UNIVERSITY OF BIRMINGHAM
University of Birmingham Research at Birmingham

Influence of initial conditions on granular dynamics near the jamming transition

Windows-Yule, C.R.K.; Rosato, A.D.; Rivas, N.; Parker, David

DOI:
10.1088/1367-2630/16/6/063016

License:
Creative Commons: Attribution (CC BY)

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (Harvard):
Windows-Yule, CRK, Rosato, AD, Rivas, N & Parker, D 2014, 'Influence of initial conditions on granular dynamics near the jamming transition', *New Journal of Physics*, vol. 16, 063016. https://doi.org/10.1088/1367-2630/16/6/063016

Link to publication on Research at Birmingham portal

Publisher Rights Statement:
Eligibility for repository : checked 23/06/2014

General rights
Unless a licence is specified above, all rights (including copyright and moral rights) in this document are retained by the authors and/or the copyright holders. The express permission of the copyright holder must be obtained for any use of this material other than for purposes permitted by law.

•Users may freely distribute the URL that is used to identify this publication.
•Users may download and/or print one copy of the publication from the University of Birmingham research portal for the purpose of private study or non-commercial research.
•User may use extracts from the document in line with the concept of 'fair dealing' under the Copyright, Designs and Patents Act 1988 (?)
•Users may not further distribute the material nor use it for the purposes of commercial gain.

Where a licence is displayed above, please note the terms and conditions of the licence govern your use of this document.

When citing, please reference the published version.

Take down policy
While the University of Birmingham exercises care and attention in making items available there are rare occasions when an item has been uploaded in error or has been deemed to be commercially or otherwise sensitive.

If you believe that this is the case for this document, please contact UBIRA@lists.bham.ac.uk providing details and we will remove access to the work immediately and investigate.

Download date: 09. Apr. 2024

iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Influence of initial conditions on granular dynamics near the jamming transition

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 New J. Phys. 16 063016

(http://iopscience.iop.org/1367-2630/16/6/063016)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 147.188.224.230
This content was downloaded on 23/06/2014 at 12:03

Please note that terms and conditions apply.

New Journal of Physics
The open access journal at the forefront of physics
Deutsche Physikalische Gesellschaft DPG | IOP Institute of Physics

Influence of initial conditions on granular dynamics near the jamming transition

C R K Windows-Yule[1], A D Rosato[2], N Rivas[3] and D J Parker[1]
[1] The University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
[2] New Jersey Institute of Technology, Newark, New Jersey 07102, USA
[3] Multi Scale Mechanics (MSM), MESA+, CTW, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands
E-mail: windowsyule@gmail.com

Received 16 March 2014, revised 23 April 2014
Accepted for publication 2 May 2014
Published 10 June 2014
New Journal of Physics **16** (2014) 063016
doi:10.1088/1367-2630/16/6/063016

Abstract
In this paper, we compare the behaviours of two vibrofluidized granular systems, identical in terms of their composition, geometry and driving parameters, differing only in their initial conditions. It is found that, by increasing the strength with which a system is initially excited, considerable differences in system composition and dynamics persist even after the driving is returned to its typical value. The observed changes in particle mobility and packing density are shown to result in marked differences in segregative behaviour for equivalent steady-state systems distinguished only by the history of their driving. The ability to significantly increase the rate of segregation in a granular system simply through the application of a short burst of intense vibration clearly has potential industrial applications.

Keywords: granular, segregation, jamming, caging, granular dynamics, hysteresis

1. Introduction

The behaviour of vibrated granular materials is of great relevance to a number of physical phenomena and industrial processes [1, 2]. Two phenomena particularly pertinent to industry

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

© 2014 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

are those of granular segregation [3] and jamming [4]. The former can, for instance, prove a useful tool in the sorting of granular materials [5], or a serious hindrance to their mixing [6]. The latter, meanwhile, may lead to the obstruction of granular flows [7] or problems during compaction processes [8]. The processes of jamming and segregation in a granular system can be influenced by numerous factors, including the amount of energy supplied to the system [9, 10], its geometry [11, 12], or the material properties of the particles involved [13–15] to name but a few. Perhaps one of the most intriguing factors regarding the behaviour of a granular system approaching the jamming transition is the *history dependence* of its final state [16]: the hysteretic nature of such systems means that changes in initial configuration may alter the final, jammed state achieved by a system.

In this paper, we investigate how initial driving conditions can affect the final steady state of a continuously driven (CD), vibrofluidized granular bed. We find that the system's history may affect not only the packing fraction of the final steady state (as may be expected from previous studies) but also significantly alter the mobility of particles within the system. The observed variations in particle mobility are particularly striking since they show no direct correlation to the bulk density of the system concerned. We discuss the possible implications of these findings, including their potentially dramatic effect on segregative processes, and attempt to provide a qualitative explanation for the observed behaviour.

In particular, we note a potential application of our findings in the reduction of the time and energy requirements associated with the separation of granular materials. A particularly pertinent example is the recently proposed use of vertical vibrations to reclaim the valuable components of obsolete electronic equipment [17]—a growing problem in modern society [18]. The current methods of recycling electronic waste are not only inefficient, but also carry a significant environmental and human cost [19]. Improvements to this emerging technology may yield a transformative influence on the manner in which electronic waste is reprocessed, providing a strong motivation for our current research.

2. Experimental details

2.1. Experimental set-up

The experimental set-up consists of a cuboidal acrylic container with a square base of dimensions $L_x = L_y = 40$ mm and height $L_z = 200$ mm affixed to an LDS V721 electrodynamic shaker. The container houses either a monodisperse granular bed of N steel spheres, each of diameter $d = 5$mm, or a similar bed to which a single 5 mm glass sphere is added. The container is subjected to sinusoidal vibrations in the vertical direction, the amplitude, A, and frequency, f, of which are controlled using feedback from an accelerometer in order to ensure constant driving. Energy is thus transferred into the granular bed through particle collisions with the steel base of the container. The relatively large size of the spheres and the choice of container materials means that the influence of interstitial air in the system may be neglected [6], as may the effects of static charge [20]. The height of the container is, for the range of driving accelerations used, adequate to minimize particle collisions with the containers' upper boundary, meaning that the system can be considered 'open'. The number of particles in the system is varied to give a range of dimensionless resting bed heights, $H \in (5, 12)$, and thus allow variation of the dissipation parameter $F_d = H(1 - \varepsilon)$ [21], where ε is the interparticle coefficient of restitution. The energy input to the system, meanwhile, may be varied through the adjustment of f and A, and hence the dimensionless acceleration $\Gamma = \frac{4\pi^2 f^2 A}{g}$ and energy parameter $S = \frac{4\pi^2 f^2 A^2}{dg}$ [22]. The alteration of these key control parameters allows various states of the granular system to be accessed [23].

