, maximum drop (i.e., minimum value), or empirical VaR (1% and 5%). On the other hand, the SSEC (Shanghai, China) is the least risky index, measured in terms of standard deviation , maximum drop, or 1% empirical VaR.

## 4.2 Results for volatility models from the GARCH family

In this section, we work with the assumption that the potential impact of COVID-19 on stock market volatility is through long-memory persistence only. Having verified that the series of daily log-returns are stationary, it is appropriate to fit each of the 80 possible model combinations from Tables 1 and 2 to the 16 stock market indices. As a result, there is a total of 1,280 estimated models. Then, to determine which model best explains the evolution of volatility in a given stock market index, we use the following filtering protocol:

1. All coefficients must be statistically significant at a 5% level, and the half-life of shocks to variance (i.e., persistence) [50] must be less than 100 days;
2. The null ($H_0$) should not be rejected for all VaR and ES tests [unconditional and conditional coverage, duration of time between violations and the mean of the shortfall violations] using in-sample data [61–64];
3. The model with the lowest Bayesian Information Criterion value is selected [65], to avoid overfitting the data.

Figure 1 shows the daily log-return time series for each of the 16 stock indices and also reports the best GARCH model for each market. The purple dashed line is the 5% conditional VaR. Periods of high volatility (in red) are the outliers obtained via Box-plot statistics. Dates when the first documented samples of COVID-19 Variants of Concern [VoC] [23] were reported are identified using the symbols [alpha ($\alpha$), beta ($\beta$), gamma ($\gamma$), delta ($\delta$), omicron (o)]. The start of the pandemic (i.e., March 11, 2020) is identified by the symbol Ⓒ.
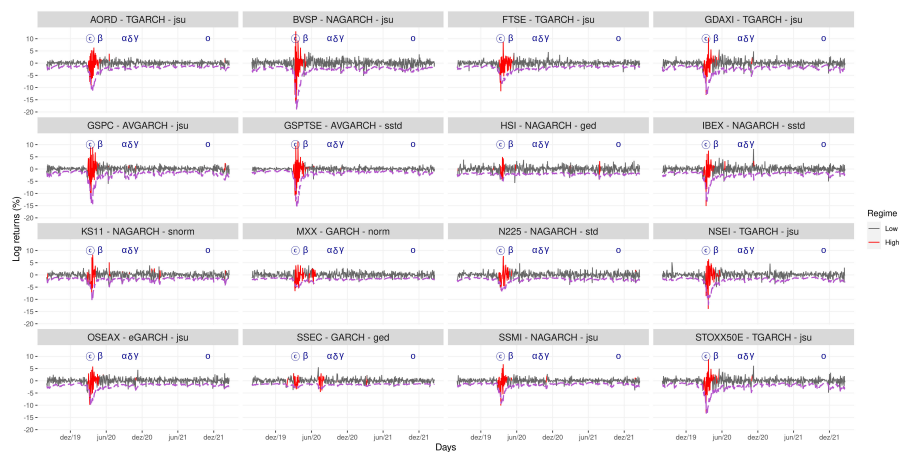


**Fig. 1** Time series of log-returns and best GARCH model for each of the stock indexes for the period including COVID-19. Note: The Ⓒsymbol indicates the beginning of the COVID-19 pandemic (March 11, 2020). The ($\alpha$), ($\beta$), ($\gamma$), ($\delta$), and (o) symbols indicate the dates when the first documented samples of COVID-19 Variants of Concern were reported.

Table 5 reports our results for the estimated long-memory models. The columns show which model combination is optimal for a given stock index

(GARCH model and PDF), the degree of persistence in volatility shocks (with a value of 1 indicating that shocks are permanent), half-life (measured in days), BIC, low and high annualized volatility, and the number of days of low and high volatility. Details on the computation of persistence and half-life for each GARCH model are given by Ghalanos [58].

**Table 5**  Results for the long-memory models, for each stock index in our sample

| Index | Model-PDF | Pers. | H2L | BIC | lowVol | highVol | Days low | Days high |
|-------|-----------|-------|-----|-----|--------|---------|----------|-----------|
| AORD | tgarch-jsu | .971 | 23.6 | -6.6 | 14.3% | 57.8% | 622 | 46 |
| BVSP | nagarch-jsu | .972 | 24.6 | -5.7 | 21.1% | 99.2% | 630 | 38 |
| FTSE | tgarch-jsu | .982 | 37.8 | -6.4 | 15.9% | 53.0% | 620 | 48 |
| GDAXI | tgarch-jsu | .980 | 33.9 | -6.2 | 18.6% | 60.4% | 626 | 42 |
| GSPC | avgarch-jsu | .977 | 29.3 | -6.5 | 16.0% | 72.7% | 626 | 42 |
| GSPTSE | avgarch-sstd | .977 | 30.3 | -7.1 | 11.4% | 68.9% | 617 | 51 |
| HSI | nagarch-ged | .878 | 5.3 | -6.0 | 18.8% | 39.5% | 636 | 32 |
| IBEX | nagarch-sstd | .984 | 43.1 | -6.0 | 19.3% | 67.1% | 632 | 36 |
| KS11 | nagarch-snorm | .933 | 10.0 | -6.3 | 16.6% | 49.2% | 618 | 50 |
| MXX | garch-norm | .959 | 16.7 | -6.3 | 15.7% | 37.6% | 612 | 56 |
| N225 | nagarch-std | .968 | 21.1 | -6.1 | 17.3% | 47.3% | 628 | 40 |
| NSEI | tgarch-jsu | .970 | 22.8 | -6.3 | 16.8% | 60.7% | 623 | 45 |
| OSEAX | egarch-jsu | .978 | 31.8 | -6.3 | 16.3% | 52.7% | 626 | 42 |
| SSEC | garch-ged | .936 | 10.5 | -6.5 | 14.3% | 27.4% | 614 | 54 |
| SSMI | nagarch-jsu | .988 | 57.9 | -6.8 | 13.2% | 47.3% | 627 | 41 |
| STOXX50E | tgarch-jsu | .980 | 33.8 | -6.3 | 18.3% | 59.2% | 625 | 43 |

