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Improving students' mathematical skills in secondary science: ideas from mathematics pedagogy

Dan Cottle

Abstract Learning science in school means also learning to use some mathematics. For many students at secondary level, even those who demonstrate an interest in the subject, this aspect can prove problematic. This article seeks to learn from the pedagogical practice of mathematics educators and examine three ways in which ideas that are used in mathematics can be adapted to support science. It concludes by expressing a hope that open professional discussion among science teachers on this topic will be provoked, leading to improved learning experiences for students. The thinking explained in this article emerged from questions raised in a series of professional development and teacher education workshops over a period of around a year with beginner and experienced science teachers.

The '*gate and key*' to the sciences, as Bacon (1928) succinctly put it over 850 years ago, is mathematics. The two are inextricably linked. Current secondary school science content in England set by the Department for Education (2015) explicitly sets out how this link is to be understood by today's teachers of science, specifying that '*students need to have been taught… the appropriate areas of mathematics relevant to the subject*'. This raises questions as to how mathematics is taught in science lessons, what problems students might have with mathematics and what can be learned by the science teaching community from subject specialist teaching of mathematics? This article explores three suggestions in answer to these questions.

1 Don't underestimate maths anxiety

Many students' experience of maths is not positive (Barton, 2018). Home influences over many years may have convinced them that mathematics is not only difficult but 'not for them'. It might be that they have known adults who have found it difficult to work out percentage discounts in shops or are confused by calculations on electricity bills. Their experience of maths lessons may have convinced them that they 'don't get it', that answers are always 'right or wrong', and that they always end up being wrong and not understanding why. They may be glad to learn other subjects where they can get away from maths and feel more comfortable. Imagine then, how they might feel when they discover there is a maths component in a science lesson: anxiety, stress, fear, boredom, apathy? These might manifest themselves in different ways, perhaps in disengagement or in poor behavioural choices. Students may also associate maths-based activities with sitting on their own in quiet classrooms looking at complicated examples on paper that they do not understand, and feeling stuck and isolated while looking around and thinking that everyone else seems to know exactly what they are doing. It is well attested that maths anxiety exists (Ashcraft, 2002) but its impact on the science classroom and science teachers is perhaps considered less often. I have observed lessons where a science teacher has carelessly made the comments, '*I hate equations*' and '*this is a difficult bit of the course*' to the class. Some teachers avoid exposing their own insecurities by avoiding teaching maths skills at all, presenting answers as though they are obvious and dismissing questions. Science teachers can also talk themselves into a dead end when demonstrating maths on the board, perhaps rearranging an equation incorrectly and then having a mental block on how to get the right answer. These sorts of behaviours by science teachers reinforce what some children have 'known' all along: '*maths is too difficult and not for me*'. The consequences can be significant for learning science. As Ashcraft (2002) found, maths anxiety causes lower performance in maths tasks that depend on working memory. The relationship between maths anxiety and performance is also bi-directional: not only does it cause poor performance, but poor performance can lead to maths anxiety, perpetuating a cycle of difficulty (Dowker, 2019).

How can maths anxiety be reduced?

Acknowledging that maths anxiety is real can lead to positive changes in classroom practice. The following strategies are inspired by Finlayson (2014) and my own experience of teaching and observing the classrooms of trainee science teachers:

- Saying helpful and positive phrases to students, particularly when they struggle, to help them focus on the process of learning and avoid inadvertently validating a negative 'right or wrong' or 'I don't get it' view of maths. Table 1 gives some suggestions.

Table 1 Teacher talk: to say or not to say

Don't say	Do say
This is hard.	We can break this problem down into smaller chunks.
It's OK, this is difficult.	Here is a simpler example.
I hate equations.	Equations really make me think hard.
That's wrong, this is the right answer.	I see how you got that but here is another way of doing it.

- Preparing solutions. Lesson plans can include not only the numerical answers to mathematical problems but also worked solutions to share with the class. This will also bolster the confidence of a slightly maths-anxious teacher.
- Supporting science teachers' knowledge of mathematical curriculum content through effective continuing professional development (CPD) is a useful investment of time.
- Explore ways of making maths activities more social to allow students to generate solutions collaboratively. For example, before plotting graphs on paper, students could build three-dimensional representations of data in groups using LEGO® (Figure 1). In doing this, mistakes can be made without committing pen to paper, peer learning can occur and quick verbal feedback can be received. Similar approaches could be used to balance chemical equations, or model nuclear decays or pyramids of biomass.