2.2. Data acquisition—positron emission particle tracking (PEPT)

Data is acquired from the experimental system using PEPT. PEPT uses a dual-headed gamma camera to track the motion of a single tracer particle in three dimensions. The tracer particle used is identical to the others in the system aside from being 'labelled' with β^+-emitting radioisotopes, making PEPT a non-invasive technique. The back-to-back 511 keV gamma rays emitted due to β^+ annihilation within the tracer material are detected by the gamma camera. The detection of multiple such gamma ray pairs allows the position of the particle to be triangulated algorithmically and, with adequately high activity, the dynamics of the particle to be tracked. In this manner, PEPT can achieve a spatial resolution on the millimetre scale and temporal resolution of the order of milliseconds [24]. By tracking the single particle motion within a steady-state system for an adequately long time it is possible, through the principle of ergodicity, to extract quantitative information pertaining to the system as a whole. Thus, PEPT can successfully be used to measure and quantify various important parameters, including particle packing fractions, granular temperatures (and indeed the spatial variations of these quantities), mean squared displacements, velocity fields and autocorrelation functions and segregation intensities, to name but a few [25–28]. For full details of the PEPT technique, and the specific manners in which the aforementioned properties may be calculated from PEPT data, please refer to [24–28].

In the current study, PEPT is employed in two ways; firstly, analysis of the motion of a single 5 mm steel particle in a monodisperse system of identical particles over a long period of time ($t > 3600$ s) allows the state and dynamics of the system to be probed. Secondly, the tracking of a single 5 mm *glass* particle within a similar bed of steel beads allows investigation of the segregative processes within the system.

2.3. Experimental procedure

The main experiment focuses on the comparison of two systems. Both are identical in terms of geometry, dimensions, particle number and material properties. In both systems, a steady state is produced through vibration at a constant frequency f_0 and amplitude A_0, corresponding to a constant acceleration Γ_0. The systems differ only in the *history* of their driving—while one system is driven at a constant, fixed amplitude A_0, the other is initially driven, at the same frequency f_0, with an increased amplitude A_i, which is subsequently decreased to A_0 and the system allowed to relax into a steady state at this reduced amplitude. Unless otherwise stated, the duration of the initial strong excitation is $t_i = 3$ s, and the transition between A_i and A_0 is effectively instantaneous. Throughout this paper, for brevity and clarity, we shall refer to the former case as a CD system, and the latter as an initially strongly driven (ISD) system. Comparison of the CD and ISD cases is made over a range of H and A_0, allowing the hysteretic effects of the initial driving strength to be analysed for a variety of system densities and energies.

In order to ensure that the driving history is the only difference between the two systems, several precautions were taken, and preliminary tests conducted. Before each run, each system was prepared in a similar manner, with beads being poured into the container and then stirred to produce a random loose-packed bed. However, to ensure that subtle changes in initial conditions did not affect the system's later behaviour, multiple runs for both the CD and ISD case with $H = 7$ and $\Gamma_0 = 13$ were conducted. Runs were also conducted for systems which had not been stirred, but were simply poured into the container. In all cases, no significant variations in behaviour were observed. In order to discount possible effects due to particle aging, each set of experiments was repeated, with the order in which the runs were conducted alternating between CD and ISD. Direct comparison was also made between CD and ISD cases using new particles. Over the parameter range tested, particle aging was not found to produce any noticeable changes in the behaviour of the system. In all cases, data is recorded over a period $3600 \leqslant \tau_{run} \leqslant 7200$ s. The run length is adjusted according to the mobility of particles for a given set of driving and dissipation parameters—the less mobile the particles, the longer it will take for the tracer to explore the entire system, and hence the longer the required length of the run. In instances where the equilibrium behaviour of the bed is investigated, the bed is initially vibrated at A_0 for 500 s before readings are taken, allowing the system to attain a steady state. In all cases, velocity fields are analysed to ensure the absence of convective motion within the system.

Although the PEPT technique is capable of quantifying the equilibrium distribution, or steady-state degree of segregation, of a binary granular system [28, 29], the averaging processes used to calculate the relevant properties mean that the time-evolution of the system cannot be easily determined. Therefore, in order to investigate segregative behaviour, runs were also conducted tracking the motion of a single glass sphere through the bed. In each instance, this glass 'intruder' is initially placed at the bottom of the system. Since the diameters of the two particle species are equal ($d_{glass} = d_{steel} = 5$ mm) yet the density of the glass sphere is considerably smaller than that of its steel counterpart ($\rho_{glass} \approx 2500$ kg m^{-3}, $\rho_{steel} \approx 7850$ kg m^{-3}), the glass sphere can be expected to make an upward transit through the bed due to buoyancy forces within the system [30]. The time, τ_{rise}, taken for the tracer to move from the container base to the free surface of the system can be considered representative of the segregation rate for a similar, truly bidisperse system [31]. For each data set, particle motion was recorded over a period of time equal to τ_{rise} plus an additional 500 s. The extra 500 s of data was used to analyse the behaviour of the intruder after its initial upward transit, which can be used as an indication of the degree of segregation one might expect from an equivalent bidisperse system. For instance, if the tracer is observed to remain at the free surface of the system for the duration of the post-rise run, one would assume that the equivalent binary system would demonstrate complete, or near-complete, segregation. If, however, the particle is observed to periodically re-enter the bulk of the system, it is more likely that the system would only achieve a partially-segregated steady state. The regularity of the tracer's excursions into the steel bed can be thought of as representative of the degree to which a similar binary system might exhibit segregation/mixing. The analysis of this additional 500 s of data also ensures that one can distinguish between buoyancy-driven upward transits representative of segregative processes and upward motion due to convective flow within the system [32]. Since the rise of an intruder through a granular bed is—in particular for denser systems where motion is dependent on particle rearrangement [33]—a stochastic processes, runs were repeated at least ten times for each set of system parameters[4]. The rise time is then taken as the average of the observed values.

Although the above analysis cannot provide quantitative information regarding the segregation of fully bidisperse granular beds, it nonetheless elucidates important general trends pertaining to the system's segregative behaviour. In order to verify these trends, comparison can be drawn with simulational data for the fully-bidisperse case, as described in the following section.

3. Simulation details

In order to allow further analysis of the effects of excitation history on the segregative behaviours of a binary granular bed, discrete particle simulations were performed using the MercuryDPM software developed at the University of Twente [34–36]. The various parameters used in simulation were chosen to emulate their experimental counterparts as closely as possible. A cuboidal domain of dimensions $L_x \times L_y \times L_z = 40 \times 40 \times 200$ mm was used, with the base and sidewalls undergoing sinusoidal motion in the vertical direction, thus providing energy to a system of 5 mm spheres. Values of the resting bed height, H, and hence particle number, N, were chosen to match the values used in experiment. However, in the simulations, the composition of the bed is a 50/50 mixture of heavy and light particles, the specific values of particle size and density chosen to match those of the steel and glass particles used in experiment.

Interparticle coefficients of restitution, ε, were set to the experimentally measured effective elasticities of Feitosa and Menon [37], specifically, $\varepsilon_{\text{glass}} = 0.83$, $\varepsilon_{\text{steel}} = 0.79$. For cross-species collisions, the coefficient of restitution was taken as the geometric average of these values; although this manner of estimating the inter-species coefficient of restitution may seem overly simple, such a relationship is in fact a direct consequence of the spring-dashpot model of particle collisions [38–40]. The particle-wall coefficient of restitution was taken as the experimentally measured value $\varepsilon_w = 0.59$, while the frictional coefficient, μ, is set to a value of 0.1 [41] for both inter-particle and particle-wall interactions. These particular values of the various dissipative parameters have been previously shown to accurately reproduce the dynamical behaviour of systems similar to that currently under investigation [28]. The contact time, t_c, was set to a value of 10^{-5} s. In order to ensure that the implemented t_c was adequately large to prevent the underestimation of collisional energy loss [42, 43], a range of repeated simulations were performed with t_c values ten times greater and ten times smaller than the typical value used. The lack of significant variation observed in these simulations implies that $t_c = 10^{-5}$ is indeed a suitable choice.