Note: Pers. and H2L denote Persistence and Half-Life of volatility, respectively.

Source: computed by the authors from Yahoo! Finance data.

Overall, the main message from the results shown in Figure 1 and Table 5 is as follows:

- While no single model is optimal for all markets, some are clearly more useful than others. The TGARCH-jsu model best captures the volatility in five stock indices, implying that volatility in several markets is fairly similar. The NAGARCH model fits the data best for six stock indices, but five different types of PDFs are used. The jsu distribution best captures innovations (errors) in nine markets: Across markets, it is the distribution that is most often optimal to describe innovations.
- The models that are optimal for our sample of indices are often different than those reported in the prior literature (see Section 1), where the most common ones are GARCH, EGARCH, and GJR-GARCH models using the Normal distribution. Since asset log-returns tend to be skewed and leptokurtic, using the Normal distribution in the maximum likelihood function could lead to an incorrectly specified model, given that it fails to capture these two important stylized facts;

- We estimate the mean persistence to be .965 [SD = .028], with a mean half-life of 27 days [SD = 13.3 days]. That is, once the shock occurs, it takes on average almost a month for half of the effect to dissipate. The SSMI and IBEX stock indices in Europe have the highest values, while the lowest values are found for the HSI, KS11, and SSEC indices in Asia;
- Low annualized volatility is 16.5% [SD=2.5%] on average across indices, while high annualized volatility is 56.2% [SD=16.6%] on average. The GSPTSE Canadian stock index has the lowest value of "low annualized volatility", while the BVSP Brazilian stock index has the highest "high annualized volatility";
- Since the beginning of the pandemic, the 16 stock indices have recorded more low-volatility days [mean = 623.9; SD = 6.5] than high-volatility days [mean = 44.1; SD = 6.5]. Days with high volatility are clustered between the beginning of the pandemic (Ⓒ) and the emergence of the beta ($\beta$) variant [in May 2020]. This result goes beyond what is presented in earlier studies (Section 1), as it shows that the emergence of other VoC did not have much effect on the volatility of the 16 stock indices.

Figure 2 shows, for each stock index, the News Impact Curve obtained from the estimated GARCH models discussed above. The NIC illustrates the effects of past positive and negative shocks $\varepsilon_{t-1}$ on the estimate of variance $\sigma_t^2$. The shape of the NIC is determined by which GARCH model is found to fit the data best, and by the model parameters that are estimated.
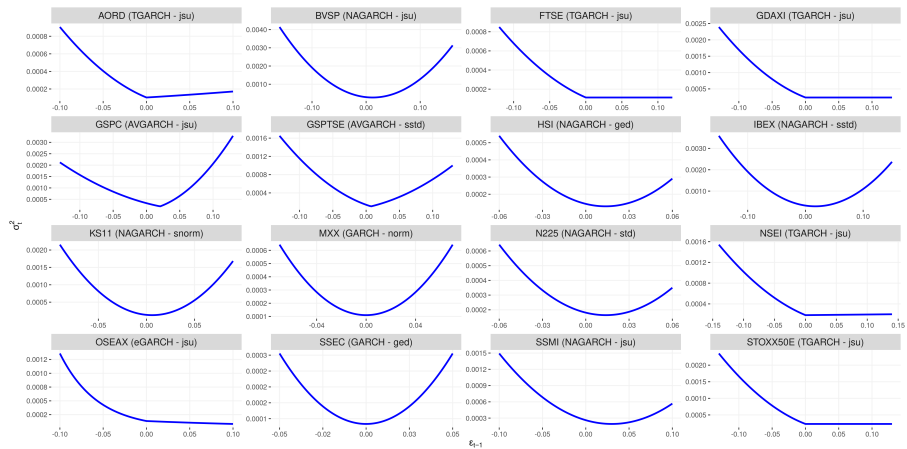


**Fig. 2** News Impact Curve obtained from GARCH models for 16 indices in our sample during the COVID-19 period

The NIC is symmetrical in only two cases, the MXX and SSEC indices. This means negative and positive past shocks have the same impact on present-day conditional variance. For the other 14 indices, the effects are asymmetrical. In the case of TGARCH and EGARCH models (e.g, AORD, OSEAX), the NIC

is rotated clockwise due to the positive $\eta_1$ coefficient, which implies that negative shocks have a greater impact on conditional variance than do positive shocks. NAGARCH models (e.g., BVSP, HSI) always have a positive $\eta_2$ coefficient, which shifts the NIC to the right. That is, negative shocks have a greater effect on variance than do positive shocks. For AVGARCH models (e.g., GSPC, GSPTSE), the effect on the rotation of the curve can vary. There is a clockwise rotation for the GSPTSE (i.e., bigger effects from negative shocks) but a counterclockwise rotation for GSPC (bigger effects from positive shocks).