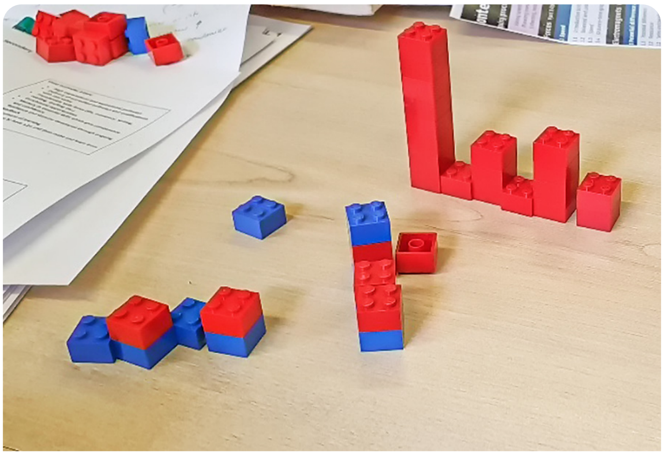

Figure LEGO used to represent a bar chart

- Use a variety of approaches to solve problems. For example, when practising rearranging formulae in physics, present a solution written by the teacher with deliberate errors and ask groups of students to identify these before writing their own correct working (e.g. Box 3).

2 Don't be afraid of repetition

Repetition is a common strategy used in the learning of mathematics (Drury, 2018). Students complete a variety of similar problems to develop competence. In recent years, the concept of 'deliberate practice' has been applied by educators to describe the process by which this repetition is organised to have the largest impact on the progress of students. However, to what extent is this just learning by rote? Is there a difference between this new deliberate practice and what has always taken place? Drury (2018) differentiates between two types of repetition that can benefit students: procedural variation and conceptual variation. Procedural variation aims to set problems for students that are at an optimum 'level of challenge' to support their improvement as they learn. It is a process of building confidence alongside competence, where the first problem of a set is easily solvable and gradually more complexity is introduced as the student attempts more. Conceptual variation involves thinking about the same problems in different ways, either in different contexts or by using alternative methods.

How might repetition be used to support learning in science?

Box 1 shows an example of a worksheet on the use of the wave equation. There is repetition here but it is not deliberate or varied since all the questions are similar. The questions are of the type commonly found in examinations and, in each, students must do the same thing in the same way.

Consider if we were to modify this set of questions utilising procedural variation. We might end up with something similar to Box 2. Question 1 in Box 2 is simple; the equation is given, the quantities are written with symbols, there are straightforward numbers and no rearrangement of the equation is required. The hope is that this will be an accessible starting point to most learners. As progress is made through questions 2

Box 1 Repetition in calculations but not deliberate practice

1. Laura reads the label on the back of her microwave oven. It says, frequency = 2450 MHz. The speed of microwaves is $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$. Calculate the wavelength.
2. A laser emits red light with a wavelength of 660 nm. The laser light travels through a vacuum. Calculate the frequency of the light.
3. Amrit plays a note on a piano with a frequency of 440 Hz. The wavelength is 78 cm. Calculate the wave velocity through the air.

Box 2 Example of procedural variation

1. Calculate v if $f=6$ Hz and $\lambda=3$ m ($v=f\lambda$).
2. Calculate f if $v=15\,\text{m}\,\text{s}^{-1}$ and $\lambda=5$ m ($v=f\lambda$).
3. Calculate λ if $f=4$ GHz and $v=3\times10^8\,\text{m}\,\text{s}^{-1}$ ($v=f\lambda$).
4. A sound wave has a frequency of 520 Hz and a wavelength of 0.65 m. Calculate the velocity ($v=f\lambda$).
5. Harjun plays a note of wavelength 0.25 m on the trumpet. He knows the speed of sound is $340\,\text{m}\,\text{s}^{-1}$ in air. Calculate the frequency.
6. Laura reads the label on the back of her microwave oven. It says frequency = 2450 MHz. The speed of microwaves is $3.00\times10^8\,\text{m}\,\text{s}^{-1}$. Calculate the wavelength.

and 3 more complexity is added: in question 2 a rearrangement is required; in question 3 the numbers use standard form and scientific prefixes typical of electromagnetic waves. In question 4 context is introduced and the students now have to pick out values from the text of the question. Question 5 adds recall of the equation and question 6 is like the original questions in Box 1. As well as building the confidence of the learner, the new sequence provides assessment information to the teacher if the student becomes 'stuck' at any point in the series.

We could extend this repetition with an example of conceptual variation (Box 3). In this question, students are not only solving the problem for themselves but also thinking about the context of the classroom and how to make it intelligible for others. They may identify different errors. As well as the obvious mistakes in units, a student may wish to align all the equals signs underneath each other or substitute the numbers into the equation before rearranging. These features may not be shared by all students and could then provoke further discussion and agreement on a correct solution.

Box 3 Example of conceptual variation

7. A teacher makes some errors when writing out the solution to the following GCSE calculation as follows:

   A laser emits red light with a wavelength of 660 nm. The laser light travels through a vacuum. Calculate the frequency of the light.

   $v=f\lambda$

   $$f=\frac{v}{\lambda}=\frac{3\times10^8}{660}=4.54\times10^5\,\text{MHz}$$

   a. How many errors can you identify?
   b. What is the correct solution?
   c. How would you write this example more clearly to explain your method to the class?