[4] For systems where significant variance was observed, a greater number of repeated runs was used.

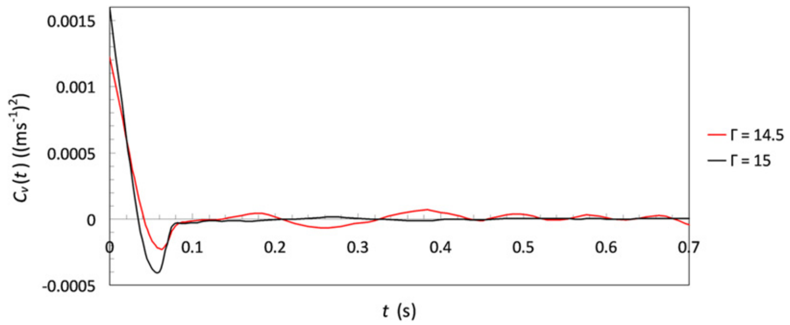

Figure 1. Experimentally acquired velocity autocorrelation function, $C_v(t)$, for continuously driven systems of resting bed height $H = 7$.

4. Results and discussion

In this section, we present results illustrating the effects of initial driving conditions on various aspects of the behaviour of a granular system. The section is split into two parts; in the first, we analyse the effects of a system's driving history on its dynamics for the case of monodisperse beds. In the second, we discuss the observed dynamics in the context of bidisperse systems, investigating their effects on segregative processes.

4.1. Monodisperse systems—caging effects and jamming

Through the analysis of data at a variety of resting bed heights, H, and driving strengths, S, it is apparent that, for the system under investigation, significant effects due to hysteresis arise only above a threshold bulk packing fraction, $\phi_h \approx 0.42$. Evidence of this transition point can be seen in figure 1, which shows a typical comparison of the time-evolution of the velocity autocorrelation function, $C_v(t)$, for systems with densities below and above the threshold value. The velocity autocorrelation function, defined as $C_v(t) = \langle v(t + t_0) \cdot v(t_0) \rangle$, shows the rate at which the velocity of a particle becomes decorrelated from its initial value. For the more strongly driven—and hence more dilute—system, $C_v(t)$ decays to zero fairly rapidly, indicating chaotic, history-independent behaviour [44]. For the denser system, one observes periodic structure in $C_v(t)$ even at relatively large times, demonstrating that the system retains some memory of its initial conditions even after multiple particle collisions [45]. Another example of the differing behaviours above and below the density threshold can be seen in figure 2, which compares mean squared displacements for the CD and the ISD cases. For densities below ϕ_h, the displacement behaviour is near identical in both cases. For densities above ϕ_h, however, clear differences in the systems' behaviours can be observed. It should be noted that the transition point remains consistent whether density variations are due to alterations in *H or S*.

The remainder of this section focuses on systems with packing densities $\phi > \phi_h$, investigating how further increases in density affect the dynamics of the system and the history-dependence exhibited. Figure 3 shows data for a system of resting bed height $H = 7$ vibrated at a constant frequency of 70 Hz with an amplitude $A_0 = 0.66$ mm. The ISD system is initially excited at the same frequency with an amplitude $A_i = 1.5A_0$. In this instance, $M(t)$ shows two

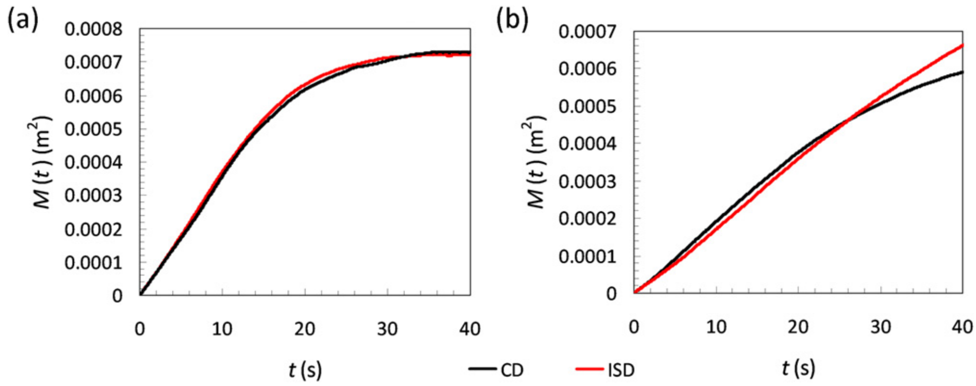

Figure 2. Comparison of experimental mean squared displacements, $M(t)$, for continuously driven (CD) and initially strongly driven (ISD) systems with (a) $H = 7$, $\Gamma_0 = 15$ ($\phi = 0.415$) and (b) $H = 7$, $\Gamma_0 = 14.5$ ($\phi = 0.428$). The ISD systems are initially excited with an acceleration $\Gamma_i = \frac{3}{2}\Gamma_0$.

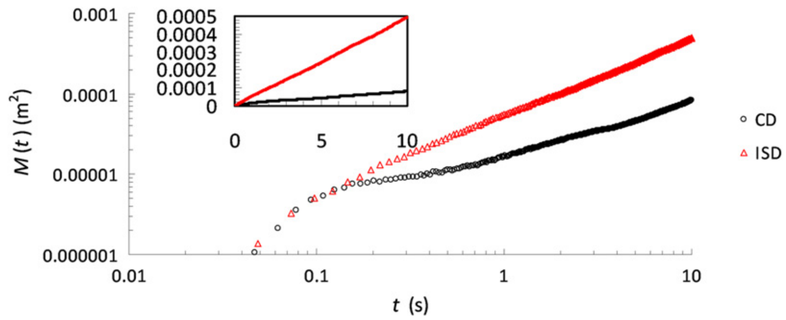

Figure 3. Experimentally measured mean squared displacement for $H = 7$, $\Gamma_0 = 13$. Data is shown for both continuously driven (CD) and initially strongly driven (ISD) systems, where $\Gamma_i = \frac{3}{2}\Gamma_0 = 19.5$.

distinct behaviours for the two systems—for the case of initially strong driving, the system demonstrates behaviour reminiscent of simple fluids, exhibiting ballistic behaviour at short times before making a transition to diffusive behaviour [46], as illustrated by the linear increase of $M(t)$ with time [47]. For the case of continuous driving, however, one observes a plateau of subdiffusive behaviour separating the ballistic and diffusive regimes, behaviour more akin to that observed in supercooled or glassy molecular systems [48]. It should be noted that care has been taken to ensure that the mean squared displacement behaviour shown and discussed here is truly representative of the behaviour of the systems concerned. In all cases, $M(t)$ is determined using data from the central region of the bed's main bulk; this region is taken as the central 50% of the dynamic bed height, $\tilde{H}$, defined as twice the system's vertical centre of mass position. The choice to use data from this domain means that the resulting $M(t)$ values will not be skewed by particles' dynamics close to the energizing base or at the bed's free surface, which are not representative of the system's bulk behaviour. The resultant $M(t)$ values are then compared with values obtained for a series of individual thin horizontal 'slices' through the system at various heights to ensure consistency.