**Table 6**  Estimated GARCH model coefficients for each stock index in our sample

| Index | $\omega$ | $\alpha$ | $\beta$ | $\eta_1$ | $\eta_2$ | skew | shape |
|---|---|---|---|---|---|---|---|
| AORD | .000321 | .114 | .886 | .743 | - | -1.056 | 1.797 |
| BVSP | .000009 | .131 | .785 | - | .658 | -1.237 | 2.605 |
| FTSE | .000204 | .077 | .925 | 1.000 | - | -.516 | 1.604 |
| GDAXI | .000344 | .129 | .887 | 1.000 | - | -.471 | 1.406 |
| GSPC | .000435 | .306 | .718 | -.300 | 1.104 | -1.168 | 1.958 |
| GSPTSE | .000283 | .198 | .794 | .118 | .586 | .697 | 8.096 |
| HSI | .000019 | .075 | .711 | - | 1.102 | - | 1.329 |
| IBEX | .000006 | .118 | .771 | - | .899 | .827 | 5.490 |
| KS11 | .000012 | .220 | .667 | - | .453 | .862 | - |
| MXX | .000005 | .109 | .851 | - | - | - | - |
| N225 | .000006 | .088 | .794 | - | .988 | - | 6.976 |
| NSEI | .000437 | .094 | .899 | .953 | - | -.968 | 1.878 |
| OSEAX | -.190702 | .109 | .978 | 1.554 | - | -.862 | 1.931 |
| SSEC | .000006 | .089 | .847 | - | - | - | 1.325 |
| SSMI | .000003 | .077 | .666 | - | 1.788 | -.825 | 1.882 |
| STOXX50E | .000338 | .128 | .886 | 1.000 | - | -.685 | 1.540 |

Note: the skew and shape are the PDF coefficients, as appropriate.

Source: computed by the authors from Yahoo! Finance data.

Finally, Table 6 presents estimated coefficients for the best-fitting GARCH models [equation 1] for each stock index, while respecting the restrictions of each model as described in Table 1 and the probability distribution functions in Table 2.

## 4.3 Results for the pure Markov-Switching models

In this section, we assume that the impact of COVID-19 on volatility is only through structural breaks and not long memory. In this framework, volatility is constant in each regime (MSwM model), as shown in equations (3)-(9). To determine the best fit, we estimate 16 pure Markov-Switching models for each stock index. The selection criteria are simply that the null the $H_0$ cannot be rejected for all VaR and ES tests applied to the in-sample data.

Figure 3 shows that the estimated models meet the selection criteria in only 11 out of 16 stock indices. Thus, using a pure Markov-Switching model cannot explain volatility in five stock markets. The purple dashed line is the 5% conditional VaR. This is a piecewise function since volatility is constant in

each of the two regimes. Unlike Figure 1, here the results suggest that periods of high volatility occurred not only with the beginning of the pandemic, but also with the emergence of subsequent VoC. This is mainly the case for the stock indices STOXX50E, GDAXI, GSPC, FTSE, and OSEAX. However, the intensity is lower than for the first wave of contagion.
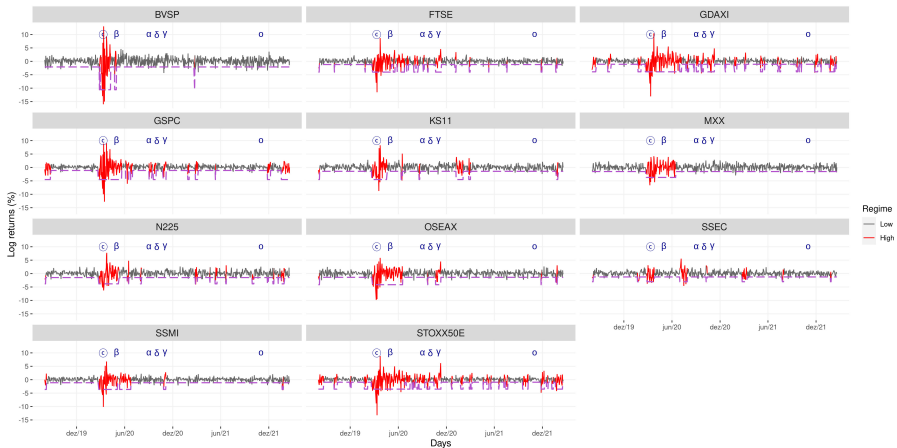


**Fig. 3** Time series of log-returns and best pure Markov-Switching model for each of the stock indexes for the period including COVID-19. Note: In five stock indices, no MSwM meets the selection criteria. The ©symbol indicates the beginning of the COVID-19 pandemic (March 11, 2020). The ($\alpha$), ($\beta$), ($\gamma$), ($\delta$), and (o) symbols indicate the dates when the first documented samples of COVID-19 Variants of Concern were reported.

Note that no country had yet started vaccinating their populations at that time (September to November 2020). The United States were the first country to initiate such a campaign, on December 14, 2020 [66].

Table 7 presents the results of the MSwM models. The table highlights the low and high annualized volatility, as well as the respective number of days in each regime. Also reported are the estimated values of remaining in the low ($p_{11}$) and high ($p_{22}$) volatility regimes once they start, and the BIC.

