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Abstract

We develop simple versions of upper bounds of the widely used systemic risk measure of ΔCoVaR that are straightforward to calculate, and may prove useful as (conservative) benchmarks in an applied context.

Keywords: ΔCoVaR; systemic risk; risk measure; probability bounds
JEL: G01; G21; G28; G32

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1 Introduction

ΔCoVaR, a widely used systemic risk measure introduced by Adrian & Brunnermeier (2016), corresponds to the Value-at-Risk (VaR) of the financial system obtained conditionally on a specific event affecting a given firm.[1] In a general framework which assumes that market and firm returns are linearly dependent, but otherwise makes only the most basic assumptions regarding the distribution of stock returns, we draw on established upper bounds for VaR using Cantelli's inequality and the one-sided Vysochanskii-Petunin inequality[2] to construct corresponding simple versions of upper bounds for ΔCoVaR. These measures are straightforward to calculate, as illustrated for the (listed) G-SIBs on the Financial Stability Board's 2020 list of global systemically important banks, and may prove useful as simple (conservative) ΔCoVaR benchmarks for applied researchers, market participants as well as financial regulators.

2 Simple ΔCoVaR bounds

In line with Adrian & Brunnermeier (2016), the ΔCoVaR of firm i is defined as the difference between the VaR of the market return conditional on firm i being in financial distress and the VaR of the market return conditional on firm i being in its median state. In line with the common framework in Benoit et al. (2017),[3] let us assume that the vector of market and firm (demeaned) returns $r_t^{'} = (r_{mt}\ r_{it})$ follows a bivariate GARCH process such that

$$r_t = H_t^{1/2} \nu_t \tag{1}$$

[1]For some recent papers using this measure, see e.g. Anginer et al. (2018a), Anginer et al. (2018b), Bakkar et al. (2019), Berger et al. (2020), Bostandzic & Weiss (2018), Brownlees et al. (2020), Brunnermeier et al. (2020), Chu et al. (2019).

[2]As derived in Barrieu & Scandolo (2015) and Mercadier & Strobel (2021), respectively; note that one-sided inequalities are most relevant in the sense of "how bad could losses be".

[3]See also Giesecke & Kim (2011), Chen et al. (2013) and Löffler & Raupach (2018) for alternative perspectives on systemic risk.

where innovation $\nu_t' = (\varepsilon_{mt}\ \xi_{it})$ is i.i.d., with $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}\left(\nu_t \nu_t'\right) = I_2$, a two-by-two identity matrix, and the conditional variance–covariance matrix H_t is defined as:

$$H_t = \begin{pmatrix} \sigma_{mt}^2 & \sigma_{it}\sigma_{mt}\rho_{it} \\ \sigma_{it}\sigma_{mt}\rho_{it} & \sigma_{it}^2 \end{pmatrix} \tag{2}$$

where σ_{mt} and σ_{it} are the conditional standard deviations and ρ_{it} the conditional correlation. The assumption that innovations ε_{mt} and ξ_{it} are independently distributed at time t implies that the dependence between firm and market returns is fully captured by the (time-varying) conditional correlation ρ_{it}.

Given Equations (1) and (2), Benoit et al. (2017) show that "the ΔCoVaR of a given financial institution i is proportional to its tail risk, as measured by its VaR"; in particular, for losses $X_i = -r_i$, one can write:[4]

$$\Delta\,\mathrm{CoVaR}_{\alpha t}(X_i) = \gamma_{it} \cdot [\mathrm{VaR}_{\alpha t}(X_i) - \mathrm{VaR}_{0.5,t}(X_i)] \tag{3}$$

where $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$, i.e. the linear projection coefficient of the market return on the firm return.

An implicit definition of the Value-at-Risk (VaR) of losses X at confidence level α is $\mathbb{P}(X \geq \mathrm{VaR}_\alpha(X)) = \alpha$ for short time-horizons, where X can be assumed as $E[X] = 0$ and $V(X) = \sigma^2$. For the most agnostic of distributional assumptions, requiring only the first two moments of losses X to exist, Barrieu & Scandolo (2015) give an upper bound of the VaR of X at confidence level α using the Cantelli (1928) inequality as

$$\mathrm{VaR}_\alpha(X) \leq \sigma\sqrt{\frac{1}{\alpha} - 1} \tag{4}$$

For applications where the additional assumption of unimodality of losses X is not overly restrictive,[5] Mercadier & Strobel (2021) provide a refined upper bound of

[4] The proof of Equation (3) is given in Benoit et al. (2013, Appendix B).