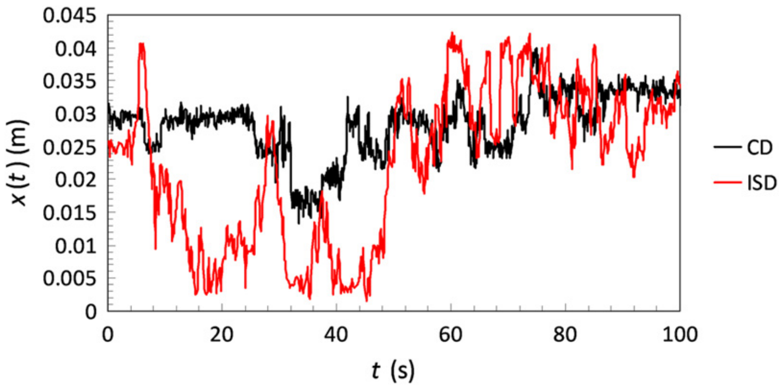

Figure 4. Time evolution of the x-position of a single tracer particle for the experimental system described in figure 3.

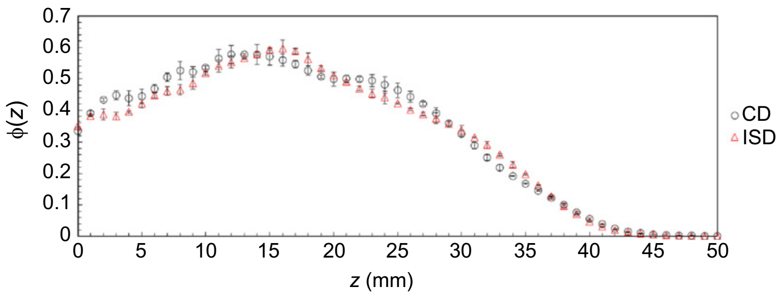

Figure 5. Experimental vertical density profiles for the system described in figure 3 .

The presence of an inflection in $M(t)$ demonstrates presence of *caging effects* in the system [49, 50], whereby particles become temporarily 'trapped' by their neighbours [51]. The presence of this caging in the CD case, and the markedly different dynamical behaviour for the ISD case, are perhaps more clearly illustrated in figure 4. For the ISD case, motion is fairly continuous, with the particle's trajectory approximating a random walk, as would be expected for a classically diffusive system. The single-particle motion for the CD case, however, demonstrates the presence of extended periods during which the tracer's position simply fluctuates about a fixed point. This motion confirms the behaviour inferred from the form of the system's mean squared displacement—particles become confined within cages formed by their neighbours, periodically managing to 'break free', before becoming caged once more by a new set of neighbours [52]. Although the dynamics of the CD and ISD systems are considerably disparate, their bulk densities—and indeed their packing profiles—are observed to remain remarkably similar, as illustrated in figure 5. Experimental packing profiles, such as those shown in figure 5, were extracted from PEPT data as follows: the experimental cell was divided into a series of equally sized segments in the vertical direction. The fractional residence time, $F(z)$, of the particle within each of these segments was then calculated. Due to the ergodicity of the steady-state systems under investigation, this residence time is directly proportional to the local packing fraction for each segment, allowing $\phi\left(z\right)$ to be calculated simply as:

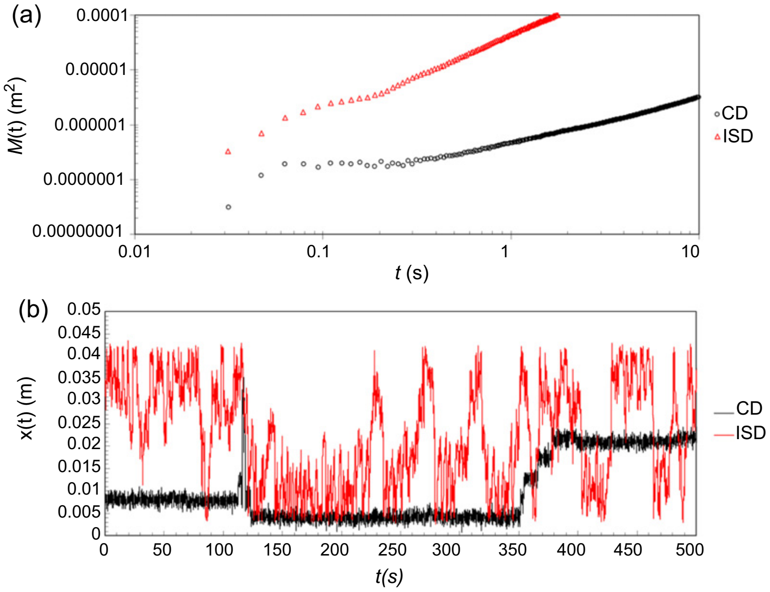

Figure 6. (a) Mean squared displacement and (b) time evolution of the *x*-position of a single tracer particle for an experimental system with H = 12 and Γ_0 = 14.5

$$\phi(z) = \frac{NF(z)\pi d^3}{6V_s} \tag{1}$$

where V_s is the segment volume. The ability to maintain the packing density of a granulate whilst altering its dynamical behaviour is potentially of significance to various applications, as will be discussed further in later sections.

As the density of the bed is increased further, the differences in the steady state dynamics of the system become more pronounced, as can be seen in figure 6. In this instance, a point of inflection in $M(t)$ can be observed for the CD *and* ISD cases (figure 6(a)), indicating the presence of caging effects in both instances. However, the comparatively increased length of the subdiffusive plateau in $M(t)$ for the CD case implies that the average duration over which particles remain caged is considerably reduced for the ISD case [53]. This observation is further evidenced by the observed single particle trajectory, a typical example of which can be seen in figure 6(b).

As the density of the system approaches its maximal value, the difference in initial driving conditions is found to play a determining role in the final state achieved by the system. Figure 7 shows data for a system of resting bed height H = 12 driven with an acceleration Γ_0 = 13. In the ISD case (Γ_i = 19.5) the system is found to maintain some degree of mobility, despite significantly slowed dynamics due to caging effects. For the CD case, however, the system is found to be *jammed*—permanently stuck in a single packing configuration [54]. It should be specifically noted that, for this case, the mean squared displacement shown is the average of multiple repeated runs with the tracer particle initially placed at various positions within the bed. Although it is known that the packing densities and stress distributions of the final, jammed state achieved by a granular material exhibit history-dependence [55, 56], it is noteworthy that, in this instance, the system's initial conditions can determine whether the system's dynamical state itself—i.e. whether the final configuration is static or mobile. It is also interesting to note

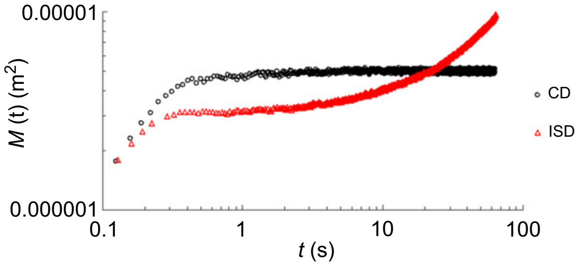

Figure 7. Experimentally acquired mean squared displacement for $H = 12$, $\Gamma_0 = 13$.

that despite being more mobile, the ISD system also appears to possess a *higher packing fraction*—the comparatively small plateau value of $M(t)$ for the ISD case implies a reduced free volume for this system [50]. This observation is somewhat surprising, as systems such as the one described here are typically found to become *less mobile* as the system's density increases [57].