Comparing the results of Tables 5 and 7, we can see that:

- The BIC values for the estimated MSwM models are much smaller than are those for the long-memory models, suggesting that they better explain the evolution of conditional volatility. This result would suggest that COVID-19 had a greater impact on the volatility regimes than on long-memory persistence;

- The average for low annual volatility is 12.9% [SD=3.0%], while for high annual volatility it is 43.3% [SD=19.9%]. Compared with long-memory models, estimated volatility in MSwM models is lower on average but standard deviations are higher. The STOXX50E European stock index has the lowest value of "low annual volatility", while the BVSP Brazilian stock index has the highest value of "high annual volatility";

**Table 7** Results of the pure Markov-switching models for 11 stock indices in our sample

| Index | low Vol | high Vol | $p_1 1$ | $p_2 2$ | Days low | Days high | BIC |
|-------|---------|----------|---------|---------|----------|-----------|-----|
| BVSP | 20.2% | 102.2% | 99.5% | 91.7% | 632 | 36 | -3,754.4 |
| FTSE | 11.5% | 38.7% | 96.2% | 87.0% | 539 | 129 | -4,182.6 |
| GDAXI | 10.6% | 37.7% | 93.5% | 87.2% | 457 | 211 | -4,037.6 |
| GSPC | 10.8% | 43.6% | 97.3% | 91.8% | 507 | 161 | -4,168.2 |
| KS11 | 13.9% | 43.5% | 98.2% | 89.7% | 589 | 79 | -4,119.9 |
| MXX | 14.7% | 35.9% | 99.8% | 98.6% | 587 | 81 | -4,169.8 |
| N225 | 14.3% | 36.9% | 97.2% | 87.2% | 571 | 97 | -4,064.6 |
| OSEAX | 13.5% | 40.0% | 98.7% | 94.2% | 562 | 106 | -4,142.0 |
| SSEC | 12.1% | 29.3% | 98.1% | 89.0% | 594 | 74 | -4,359.7 |
| SSMI | 10.8% | 35.1% | 98.5% | 93.2% | 571 | 97 | -4,423.3 |
| STOXX50E | 9.0% | 33.9% | 91.5% | 88.6% | 396 | 272 | -4,047.8 |

Source: computed by the authors from Yahoo! Finance data.

- There are considerably more low volatility days [mean = 545.9; SD = 68.4] than high volatility days [mean = 122.1; SD = 68.4] in MSwM models, even considering the lower average value compared to long memory models.[1].

However, it is important to note that the MSwM models do not work for all stock indices. Indeed, we fail to find a suitable MSwM model for five out of 16 markets. Thus, a combination of Markov-switching and long memory seems to be appropriate in order to obtain the best possible model.

## 4.4 MS-GARCH models results

In this section, we consider the possibility that COVID-19 has had an impact on volatility through structural breaks and/or long memory persistence. We do not restrict the possible impact to just one or the other, but rather we let the data lead to the best fitting model. This leads to a total of 900 possible combinations as explained in Section 3. We estimate the 900 models for each of the 16 stock indices, which generates a total of 14,400 estimated model outcomes.

To determine the best model to explain volatility in each stock index, we apply the following filtering protocol:

1. All coefficients must be statistically significant at a 5% level;
2. The number of low-volatility days must be greater than or equal to the number of high-volatility days, to be consistent with the evidence shown in subsections 4.2 and 4.3;
3. The null ($H_0$) for all VaR and ES tests should not be rejected using in-sample data;

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[1]This difference in mean values is a consequence of the use of the first order homogeneous ergodic Markov chain, which can better capture the regime-switching moments throughout the historical series than the identification of outliers via Box-plot statistics, as with the long memory models

4. The annual volatilities for both regimes must not be 0, and the annual volatility of the first regime ("low") must be less than that of the second regime ("high");
5. The probabilities of staying in the original regime, $p_{11}$ and $p_{22}$, must be higher than 85%;
6. The half-life of volatility persistence should be less than 100 days, and the volatility half-life of the first regime must be less than that of the second regime;
7. The model with the highest mean $(p_{11}, p_{22})$ should be selected because the higher this value, the more persistent the regime. Otherwise, regime-switching might occur too often, which would not be plausible for MS-GARCH models, as even a single-regime GARCH model could perform better to explain periods of high and low volatility in stock indices [67];

Figure 4 shows the best-fitting models that meet the filtering protocol for each of the 16 stock indices. The purple dashed line is the 5% conditional VaR. The sub-figure headers show that the eGARCH model often performs best ($n = 16$), as does the sstd distribution for innovations ($n = 10$). As with Figure 3, there are periods of high volatility with the emergence of the alpha, delta, and gamma VoC for some of the the stock indices (AORD, FTSE, GDAXI, KS11, NSEI, and SSMI). However, the intensity of the volatility spike is less than the spike caused by the first wave of the contagion.
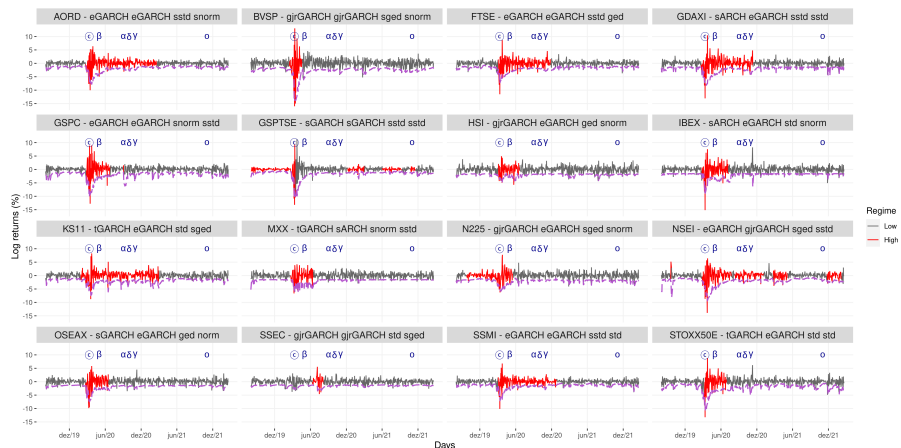