[5] Mercadier & Strobel (2021, fn. 3) reports unimodality tests for stock returns, for a sample of 1748 firms in 44 countries covering the period 1991q1–2020q1; the hypothesis of unimodality was not rejected in 96% of all cases at the quarterly level using conditional, i.e. GARCH(1,1) filtered, firm returns, with analogous results obtained for unconditional firm returns.

the VaR of X using the one-sided Vysochanskii-Petunin inequality, for usual values of confidence levels that satisfy $\alpha \leq 1/6$, as

$$\mathrm{VaR}_{\alpha}(X) \leq \sigma \sqrt{\frac{4}{9\alpha} - 1} \tag{5}$$

Drawing on these results, we can use equation (3) and inequality (4), assuming that $\mathrm{VaR}_{0.5}(X) \geq 0$ in line with Adrian & Brunnermeier (2016), to provide an upper bound of the ΔCoVaR using Cantelli's inequality as follows:

$$\Delta\,\mathrm{CoVaR}_{\alpha t}(X_i) \leq \rho_{it} \cdot \sigma_{mt} \cdot \sqrt{\frac{1}{\alpha} - 1} := \Delta\,\mathrm{CoVaR}^{cant}_{\alpha t}(X_i) \tag{6}$$

If losses X_i can be assumed to be unimodal, for usual values of confidence levels that satisfy $\alpha \leq 1/6$, we can use equation (3) and inequality (5), again assuming that $\mathrm{VaR}_{0.5}(X) \geq 0$ as above, to refine this upper bound using the one-sided Vysochanskii-Petunin inequality as follows:

$$\Delta\,\mathrm{CoVaR}_{\alpha t}(X_i) \leq \rho_{it} \cdot \sigma_{mt} \cdot \sqrt{\frac{4}{9\alpha} - 1} := \Delta\,\mathrm{CoVaR}^{osvp}_{\alpha t}(X_i) \tag{7}$$

The measures $\Delta\,\mathrm{CoVaR}^{osvp}_{\alpha t}(X_i)$ and $\Delta\,\mathrm{CoVaR}^{cant}_{\alpha t}(X_i)$ represent upper bounds of the ΔCoVaR when market and firm returns are assumed to be linearly dependent, and unimodality of firm returns can either be assumed or more agnostic assumptions prevail. They are proportional to the product of the correlation coefficient between market and firm returns ρ_{it} and the standard deviation of market returns σ_{mt}, with the respective proportionality coefficients being nonlinear functions of the confidence level α. As a consequence, they are straightforward to calculate, and may prove useful as simple (conservative) ΔCoVaR benchmarks in an applied context.

3 Empirical illustration

To illustrate, we calculate our simple benchmarks for ΔCoVaR as well as the regular measure for the 29 (listed) G-SIBs on the Financial Stability Board (FSB)'s 2020 list of global systemically important banks,[6] using daily stock return data extracted from Bloomberg L.P. Rather than estimating the ΔCoVaR with a quantile regression, as proposed by Adrian & Brunnermeier (2016), we follow Benoit et al. (2013, Appendix F) and implement a GARCH-DCC model, using a coefficient α of 5% and setting the threshold C equal to the unconditional market daily VaR.[7] To construct our simple benchmarks $\Delta\,\mathrm{CoVaR}_{\alpha t}^{osvp}(X_i)$ and $\Delta\,\mathrm{CoVaR}_{\alpha t}^{cant}(X_i)$, we draw on the same GARCH-DCC model to calculate the required second-order moments.