We now attempt to provide an explanation for the various phenomena detailed above. Previous work [58] has shown that there exists a range of packing fractions near the RCP limit for which both jammed and unjammed states may exist. Thus it is perhaps not overly surprising that one might, as is the case in this study, observe various other dissimilar dynamics in similarly dense systems. The question remains, however, as to what, in the present system, is the origin of these distinct dynamical behaviours? One possible solution to this quandary is the existence of differing degrees of spatial heterogeneity in the CD and ISD systems. Recent work by Watanabe and Tanaka [59] suggests that the slowing of granular dynamics as a system approaches jamming may be due to the presence of 'medium range crystalline order', or MRCO [60]—long-lived crystalline clusters within the granular bed. The work of Watanabe and Tanaka finds that the slowing of dynamics observed with increasing system density, ϕ, may be described solely by the degree of MRCO within the bed. In other words, the commonly observed correlation between increasing density and slowing dynamics may only be an *indirect* link—MRCO causes slow dynamics, the presence of MRCO is more likely in denser systems. Thus, it stands to reason that for two systems with equal bulk densities but differing degrees of MRCO clustering, one may indeed observe dissimilar dynamics. Based on this hypothesis, we propose a tentative explanation for the behaviour observed in this study: for the case of the ISD system, the initial, harsh driving can be expected to provide an initially less clustered system [11, 61]. The sudden drop to A_0, in a manner analogous to the rapid quenching necessary for the supercooling of molecular liquids [62], results in a more homogeneous final state with reduced (or even entirely absent) MRCO, and hence a more mobile system.

Such a hypothesis can also explain the increased packing for the ISD case as observed in figure 7. One may consider a granular material as a system of randomly packed particulate clusters [63]. This random packing of clusters will result in the presence of defects in the bed, resulting in an increased void space and hence a reduced average system packing compared to a system composed of individual grains in the absence of clustering. Hence the proposed reduction in clustering due to initial strong driving may indeed, under certain circumstances, be expected to lead to a more densely packed final state. Even if clustering is not eliminated entirely, a reduction in the size of clusters may also result in an improved packing.

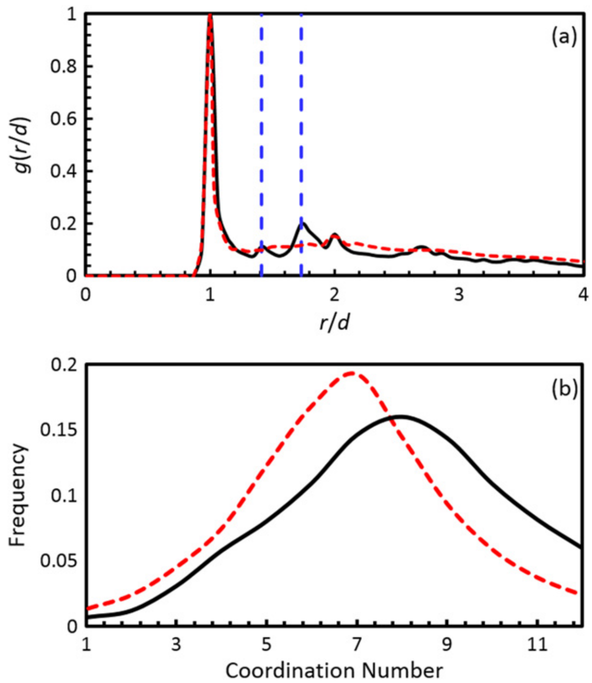

Figure 8. Evidence of differing degrees of crystalline structure for simulated continuously driven (CD) and initially strongly driven (ISD) systems: (a) compares the radial distribution function, $g\left(r/d\right)$ for the two cases with $\Gamma_0 = 14$ and H=10, while (b) shows coordination number distributions for $\Gamma_0 = 13$ and $H = 8$. In both images, the solid black line corresponds to the CD system, while the dotted red line represents the ISD system. The data in (a) is normalized such that the maxima in $g\left(r/d\right)$ are equal to unity for both driving protocols, allowing an easier comparison.

In order to ascertain whether MRCO-like clustering does indeed play a rôle in our observations, analysis of the various systems' structural properties was undertaken via the study of their coordination numbers and radial distribution functions. The radial distribution function, $g\left(r/d\right)$, represents the average number of particle centres, $N_{ave.}$, within a spherical shell of thickness Δr at a distance r/d from the centre of a given particle, normalized by the volume of the shell. Thus, $g\left(r/d\right)$ can be determined as:

$$g\left(r/d\right) = \frac{N_{ave.}}{4\pi\left(r/d\right)^2\Delta\left(r/d\right)}. \tag{2}$$

The existence of distinct peaks in $g\left(r/d\right)$, for instance those at $r/d = \sqrt{2}$ and $r/d = \sqrt{3}$ as can be seen in figure 8(a) for the CD case, signifies the presence of crystalline structure within the system [64]. Notable also from this image is the *absence* of such peaks in the equivalent ISD case. This observation shows definitively that the method of driving can indeed influence the degree of crystalline microstructure within the bed as hypothesized, and thus provides strong support for the possibility that this difference in microstructure may explain the other observed dissimilarities between CD and ISD systems. Evidence for structural differences between the two cases can also be seen from the coordination number distribution, an example of which is

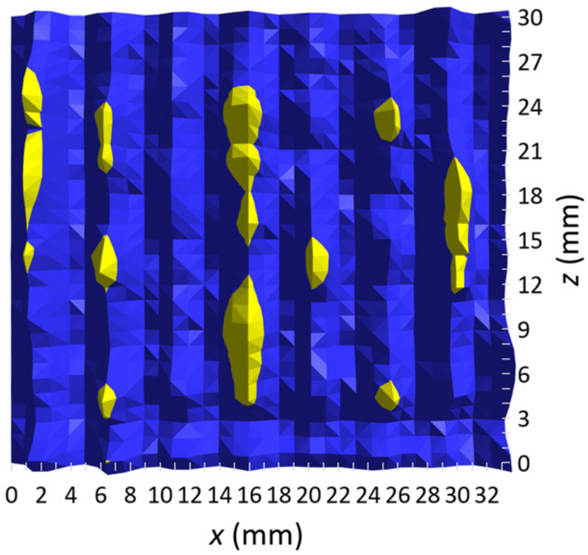

Figure 9. Two dimensional packing density field for the central region of a continuously-driven experimental system for which $\Gamma_0 = 14$ and $H = 10$. Areas in which the local density exceeds the random close-packed value of $\phi = 0.63$ are highlighted in yellow.