**Fig. 4** Time series of log-returns and best Markov-Switching-GARCH combination model for each of the stock indexes for the period including COVID-19. Note: Two GARCH specifications are reported for each stock index, one for each volatility regime. The ©symbol indicates the beginning of the COVID-19 pandemic (March 11, 2020). The $(\alpha)$, $(\beta)$, $(\gamma)$, $(\delta)$, and (o) symbols indicate the dates when the first documented samples of COVID-19 Variants of Concern were reported.

Figure 4 also shows that there is no high volatility regime that is recorded after the emergence of the omicron variant, except for the GSPTSE and NSEI

indices (and once more, with lower intensity). For all stock indices except one (GSPTSE), the best-fitting GARCH model for low volatility is different than the best model for high volatility.

Figure 5 shows the News Impact Curves obtained for each stock index. In each sub-figure there are two curves, one for the low-volatility regime and one for the high-volatility regime. The NIC displays the effects of past positive and negative shocks $\varepsilon_{t-1}$ on the conditional variance estimate $\sigma_t^2$. Based on the same estimation results, Table 8 reports details for the models that best fit the data for each stock index. [68] provides details on how to calculate the results in the table.
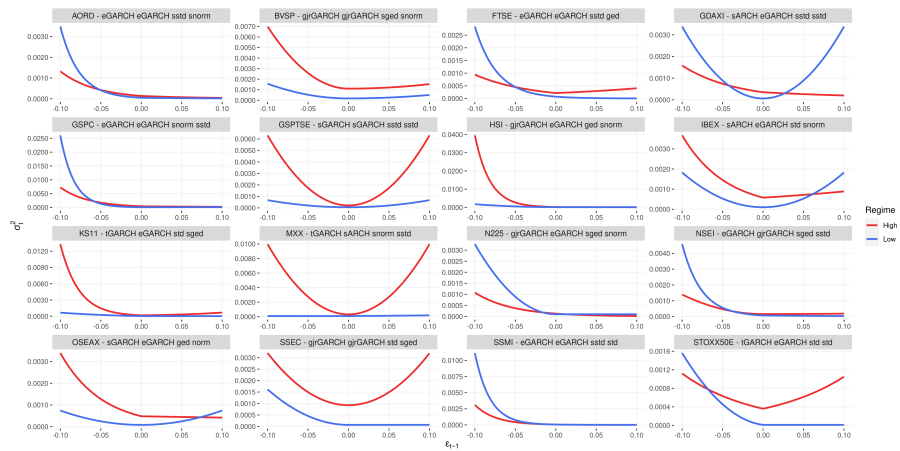


**Fig. 5** News Impact Curve obtained from MS-GARCH models for 16 indices in our sample during the COVID-19 period

The shape and curvature of the NIC are determined by the the persistence/half-life of each model and the coefficients described in equation (1). In particular, we should look at coefficients that capture the effects of sign ($\eta_1$) and magnitude ($\alpha$) on volatility since their interpretation varies according to the selected function.

If we consider persistence/half-life as shown in Table 8, the values for low volatility regimes are lower than those for high volatility regimes for all stock indices. This is as expected given the restrictions of the filtering protocol. Moreover, negative shocks cause more significant impacts on volatility than do positive shocks, except for models that stipulate symmetrical shock effects (sARCH and sGARCH).

Interestingly, the intensity of shocks is not always more significant in high volatility regimes. In the case of AORD stock index, for instance, the best model for both regimes is the eGARCH. Here, negative shocks are more important in the low volatility regime. However, the half-life of these shocks is almost six times smaller than shocks in the high volatility regime. Results are similar for the FTSE, GSPC, and SSMI indices.