3 Don't confuse students

How would you explain to a student a method for rearranging a formula? Would it be the same as the explanation given by a colleague in your science department? How about if you were to compare your explanations with a teacher in the maths department of your school? It is not uncommon for a 15-year-old student in England to have three different teachers of science concurrently plus two teachers of maths. I suspect that significant differences would emerge if they were all asked to explain the same simple rearrangement. Perhaps you could try this out in your school? I provide the problem in Box 3 to use for this purpose as well as an example of conceptual variation. It is not that a variety of approaches to rearranging equations is necessarily a bad thing, but rather that there are benefits in agreeing a consistent approach to a fundamental shared mathematical skill like this that can support student understanding. Mathematics educators typically use the 'balance method'. Briefly described, this has two main principles (Boohan, 2016): in rearranging an equation always do the same to both sides of an equals sign and learn the inverse of every operator. The balance method for rearranging $V=IR$ is written out in Table 2.

Interestingly, the balance method sometimes provokes controversy among science teachers as it may seem labour intensive to explicitly cancel out the two R terms in line 2. The main alternative method, 'cross multiplying' (Table 2), although at first glance similar, misses out the intermediate step that can link this operation to basic number work preceding algebra in the maths curriculum. Using the balance method, you explicitly divide both sides of the equals sign by the same factor, R. When cross multiplying, the description is to move the R 'down' to a divide on the left-hand side. It forms a short cut to the answer that can lead to a feeling of mystery for those students who are not

Table 2 The balance and cross multiply methods for rearranging $V=IR$

'Balance method' to rearrange $V=IR$ to make I the subject	'Cross multiply' to rearrange $V=IR$ to make I the subject
$V=IR$	$V=IR$
$\frac{V}{R}=\frac{I\cancel{R}}{\cancel{R}}$	$\frac{V}{R}=I\cancel{R}$
$\frac{V}{R}=I$	$\frac{V}{R}=I$
$I=\frac{V}{R}$	$I=\frac{V}{R}$
	In some cases, a 'cross' would be formed when the process was applied to both sides of the equals sign simultaneously.

familiar with it and whose understanding relies more on the application of the well-established rules learned from an early age in their maths studies. Whatever the views of individual teachers of maths or science, it is possible to justify and then agree a method to use for simple mathematical operations within the context of a school science department, or even a combined mathematics and science department, such that every teacher approaches it in the same way. This consistency builds confidence in students, with assurance that their maths skills can be applied correctly in different curriculum areas.

Another area of difference among teachers is the question of when to introduce numbers into a calculation. Is it before or after an equation has been rearranged? Recent literature, for example Boohan (2016), suggests to the surprise of many teachers that it is not at the end of the calculation but as soon as possible. Justification for this is that it allows students to grasp the relationship between the quantities and that it allows patterns to emerge more easily, for example, powers of 10 or different units. Whatever our personal opinions on this, students would benefit from agreement within a department on a common approach.

Finally, and perhaps most controversially, confusion can arise from the use of formula triangles of the type shown in Figure 2 that many teachers use to support students in rearranging equations.

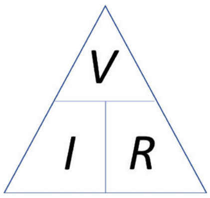

Figure 2 A formula triangle

Triangles are used to relate three quantities, in this case potential difference, current and resistance. The intention is to provide a quick way to obtain the formula to calculate any one of the three that may be unknown when the other two are known. The method is to place a finger over the quantity desired, for example, cover I to see V/R, which helps students to remember that $I = V/R$ and so on.

For a full explanation of the problems associated with this approach see Southall (2016). For now, I point out my two main concerns. Firstly, the triangle approach only works for a small subset of situations where a rearrangement is required. For example, the equation for kinetic energy $E = \frac{1}{2}mv^2$, which contains five quantities, including the factor of $\frac{1}{2}$ and the square of v, would force a student who has so far relied on the formula triangle to learn a new and different method. If the balance method had been used consistently there would be no need, this would simply be a new example of a familiar concept and would serve to reinforce prior knowledge rather than expose the limits of it. Secondly, a 'mixed economy' in a science department, where some teachers use formula triangles extensively and others do not, may have a negative effect on the maths confidence of students if they perceive that there is disagreement among their teachers on the correct way to rearrange.

The method for rearranging equations is just one area where there is potential for significant difference in approach between science teachers and more widely between different curriculum areas in a school. The way to plot and use a graph is another, as is the use of standard form.

Ways forward

Some student difficulties in learning to use mathematics as a part of secondary science have been discussed in this article with ideas suggested to try out in science lessons. If there is one wider learning point to emerge from this, however, it is the amount there is to be gained by initiating an open professional discussion in a science department about the learning of mathematics. A broader range of perspectives and expertise could be considered by including colleagues from mathematics and perhaps geography, computer science and design and technology. As this is done, perhaps formally in a departmental meeting or informally over a staffroom cup of coffee, my own experience is that teachers will increasingly be actively supportive of one another as they understand the pedagogical practice of their peers. I suggest also that this would strengthen the maths confidence of teachers, the knowledge base of maths teaching in science would become more evidence based and, instead of accidentally contradicting each other, a consistent approach by teachers would lead to improved understanding for students.

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