shown in figure 8(b). For a system of monosized, randomly-packed particles, one might expect the coordination number distribution to approximate a Gaussian [65], as is observed for the ISD case in the aforementioned figure. The CD system, however, demonstrates a strong deviation from the Gaussian form, displaying a marked shift towards higher coordination numbers, indicative once again of increased ordering within the system [66]. The localized nature of the crytalline structure is confirmed by subdividing the computational volume into a series of individual segments, and determining the number of sphere centres within each partitioned region. The spatial variation in this parameter provides evidence that we are indeed dealing with a localized crystalline clustering as would be expected for the case of MRCO. A similar analysis of the experimental solids fraction distribution yields comparable results, as may be seen from figure 9. A final interesting result pertaining to the structural properties of the system is that the spatial extent of this ordering is found to increase for denser systems, i.e. those more closely approaching the jammed state. This is true for both the CD and ISD protocols, although the degree of order is always markedly reduced in the ISD case, as may be expected from our previous discussion. It should be noted that the system behaviours discussed here are found to hold for both monodisperse and binary systems; the importance of the latter part of this statement will become clear in the following section.

The above findings contrast somewhat with those of Charbonneau *et al* [67], who find the length scales associated with ordering not to grow for binary systems approaching the glass transition. The differences between the findings of this study and our own may perhaps be explained by the presence of walls in our current set-up, which have a known influence on ordering within granular systems [68], or the fact that the system described here is bidisperse-by-density, while the particles used by Charbonneau *et al* differ instead in their size. The precise origin of this discrepancy is a subject worthy of further research.

It is worth mentioning here that the *rate at which* the initial driving amplitude A_i is reduced to A_0 was also found to affect the dynamical behaviour of the system. In all cases discussed above, the transition between A_i and A_0 was effectively instantaneous. However, runs were also conducted in which the initial driving amplitude was linearly reduced to the final amplitude over a period of 5 s. The resultant steady state dynamics achieved in this manner were found to be highly variable dependent on the control parameters of the system in question: in some instances, the steady state dynamics were found to be effectively identical to those observed for the CD case; in other instances, the degree of particle mobility more closely resembled the case of the instantaneously reduced amplitude, or fell somewhere in between the values expected for the original CD and ISD cases. Although the data currently available is insufficient to reliably suggest any concrete trends, it is still possible to explain these findings in terms of the framework presented above; for an initially homogeneous granular system undergoing relaxation or 'cooling', the formation of clusters takes a finite time [69]. If the timescale for the relaxation (τ_r) of the system from its initial, strongly driven state to the final, weakly driven state is smaller than or comparable to the timescale for cluster formation, τ_{cluster}, then one might well expect a reduced degree of clustering in the resultant system. As the rate at which A_i is reduced decreases, and hence τ_r increases, more significant clustering is likely to occur, resulting in dynamics more reminiscent of the CD system. Conversely, alteration of the system's control parameters may also result in variations in τ_{cluster}, which may well account for the varied behaviour discussed above even at a constant rate of decrease of A.

4.2. Rise time and segregation

Recent work [28] has demonstrated that, for a vibrofluidized binary granular system, denser beds will exhibit *more complete* segregation, due to a decrease in void space, yet also display a *reduced rate* of segregation due to their slower dynamics. Thus, the findings of this current study are potentially highly relevant to various industrial processes where segregation is a prerequisite—the ability to increase the mobility of systems without necessarily decreasing their packing density means that more complete segregation may be achieved more quickly, potentially resulting in significant time and energy savings.

Figure 10 shows the time-evolution of the vertical position of a single glass tracer within a bed of steel beads. Aside from the inclusion of the tracer within the system, the set-up corresponding to 10(a) is identical to that described in figure 3. As might be expected from the preceding discussion, not only does the ISD system shows a more rapid progression of the light intruder towards the top of the system (indicative of accelerated segregative processes), but the transition also seems to be comparatively smoother—the CD system makes its upward transit in a series of discrete, indicative of caging effects. The behaviour shown in the figure is typical of that observed over a numerous runs.

Figure 10(b) shows the ascent of a glass intruder for a system with $H = 12$, $\Gamma_0 = 13$. Here, the effects of caging become apparent for the ISD case, causing an increase in the rise time compared to the previous, more dilute system. The CD case, meanwhile, was found to show no upward motion even after being vibrated for 7200 s. To ensure the presence of a jammed state, repeated runs were taken with the particle in various initial positions within the system; the resulting data, as well as visual inspection, confirmed the system to be jammed in all cases.

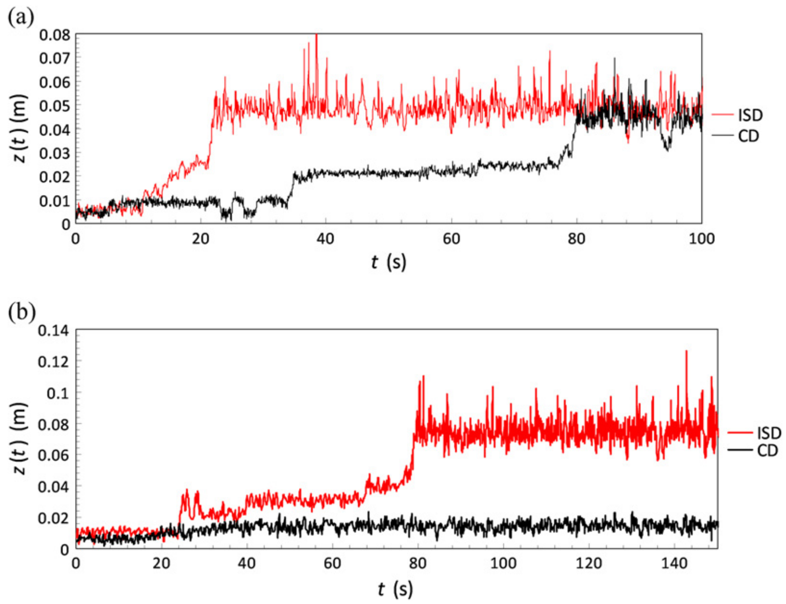

Figure 10. Experimental data showing the time evolution of the vertical (z) position of a single glass ($\rho = 2500$ kg m^{-3}) particle in a bed of steel ($\rho = 7850$ kg m^{-3}) spheres. The control parameters are (a) $H = 7$, $\Gamma_0 = 13$ and (b) $H = 12$, $\Gamma_0 = 13$.

As discussed in section 2.3, the effects of driving history on segregative behaviour inferred from the experimentally observed rise times can be substantiated through comparison with data produced using simulations of an equivalent, fully-bidisperse system (see section 3).