**Table 8** Estimated results and coefficients for the MS-GARCH models for all stock indices in our sample

| Index | Model-PDF | Pers. | H2L | ω | α | β | $\eta_1$ | skew | shape | BIC | $\overline{P}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AORD | egarch-sstd | .888 | 5.9 | -1.099151 | .114 | .888 | 1.725 | .663 | 8.312 | -4,365.3 | 0.997 |
|  | egarch-snorm | .980 | 34.5 | -0.178424 | -.062 | .980 | 3.166 | .697 | - |  |  |
| BVSP | gjrgarch-sged | .963 | 18.2 | 0.000007 | .020 | .880 | 1.617 | .804 | 1.591 | -3,774.0 | 0.977 |
|  | gjrgarch-snorm | .989 | 64.9 | 0.000019 | .127 | .628 | 1.156 | .552 | - |  |  |
| FTSE | egarch-sstd | .879 | 5.4 | -1.150173 | .022 | .879 | 13.621 | .804 | 3.738 | -4,236.6 | 0.997 |
|  | egarch-ged | .970 | 23.0 | -0.254639 | -.146 | .970 | .402 | - | 1.211 |  |  |
| GDAXI | sarch-sstd | .332 | 0.6 | 0.000063 | .332 | - | - | .805 | 4.915 | -4,073.2 | 0.997 |
|  | egarch-sstd | .980 | 34.1 | -0.161594 | -.086 | .980 | 2.179 | 1.013 | 3.069 |  |  |
| GSPC | egarch-snorm | .864 | 4.7 | -1.276956 | .289 | .864 | .910 | .683 | - | -4,287.8 | 0.993 |
|  | egarch-sstd | .956 | 15.2 | -0.353842 | -.210 | .956 | 1.544 | .873 | 4.334 |  |  |
| GSPTSE | sgarch-sstd | .935 | 10.4 | 0.000004 | .062 | .874 | - | .766 | 7.626 | -4,666.9 | 0.986 |
|  | sgarch-sstd | .989 | 64.9 | 0.000006 | .610 | .380 | - | .553 | 7.052 |  |  |
| HSI | gjrgarch-ged | .813 | 3.3 | 0.000024 | .040 | .738 | 1.002 | - | 1.311 | -3,970.2 | 0.994 |
|  | egarch-snorm | .922 | 8.5 | -0.687551 | -.266 | .922 | 1.460 | .373 | - |  |  |
| IBEX | sarch-std | .173 | 0.4 | 0.000099 | .173 | - | - | - | 4.769 | -3,963.9 | 0.996 |
|  | egarch-snorm | .920 | 8.4 | -0.609117 | -.248 | .920 | .622 | .845 | - |  |  |
| KS11 | tgarch-std | .757 | 2.5 | 0.001346 | .101 | .757 | 1.000 | - | 99.336 | -4,134.7 | 0.996 |
|  | egarch-sged | .840 | 4.0 | -1.341366 | .443 | .840 | .513 | .787 | 1.479 |  |  |
| MXX | tgarch-snorm | .036 | 0.2 | 0.009100 | .022 | .003 | -.995 | .990 | - | -4,106.6 | 0.997 |
|  | sarch-sstd | .966 | 19.8 | 0.000316 | .966 | - | - | .971 | 5.003 |  |  |
| N225 | gjrgarch-sged | .532 | 1.1 | 0.000055 | .079 | .383 | 1.000 | .909 | 1.492 | -4,066.6 | 0.994 |
|  | egarch-snorm | .978 | 31.2 | -0.197241 | -.010 | .978 | 23.279 | .999 | - |  |  |
| NSEI | egarch-sged | .921 | 8.5 | -0.743769 | .148 | .921 | 1.486 | .928 | 1.931 | -4,180.8 | 0.979 |
|  | gjrgarch-sstd | .964 | 18.9 | 0.000006 | .030 | .914 | 1.042 | .484 | 5.416 |  |  |
| OSEAX | sgarch-ged | .908 | 7.2 | 0.000008 | .066 | .842 | - | - | 1.249 | -4,155.6 | 0.992 |
|  | egarch-norm | .919 | 8.3 | -0.627279 | -.185 | .919 | 1.150 | - | - |  |  |
| SSEC | gjrgarch-std | .894 | 6.2 | 0.000008 | .039 | .819 | 1.002 | - | 7.897 | -4,304.1 | 0.980 |
|  | gjrgarch-sged | .980 | 34.3 | 0.000024 | .000 | .753 | 402.016 | .921 | 0.725 |  |  |
| SSMI | egarch-sstd | .817 | 3.4 | -1.792510 | .077 | .817 | 4.151 | .740 | 8.069 | -4,476.2 | 0.995 |
|  | egarch-std | .983 | 40.0 | -0.166350 | .037 | .983 | 7.331 | - | 4.777 |  |  |
| STOXX50E | tgarch-std | .845 | 4.1 | 0.000493 | .182 | .845 | 1.000 | - | 4.336 | -4,126.4 | 0.994 |
|  | egarch-std | .955 | 14.9 | -0.367268 | -.194 | .955 | 0.029 | - | 19.837 |  |  |

Note: for all columns except Index and BIC, top/bottom models stands for low/high volatility models. Pers, H2L and $\overline{P}$ means Persistence, Half-Life and Probability mean $(p_{11}, p_{22})$.

Source: computed by the authors from Yahoo! Finance data.

Some care is required to properly interpret the different models. For eGARCH models, if we multiply $|\alpha|$ by $-\eta_1$, we obtain the $\theta$ coefficient of the original model due to [41], which captures the effect of the signal on volatility. Thus, for the SSMI stock index, the low volatility regime [-.321] causes a more significant impact than the high volatility regime [-.273] when the return is negative. Results are similar for the AORD and FTSE indices.

As for the GSPC index, although $\eta_1$ is smaller in the low volatility regime, its $|\alpha|$ is more significant than in the high volatility regime. This result indicates that a shock has a greater impact on volatility in the low-volatility regime for log-returns below -5%. However, it has a half-life that is about three times shorter than for shocks under high-volatility.

In the case of the N225 index, we can use the expression $\alpha \left(1 + \eta_1\right)^2$ to obtain the $g_1 + g_2$ coefficient found in the original model due to [42] when $\varepsilon_t < 0$. The leverage effect in low volatility [.317] is stronger than in high volatility [-.230], where the EGARCH model fits the data better, but the half-life is about 28 times shorter.