Figure 1 focusses on JPMorgan Chase & Co. (JPM), HSBC Holdings (HSBA) and Mitsubishi UFJ Financial Group (MITF), i.e. the most systemically relevant G-SIBs in the US, Europe and Asia, respectively, highlighting that our simple ΔCoVaR benchmarks closely track the regular ΔCoVaR measure for both the core global financial crisis (GFC) period of 1/1/2008–7/15/2009 and the early Covid period of 10/1/2019–10/1/2020.[8]

We note that the measure based on the one-sided Vysochanskii-Petunin inequality provides significantly tighter upper bounds for ΔCoVaR; this is further illustrated by Figure 2, which presents box plots for the corresponding ratios $\Delta\,\mathrm{CoVaR}^{osvp}/\Delta\,\mathrm{CoVaR}$ and $\Delta\,\mathrm{CoVaR}^{cant}/\Delta\,\mathrm{CoVaR}$, as calculated on a daily basis over the period 1/1/2001–

[6] See FSB (2020); they are: Citigroup (C), HSBC Holdings (HSBA), JP Morgan Chase & Co (JPM), Bank of America (BAC), Bank of China (BCL), Barclays (BARC), BNP Paribas (BNP), Deutsche Bank (DBK), Industrial & Commercial Bank of China (ITL), Mitsubishi UFJ Financial Group (MITF), China Construction Bank (CON), Agricultural Bank of China (ABC), Bank of New York Mellon (BK), Credit Suisse Group (CSGN), Goldman Sachs Group (GS), Credit Agricole (CRDA), ING Groep (INGA), Mizuho Financial Group (MIZH), Morgan Stanley (MS), Royal Bank of Canada (RY), Banco Santander (SAN), Societe Generale (SGE), Standard Chartered (STAN), State Street (STT), Sumitomo Mitsui Financial Group (SMFI), Toronto-Dominion Bank (TD), UBS Group (UBSG), Unicredit (UCG), Wells Fargo & Cu (WFC) (note that Groupe BPCE is not listed).

[7] Calculations are carried out using MATLAB R2020a, drawing in part on code provided by Benoit et al. (2013) via www.runmycode.org.

[8] Similar results are obtained for the other G-SIBs on the FSB's 2020 list; these are available in the (online) technical appendix.

Figure 1: Simple benchmarks vs regular ΔCoVaR measure (for $\alpha = 0.05$), major G-SIBs (JPM, HSBA & MITF), GARCH-DCC model, for GFC & Covid periods