Figures 11(a) and (b) show the time-evolving ratios of the vertical centres of mass, $Z(t)$, for the heavy and light components of the system. The ratio can be thought of as a measure of the degree to which a system exhibits segregation. In the current case, $\frac{Z_{glass}}{Z_{steel}} = 1$ corresponds to a perfectly mixed system, while $\frac{Z_{\mathrm{glass}}}{Z_{\mathrm{steel}}} \approx 2.5$ implies a completely segregated system. It is primarily worth noting that simulations successfully reproduce the driving history dependence observed in experiment. Importantly, this confirms that the differences observed cannot simply be explained by factors such as particle aging, static charge, interstitial air or variations in humidity, as these were not included in simulations. Indeed, it was ensured that *all parameters* other than initial driving were held constant for each pair of data sets.

As may be expected, the segregative behaviour demonstrated in figure 11 shows certain important parallels with the rise time dynamics illustrated in figure 10. For comparatively strongly-driven (and hence dilute) systems, one observes a relatively smooth, rapid transition of the granular bed to the fully-segregated state for the ISD case with a more step-like increase for the CD case, mirroring rise time behaviour such as that shown in figure 10(a). For less strongly excited (and therefore denser) systems, the observed segregation is more similar to that shown in figure 10(b)—a stepwise increase in segregation for the ISD case, while the CD case remains seemingly paralyzed. It should be noted that the precise Γ_0 values for which the specific segregative behaviours of the system occur vary slightly between simulation and experiment. This is understandable due to differences in bed composition, as well as the possibility of imperfect agreement between the dissipative parameters implemented and the true experimental

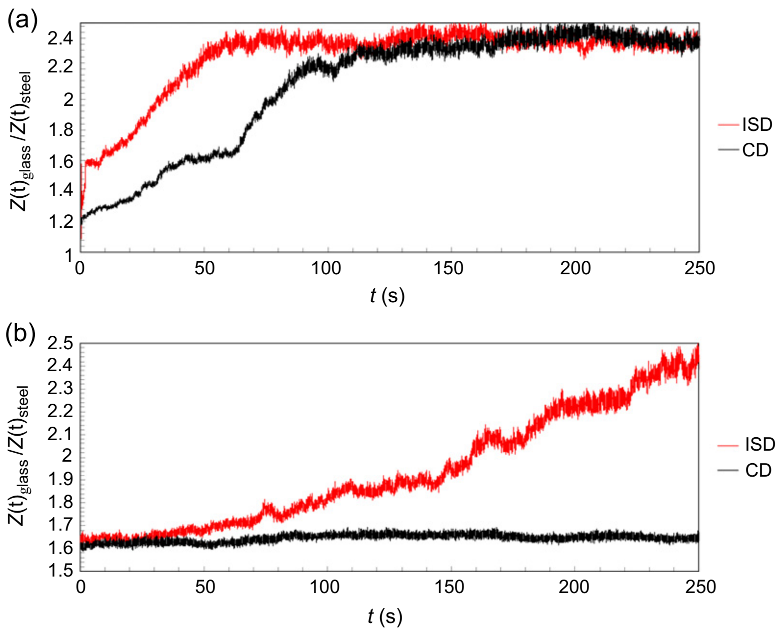

Figure 11. Ratios of the vertical centres of mass, $Z(t)$, for the light (glass) and heavy (steel) components of a simulated bidisperse granular system with (a) $H = 12$, $\Gamma_0 = 15$ and (b) $H = 12$, $\Gamma_0 = 13$.

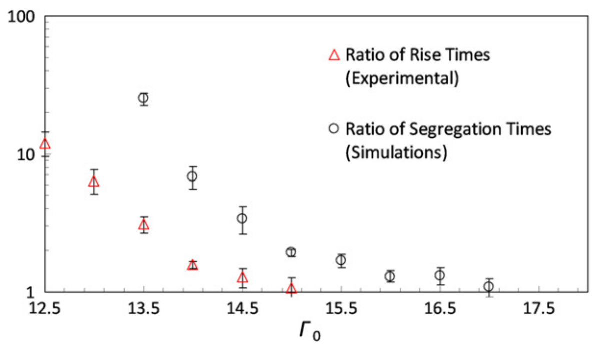

Figure 12. Ratio of the timescales for segregation and rise time comparing initially strongly driven to continuously driven systems. Comparison is made over a range of driving strengths.

values. Nonetheless, the observed trends in system behaviour observed with decreasing Γ are closely reproduced by simulations. The close correspondence between the simulated segregation times and experimental rise times, and their variation with the relevant control parameters, provides strong evidence that the observed variation in system dynamics and segregative behaviours due to initial driving conditions is indeed a robust phenomenon.

Moreover, the simulational data verifies that differences in initial driving conditions may indeed alter the subsequent rate of segregation in a binary granular system. Figure 12 shows how the relative amount of time required for a system to achieve its equilibrium distribution (i.e. its steady-state, maximal degree of segregation) for the CD and ISD cases varies with Γ_0. For relatively large Γ_0 values, corresponding to comparatively well fluidized, chaotic systems, little difference is observed in the measured segregation timescale, τ_{seg}, between the CD and ISD systems. As Γ_0 decreases, however, the ISD system shows a marked increase in segregation rate compared to its CD counterpart. For both τ_{seg} and its equivalent in the tracer limit, τ_{rise}, there exists a region of phase space for which segregative behaviour is still observed for the ISD case, while the CD case remains seemingly jammed, i.e. the ratio of timescales diverges to infinity.

5. Conclusions

It has been shown that the final steady state achieved by a vibrated granular system can be highly dependent on the details of its original driving. This history-dependence is shown to affect both packing density and particle mobility, and hence also significantly impacts segregative behaviours. In certain cases, simple differences in initial driving were found to increase the rate of segregation within otherwise identical systems by more than a factor of ten. In other instances, driving history was found to provide the difference between a jammed and a mobile final state. The hysteretic nature of systems such as those detailed in this paper may, potentially, be exploited to provide significant improvements to the efficiency (both in terms of time and energy) of various industrial processes where segregation is required. The observations of this study also provide great scope for future research-the history-dependent dynamics may be affected by a wide range of parameters, of which only a few have been explored in the current work. Thus, with more research, even greater improvements to the efficiency of segregative processes may be achieved.

Acknowledgments

The authors would like to thank Dr Anthony Thornton, Dr Thomas Weinhart and Professor Stefan Luding of the University of Twente for access to, and support using, the computer facilities on which the simulations in this paper were performed. This work was made possible thanks to the financial support provided by the Hawkesworth Scholarship, set up by the late Dr Michael Hawkesworth.