To conclude, averaging across markets, low-regime volatility is 15.6% [SD=2.8%] with 508.6 days [SD=96.9] low-volatility days. In contrast, high-regime volatility is 37.9% [SD=16.3%] on average with 159.4 high-volatility days [96.9]. The BIC values of the MS-GARCH models are also much smaller than those for the long-memory models, which supports the earlier finding that a Markov-Switching component is important to model stock market volatility, and that COVID-19 had an impact on volatility through the probabilities and intensities of the two regimes.

## 4.5  Which model should we choose?

Since the Markov-Switching (MS) models have a much smaller BIC than do the models from the long-memory GARCH family, it is fair to conclude that they perform better to explain the volatility of stock indices. The next step is to choose the best MS model for each stock index, by comparing the results shown in Tables 7 and 8.

If we adopt the BIC as a decision criterion, the MSwM is the best model for the MMX and SSEC indices, while the MS-GARCH model is the best for the other 14 indexes. On the other hand, if we use as a criterion the mean $(p_{11}, p_{22})$, then the MS-GARCH model is the best for all indices. Also note that while there was at least one MS-GARCH model specification that respected the filtering protocol for all stock indices, the MSwM failed to yield an acceptable model for five of the indices. Therefore, MS-GARCH models seem to be the best at explaining the fundamental structure of stock index volatility during the COVID-19 period.

## 4.6  The relationship between COVID-19 cases and stock index volatility

Figures 6 and 7 show the evolution in confirmed COVID-19 cases for each of the 16 countries in our sample (as shown in Table 3) for 2020 and 2021-2022, respectively. We collected the data using the *tidycovid19* R library [69]. We split the data in two for purposes of generating the figures, because the scale of transmission was higher in 2021-2022 due to the high transmissibility of the omicron variant [70].

In the case of the Eurozone (Stoxx50E index), we build the curve representing the growth in COVID-19 cases by aggregating data from the 19 countries that compose it.[2]

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[2]The countries are: Belgium, Germany, Ireland, Spain, France, Italy, Luxembourg, the Netherlands, Austria, Portugal, Finland, Greece, Slovenia, Cyprus, Malta, Slovakia, Estonia, Latvia, and Lithuania.
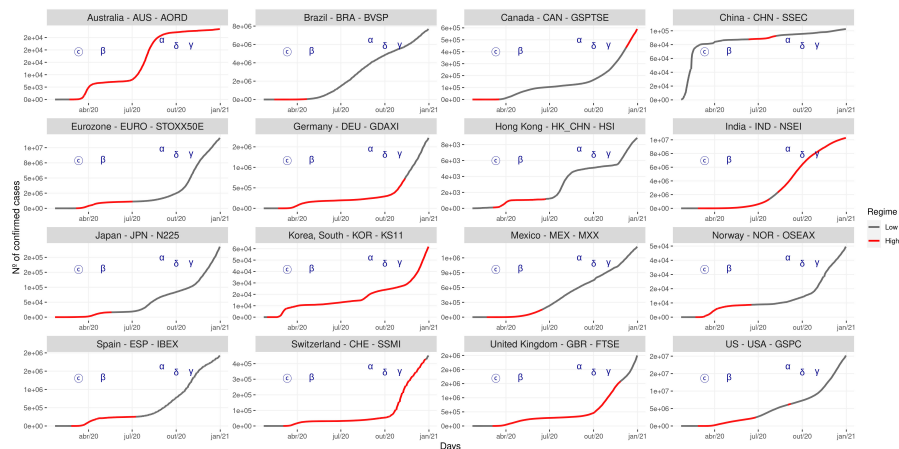
**Fig. 6** Confirmed COVID-19 cases and stock index volatility regime in 2020

In each sub-figure, the curve representing the growth in COVID-19 cases is colored black or red depending on the stock market index volatility regime (low or high), as determined by the MS-GARCH models presented in subsection 4.4. As before, we mark with a symbol the date of the first documented samples of each Variant of Concern [VoC] [23].[3]

A visual inspection makes it clear that the relationship between COVID-19 cases and volatility regimes was quite different in 2020 as opposed to 2021-2022.

In 2020, many stock indices were in the high volatility regime for most of the year as the number of new cases increased (AORD, GDAXI, NSEI, KS11, SSMI, and FTSE). Yet, most stock indices were in the low volatility regime

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[3]These VoC are [alpha ($\alpha$), beta ($\beta$), gamma ($\gamma$), delta ($\delta$), omicron (o)], and the beginning of the pandemic (March 11, 2020) is identified by the symbol Ⓒ.
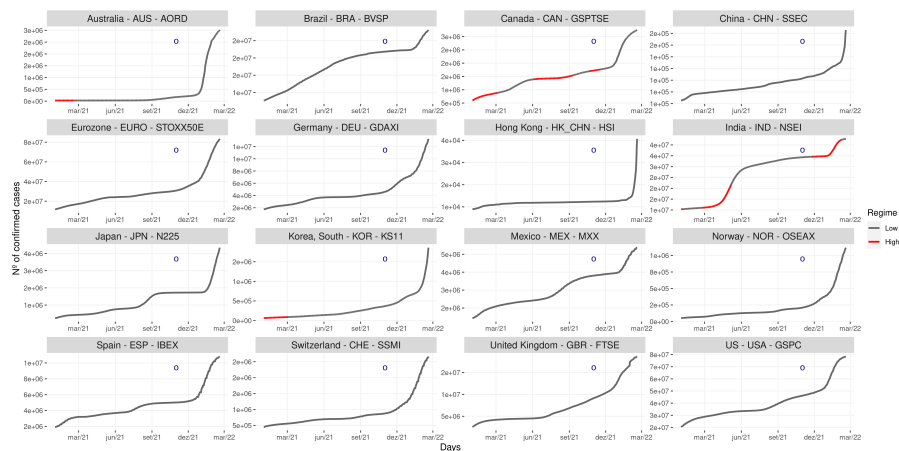