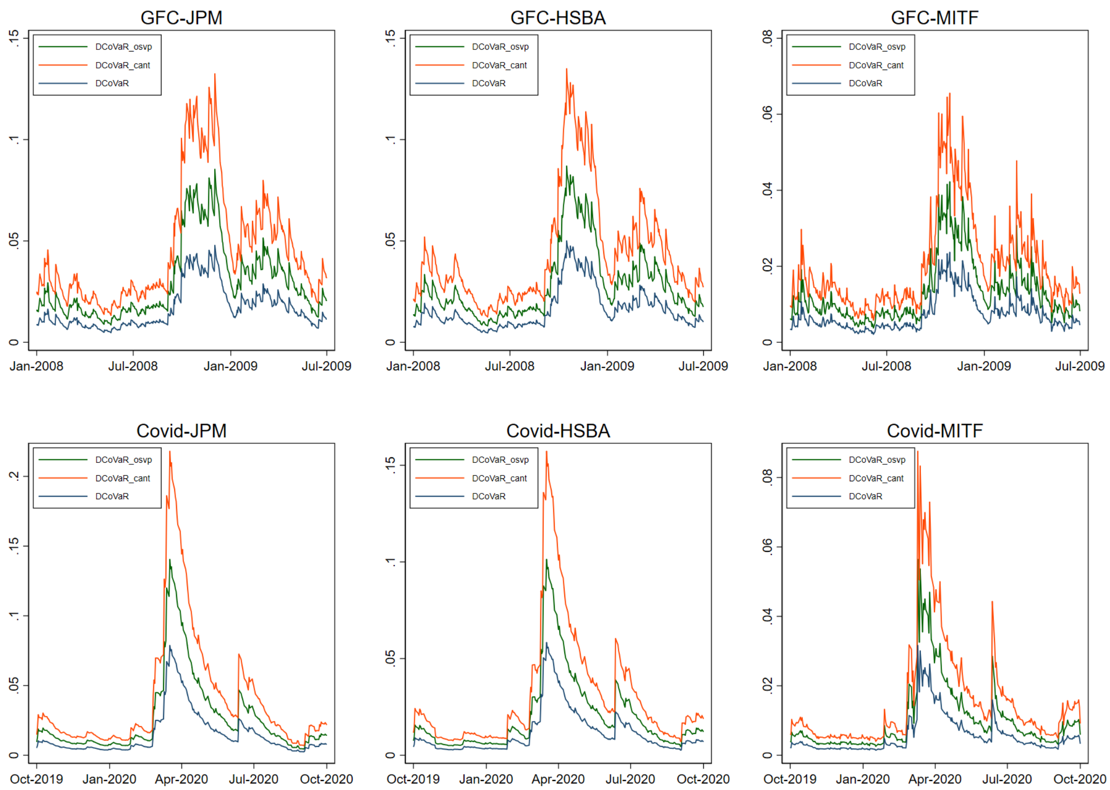
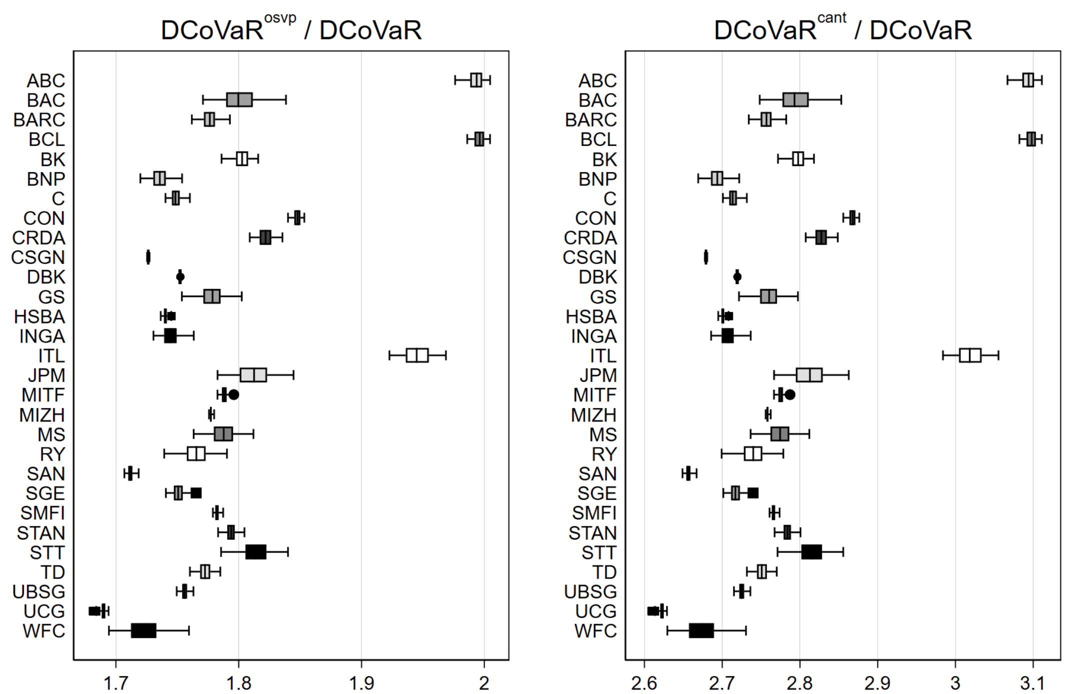


Figure 2: Box plots for ratios of simple benchmarks vs regular ΔCoVaR measure, FSB 2020 G-SIB list, for period 1/1/2001–12/31/2020



12/31/2020 for each of the 29 (listed) G-SIBs on the FSB's 2020 list.

4 Conclusion

We develop simple versions of upper bounds of the widely used systemic risk measure of ΔCoVaR, drawing on the common framework for systemic risk measures introduced by Benoit et al. (2017), in combination with upper bounds for VaR using Cantelli's inequality and the one-sided Vysochanskii-Petunin inequality. Relying on only the most basic assumptions regarding the distribution of stock returns, these simple (conservative) ΔCoVaR benchmarks are straightforward to calculate, as illustrated for the (listed) G-SIBs on the FSB's 2020 list of global systemically important banks, and may prove useful for applied researchers, market participants as well as financial regulators.

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