References

[1] Jaeger H M, Nagel S R and Behringer R P 1996 *Phys. Today* **49** 32
[2] Pak H K, van Doorn E and Behringer R P 1994 *Phys. Rev. Lett.* **74** 4643
[3] Ahmad K and Smalley I J 1973 *Powder Technol.* **8** 69
[4] Cates M E, Wittmer J P, Bouchaud J-P and Claudin P 1998 *Phys. Rev. Lett.* **81** 1841
[5] Yang S C 2006 *Powder Technol.* **164** 65
[6] Zeilstra C, van der Hoef M A and Kuipers J A M 2008 *Phys. Rev.* E **77** 031309
[7] To K, Lai P-Y and Pak H K 2001 *Phys. Rev. Lett.* **86** 71
[8] Levy A and Kalman H (ed) 2001 *Handbook of Conveying and Handling of Particulate Solids* (Amsterdam: Elsevier)
[9] Hong D C, Quinn P V and Luding S 2001 *Phys. Rev. Lett.* **86** 3423
[10] Song C, Wang P and Makse H A 2008 *Nature* **453** 629
[11] Rosato A D and Yacoub D 2000 *Powder Technol.* **109** 255–61

[12] Knight J B, Jaeger H M and Nagel S R 1993 *Phys. Rev. Lett.* **70** 3728
[13] Ottino J M and Khakhar D V 2000 *Annu. Rev. Fluid Mech.* **32** 55
[14] Brito R, Enríquez H, Godoy S and Soto R 2008 *Phys. Rev.* E **77** 061301
[15] Bocquet L, Errami J and Lubensky T C 2002 *Phys. Rev. Lett.* **89** 184301
[16] McCoy B J and Madras G 2004 *Phys. Rev.* E **70** 051311
[17] Mohabuth N, Hall P and Miles N 2007 *J. Miner. Eng.* **20** 926–32
[18] Kuehr R and Williams E 2003 *Computers and the Environment: Understanding and Managing their Impacts* (Dordrecht: Kluwer)
[19] Sthiannopkao S and Wong M H 2013 *Sci. Total Environ.* **463** 1147
[20] Shi Q, Sun G, Hou M and Lu K 2007 *Phys. Rev.* E **75** 061302
[21] Luding S, Herrman J and Blumen A 1994 *Phys. Rev.* E **50** 3100
[22] Eshuis P, van der Weele K, van der Meer D and Lohse D 2005 *Phys. Rev. Lett.* **95** 258001
[23] Eshuis P, van der Weele K, van der Meer D, Bos R and Lohse D 2007 *Phys. Fluids* **19** 1223301
[24] Parker D J, Forster R N, Fowles P and Takhar P S 2002 *Nucl. Instrum. Methods Phys. Res. Sect.* A **477** 540
[25] Wildman R D, Huntley J M, Hansen J-P, Parker D J and Allen D A 2000 *Phys. Rev.* E **62** 3826–35
[26] Wildman R D, Huntley J M and Parker D J 2001 *Phys. Rev.* E **63** 061311
[27] Wildman R D, Hansen J-P and Parker D J 2002 *Phys. Fluids* **14** 232
[28] Windows-Yule C R K, Weinhart T, Parker D J and Thornton A R 2014 *Phys. Rev. Lett.* **112** 098001
[29] Wildman R D and Parker D J 2002 *Phys. Rev. Lett.* **88** 064301
[30] Huerta D A and Ruiz-Suárez J C 2004 *Phys. Rev. Lett.* **92** 114301
[31] Brey J J, Ruiz-Montero M J and Moreno F 2005 *Phys. Rev. Lett.* **95** 098001
[32] Vanel L, Rosato A D and Dave R N 1997 *Phys. Rev. Lett.* **78** 1255–8
[33] Rosato A, Strandburg K J, Prinz F and Swendsen R H 1987 *Phys. Rev. Lett.* **58** 1038–40
[34] Thornton A R, Weinhart T, Luding S and Bokhove O 2012 *Int. J. Mod. Phys.* C **23** 1240014
[35] Thornton A R, Weinhart T, Ogarko V and Luding S 2013 *Computer Methods Mater. Sci.* **13** 197
[36] www.MercuryDPM.org
[37] Feitosa K and Menon N 2002 *Phys. Rev. Lett.* **88** 198301
[38] Cundall P A and Strack O D L 1979 *Geotechnique* **29** 47
[39] Luding S 2008 *Granul. Matter* **10** 235
[40] Weinhart T, Thornton A R, Luding S and Bokhove O 2012 *Granul. Matter* **14** 531
[41] Louge M Y 1999 Cornell University, Ithaca, NY (http://mae.cornell.edu/microgravity/impact-table.html)
[42] Luding S, Clément E, Blumen A, Rajchenbach J and Duran J 1994 *Phys. Rev.* E **50** 4113
[43] Luding S, Clément E, Blumen A, Rajchenbach J and Duran J 1994 *Phys. Rev.* E **50** R1762
[44] Baxter G W and Olafsen J S 2007 *Phys. Rev. Lett.* **99** 028001
[45] Campbell C S 1997 *J. Fluid Mech.* **348** 85
[46] Abate A R and Durian D J 2006 *Phys. Rev.* E **74** 031308
[47] Wildman R D, Huntley J M and Hansen J-P 1999 *Phys. Rev.* E **60** 7066
[48] Liu A J and Nagel S R (ed) 2001 *Jamming and Rheology: Constrained Dynamics on Microscopic and Macroscopic Scales* (New York: Taylor and Francis)
[49] Keys A S, Abate A R, Glotzer S C and Durian D J 2007 *Nat. Phys.* **3** 260
[50] Reis P M, Ingale R A and Shattuck M D 2007 *Phys. Rev. Lett.* **98** 188301
[51] Pouliquen O, Belzons M and Nicolas M 2003 *Phys. Rev. Lett.* **91** 014301
[52] Choi J, Kudrolli A, Rosales R R and Bazant M Z 2004 *Phys. Rev. Lett.* **92** 174301
[53] Marty G and Dauchot O 2005 *Phys. Rev. Lett.* **94** 015701
[54] Liu A J and Nagel S R 1998 *Nature* **396** 21
[55] Zhang H P and Makse H A 2005 *Phys. Rev.* E **72** 011301
[56] Nowak E R, Knight J B, Ben-Naim E, Jaeger H M and Nagel S R 1998 *Phys. Rev.* E **57** 1971
[57] Sellitto M and Arenzon J J 2000 *Phys. Rev.* E **62** 7793
[58] O'Hern C S, Langer S A, Liu A J and Nagel S R 2002 *Phys. Rev. Lett.* **88** 075507

[59] Watanabe K and Tanaka H 2008 *Phys. Rev. Lett.* **100** 158002
[60] Kawasaki T, Araki T and Tanaka H 2007 *Phys. Rev. Lett.* **99** 215701
[61] Lim E W C 2010 *AIChE J.* **56** 2588
[62] Silbert L E, Ertas D, Grest G S, Halsey T C and Levine D 2002 *Phys. Rev.* E **65** 051307
[63] Gavrilov K L 1997 *Phys. Rev.* E **58** 2107
[64] Reis P M, Ingale R A and Shattuck M D 2006 *Phys. Rev. Lett.* **96** 258001
[65] Cumberland D J , Crawford R J, William J C and Allen T (ed) 1987 *The Packing of Particles (Handbook of Powder Technology)* vol 6 (Amsterdam: Elsevier)
[66] McGeary R K 1967 Mechanical packing of spherical particles *Vibratory Compacting* ed H H Hausner, K R Roll and P K Johnson (New York: Plenum) pp 209–18
[67] Charbonneau B, Charbonneau P and Tarjus G 2012 *Phys. Rev. Lett.* **108** 035701
[68] Benenati R F and Brosilow C B 1962 Void fraction distribution in beds of spheres *AIChE J.* **8** 359
[69] Luding S and Herrmann H J 1999 *Chaos* **9** 673