**Fig. 7** Confirmed COVID-19 cases and stock index volatility regime between January 2021 and February 2022

most the year, despite the emergence of new VoC and increasing cases of COVID-19 (BVSP, GSPTSE, SSEC, STOXX50E, HSI, N225, MXX, OSEAX, IBEX, and GSPC). Thus, countries with distinct social structures, lifestyles, and cultural backgrounds tend to adopt different policies to delay the spread of COVID-19, trying to balance their culture and policy [71].

Between January 2021 and February 2022, vaccination rates advanced in all countries in our sample. All stock indices stayed in the low volatility regime most of the year or the entire year, despite the high transmissibility of the omicron variant that initiated a new wave of contagion. In only two countries, Canada [GSPTSE] and India [NSEI], do we see a link between the omicron variant and a high volatility state. In Australia [AORD] and South Korea [KS11], there are brief periods of a high volatility regime early in 2021, but these do not coincide with omicron.

Overall, the results shown in Figures 6 and 7 strengthen the evidence from Figure 4, namely that the greatest occurrence of high volatility regimes across stock indices happened between the time of the emergence of the COVID-19 pandemic and the appearance of the Beta variant. In contrast, the emergence of new variants had a much smaller impact on stock market volatility.

# 5  Final remarks

The purpose of this study was to find, for each of 16 countries, the best model to describe the impact of COVID-19 on stock index volatility. Our empirical approach allows for flexibility to assess whether the impact caused long-memory persistence, structural breaks, or both.

To our knowledge, this is the most comprehensive empirical study to date on the impact of COVID-19-related shocks to financial market volatility, in terms of scope (number of country stock indices) and model flexibility (number of specifications allowing for different volatility processes and innovation distributions). In particular, our approach allows for combining long memory persistence with Markov-switching for different volatility regimes. We find that both features are empirically important in our sample.

First, for each market index we fit price log-return time series data to the best individual GARCH model (out of many possible candidates). We find the best model using three steps: statistical significance, tests that ensure that we respect Value at risk and Expected shortfall exceedance rates, and attaining the lowest Bayesian Information Criterion value.

We find that while no single GARCH specification best captures volatility in all 16 stock indices, ten stock indices are well described by either the Threshold GARCH (TGARCH) or the NAGARCH model. As for innovations, for nine stock indices the best assumption is the Johnson reparameterized SU distribution, which is a transformation of the Normal distribution that allows for greater kurtosis.

For most stock indices, volatility is persistent with a half-life of shocks around 20-40 days (i.e., 1-2 months). A notable exception is the HSI (Hong

Kong) with a half-life of only 5.3 days. In general, volatility is more persistent in European markets and less persistent in Asian markets.

In addition, we find that high volatility is clustered in the period between the beginning of the COVID-19 pandemic and the emergence of the Beta variant. Indeed, other variants of concern did not have sizable impacts on volatility. We obtain News Impact Curves from the different GARCH models to assess the impact of past shocks on volatility. For 14 of 16 stock indices, this curve is asymmetrical, implying that negative shocks increase volatility more than do positive shocks.

The Markov-switching models allow for sharp breaks in volatility, thus distinguishing between regimes. Among our findings, the BVSP (Brazil) index has the highest overall volatility but interestingly, spends the most time in the low-volatility regime and the fewest days in the high-volatility regime. In contrast, the STOXX50E (Euro zone) index has the lowest overall volatility, but spends the most days in the high-volatility regime and the fewest in the low-volatility regime.

Therefore, the results showed that COVID-19 affected long-memory persistence as well as regime-switching in the volatility of major world stock indices. The impacts on volatility differed between stock indices, which required specific model-distribution configurations to better capture the stylized facts described in the literature.

To this end, we combine the two frameworks. We find that the EGARCH specification is best suited for most markets (16 indices), as is the skewed student-$t$ distribution (10 indices). As in the baseline GARCH analysis, the NIC is asymmetrical for most indexes, with negative shocks having a more significant impact on volatility than positive shocks.

Moreover, we show that the News Impact Curves are quite different in high and low volatility regimes. In all indices, the NIC is steeper in one regime than the other. Interestingly, it is not always steeper in the high volatility regime. For nine markets, the NIC is steeper in the high regime, but in seven markets, it is steeper in the low volatility regime. Overall, the combined MS-GARCH model performs better than using only a MSwM or single GARCH model.

We find that the main impact of COVID-19 on stock index volatility occurred in the first wave of contagion. Although there is some evidence of other effects of COVID-19 on stock index volatility linked to the emergence of new variants, they were less intense or even hardly detectable.

In methodological terms, our study also contributes by presenting new filtering protocols, written in freely available software. They allow for broader, more sophisticated, and efficient model selection to apply to different assets such as stock indices.

# Declarations

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- **Availability of data and materials**: Data are obtained by the code available in the supplementary materials.
- **Code availability**: the algorithm written for this research is available in the supplementary materials.
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